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First Semester M.Tech. Degree Examination, June/July 2011
Applied Mathematics

20

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Use $\frac{dv}{dt} = g - \frac{c}{m}v$ to compute velocity 'v' prior to opening the chute. The drag coefficient is equal to 12.5 kg/s. Given that g = 9.8, v = 0 at t = 0. Tabulate the values of v for t = =2(2)14. **(10 Marks)**

b. Find the root of the equation $xe^x = 1$ by bisection method. Stopping criterion of $E_S = 0.05\%$. **(10 Marks)**

2 a. Find the root of the equation $xe^x = \cos x$ lies between 0.4 and 0.6, correct to 4 decimal places by Regula - falsi method. **(08 Marks)**

b. Determine the real root of $f(x) = -11 - 22x + 17x^2 - 2.5x^3$, using the Secant method to a value of E_S corresponding to 3 significant figures, by taking $x_0 = 0$, $x_1 = 0.5$. **(12 Marks)**

3 a. Perform 3 iterations of the Muller's method to find the root of the equation $x^3 - 5x + 1$ with $x_0 = 0$, $x_1 = 0.5$ and $x_2 = 1$. **(10 Marks)**

b. Apply the Graeff's roots squaring method to find the roots of $x^3 + 2 - 2x = 0$, correct to two decimal places. **(10 Marks)**

4 a. From the following table of values of x and y, obtain dy/dx & d^2y/dx^2 for x = 1.2 and dy/dx & d^2y/dx^2 for x = 2.2 **(10 Marks)**

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using th Romberg's method, correct to 4 decimal places, with n = 0.5, n = 0.25 and n = 0.125. **(10 Marks)**

5 a. Solve the following system of equations by the Gauss-Jordan method:

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$2x_1 - x_2 + 2x_3 - x_4 = -5$$
$$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$
$$x_1 - 2x_2 - 3x_3 + 2x_4 = 5$$

(10 Marks)

b. Solve $\begin{bmatrix} 4 & 2 & 14 & 14 \\ 2 & 17 & -5 & -101 \\ 14 & -5 & 83 & 155 \end{bmatrix}$ by Cholesk's method **(10 Marks)**

6 a. Determine the largest eigen value and the corresponding eigen vector of a matrix $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ by power method with $x_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. **(10 Marks)**

b. If $T : U \rightarrow V$ is a linear transformation then prove that
i) $T(0) = 0'$, where 0 & 0′ are zero vectors of U & V respectively.
ii) $T(-\alpha) = -T(\alpha) \quad \forall \alpha \in U$
iii) $T(c_1\alpha_1 + c_2\alpha_2 + \ldots\ldots + c_n\alpha_n) = c_1T(\alpha_1) + c_2T(\alpha_2) + \ldots\ldots + c_nT(\alpha_n)$. **(10 Marks)**

7 a. Find the matrix of the linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (x + y, y + z)$ relative to $B_1 = \{(1, 1, 1)\ (1\ 0\ 0)\ (1\ 1\ 0)\}$ and $B_2 = \{(1, 0)\ (0, 1)\}$. **(10 Marks)**

b. If $S = \{u_1, u_2, \ldots\ldots u_p\}$ is an orthogonal set of non zero vectors in R^n, then show that S is linearly independent and express $v = [6\ \ 1\ -8]$ as a linear combination of $u_1 = [3\ \ 1\ \ 1]'$, $u_2 = [-1\ \ 2\ \ 1]'$ and $u_3 = [-½\ \ -2\ \ 7/2]'$. **(10 Marks)**

8 a. Explain the Gram – Schmidt process to find the orthogonal set and construct an orthogonal set for $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **(10 Marks)**

b. Find the least square solution of the system $Ax = b$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ **(06 Marks)**

c. Let $u = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ be two vectors. Find the orthogonal projection of u on v and component of u orthogonal to v. **(04 Marks)**

