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Elements of Electrical Engineering

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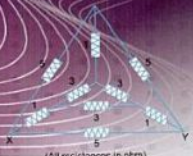

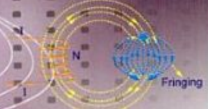

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D.C. Circuits

1.1 Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy gives rise to complicated electrical circuits, generally referred as **networks**. The terms **circuit** and **network** are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit. This chapter includes various techniques of analysing d.c. circuits.

The chapter starts with explaining the fundamentals of electricity along with the detail discussion of the effect of temperature on resistance. The chapter also includes the discussion of the characteristics of series-parallel circuits, star-delta and delta-star transformations and Kirchhoff's laws and its applications.

1.2 The Structure of Matter

In the understanding of fundamentals of electricity, the knowledge of the structure of matter plays an important role. The matter which occupies the space may be soild, liquid or gaseous. The molecules and atoms, of which all substances are composed are not at all elemental, but are themselves made up of simpler entities. We know this because we, up to certain extent, are successful in breaking atoms and studying the resulting products. For instance, such particles are obtained by causing ultraviolet light to fall on cold metal surfaces, such particles are spontaneously ejected from the radioactive elements. So these particles are obtained from many different substances under such widely varying conditions. It is believed that such particles are one of the elemental constituents of all matter, called **electrons.**

Infact, according to the **modern electron theory**, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron,** the **proton** and the **electron**. The proton is defined as positively charged while the electron is defined as negatively charged. The neutron is uncharged i.e. neutral in nature possessing no charge. The mass of neutron and proton is same while the electron is very light, almost 1/1840th the mass of the neutron and proton. The following table gives information about these three particles.

Fundamental particles of matter	Symbol	Nature of charge possessed	Mass in kg.
Neutron	n	0	1.675×10^{-27}
Proton	p+	+	1.675×10^{-27}
Electron	e^-	–	9.107×10^{-31}

Table 1.1

1.2.1 Structure of an Atom

All of the protons and neutrons are bound together into a compact nucleus. **Nucleus** may be thought of as a central sun, about which electrons revolve in a particular fashion. This structure surrounding the nucleus is referred as the **electron cloud.**

In the normal atom the number of protons equal to the number of electrons. An atom as a whole is electrically neutral. The electrons are arranged in different **orbits**. The nucleus exerts a force of attraction on the revolving electrons and hold them together. All these different orbits are called **shells** and possess certain energy. Hence these are also called **energy shells** or **quanta**. The orbit which is closest to the nucleus is always under the tremendous force of attraction while the orbit which is farthest from the nucleus is under very weak force of attraction.

Key Point : *The electron or the electrons revolving in farthest orbit are hence loosely held to the nucleus. Such a shell is called the **valence shell**. And such electrons are called **valence electrons**.*

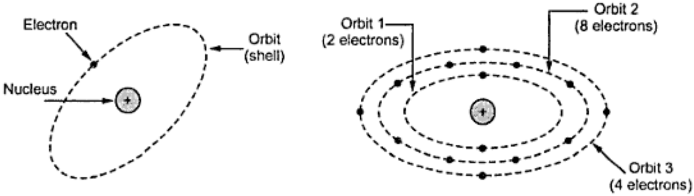
In some atoms such valence electrons are so loosely bound to the nucleus that at room temperature the additional energy imparted to the valence electrons causes them to escape from the shell and exist as **free electrons**. Such free electrons are basically responsible for the flow of electric current through metals.

Key Point : *More the number of free electrons, better is the metal for the conduction of the current. For example, copper has 8.5×10^{28} free electrons per cubic meter and hence it is a good conductor of electricity.*

The electrons which are revolving round the nucleus, not revolve in a single orbit. Each orbit consists of fixed number of electrons. In general, an orbit can contain a maximum of $2n^2$ electrons where n is the number of orbit. So first orbit or shell can occupy maximum of 2×1^2 i.e. 2 electrons while the second shell can occupy maximum of 2×2^2 i.e. 8 electrons and so on. The exception to this rule is that the valence shell can occupy maximum 8 electrons irrespective of its number. Let us see the structure of two different atoms.

1) Hydrogen : This atom consists of one proton and one electron revolving around the nucleus. This is the simplest atom. This is shown in the Fig. 1.1 (a). The dot represents an electron while nucleus is represented by a circle with the positive sign inside it.

2) Silicon : This atom consists of 14 electrons. These revolve around the nucleus in three orbits. The first orbit has maximum 2 electrons, the second has maximum 8 electrons and the third orbit has remaining 4 electrons. This is shown in the Fig. 1.1 (b).



(a) Hydrogen atom **(b) Silicon atom**

Fig. 1.1

The 4 electrons located in the farthest shell are loosely held by the nucleus and generally available as free electrons. If by any means some of the electrons are removed, the negative charge of that atom decreases while positively charged protons remain same. The resultant charge on the atom remains more positive in nature and such element is called **positively charged.** While if by any means the electrons are added, then the total negative charge increases than positive and such element is called **negatively charged.**

1.3 Concept of Charge

In all the atoms, there exists number of electrons which are very loosely bound to its nucleus. Such electrons are free to wonder about, through the space under the influence of specific forces. Now when such electrons are removed from an atom it becomes positively charged. This is because of loosing negatively charged particles i.e. electrons from it. As against this, if excess electrons are added to the atom it becomes negatively charged.

Key Point : *Thus total deficiency or addition of excess electrons in an atom is called its charge and the element is said to be charged.*

The following table shows the different particles and charge possessed by them.

Particle	Charge possessed in Coulomb	Nature
Neutron	0	Neutral
Proton	1.602×10^{-19}	Positive
Electron	1.602×10^{-19}	Negative

Table 1.2

1.3.1 Unit of Charge

As seen from the Table 1.2 that the charge possessed by the electron is very very small hence it is not convenient to take it as the unit of charge.

The unit of the measurement of the charge is **Coulomb**.

The charge on one electron is 1.602×10^{-19}, so one coulomb charge is defined as the charge possessed by total number of ($1/\ 1.602\times10^{-19}$) electrons i.e. 6.24×10^{18} number of electrons.

Thus, $$1 \text{ coulomb} = \text{charge on } 6.24\times10^{18} \text{ electrons}$$

From the above discussion it is clear that if an element has a positive charge of one coulomb then that element has a deficiency of 6.24×10^{18} number of electrons.

