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First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015

Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Construct QR decomposition for the matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

(08 Marks)

b. Find the pseudo inverse of matrix A and verify all the properties of pseudo inverse. **(12 Marks)**

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}$$

2 a. Find singe value decomposition of matrix $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. **(10 Marks)**

b. Find a least-squares solution of Ax = b of the following equations.

$x_3 + 2x_4 = 1$

$x_1 + 2x_2 + 2x_3 + 3x_4 = 2$

$x_1 + 3x_2 + 2x_3 = 4.$

(10 Marks)

3 a. Find the extremum of functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$. **(06 Marks)**

b. Find the extremum of functional

$$v[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')dx \quad y(0) = 1 \quad y(1) = 2.$$

(06 Marks)

c. Show that the extremum of functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{x} dx$ is a family of circles and also determine centre and radius of circle which is passing through (1, 0) and (3, 5). **(08 Marks)**

4 a. Find the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x} [(y'')^2 - 2(y')^2 + y^2 - 2y\sin x]dx.$$

(08 Marks)

b. Find a function y(x) for which $\int_0^1 (x^2 + y'^2)dx$ is stationary given that $\int_0^1 y^2 dx = 2$; $y(0) = y(1) = 0$.

(12 Marks)

5 a. An infinitely long string having one end at $x = 0$ is initially at rest on the x – axis. The end $x = 0$ undergoes a periodic transverse displacement described by $A_0 \text{Sin } wt$, $t > 0$. Find the displacement of any point on the string at any time t. Solve by Laplace transform method. **(10 Marks)**

b. Solve the heat conduction problem described by $\text{PDE} : K\frac{\partial^2 u}{0x^2} = \frac{\partial u}{\partial t}$ $\quad 0 < x < \infty$ t 70 with condition $u(0, t) = u0 \quad t \geq 0$; $u(x, 0) = 0 \quad 0 < x < \infty$; u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. **(10 Marks)**

6 a. Solve the following potential problem in the semi-infinite strip described by $u_{xx} + u_{yy} = 0$, $0 < x < \infty$, and $0 < y < a$ subject to $u(x, 0) = f(x)$, $x(x, a) = 0$, $u(x, y) = 0$, $0 < y < a$, $0 < x < \infty$ and $\frac{\partial u}{\partial x}$ tends to zero as $x \to \infty$. **(10 Marks)**

b. One dimensional infinite solid $-\infty < x < \infty$, is initially at temperature $F(x)$. For times $t > 0$, heat is generated with in solid at a rate of $g(x, t)$ units. Determine the temperature in the solid for $t > 0$. **(10 Marks)**

7 a. Use simplex method to solve the following equation

Maximize $Z = 5x_1 + 2x_2$

Subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

with all variable non negative. **(10 Marks)**

b. Determine the symmetric dual of the program. Show that both primal and dual program have the same optimal value for Z.

Maximize $Z = 2x_1 + x_2$

Subject to $x_1 + 5x_2 \leq 10$

$x_1 + 3x_2 \leq 6$

$2x_1 + 2x_2 \leq 8$

with all variables non negative. **(10 Marks)**

8 a. Solve the following program by use of Lagrange-multipliers:

Minimize $Z = x_1 + x_2 + x_3$

Subject to $x_1^2 + x_2 = 3$

$x_1 + 3x_2 + 2x_3 = 7$. **(10 Marks)**

b. Solve the following programme by use of the Kuhn-Tucker conditions:

Minimize $Z = x_1^2 + 5x_2^2 + 10x_3^2 - 4x_1x_2 + 6x_1x_3 - 12x_2x_3 - 2x_1 + 10x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \geq 4$

with all variables nonnegative. **(10 Marks)**

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14ECS12

First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015
Antenna Theory and Design

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Derive the equations for, i) antenna impedance, for a short dipole ii) radiation efficiency. **(10 Marks)**

b. Explain the following terms as related to antenna systems:
i) Radiation pattern ii) Polarization iii) Directivity and gain. **(10 Marks)**

2 a. Explain with suitable Maxwell's equations, solution for radiation problems. **(10 Marks)**

b. Calculate the exact directivity for the 3-dimensional sources having the following patterns:
i) $u = u_m \sin\theta \sin^2\phi$
ii) $u = u_m \sin\theta \sin^3\phi$
iii) $u = u_m \sin^2\theta \sin^3\phi$
Take $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$
Find also the approximate directivity from the half power beamwidth. **(10 Marks)**

3 a. Describe the Yagi Uda array antenna. List design steps using a specific example. **(08 Marks)**

b. Derive an expression for an array factor of N number of elements which are isotropic. The array is linear and equally spaced and uniformly excited. What are the conclusions that can be drawn out of array factor expressions? **(12 Marks)**

4 a. Explain the design considerations for the axial mode helical antenna. **(10 Marks)**

b. What are traveling-wave antennas? Give example with suitable figures. **(10 Marks)**

