

USN

08MMD/MDE/MEA/MCM/MAE/MAU12

**First Semester M.Tech. Degree Examination, May/June 2010**  
**Finite Element Methods**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions.**

1. a. Outline the basic steps in the finite element method, for engineering analysis of an elastic body. (06 Marks)
- b. A cantilever beam of span 'l' is subjected to a point load 'p' at the free end. The Young's modulus of elasticity of the beam material is 'E' and moment of inertia of the section is 'I'. Derive an equation for deflection by using Rayleigh-Ritz method. (14 Marks)
2. a. Derive shape functions for one dimensional linear bar element in terms of global coordinates. (06 Marks)
- b. The two-element truss is subjected to an external loading as shown in Fig.Q2(b). The elements have modulus of elasticity  $E_1 = E_2 = 10 \times 10^6 \text{ N/mm}^2$  and roll-sectional area  $A_1 = A_2 = 1.5 \text{ mm}^2$ . Find the displacement components of node 3 and one elemental stresses. (14 Marks)

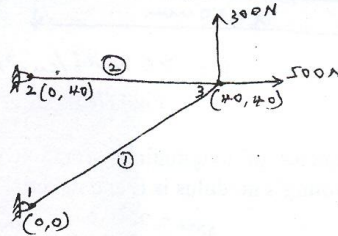


Fig.Q2(b)

(14 Marks)

3. a. Evaluate the shape functions  $N_1$ ,  $N_2$  and  $N_3$  for a triangular element shown in Fig.Q3(a) at point P, having coordinates 3.85 and 4.8. (08 Marks)

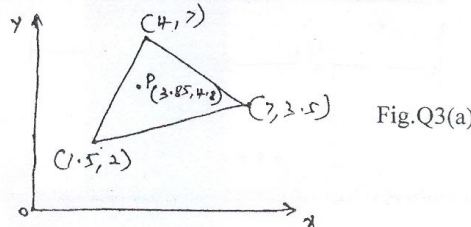


Fig.Q3(a)

- b. Derive the shape functions in local coordinates for a four-noded quadrilateral element (QVAD4). (12 Marks)
4. a. Explain continuity conditions to be satisfied by the interpolation model. (05 Marks)
- b. Write a note on axisymmetric problem. (05 Marks)
- c. Formulate matrix [B] for an axisymmetric ring element. (10 Marks)
5. a. Bring out the expression for potential energy functional for a general three dimensional elastic body. (06 Marks)
- b. Derive shape functions for 3D four-noded tetrahedral element (TET4). (14 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 6 The Fig.Q6 shows a beam subjected to a transverse load applied at the midspan. Using two beam elements, obtain a solution for the midspan deflection. (20 Marks)

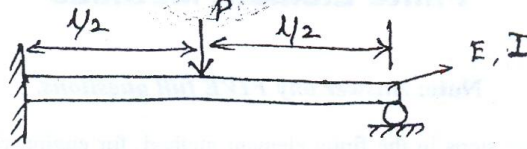


Fig.Q6

- 7 a. Derive the differential equation governing the heat conduction in an orthotropic solid body. (06 Marks)  
 b. Find the temperature distribution in 1-D fin shown in Fig.Q7(b). Consider minimum two elements in finite element mesh. (14 Marks)

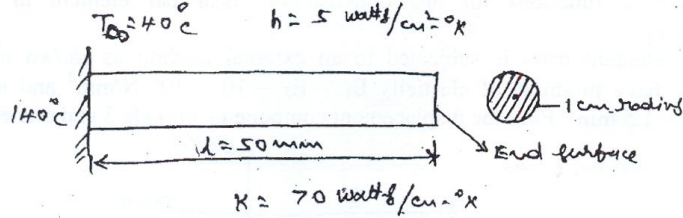


Fig.Q7(b)

- 8 Find the natural frequencies of longitudinal vibration of the unconstrained stepped bar shown in Fig.Q8. The Young's modulus is E and material density is  $\rho$ . (20 Marks)