Key Point: *Thus, addition or removal of electrons causes the change in the nature of the charge possessed by the element.*

1.4 Concept of Electromotive Force and Current

It has been mentioned earlier that the free electrons are responsible for the flow of electric current. Let us see how it happens.

To understand this, first we will see the enlarged view of the inside of a piece of a conductor. A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions as shown in the Fig. 1.2.

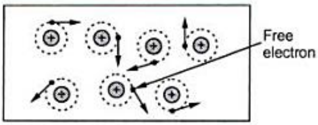


Fig. 1.2 Inside the piece of a conductor

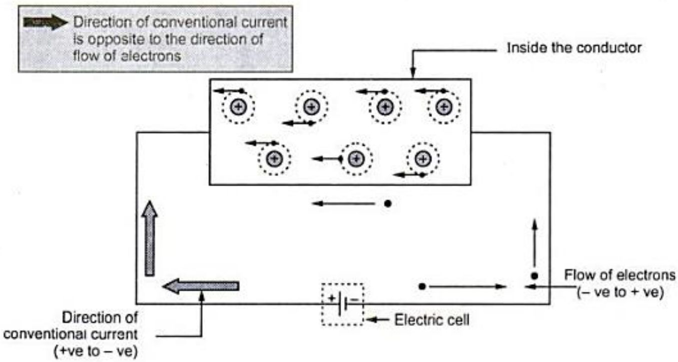


Fig. 1.3 The flow of current

The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected across the two ends of a conductor. Such physical phenomenon is represented in the Fig. 1.3.

Key Point: *An electrical effort required to drift the free electrons in one particular direction, in a conductor is called* ***Electromotive Force*** *(e.m.f.)*

The metal consists of particles which are charged. The like charges repel while unlike charges attract each other. But as external electric effort is applied, the free electrons as are negatively charged, get attracted by positive of the cell connected. And this is the reason why electrons get aligned in one particular direction under the influence of an electromotive force.

Key Point: *The electric effort i.e. e.m.f. is maintained across the positive and negative electrodes of the cell, due to the chemical action inside the solution contained in the cell.*

Atoms, when they loose or gain electrons, become charged accordingly and are called **ions**. Now when free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor. So there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an **Electric Current**.

The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called **direction of conventional current**.

Key Point: *Direction of conventional current is from positive to negative terminal while direction of flow of electrons is always from negative to positive terminal, through the external circuit across which the e.m.f. is applied.*

We are going to follow direction of the conventional current throughout this book. i.e. from positive to negative terminal, of the battery through the external circuit.

1.5 Relation between Charge and Current

The current is flow of electrons. Thus current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second. So the **flow of charge per unit time** is used to quantify an electric current.

Key Point: *So current can be defined as rate of flow of charge in an electric circuit or in any medium in which charges are subjected to an external electric field.*

The charge is indicated by **Q** coulombs while current is indicated by I. The unit for the current is **Amperes** which is nothing but coulombs/sec. Hence mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$I = \frac{Q}{t} \quad \text{amperes}$$

Where I = Average current flowing

Q = Total charge transferred

t = Time required for transfer of charge.

Definition of 1 Ampere : *A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.*

Now 1 coulomb is 6.24×10^{18} number of electrons. So 1 ampere current flow means flow of 6.24×10^{18} electrons per second across a section taken any where in the circuit.

1 Ampere current = Flow of 6.24×10^{8} electrons per second

1.6 Concept of Electric Potential and Potential Difference

When two similarly charged particles are brought near, they try to repel each other while dissimilar charges attract each other. This means, every charged particle has a tendency to do work.

Key Point *: This ability of a charged particle to do the work is called its* ***electric potential****. The unit of electric potential is volt.*

The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge i.e. positive charge of one coulomb from infinity to that point.

Mathematically it is expressed as,

$$\text{Electrical Potential} = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

Let us define now the potential difference.

It is well known that, flow of water is always from higher level to lower level, flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on, also exists in electric circuits. In electric circuits flow of current is always from higher electric potential to lower electric potential. So we can define potential difference as below :

Key Point: *The difference between the electric potentials at any two given points in a circuit is known as Potential Difference (p.d.). This is also called voltage between the two points and measured in volts. The symbol for voltage is V.*

For example, let the electric potential of a charged particle A is say V_1 while the electric potential of a charged particle B is say V_2. Then the potential difference between the two particles A and B is $V_1 - V_2$. If $V_1 - V_2$ is positive we say that A is at higher potential than B while if $V_1 - V_2$ is negative we say that B is at higher potential than A.

Key Point: *The potential difference between the two points is one volt if one joule of work is done in displacing unit charge (1 coulomb) from a point of lower potential to a point of higher potential.*

Consider two points having potential difference of V volts between them, as shown in the Fig. 1.4. The point A is at higher potential than B. As per the definition of volt, the V joules of work is to be performed to move unit charge from point B to point A.

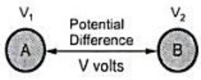


Fig. 1.4

Thus, when such two points, which are at different potentials are joined together with the help of wire, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. Hence, to maintain the flow of electrons i.e. flow of electric current, there must exist a potential difference between the two points.

Key Point: *No current can flow if the potential difference between the two points is zero.*

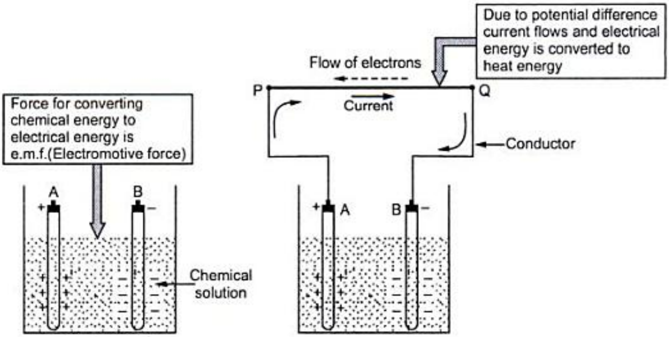
1.7 Electromotive Force and Potential Difference

Earlier we have seen the concept of e.m.f. The e.m.f. is that force which causes the flow of electrons i.e. flow of current in the given circuit. Let us understand its meaning more clearly.

Consider a simple cell shown in Fig. 1.5 (a). Due to the chemical reaction in the solution the terminal 'A' has acquired positive charge while terminal 'B' has acquired negative charge.