5 a. What are log-periodic antennas? Explain a method of constructing a log-periodic dipole array. **(10 Marks)**

b. Define parabolic reflector. What are the parabolic reflector antenna principles? Discuss GO/Aperture distribution method for analyzing reflector antennas. **(10 Marks)**

6 a. Derive Pocklington's integral equation in the (MOM) method of moments. **(10 Marks)**

b. With relevant mathematical details, explain wedge diffraction theory. **(10 Marks)**

7 a. Describe the characteristics involved for feeding antenna efficiently, if high aperture efficiency is desired in practice. **(10 Marks)**

b. Explain antenna pattern synthesis by woodword Lawson sampling method. **(10 Marks)**

8 Write an explanatory note on the following:
a. Taylor line source method.
b. Phased arrays.
c. Spiral antennas.
d. Cylindrical parabolic antennas. **(20 Marks)**

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14ECS13

First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015

Probability and Random Process

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define the terms : (i) Sample space (ii) Event (iii) Mutually exclusive events (iv) Independent event. Give one example for each. **(06 Marks)**

b. State and prove Baye's rule. **(06 Marks)**

c. What is MAP rule? With equations explain how you will decide signal '1' was sent in optical fibre communication system? **(08 Marks)**

2 a. A software manufacturer knows that one out of 10 software games that the company markets will be a financial success. The manufacturer selects 10 new games to market. What is the probability that (i) Exactly one game will be a financial success? (ii) At least two games will be a financial success. **(05 Marks)**

b. A certain random variable has a probability density function of the form $f_x(x) = Ce^{-2x}u(x)$. Find the following : (i) the constant C (ii) $P_r(x>2)$ (iii) $P_r(x<3)$ (iv) $P_r(x<3 \mid x>2)$ **(06 Marks)**

c. A Gaussian random variable has a PDF of the form, $f_x(x) = \frac{1}{\sqrt{50\pi}} \exp\left(\frac{-(x-10)^2}{50}\right)$.

Write each of the following probabilities in terms of 2 functions.

i) $P_r(x>17)$ ii) $P_r(x<-2)$ iii) $P_r(|x-10|>7)$ iv) $P_r(|x-4|<7)$ **(06 Marks)**

d. Write the pdf of the following random variables:

i) Gamma Random variable. ii) Cauchy Random variable. **(03 Marks)**

3 a. Define "Central moment" of a random variable X. Find the first four central moments of a Laplace random variable with a PDF given by $f_X(x) = \frac{b}{2}\exp(-b|x|)$. Also find the coefficient of skewness and coefficient of Kurtosis. **(07 Marks)**

b. Define the term "Conditional expected value". Consider a Gaussian random variable of the form $f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$. Find $E(X/X>0)$. **(07 Marks)**

c. Obtain the mean value, second moment & variance of Poisson random variable. **(06 Marks)**

4 a. Define characteristic function of a random variable X. For any random variable whose characteristic function is differentiable at w = 0, prove that $E(X) = -j\frac{d}{dw}(\phi_x(w))|_{w=0}$ **(07 Marks)**

b. Define probability generating function. Consider the binomial random variable whose PMF is $P_x(K) = nC_K P^K (1-P)^{n-K}$, K = 0, 1, 2,…n

i) Find the probability generating function.

ii) Using probability generating function find the factorial moments and hence obtain mean, second moment and variance. **(07 Marks)**

c. Define the terms: i) Signal-to-Quantization Noise Ratio (SQNR). ii) Shannon entropy. Suppose a source sends symbols from a three-letter alphabet with $X \in \{a, b, c\}$ and $P_a = \frac{1}{2}$, $P_b = \frac{1}{4}$ and $P_c = \frac{1}{4}$ are the source symbol probabilities. i) Determine the entropy of this source ii) Give a source code that has an average code word length that matches the entropy. **(06 Marks)**

5 a. Suppose a pair of random variables has the joint PDF given by,

$$f_{x,y}(x,y)=\frac{1}{2\pi}\exp\left(-\frac{(x^2+y^2)}{2}\right),\ \text{Find } P_r(x^2+y^2<1).$$ **(06 Marks)**

b. A certain pair of random variables has a joint PDF given by,

$$f_{x,y}(x,y)=\frac{2abc}{(ax+by+c)^2}u(x)u(y)$$ for some positive constants a, b and c. Find

i) The marginal PDF's $f_x(x)$ and $f_y(y)$.

ii) The conditional PDF's $f_{x|y}(x \mid y)$ and $f_{y|x}(y \mid x)$. **(06 Marks)**

c. Define the terms:

i) Conditional entropy of a discrete random variable X with the condition that Y = y.

ii) Mutual information between two discrete random variables x and y.

iii) Channel capacity of a discrete communication channel.