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10MMD/MDE/MCM/MEA/MAE12

First Semester M.Tech Degree Examination, June/July 2011

Finite Element Method

22

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
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1 a. List important steps in linear static finite element analysis of a structure. **(05 Marks)**

b. Explain convergence criteria. **(05 Marks)**

c. Determine the displacement of nodes of the spring system shown in Fig.Q1(c), using variational approach (PMPE). **(06 Marks)**

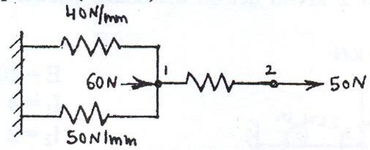
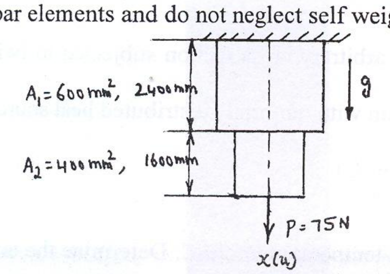


Fig.Q1(c)

d. Explain Galerkin's weighted residual method. **(04 Marks)**

2 a. Find the nodal displacements and element stresses for a stepped bar shown in Fig.Q2(a). Use 2 noded bar elements and do not neglect self weight. **(10 Marks)**



$\rho g = 78500\ N/m^3$
E = 200 GPa

Fig.Q2(a)

b. Derive element stiffness matrix for 2 noded plane truss. **(06 Marks)**

c. Write shape functions for 3 noded bar element and plot them. **(04 Marks)**

3 a. For four node quadrilateral element :

i) Write shape functions

ii) Derive expressions for Jacobian and strain displacement matrix. **(14 Marks)**

b. Obtain load vector for the triangle element shown in Fig.Q3(b). Thickness = 2 mm.

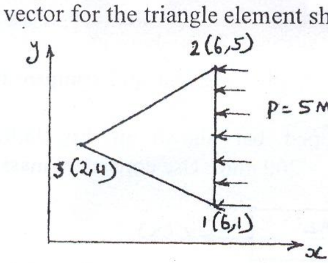


Fig.Q3(b)

(06 Marks)

4 For an axisymmetric triangular element, derive :

a. Jacobian

b. Element strain displacement matrix

c. Element stiffness matrix

d. Consistent load vector due to body force. **(20 Marks)**

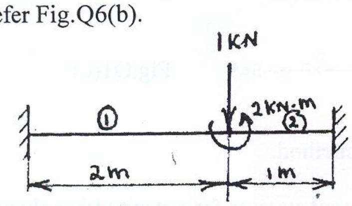
5 a. For a four noded tetrahedral element :
i) Sketch the element in NCS (ξ, n, ζ)
ii) Write displacement model in NCS
iii) Derive shape functions
iv) Jacabian **(12 Marks)**

b. Sketch and write Lagrange shape functions of a linear hexahedral element. **(05 Marks)**

c. For quadratic tetrahedral element, write displacement model. **(03 Marks)**

6 a. For a 2 noded, 1- D beam element :
i) Derive and sketch hermite shape functions
ii) Element curvature matrix.
iii) Consistent load vector due to UDL (P). **(09 Marks)**

b. Determine the deflection and slope at 2 m from the left end of a shaft mounted in bearings. A load of 1 kN and a moment of 2 kN-m act on the shaft. Model the bearings as fixed supports. Refer Fig.Q6(b). **(11 Marks)**



$E = 200$ GPa
$I_1 = 4 \times 10^{-6}$ m^4
$I_2 = 2 \times 10^{-6}$ m^4.

Fig.Q6(b)

7 a. Compare differential equations of :
i) Torsion of a prismatic rod of arbitrary cross section subjected to twisting moment M and
ii) Steady state 2D heat conduction with uniformly distributed heat source Q. **(04 Marks)**

b. The wall of a room is built using :
i) Particle board ($K_1 = 0.164$ W/m°C)
ii) Insulation ($K_2 = 0.125$ W/m°C)
iii) Brick ($K_3 = 2.25$ W/m°C).