If now a piece of conductor is connected between the terminals A and B then flow of electrons starts through it. This is nothing but the flow of current through the conductor. This is shown in the Fig. 1.5 (b). The electrons will flow from terminal B to A and hence direction of current is from A to B i.e. positive to negative as shown.

One may think that once the positive charge on terminal A gets neutralised due to the electrons, then flow of electrons will stop. Both the terminals may get neutralised after some time. But this does not happen practically. This is because chemical reaction in the solution maintains terminal A positively charged and terminal B as negatively charged. This maintains the flow of current. The chemical reaction converts chemical energy into electric energy which maintains flow of electrons.



(a) Cell **Fig. 1.5** **(b) Current due to a cell**

> **Key Point** : *This force which requires to keep electrons in motion i.e. to convert chemical or any other form of energy into electric energy is known as* ***Electromotive Force (e.m.f.) denoted by*** *E. The unit of e.m.f. is* ***volt*** *and unless and untill there is some e.m.f. present in the circuit, a continuous flow of current is not possible.*

Consider two points P and Q as shown in the Fig. 1.5 (b), then the current is flowing from point P to point Q. This means there exists a potential difference between the points P and Q. This potential difference is called voltage denoted as V and measured in volts.

In other words we can explain the difference between e.m.f. and p.d. as below. In the cell two energy transformations are taking place simultaneously. The one is chemical energy because of solution in cell is getting converted to electrical energy which is basic cause for flow of electrons and hence current. The second is when current flows, the piece of metal gets heated up i.e. electrical energy is getting converted to heat energy, due to flow of current.

In the first transformation electrical energy is generated from other form of energy. The force involved in such transformation is electromotive force. When current flows, due to which metal gets heated up i.e. due to existence of potential difference between two points, voltage is existing. And in such case electrical energy gets converted to other form of energy. The force involved in such transformation is nothing but the potential difference or voltage. Both e.m.f. and potential difference are in generally referred as voltage.

1.8 Resistance

The current in the electrical circuit not only depends on e.m.f. or p.d. but also on the circuit parameters. For example if lamp is connected in a circuit, current gets affected and lamp filament becomes hot radiating light. But if contact at one end is loose, current decreases but sparking occurs at loose contact making it hot. If two lamps are connected one after the other, brightness obtained is less than that obtained by a single lamp. These examples show that current, flow of electrons depends on the circuit parameters and not only the e.m.f. alone.

> **Key Point**: *This property of an electric circuit tending to prevent the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.*

The concept of resistance is analogous to the friction involved in the mechanical motion. Every metal has a tendency to oppose the flow of current. Higher the availability of the free electrons, lesser will be the opposition to the flow of current. The conductor due to the high number of free electrons offer less resistance to the flow of current. The opposition to the flow of current and conversion of electrical energy into heat energy can be explained with the help of atomic structure as below.

When the flow of electrons is established in the metal, the ions get formed which are charged particles as discussed earlier. Now free electrons are moving in specific direction when connected to external source of e.m.f. So such ions always become obstruction for the flowing electrons. So there is collision between ions and free flowing electrons. This not only reduces the speed of electrons but also produces the heat. The effect of this is nothing but the reduction of flow of current. Thus the material opposes the flow of current.

The resistance is denoted by the symbol 'R' and is measured in **ohm** symbolically represented as Ω . We can define unit ohm as below.

> **Key Point**: *1 Ohm : The resistance of a circuit, in which a current of 1 Ampere generates the heat at the rate of one joules per second is said to be 1 ohm.*

Now $\quad$ 4.186 joules = 1 calorie

hence $\quad$ 1 joule = 0.24 calorie

Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

Earlier we have seen that some materials possess large number of free electrons and hence offer less opposition to the flow of current. Such elements are classified as the **'Conductors'** of electricity. While in some materials the number of free electrons are very less and hence offering a large resistance to the flow of current. Such elements are classified as the **'Insulators'** of electricity.

Examples of good conductors are silver, copper, aluminium while examples of insulators are generally non metals like glass, rubber, wood, paper etc.

Let us see the factors affecting the resistance.

1.8.1 Factors Affecting the Resistance

1. Length of the material : The resistance of a material is directly proportional to the length. The resistance of longer wire is more. Length is denoted by '*l*' .

2. Cross-sectional area : The resistance of a material is inversely proportional to the cross-sectional area of the material. More cross-sectional area allowed the passage of more number of free electrons, offering less resistance. The cross sectional area is denoted by 'a'.

3. The type and nature of the material : As discussed earlier whether it consists more number of free electrons or not, affects the value of the resistance. So material which is conductor has less resistance while an insulator has very high resistance.

4. Temperature : The temperature of the material affects the value of the resistance. Generally the resistance of the material increases as its temperature increases. Generally effect of small changes in temperature on the resistance is not considered as it is negligibly small.

So for a certain material at a certain temperature we can write a mathematical expression as,

$$R \propto \frac{l}{a}$$

and effect of nature of material is considered through the constant of proportionality denoted by ρ (rho) called **resistivity or specific resistance** of the material. So finally,

$$\boxed{R = \frac{\rho l}{a}}$$

Where l = Length in metres

a = Cross-sectional area in square metres

ρ = Resistivity in ohms-metres

R = Resistance in ohms

1.9 Resistivity and Conductivity

The resistivity or specific resistance of a material depends on nature of material and denoted by ρ(rho). From the expression of resistance it can be expressed as,

$$\boxed{\rho = \frac{Ra}{l} \quad \text{i.e.} \quad \frac{\Omega\text{-}m^2}{m} \quad \text{i.e.} \quad \Omega\text{-}m}$$

It is measured in Ω - m.

Definition : *The resistance of a material having unit length and unit cross-sectional area is known as its specific resistance or resistivity.*

The Table 1.3 gives the value of resistivity of few common materials.

Name of material	ρ in Ω - m
International Standard Copper	1.72×10^{-8}
Aluminium Cast	2.6×10^{-8}
Bronze	3.6×10^{-8}
Iron-Wrought	10.7×10^{-8}
Carbon Graphite	4.6×10^{-8}
Gold	2.36×10^{-8}
Silver Annealed	1.58×10^{-8}
Lead	22×10^{-8}

Table 1.3

Key Point : *A material with highest value of resistivity is the best insulator while with poorest value of resistivity is the best conductor.*

1.9.1 Conductance (G)

The conductance of any material is reciprocal of its resistance and is denoted as G. It is the indication of ease with which current can flow through the material. It is measured in siemens.

