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10MMD/MDE/MCM/MEA/MAR11

First Semester M.Tech. Degree Examination, June 2012

Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Define with suitable examples :
i) Significant figure ii) Round off error
iii) Truncation error iv) Inherent error. **(10 Marks)**

b. The differential equation governing velocity of falling parachutist is given by $\frac{dv}{dt} = g - \left(\frac{c}{m}\right)v$ where g – acceleration due to gravity, m – mass of parachutist, c – drag coefficient. Obtain analytical solution at t = 10 sec. Also obtain finite difference approximation in steps of 2 sec till t = 10 assuming initial velocity v(0) = 0.
Data : g = 9.8, m = 68.1, c = 12.5. **(10 Marks)**

2 a. A real root of the equation $x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the Secant method and the Regula-Falsi method to obtain this root. **(10 Marks)**

b. Use both the standard and modified Newton-Raphson methods to evaluate the multiple root of $f(x) = x^3 - 5x^2 + 7x - 3$ with the initial value $x_0 = 0$. (Carryout three approximations in each case). **(10 Marks)**

3 a. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use $p_0 = -0.9$ and $q_0 = 0.9$ as initial approximations. **(10 Marks)**

b. Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Graeffe's root squaring method. **(10 Marks)**

4 a. Given the following table of values, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1 and x = 1.6, using suitable interpolation formula :

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

(10 Marks)

b. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places. Take h = 0.5, 0.25 and 0.125. **(10 Marks)**

5 a. Using partition method, solve the system of equations $x_1 + x_2 + x_3 = 1$, $4x_1 + 3x_2 - x_3 = 6$ and $3x_1 + 5x_2 + 3x_3 = 4$. **(10 Marks)**

b. State the important steps involved in the Cholesky method to solve the system of linear equations. $x_1 + 2x_2 + 3x_3 = 5$, $2x_1 + 8x_2 + 22x_3 = 6$ and $3x_1 + 22x_2 + 82x_3 = -10$. **(10 Marks)**

6 a. Using the Jacobi method, find all the eigen values and the corresponding eigen vectors of the matrix, $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. **(10 Marks)**

b. Using the Householder's transformation reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into a tridiagonal matrix. **(10 Marks)**

7 a. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, define a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(x) = Ax = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$.

i) Find T(u), the image of u under T
ii) Find $x \in \mathbb{R}^2$ whose image under T is b. **(06 Marks)**

b. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then T is one-to-one if and only if the equation $T(x) = 0$ has only the trivial solution. **(06 Marks)**

c. In a certain city about 7% of a city population moves to the surrounding suburbs each year and 3% of suburban population moves into the city. In 2009 there were 800,000 residents in the city and 500,000 in the suburbs. Set up a difference equation that describes the situation, where x_0 is initial population in 2009. Then estimate the population in the city and in the suburbs two years later, in 2011. **(08 Marks)**

8 a. Find a least-squares solution of the system $AX = b$ for $A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$. **(07 Marks)**

b. The distance from a point y in $\mathbb{R}^n$ to a subspace W is defined as the distance from y to the nearest point in W. Find the distance from y to W = span $\{u_1, u_2\}$, where

$$y = \begin{bmatrix} -1 \\ -5 \\ 10 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

(05 Marks)

c. Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. **(08 Marks)**

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10MMD/MDE/MCM/MEA/MAE12

First Semester M.Tech. Degree Examination, June 2012
Finite Element Method

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Using matrix notations, develop an expression for the potential energy functional for an elastic solid body subjected to body force, surface traction and concentrated loads. **(10 Marks)**

b. Using Rayleigh-Ritz method, determine the maximum deflection at the end point of the cantilever beam subjected to a load 'P' at the end. **(10 Marks)**

2 a. In Fig.Q.2(a), a load $P = 60 \times 10^3$N is applied as shown. Determine the displacement field, stress and support reactions in the body. Take $E = 20 \times 10^3$ N/mm^2. **(10 Marks)**