The interior wall is maintained at a temperature of 21°C. Determine the temperature at wall interfaces and the rate of heat loss from the room per m^2. **(16 Marks)**

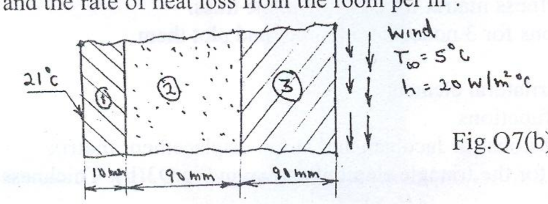


Fig.Q7(b)

8 a. Derive consistent mass matrix for 2 noded bar element and compare it with lumped mass matrix. **(06 Marks)**

b. Evaluate eigen values for the stepped bar shown in Fig.Q8(b). $E = 200$ GPa, $\rho = 800.2$ kg/m^3, $A_1 = 400$ mm^2 and $A_2 = 200$ mm^2. Use consistent mass matrix. **(14 Marks)**

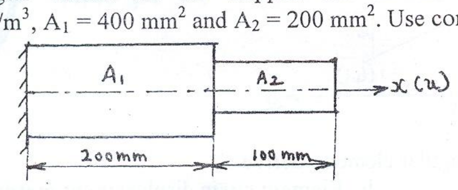


Fig.Q8(b)

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10MMD/MDE14

First Semester M.Tech. Degree Examination, June/July 2011

Experimental Stress Analysis

Time: 3 hrs. Max. Marks:100

27

Note: ***Answer any FIVE full questions.***

1 a. Define gauge factor of a strain gauge. Derive an expression for gauge factor for an electrical resistance strain gauge. **(10 Marks)**

b. Explain the procedure of mounting a strain gauge over a component. List the factors while choosing an adhesive. **(10 Marks)**

2 a. A delta rosette yields the following strain indications $\in_a = -845$ μcm/cm; $\in_b = 1220$ μcm/cm; $\in_c = 710$ μcm/cm. Calculate the maximum principal strain direction, the principal stress and the maximum shear stress. Take $E = 2 \times 10^6$ kg/cm^2, $\gamma = 0.285$ **(20 Marks)**

3 a. Derive an expression for stress optic law in 2D photo elasticity. **(10 Marks)**

b. Explain with neat sketch the experimental procedure for determination of fractional fringe order by Jardy's method of compensation. **(10 Marks)**

4 a. Explain shear difference method of stress separation in 2D photo elasticity. **(10 Marks)**

b. Explain the significance of Isoclinics and Isochromatics in photo elasticity and also show Isoclinic fringes and Isochromatic fringes framed in a plane Poloriscope. **(10 Marks)**

5 a. Sketch and explain scattered light polariscope. **(10 Marks)**

b. Explain stress freezing technique used in 3D photo elasticity. **(10 Marks)**

6 a. Explain any method of separation of principal stress in birefeingent coating method. **(10 Marks)**

b. Explain any method of Moire technique for out of plane problems. **(10 Marks)**

7 a. Explain any two crack detection techniques in brittle coating method. **(10 Marks)**

b. Explain recording and reconstruction process with necessary layout of optics in holography. **(10 Marks)**

8 a. Write short notes : **(20 Marks)**

i) Wheat stone bridge sensitivity.
ii) Calibration techniques in photo elasticity.
iii) Brittle coating technique.
iv) Principle of holography.