So

$$G = \frac{1}{R} = \frac{a}{\rho l} = \frac{1}{\rho}\left(\frac{a}{l}\right) = \sigma\left(\frac{a}{l}\right)$$

1.9.2 Conductivity

The quantity (1/ρ) is called **conductivity,** denoted as σ (sigma). **Thus the conductivity is the reciprocal of resistivity. It is measured in siemens/ m.**

Key Point : *The material having highest value of conductivity is the best conductor while having poorest conductivity is the best insulator.*

➠ **Example 1.1 :** *The resistance of copper wire 25 m long is found to be 50 Ω. If its diameter is 1mm, calculate the resistivity of copper.*

Solution :

$l = 25$ m, d = 1 mm, R = 50 Ω

$$a = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(1^2) = 0.7853\ mm^2$$

Now $$\rho = \frac{Ra}{l} = \frac{50 \times 0.7853 \times 10^{-6}}{25} \quad \ldots 1\ mm = 10^{-3}\ m$$

$$= 1.57 \times 10^{-6}\ \Omega - m = \mathbf{1.57\ \mu\Omega - m}$$

Example 1.2 : *Calculate the resistance of a 100 m length of wire having a uniform cross-sectional area of 0.02 mm² and having resistivity of 40 μΩ - cm.*

If the wire is drawn out to four time its original length, calculate its new resistance.

Solution : l = 100 m, a = 0.02 mm² and ρ = 40 μΩ - cm

Now $$R = \frac{\rho l}{a} \qquad \text{express a in } m^2 \text{ and } \rho \text{ in } \Omega - m$$

$$= \frac{40 \times 10^{-6} \times 10^{-2} \times 100}{0.02 \times 10^{-6}} = \mathbf{2000\ \Omega}$$

The wire is drawn out such that $l' = 4l$

But the volume of the wire must remain same before and after drawing the wire, which is the product of length and area.

$$\therefore \quad \text{Volume} = a \times l = a' \times l'$$

$$\therefore \quad a' = \frac{a \times l}{l'} = \frac{a \times l}{4l} = \frac{a}{4}$$

$$\therefore \quad R' = \text{new resistance} = \frac{\rho l'}{a'} = \frac{\rho(4l)}{\left(\frac{a}{4}\right)}$$

$$= 16\left(\frac{\rho l}{a}\right) = 16\ R = \mathbf{32000\ \Omega}$$

1.10 Effect of Temperature on Resistance

The resistance of the material increases as temperature of a metal increases. Let us see the physical phenomenon involved in this process.

Atomic structure theory says that under normal temperature when the metal is subjected to potential difference, ions i.e. unmovable charged particles get formed inside the metal. The electrons which are moving randomly, get aligned in a particular direction as shown in the Fig. 1.6.

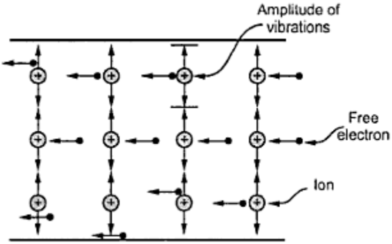


Fig. 1.6 Vibrating ions in a conductor

At low temperatures, the ions are almost stationary. But as temperature increases, the ions gain energy and start oscillating about their mean position. Higher the temperature, greater is the amplitude. Such vibrating ions cause obstruction to the flowing electrons. Similarly due to high amplitude of oscillating ions, chances of collision of electrons are more. Due to collision and obstruction due to higher amplitude of oscillations of ions, **the resistance of material increases as temperature increases.**

But this is not true for all materials. In some cases, the resistance decreases as temperature increases.

Key Point: *So effect of temperature on the resistance depends on nature of material.*

Let us see the effect of temperature on resistance of various category of materials.

1.10.1 Effect of Temperature on Metals

The resistance of all the pure metals like copper, iron, tungsten etc. increases linearly with temperature. For a copper, its resistance is 100 Ω at 0° then it increases linearily upto 100 °C. At a temperature of – 234.5 °C it is almost zero. Such variation is applicable to all the pure metals in the range of 0 °C to 100 °C. This is shown in the Fig. 1.7.

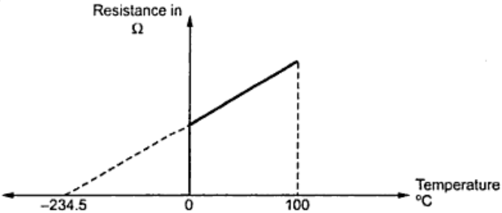


Fig. 1.7 Effect of temperature on metals

1.10.2 Effect of Temperature on Carbon and Insulators

The effect of temperature on carbon and insulators is exactly opposite to that of pure metals. **Resistance of carbon and insulators decreases as the temperature increases.** This can be explained with the help of atomic theory as below :

Insulators do not have enough number of free electrons and hence they are bad conductor of electricity. Now what happens in conductor is due to increase in temperature vibrations of ions increase but it does not increase number of free electrons. While in carbon and insulators due to increase in temperature, no doubt vibrations of ions increases but due to high temperature few electrons from atoms gain extra energy and made available as free electrons. So as number of free electrons increase though vibrations of ions increases overall difficulty to the flow of electrons reduces. This causes decrease in resistance.

Key Point : *So in case of carbon and insulating materials like rubber, paper and all electrolytes, the resistance decreases as the temperature increases.*

1.10.3 Effect of Temperature on Alloys

The resistance of alloys increases as the temperature increases but rate of increase is not significant. In fact the alloys like Manganin (alloy of copper, managanese and nickel), Eureka (alloy of copper and nickel) etc. show almost no change in resistance for considerable change in the temperature. Due to this property alloys are used to manufacture the resistance boxes.