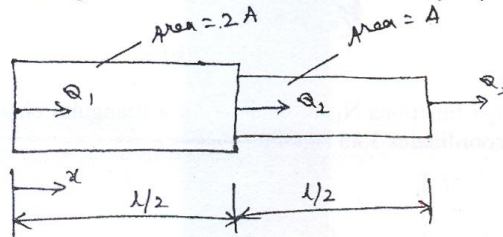


Fig.Q8

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08MMD/MDE/MEA13

First Semester M.Tech. Degree Examination, May/June 2010

## Theory of Elasticity

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Derive Cauchy's stress relations for the resultant normal and shear stresses on an arbitrary plane. (10 Marks)
- b. The state of stress at a point is characterized by the stress tensor,
- $$[\tau_{ij}] = \begin{bmatrix} 10 & 5 & 6 \\ 5 & 8 & 10 \\ 6 & 10 & 6 \end{bmatrix} \text{ MPa}$$
- Calculate the principal stresses. (10 Marks)
- 2 a. Derive the equations of equilibrium for a three dimensional stress state. (10 Marks)
- b. The strain components at a point with respect to xyz co-ordinate system are  $\epsilon_x = 0.1$ ,  $\epsilon_y = 0.2$ ,  $\epsilon_z = 0.3$ ,  $\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0.16$ . If the co-ordinate axes are rotated about the z-axis through  $45^\circ$  in the anticlockwise direction, determine the new strain components. (10 Marks)
- 3 a. Derive the conditions of compatibility for strains in three dimensions and explain their significance. (12 Marks)
- b. The state of strain at a point is given by  $\epsilon_x = 0.001$ ,  $\epsilon_y = -0.003$ ,  $\epsilon_z = 0$ ,  $\gamma_{xy} = 0$ ,  $\gamma_{yz} = 0.003$  and  $\gamma_{xz} = -0.002$ . If the material is having  $E = 210$  GPa and  $G = 82$  GPa, determine the stress matrix at this point. (08 Marks)
- 4 a. Explain Airy's stress function. Prove that  $\phi = Ax^2 + By^2$  is a Airy's stress function and examine the stress distribution represented by it. (08 Marks)
- b. Derive the equations for stress components for bending of a narrow cantilever beam of rectangular cross section having depth 'd', width 'b', length 'l' and subjected to a vertical downward load 'P' at its free end. (12 Marks)
- 5 a. For a thick cylinder subjected to internal and external pressure having internal and external radii 'a' and 'b' respectively, obtain the stress components  $\sigma_r$  and  $\sigma_\theta$  using the stress function approach. (12 Marks)
- b. A thick cylinder of inner radius 100 mm and outer radius 150 mm is subjected to an internal pressure of 12 MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces. (08 Marks)
- 6 a. Derive the expressions for the radial and circumferential stress in the case of a thin circular disc with a concentric hole and a temperature distribution symmetrical about its axis. (10 Marks)
- b. A bare steam pipe 100 mm inside diameter and 2.5 mm thick, carries dry and saturated steam at 1 MPa. Calculate radial and hoop stresses induced in the pipe at its inner and outer periphery. Take co-efficient of thermal expansion as  $10 \times 10^{-6} / ^\circ\text{C}$  and  $E = 200$  GPa. Ambient temperature is  $30^\circ\text{C}$ . (10 Marks)

- 7 a. For a prismatic bar with elliptical cross section subjected to torsional moment, determine the torsional stiffness, maximum shearing stress and its location. (12 Marks)
- b. An elliptical shaft of axes  $2a = 0.1$  m,  $2b = 0.05$  m and  $G = 80$  GPa is subjected to a twisting moment of 3770 N.m. Determine the maximum shearing stress, the angle of twist per unit length and torsional stiffness. (08 Marks)
- 8 a. Write a short note on Euler's buckling load. (05 Marks)
- b. A simply supported beam of length 'L' and supported with a concentrated load Q at its centre is shown in figure Q8. Obtain an expression for buckling load and prove that maximum deflection =  $\frac{QL^3}{48EI} \frac{3(\tan u - u)}{u^3}$ , where  $u = \frac{kL}{2}$  and  $k = \frac{P}{EI}$ . (15 Marks)

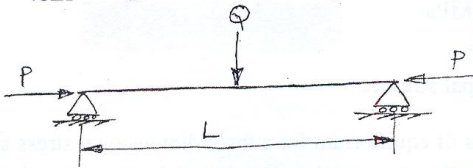


Fig. Q8

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