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USN | | | | | | | | | | 08MMD/MDE/MCM/MEA/MAR11

First Semester M.Tech. Degree Examination, June/July 2011
Applied Mathematics

21

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. The task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm respectively, compute i) the true error and ii) the true percent relative error for each case. **(07 Marks)**

b. Explain : i) Accuracy and precision ii) Round-off errors. **(06 Marks)**

c. A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. To compute velocity prior to opening the chute. The drag coefficient is equal to 12.5 kg/s. Use $v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$ **(07 Marks)**

2 a. Use the false-position method to determine the root of the equation
$$f(t) = \frac{667.38}{c}\left(1 - e^{-0.146843c}\right) - 40.$$
Perform two iterations, take $x_t = 12$ and $x_4 = 16$ as initial approximations. **(07 Marks)**

b. Use the Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$, employing an initial guess of $x_0 = 0$. **(07 Marks)**

c. Perform three iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$. **(06 Marks)**

3 a. Use the Millers method with guesses of x_0, x_1 and x_2 are 4.5, 5.5 and 5 respectively, to determine a root of the equation $f(x) = x^3 - 13x - 12$. **(07 Marks)**

b. Employ the Bairstow's method to determine the roots of the polynomial
$$f(x) = x^5 - 3.5x^4 + 2.75x^3 + 2.125x^2 - 3.875x + 1.25$$
Use initial guesses of r = s = -1 and iterate to a level of $\xi = 1\%$. **(07 Marks)**

c. Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$, using the Graeffe's root squaring method. **(06 Marks)**

4 a. Use the Trapezoidal rule of numerically integrate
$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$, from a = 0 to b = 0.8. **(10 Marks)**

b. Find the derivative of $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$, at x = 0.5, using the high-accuracy formula. (Forward difference of accuracy and backward difference of accuracy only. Take h = 0.25) **(10 Marks)**

5 a. Use the Gauss-Jordan technique to solve the system of equations
$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$; $0.1x_1 + 7x_2 - 0.3x_3 = -19.3$; $0.3x_1 - 0.2x_2 + 10x_3 = 71.4$ **(10 Marks)**

b. Solve the system of equations
$2x_1 - x_2 - 0x_3 = 7$; $-x_1 + 2x_2 - x_3 = 1$; $0x_1 - x_2 + 2x_3 = 1$
using the Gauss-Seidel method. Take the initial approximation as $x^{(0)} = 0$ and perform three iterations. **(10 Marks)**

6 a. Using the Jacobi method, find all the eigen values and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$ **(10 Marks)**

b. Find the smallest eigenvalue in magnitude of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, using the four iterations of the inverse power method. **(10 Marks)**

7 a. Let L : R3 → R^2 defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now let S = { v_1, v_2, v_3 } and T = { w_1, w_2 } , where $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

and $w_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Find the matrix L with respect to S and T. **(10 Marks)**

b. The mapping $T : P_2 \to P_2$ defined by $T(a_0 + a_1t + a_2t^2) = a_1 + 2a_2t$, is a linear transformation.
i) Find the B – matrix for T, when B is the basis $\{1, t, t^2\}$.
ii) Verify that $[T(p)]_B = [T]_B\,[p]_B$ for each p in P_2. **(10 Marks)**

8 a. For any two positive numbers, say 4 and 5 and for vectors $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in R^2, show that $(u, v) = 4u_1v_1 + 5u_2v_2$ is an inner product. **(10 Marks)**

b. Find the least squares line $y = \beta_0 + \beta_1 x$, that best fits the data (-2, 3), (-1, 5), (0, 5), (1, 4), (2, 3). Suppose the errors in measuring the y-values of the least two data points are greater than for the other points. Weight these data half as much as the rest of the data. **(10 Marks)**

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08MMD/MDE/MCM/MEA/MAE/MAU12

First Semester M.Tech. Degree Examination, June/July 2011
Finite Element Method

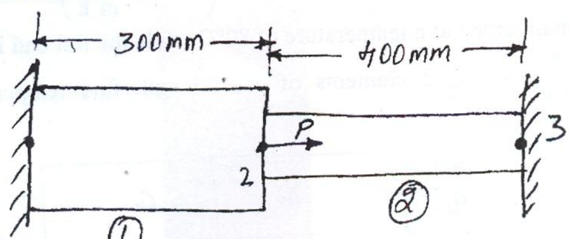
23

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Explain the weighted residual method. **(05 Marks)**
b. Explain the basic procedure of FEM. **(10 Marks)**
c. Write a note on Potential Energy. **(05 Marks)**

2 a. Derive the stiffness matrix for truss. **(10 Marks)**
b. Consider the bar shown in Fig. Q2(b). An axial load $P = 200 \times 10^3$ N is applied as shown. Determine :
i) The nodal displacements
ii) The stress in each material. **(10 Marks)**



Aluminium	Steel
$A_1 = 2400$ mm^2	$A_2 = 600$ mm^2
$E_1 = 70 \times 10^9$ N/m^2	$E_2 = 200 \times 10^9$ N/m^2.