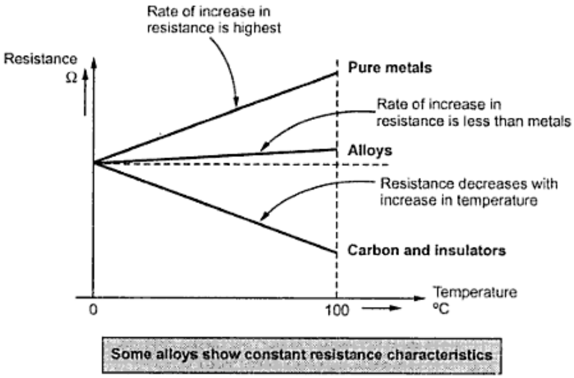


Fig. 1.8 (a) Effect of temperature on resistance

The Fig. 1.8 (a) shows the effect of temperature on metals, insulating materials and alloys.

The study of this, is very useful in finding out the temperature rise of cables, different windings in machines etc. Such study is possible by introducing the factor called resistance temperature coefficient of the material.

1.10.4 Effect of Temperature on Semiconductors

The materials having conductivity between that of metals and insulators are called semiconductors. The examples are silicon, germanium etc.

Key Point : *Semiconductors have negative temperature coefficient of resistivity hence as temperature increases, their resistance decreases.*

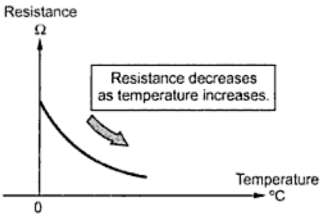


Fig. 1.8 (b) Effect of temperature on semiconductors

At normal temperature, the resistance of semiconductors is high. But as temperature increases, their resistance decreases with fast rate as shown in the Fig. 1.8(b). At absolute zero temperature, the semiconductors behave as perfect insulators. At higher temperature, more valence electrons acquire the energy and become free electrons. Due to increased number of free electrons, resistance of semiconductors decreases as temperature increases.

1.11 Resistance Temperature Coefficient (R.T.C.)

From the discussion uptill now we can conclude that the change in resistance is,

1) Directly proportional to the initial resistance.

2) Directly proportional to the change in temperature.

3) Depends on the nature of the material whether it is a conductor, alloy or insulator.

Let us consider a conductor, the resistance of which increases with temperature linearily

Let R_0 = Initial resistance at 0 °C

R_1 = Resistance at t_1 °C

R_2 = Resistance at t_2 °C

As shown in the Fig. 1.9, $R_2 > R_1 > R_0$.

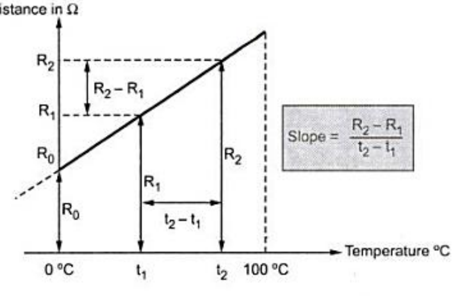


Fig. 1.9 Graph of resistance against temperature

Key Point: *The change in resistance with temperature is according to the factor called resistance temperature coefficient (R.T.C.) denoted by α.*

Definition of R.T.C. : The resistance temperature coefficient at t °C is the ratio of change in resistance per degree celcius to the resistance at t °C.

$$\text{R.T.C. at t °C} = \frac{\Delta R \text{ per °C}}{R_t} = \alpha_t$$

From the Fig. 1.9, change in resistance = $R_2 - R_1$

change in temperature = $t_2 - t_1$

$\therefore$ change in resistance per °C = $\frac{\Delta R}{\Delta t} = \frac{R_2 - R_1}{t_2 - t_1}$

Hence according to the definition of R.T.C. we can write α_1 i.e. R.T.C. at t_1 °C as,

$$\alpha_1 = \frac{\text{Change in resistance per °C}}{\text{Resistance at } t_1 \text{ °C}} = \frac{(R_2 - R_1 / t_2 - t_1)}{R_1}$$

Similarly R.T.C. at 0 °C i.e. α_0 can be written as,

$$\alpha_0 = \frac{(R_1 - R_0 / t_1 - 0)}{R_0}$$

But $\frac{R_2 - R_1}{t_2 - t_1} = \frac{R_1 - R_0}{t_1 - 0}$ = slope of the graph

Hence R.T.C. at any temperature t °C can be expressed as,

$$\alpha_t = \frac{\text{Slope of the graph}}{R_t}$$

1.11.1 Unit of R.T.C.

We know, $\alpha_t = \dfrac{\text{Change in resistance per °C}}{\text{Resistance at t °C}} \Rightarrow \dfrac{\Omega / °C}{\Omega} \Rightarrow /°C$

> Thus unit of R.T.C. is **per degree celcius** i.e. / °C

1.11.2 Use of R.T.C. in Calculating Resistance at t °C

Let
$$\alpha_0 = \text{R.T.C. at } 0\ °C$$
$$R_0 = \text{Resistance at } 0\ °C$$
$$R_1 = \text{Resistance at } t_1\ °C$$

Then
$$\alpha_0 = \frac{(R_1 - R_0 / t_1 - 0)}{R_0} = \frac{R_1 - R_0}{t_1 R_0}$$

$$\therefore \quad R_1 - R_0 = \alpha_0 t_1 R_0$$

$$\therefore \quad R_1 = R_0 + \alpha_0 t_1 R_0 = R_0 (1 + \alpha_0 t_1)$$

Thus resistance at any temperature can be expressed as,

$$\boxed{\mathbf{R_t = R_0 (1 + \alpha_0 t)}}$$

So knowing R_0 and α_0 at 0 °C, the resistance at any t °C can be obtained.

Alternatively this result can be expressed as below,

Let
$$R_1 = \text{Resistance at } t_1\ °C$$
$$R_t = \text{Resistance at } t\ °C$$
$$\alpha_1 = \frac{(R_t - R_1 / t - t_1)}{R_1} \quad \text{... from definition}$$

$$\therefore \quad \alpha_1 R_1 (t - t_1) = R_t - R_1$$

$$\therefore \quad R_t = R_1 [1 + \alpha_1 (t - t_1)]$$

Where $t - t_1$ = Change in temperature = Δt

In general above result can be expressed as,

$$R_{final} = R_{initial} [1 + \alpha_{initial} (\Delta t)]$$

So if initial temperature is t_1 and final is t_2, we can write,

$$\boxed{\mathbf{R_2 = R_1 [1 + \alpha_1 \Delta t]}}$$

> **Key Point**: *Thus knowing resistance and R.T.C. of the material at any one temperature, the resistance of material at any other temperature can be obtained.*

1.11.3 Effect of Temperature on R.T.C.

From the above discussion, it is clear that the value of R.T.C. also changes with the temperature. As the temperature increases, its value decreases. For any metal its value is maximum at 0 °C .