If the transition probability matrix is, $Q=\begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.1 & 0.8 & 0.1\\ 0.1 & 0.1 & 0.8\end{bmatrix}$ in a 3-letter communication channel and $P_i=\frac{1}{3}$, i = 1, 2, 3. Determine the mutual information for the channel. **(08 Marks)**

6 a. For a set of N random variables $X_1, X_2, \ldots X_N$, define joint PMF, CDF. **(02 Marks)**

b. For a random vector $X=[X_1, X_2, \ldots X_N]^T$ define the correlation matrix. Prove the following: For a linear transformation of vector random variables of the form $Y = AX + b$ the means of X and Y are related by $\mu_y = A\mu_X + b$. Also, the correlation matrices of x and y are related by $R_{yy} = AR_{xx}A^T + A\mu_x b^T + b\mu_x^T A^T + bb^T$ and the covariance matrices of X and Y are related by $C_{YY} = AC_{XX}A^T$. **(08 Marks)**

c. Define the joint Gaussian PDF for a vector X of N random variables. Suppose X is a two element vector and the mean vector and covariance vector are given by their general forms,

$$\mu_X=\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix} \text{ and } C_{XX}=\begin{bmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{bmatrix},$$ Obtain the Joint Gaussian PDF of vector X. **(10 Marks)**

7 a. Define the terms (i) Random process (ii) Autocorrelation function. **(06 Marks)**

b. When you say that a random process is, (i) Striet sense stationary (ii) Wide sense stationary (WSS)? **(06 Marks)**

c. Consider the random process, $X(t) = A\cos(w_0 t) + B\sin(w_0 t)$, where A and B are independent, zero-mean Gaussian random variables with equal variances of σ^2. Find the mean and autocorrelation functions of this process. Is the process WSS? **(08 Marks)**

8 a. Define and explain (i) Markov process (ii) Transition probability matrix of a Markov chain. Explain the Markov chain of queueing system. **(10 Marks)**

b. Starting with the three properties of Poisson counting process obtain the PMF for the Poisson counting process in the form $P(X(t)=i)=P_X(i\,;t)=\frac{(\lambda t)^i e^{-\lambda t}u(t)}{i!}$ where the process X(t) counts the number of telephone calls arriving at a certain switch in a public telephone network. **(10 Marks)**

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First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015
Advanced Digital Communication

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. With neat block schematic diagram, explain the operation of binary PSK transmitter and coherent binary receives. Derive the equation for average probability of symbol error. **(10 Marks)**

b. An FSK system transmits binary data at the rate of 2.5×10^6 bits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit '1' or '0' is 1μ volt. Determine the average probability of symbol error assuming coherent detection. **(05 Marks)**

c. Draw the waveform for the MSK signal s(t) for the input sequence 1101000. Assume that at time, t = 0, the phase $\theta(0) = 0$. **(05 Marks)**

2 a. Draw the block diagram of receiver for coherent M-ary psk and explain its operation. Evaluate the average probability of symbol error for $M \geq 4$. **(10 Marks)**

b. Compare the performance of binary and quaternary modulation schemes. **(10 Marks)**

3 a. Explain the distance properties of convolution codes. **(08 Marks)**

b. A convolutional code is described by $g_1 = [1\ \ 0\ \ 1]$, $g_2 = [1\ \ 1\ \ 1]$, $g_3 = [1\ \ 1\ \ 1]$.
 i) Draw the encoder corresponding to this code.
 ii) Draw the state transition diagram for this code.
 iii) Draw the trellis diagram for this code.
 iv) Find the transfer function and the free distance of the code. **(12 Marks)**

4 a. With an example, illustrate feedback convolutional decoding algorithm. **(10 Marks)**

b. Explain the distortion criterion for optimizing the equalizer coefficients. **(10 Marks)**

5 a. Draw the block diagram of predictive-decision feedback equalizer. Discuss the performance characteristics of this equalizer. **(10 Marks)**

b. Consider the discrete-time equivalent channel consisting of two taps 'f_0' and 'f_1'. Find:
 i) The minimum MSE, J_{min} for this channel.
 ii) J_{min}, when $|f_0| = |f_1| = \sqrt{\frac{1}{2}}$.
 iii) Corresponding output SNR. **(10 Marks)**

6 a. Discuss the convergence properties of LMS algorithm. **(06 Marks)**

b. Explain stochastic gradient algorithm. **(08 Marks)**

c. Draw the block diagram and explain the model of spread spectrum digital communication system. **(06 Marks)**

7 a. Discuss the error rate performance of DSSS decoder. **(10 Marks)**

b. A CDMA system consists of 15 equal-power users that transmit information at a rate of 10,000 bits/sec, each using a DS spread-spectrum signal operating at a chip-rate of 1MHz. The modulation is binary PSK.

i) Determine the E_0/J_0, where, 'J_0' is the spectral density of the combined interference.

ii) What is the processing gain?

iii) How much should the processing gain be increased to allow for doubling the number of users without affecting the output SNR? **(06 Marks)**

c. Define the following: i) Follower Jammer; ii) Tactical transmission system (TATS). **(04 Marks)**

8 a. With the help of neat block diagram, explain the operation of FH-spread spectrum system. **(08 Marks)**

b. Discuss the effect of signal characteristics on the choice of a channel model. **(06 Marks)**

c. Explain the following diversity methods employed for fading multi-path channels:
i) Rayleigh ii) Rician iii) Nakagam's. **(06 Marks)**

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