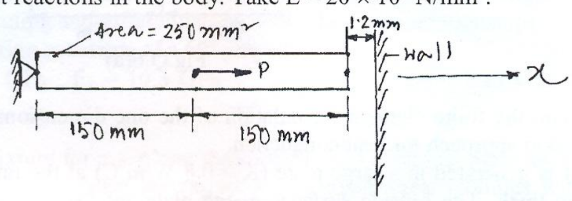


Fig.Q.2(a)

b. For the two-bar truss shown in Fig.Q.2(b), determine the displacements of node 2 and the stress in element 2-3. **(10 Marks)**

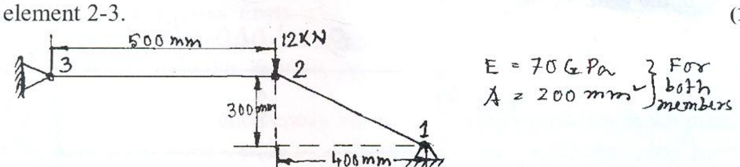


Fig.Q.2(b)

3 a. Explain the isoparametric representation for CS7 element. **(06 Marks)**

b. Evaluate the shape functions N_1, N_2 and N_3 at the interior point P for the triangular element shown in Fig.Q.3(a). **(06 Marks)**

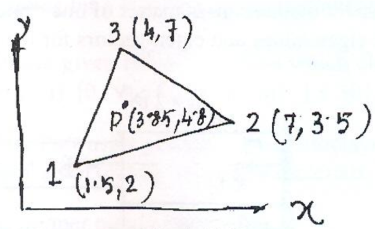


Fig.Q.3(a)

c. Derive the shape functions of eight noded quadrilateral element. **(08 Marks)**

4 a. Derive the material property matrix for axisymmetric problem. **(06 Marks)**

b. Evaluate the Jacobian matrix for axisymmetric triangular element. **(08 Marks)**

c. Explain the steps to develop stiffness matrix for four-node axisymmetric quadrilateral element. **(06 Marks)**

5 a. Write short notes on :
   i) Serendipity family
   ii) Hexahedral elements
   iii) Lagrange family. **(09 Marks)**
  b. Derive the shape function of four noded tetrahedral elements. **(06 Marks)**
  c. Explain the isoparametric representation of hexahedral elements. **(05 Marks)**

6 a. Derive the hermite shape function of beam element and draw the shape function. **(10 Marks)**
  b. For the beam shown in Fig.Q.6(a), determine deflection under the given load. Take $E = 2 \times 10^8$ kN/m^2 and $I = 4 \times 10^{-6}$ m^4. **(10 Marks)**

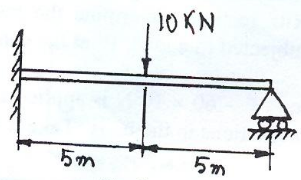


Fig.Q.6(a)

7 a. Explain the finite element formulation of the one dimensional heat transfer problem using Galerkin approach for heat conduction. **(10 Marks)**
  b. Heat is generated in a large plate (K = 0.8 W/m°C) at the rate of 4000 W/m^3. The plate is 25cm thick. The outside surfaces of the plate are exposed to ambient air at 30°C with a convective heat-transfer co-efficient of 20 W/m^2°C. Determine the temperature distribution in the wall. **(10 Marks)**

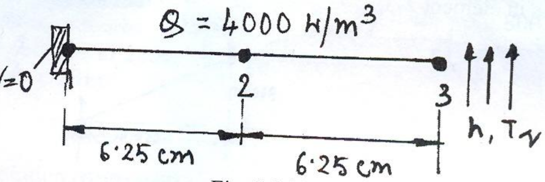
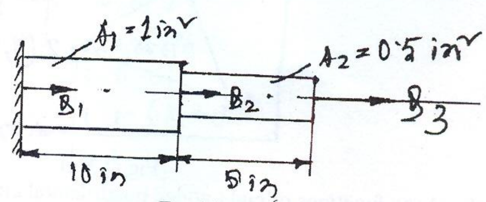


Fig.Q.7(b)

8 a. Derive an expression of the generalized eigenvalue problem for solid body with distributed mass. **(08 Marks)**
  b. Derive a consistent element mass matrix of one dimensional bar element. **(04 Marks)**
  c. Determine the eigenvalues and eigen vectors for the stepped bar shown in Fig.Q.8(c). **(08 Marks)**



$E = 30 \times 10^6$ PSi

Specific weight f = 0.283 Ib/in^3

Fig.Q.8(c)

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