From the result of section 1.11.2 we can write,

$$R_2 = R_1 \; [1 + \alpha_1 (t_2 - t_1)] \qquad \ldots (1)$$

where R_1 and α_1 are resistance and R.T.C. at t_1 °C and R_2 is resistance at t_2 °C.

If the same resistance is cooled from t_2 to t_1 °C and if α_2 is R.T.C. at t_2 °C then,

$$R_1 = R_2 \; [1 + \alpha_2 (t_1 - t_2)] \qquad \ldots (2)$$

Dividing equation (1) by R_2,

$$\therefore \quad 1 = \frac{R_1}{R_2} \; [1 + \alpha_1 (t_2 - t_1)]$$

$$\therefore \quad \frac{R_2}{R_1} = 1 + \alpha_1 (t_2 - t_1) \qquad \ldots (3)$$

Dividing equation (2) by R_1,

$$1 = \frac{R_2}{R_1} \; [1 + \alpha_2 (t_1 - t_2)]$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1}{1+\alpha_2 (t_1 - t_2)} \qquad \ldots (4)$$

Equating (3) and (4) we can write,

$$1 + \alpha_1 (t_2 - t_1) = \frac{1}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \alpha_1 (t_2 - t_1) = \frac{1}{1+\alpha_2 (t_1 - t_2)} - 1 = \frac{1-1-\alpha_2(t_1 - t_2)}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \alpha_1 (t_2 - t_1) = \frac{-\alpha_2 (t_1 - t_2)}{1+\alpha_2 (t_1 - t_2)} = \frac{\alpha_2(t_2 - t_1)}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \boxed{\alpha_1 = \frac{\alpha_2}{1+\alpha_2 (t_1 - t_2)} = \frac{1}{\frac{1}{\alpha_2} + (t_1 - t_2)}}$$

or

$$\boxed{\alpha_2 = \frac{\alpha_1}{1+\alpha_1 (t_2 - t_1)} = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}}$$

Using any of the above expression if α at any one temperature t_1°C is known then α at any other temperature t_2 can be obtained.

If starting temperature is $t_1 = 0\ ^\circ C$ and α at $t\ ^\circ C$ i.e. α_t is required then we can write,

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0(t-0)} = \frac{\alpha_0}{1+\alpha_0 t}$$

This is very useful expression to obtain R.T.C. at any temperature t °C from α_0.

1.11.4 Effect of Temperature on Resistivity

Similar to the resistance, the specific resistance or resistivity also is a function of temperature. For pure metals it increases as temperature increases.

So similar to resistance temperature coefficient we can define temperature coefficient of resistivity as fractional change in resistivity per degree centigrade change in temperature from the given reference temperature.

i.e. if ρ_1 = resistivity at $t_1\ ^\circ C$

ρ_2 = resistivity at $t_2\ ^\circ C$

then temperature coefficient of resistivity at t_1 °C can be defined as,

$$\alpha_{t1} = \frac{(\rho_2 - \rho_1)/(t_2 - t_1)}{\rho_1}$$

Similarly we can write the expression for resistivity at time t °C as,

$$\rho_t = \rho_0(1+\alpha_0 t)$$

$$\rho_{t2} = \rho_{t1}[1+\alpha_{t1}(t_2-t_1)]$$

Example 1.3 : *A certain winding made up of copper has a resistance of 100 Ω at room temperature. If resistance temperature coefficient of copper at 0 °C is 0.00428 / °C, calculate the winding resistance if temperature is increased to 50 °C. Assume room temperature as 25 °C.*

Solution : $t_1 = 25\ ^\circ C$, $R_1 = 100\ \Omega$, $t_2 = 50\ ^\circ C$, $\alpha_0 = 0.00428\ /^\circ C$

Now $\alpha_t = \dfrac{\alpha_0}{1+\alpha_0 t}$

∴ $\alpha_1 = \dfrac{\alpha_0}{1+\alpha_0 t_1} = \dfrac{0.00428}{1+0.00428 \times 25} = 0.003866\ /^\circ C$

Use $R_2 = R_1[1+\alpha_1(t_2-t_1)] = 100[1+0.003866(50-25)]$

$= \mathbf{109.6657\ \Omega}$ ... Resistance at 50 °C

Example 1.4 : *The resistance of a wire increases from 40 – ohm at 20 °C to 50 – ohm at 70 °C. Find the temperature co-efficient of resistance at 0 °C.*

Solution : $R_1 = 40\ \Omega$, $t_1 = 20\ ^\circ C$, $R_2 = 50\ \Omega$, $t_2 = 70\ ^\circ C$

Now, $R_2 = R_1 [1 + \alpha_1 \Delta t]$

$\therefore \quad 50 = 40 [1 + \alpha_1 (70 - 20)]$ i.e. $\frac{5}{4} = 1 + \alpha_1 (50)$

$\therefore \quad 50\ \alpha_1 = 0.25$

$\therefore \quad \alpha_1 = 5 \times 10^{-3} /^\circ C$ i.e. at $t_1 = 20\ ^\circ C$

Now, $\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$ i.e. $\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$

$\therefore \quad 5 \times 10^{-3} = \frac{\alpha_0}{1 + \alpha_0 \times 20}$ i.e. $1 + 20\ \alpha_0 = 200\ \alpha_0$

$\therefore \quad 180\ \alpha_0 = 1$

$\therefore \quad \alpha_0 = \mathbf{5.55 \times 10^{-3} /^\circ C}$ ... Temperature coefficient at 0 °C.