Fig. Q2(b)

3 a. Derive B matrix (strain –displacement matrix) for 2D triangular element. **(12 Marks)**
b. Determine the Jacobian of the transformation J for the triangular element shown in Fig. Q3(b). **(08 Marks)**

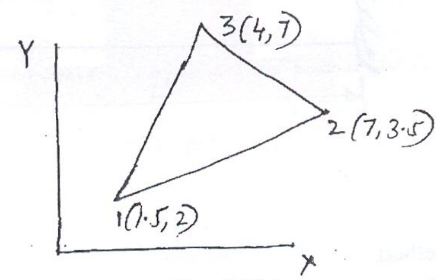
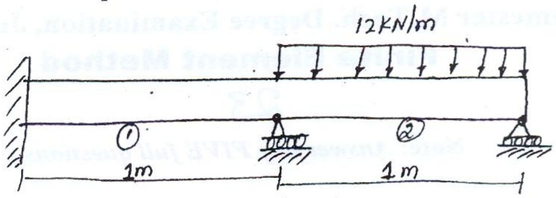


Fig. Q3(b)

4 a. Find J matrix for 4 noded quadrilateral elements. **(10 Marks)**
b. Obtain B matrix in case of axisymmetric triangular element. **(10 Marks)**

5 a. Find J matrix for a 3-D domain, using tetrahedral element. **(10 Marks)**

b. For the beam and loading shown in Fig. Q5(b), determine the slopes at 2 and 3 and the vertical deflection at the midpoint of the distributed load. **(10 Marks)**



$E = 200$ GPa ; $I = 4 \times 10^6$ mm^4

Fig. Q5(b)

6 a. Explain the types of boundary conditions in heat transfer problems. **(06 Marks)**

b. Derive the conductivity matrix for 1-D heat problem using Galerkin method. **(14 Marks)**

7 a. The circular rod shown in Fig. Q7(a), has an outside diameter of 60 mm, length 1 m and is perfectly insulated on its circumference. The left half of the cylinder is aluminium $\left(k = 200 \dfrac{W}{m^\circ k}\right)$ and the right half is copper $\left(k = 389 \dfrac{W}{m^\circ k}\right)$. The extreme right end of the cylinder is maintained at a temperature of 80°C while the left end is subjected to a heat input rate $4000 \dfrac{W}{m^\circ k}$. Using 2 elements of equal length find temperature distribution in the cylinder. **(12 Marks)**

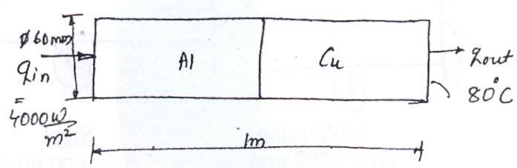


Fig. Q7(a)

b. Consider a uniform corss- section bar as shown in Fig. Q7(b) of length L made up of a material whose Young's modulus and density are given by E and ρ. Estimate the natural frequencies of axial vibration of the bar using both consistent and lumped mass matrices. **(08 Marks)**

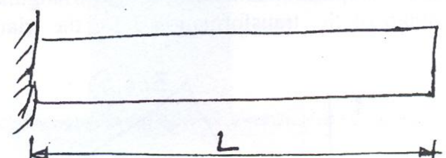


Fig. Q7(b)

8 Write short notes on :

a. Element types
b. Gauss elimination method
c. Numerical integration
d. Properties of stiffness matrix. **(20 Marks)**

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