Example 1.5 : *A specimen of copper has a resistivity (ρ) and a temperature coefficient of 1.6×10^6 ohm - cm at 0 °C and 1/254.5 at 20 °C respectively. Find both of them at 60 °C.*

Solution : $\rho_0 = 1.6 \times 10^{-6}\ \Omega\text{-cm}$, $\alpha_1 = \frac{1}{254.5} /^\circ C$ at 20 °C

Now $\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$

$\therefore \quad \alpha_1 = \frac{\alpha_0}{1+\alpha_0 \times 20}$

$\therefore \quad \frac{1}{254.5} = \frac{\alpha_0}{1+20\alpha_0}$

$\therefore \quad 1 + 20\ \alpha_0 = 254.5\ \alpha_0$

$\therefore \quad \alpha_0 = \frac{1}{234.5} /^\circ C$ at 0 °C

$\therefore \quad \alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60} = \frac{1/234.5}{1+\frac{60}{234.5}} = \mathbf{\frac{1}{294.5}} /^\circ C$ ... at 60 °C

$\rho_t = \rho_0 (1 + \alpha_0 t)$

$\therefore \quad \rho_{60} = 1.6 \times 10^{-6} \left(1 + \frac{1}{234.5} \times 60\right) = \mathbf{2 \times 10^{-6}\ \Omega\text{-cm}}$

Example 1.6 : *A resistance element having cross sectional area of 10 mm^2 and a length of 10 mtrs. takes a current of 4 A from a 220 V supply at ambient temperature of 20 °C. Find out, i) the resistivity of the material and ii) current it will take when the temperature rises to 60 °C. Assume α_{20} = 0.0003 /°C.*

Solution : $a = 10\ mm^2 = 10 \times 10^{-6}\ m^2,\ V = 220\ V,\ l = 10\ m,\ I = 4\ A,\ t_1 = 20\ °C$

and $\alpha_{20} = 0.0003/°C,\ R_1 = \frac{V}{I} = \frac{220}{4} = 55\ \Omega$

Now, $R_1 = \frac{\rho_1 l}{a}$ i.e. $55 = \frac{\rho_1 \times 10}{10 \times 10^{-6}}$

$\therefore$ $\rho_1 = 0.000055\ \Omega - m = 55\ \mu\Omega - m$ ... at 20 °C

i) ρ_2 at $t_2 = 60$ °C, $\rho_2 = \rho_1 [1 + \alpha_1 (t_2 - t_1)]$

$= 0.000055\ [1 + 0.0003\ (60 - 20)]$

$= \mathbf{55.66\ \mu\Omega - m}$

ii) $R_2 = \frac{\rho_2 l}{a}$

$\therefore$ $R_2 = \frac{55.66 \times 10^{-6} \times 10}{10 \times 10^{-6}} = 55.66\ \Omega$

$\therefore$ $I = \frac{V}{R_2} = \frac{220}{55.66} = \mathbf{3.9525\ A}$ ... at 60 °C

Example 1.7 : *A coil has a resistance of 18 ohm at 20 °C and 22 ohm at 50 °C. Find the rise in the temperature when resistance becomes 24 ohm. The room temperature is 18 °C.*

Solution : $R_1 = 18\ \Omega,\ t_1 = 20\ °C,\ R_2 = 22\ \Omega$ and $t_2 = 50\ °C$

Now, $R_2 = R_1[1 + \alpha_1 (t_2 - t_1)]$ i.e. $22 = 18\ [1 + \alpha_1 (50 - 20)]$

Solving, $\alpha_1 = 0.007407$ /°C

Now $R_3 = 24\ \Omega$ and $R_3 = R_1\ [1 + \alpha_1\ (t_3 - t_1)]$

$\therefore$ $24 = 18\ [1 + 0.007407\ (t_3 - 20)]$

$\therefore$ $0.3333 = 0.007407\ (t_3 - 20)$

$\therefore$ $t_3 - 20 = 45$

$\therefore$ $t_3 = \mathbf{65\ °C}$

So room temperature is 18 °C given

$\therefore$ Temperature rise = 65 − 18 = **47 °C**

1.11.5 R.T.C. of Composite Conductor

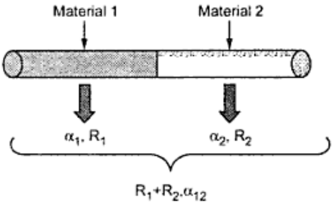


Fig. 1.10 Composite conductors

In many practical cases, it is necessary to manufacture conductors using two different types of materials, to achieve special requirements. Such a composite conductor is shown in the Fig. 1.10.

The material 1 has R.T.C. α_1 and its contribution in composite conductor is R_1.

The material 2 has R.T.C. α_2 and its contribution in composite conductor is R_2.

The combined composite conductor has resistance $R_1 + R_2$ while its RTC is neither α_1 nor α_2 but it is α_{12}, different than α_1 and α_2.

Analysis of Composite conductor : The analysis includes the calculation of α_{12} from α_1 and α_2.

Let R_1 = Resistance of material 1 at t_1 °C

R_2 = Resistance of material 2 at t_1 °C

t_2 = New temperature attained by composite conductor

R_{1t} = Resistance of material 1 at t_2 °C

R_{2t} = Resistance of material 2 at t_2 °C

R_{12} = Resistance of composite material at t_1 °C

R_{12t} = Resistance composite material at t_2 °C

It is known that,

$$R_{1t} = R_1 [1 + \alpha_1 (t_2 - t_1)] \quad ...(5)$$

$$R_{2t} = R_2 [1 + \alpha_2 (t_2 - t_1)] \quad ...(6)$$

$$R_{12t} = R_{12} [1 + \alpha_{12} (t_2 - t_1)] \quad ...(7)$$

Where α_{12} = R.T.C. of composite conductor at t_1 °C

> **Key Point** *: The overall circuit is* ***series*** *connection of resistances at all temperatures, from electrical point of view.*

$$R_{12} = R_1 + R_2 \quad \text{... at } t_1 \text{ °C} \quad ...(8)$$

And $$R_{12t} = R_{1t} + R_{2t} \quad \text{... at } t_2 \text{ °C} \quad ...(9)$$

Using equation (8) and equation (9) in equation (7),

$$[R_{1t} + R_{2t}] = [R_1 + R_2] [1 + \alpha_{12} (t_2 - t_1) \quad ...(10)$$

Using equation (5) and equation (6) in equation (10),

$$R_1 [1 + \alpha_1 (t_2 - t_1)] + R_2 [1 + \alpha_2 (t_2 - t_1)] = [R_1 + R_2] [1 + \alpha_{12} (t_2 - t_1)]$$

$$\therefore R_1 + R_1 \alpha_1 (t_2 - t_1) + R_2 + R_2 \alpha_2 (t_2 - t_1) = R_1 + R_1 \alpha_{12} (t_2 - t_1) + R_2 + R_2 \alpha_{12} (t_2 - t_1)$$

Cancelling R_1 and R_2 from both sides,

$$R_1 \alpha_1 (t_2 - t_1) + R_2 \alpha_2 (t_2 - t_1) = R_1 \alpha_{12} (t_2 - t_1) + R_2 \alpha_{12} (t_2 - t_1)$$

Cancelling $(t_2 - t_1)$ from both sides,

$$\alpha_{12} (R_1 + R_2) = R_1\alpha_1 + R_2\alpha_2 = \alpha_{12}(R_1 + R_2) \quad ...(11)$$

$$\therefore \quad \boxed{\alpha_{12} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}} \quad ...(12)$$

Thus α_{12} which is R.T.C. of composite conductor can be obtained at t_1 °C. Once this is known, α_{12} at any other temperature can be obtained as,

$$\boxed{\alpha_{12t} = \frac{\alpha_{12}}{1 + \alpha_{12}\Delta t} \quad \text{where } \Delta t \text{ is temperature rise}}$$

Prove that : $\dfrac{R_2}{R_1} = \dfrac{\alpha_1 - \alpha_{12}}{\alpha_{12} - \alpha_2}$

Divide the equation (11) by R_1,

$$\alpha_1 + \frac{R_2}{R_1}\alpha_2 = \alpha_{12} + \alpha_{12}\frac{R_2}{R_1}$$

$$\therefore \quad \alpha_1 - \alpha_{12} = \left(\alpha_{12}\frac{R_2}{R_1}\right) - \left(\frac{R_2}{R_1}\alpha_2\right)$$

$$\therefore \quad \alpha_1 - \alpha_{12} = \frac{R_2}{R_1}[\alpha_{12} - \alpha_2]$$

$$\therefore \quad \boxed{\frac{R_2}{R_1} = \frac{\alpha_1 - \alpha_{12}}{\alpha_{12} - \alpha_2}} \quad \text{... Proved}$$

Example 1.8 : *At a particular temperature the two resistances are 60 Ω and 90 Ω having temperature coefficients of 0.0037 /°C and 0.005 /°C respectively. Calculate the temperature coefficient of composite conductor at the same temperature, obtained by combining above two resistances in series.*

Solution : $R_1 = 60\ \Omega$, $R_2 = 90\ \Omega$, $\alpha_1 = 0.0037$ /°C, $\alpha_2 = 0.005$ /°C

Using the result derived,

$$\alpha_{12} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} = \frac{(60 \times 0.0037) + (90 \times 0.005)}{(60 + 90)} = \mathbf{0.00448\ /°C}$$

Example 1.9 : *Two coils A and B have resistances 60 Ω and 30 Ω respectively at 20 °C. The resistance temperature coefficients for the two coils at 20 °C are 0.001 /°C and 0.004 / °C. Find the resistance of their series combination at 50 °C.*

Solution : The given values are,

For coil A, $R_{A1} = 60\ \Omega, \quad t_1 = 20\ °C, \quad \alpha_{A1} = 0.001\ /°C$

For coil B, $R_{B1} = 30\ \Omega, \quad t_1 = 20\ °C, \quad \alpha_{B1} = 0.004\ /°C$

Now $R_{A2} = R_{A1}\ [1 + \alpha_{A1}(t_2 - t_1)]$

$\therefore \quad R_{A2} = 60\ [1 + 0.001\ (50 - 20)] = 61.8\ \Omega$

This is resistance of coil A at 50 °C.

And $R_{B2} = R_{B1}\ [1 + \alpha_{B1}\ (t_2 - t_1)]$

$= 30\ [1 + 0.004 \times (50 - 20)] = 33.6\ \Omega$

This is resistance of coil B at 50 °C.

$\therefore$ Resistance of their series combination at 50 °C $= R_{A2} + R_{B2} = 61.8 + 33.6$

$= \mathbf{95.4\ \Omega}$

Example 1.10 : *Two coils A and B have resistances 100 Ω and 150 Ω respectively at 0 °C are connected in series. Coil A has resistance temperature coefficient of 0.0038 /°C while B has 0.0018 /°C. Find the resistance temperature coefficient of the series combination at 0 °C.*

Solution : At 0 °C, the series combination is $= R_A + R_B = 100 + 150 = 250\ \Omega$

Now $R_t = R_0\ (1 + \alpha_0\ t)$ i.e. $(R_{AB})_t = (R_{AB})_0\ [1 + \alpha_{AB0}\ t]$

where R_{AB} is a resistance of series combination.

α_{AB} is resistance temperature coefficient of series combination.

Now $(R_A)_t = (R_A)_0\ [1 + \alpha_{A0}\ t]$ and $(R_B)_t = (R_B)_0\ [1 + \alpha_{B0}\ t]$

$\therefore \quad (R_{AB})_t = (R_A)_t + (R_B)_t = (R_A)_0\ [1 + \alpha_{A0}\ t] + (R_B)_0\ [1 + \alpha_{B0}\ t]$

Substituting in above,

$(R_A)_0\ [1 + \alpha_{A0}\ t] + (R_B)_0\ [1 + \alpha_{B0}\ t] = (R_{AB})_0\ [1 + \alpha_{AB0}\ t]$

$(R_A)_0 = 100\ \Omega,\ \alpha_{A0} = 0.0038$

$(R_B)_0 = 150\ \Omega,\ \alpha_{B0} = 0.0018$

$(R_{AB})_0 = 250\ \Omega$

$\therefore\ 100\ [1 + 0.0038\ t] + 150\ [\ 1 + 0.0018\ t] = 250\ [\ 1 + (\alpha_{AB})_0\ t]$

$\therefore \quad 100 + 0.38t + 150 + 0.27t = 250 + 250\ \alpha_{AB0}\ t$

$\therefore \quad 0.65\ t = 250\ \alpha_{AB0}\ t$

$\therefore \quad \alpha_{AB0} = \mathbf{0.0026\ /^\circ C.}$

This is the resistance temperature coefficient of the series combination at 0 °C.

Note : The example may be solved using the result derived as,

$$\alpha_{AB} = \frac{R_A \alpha_A + R_B \alpha_B}{R_A + R_B} = 0.0026\ /^\circ C$$

But in examination, such examples must be solved using basic procedure as used above and not by using direct expression derived.

Example 1.11 : *At any given temperature, two material A and B have resistance temperature coefficients of 0.004 and 0.0004 respectively. In what proportion resistances made up of A and B joined in series to give a combination having resistance temperature coefficient of 0.001 per °C ?*

Solution : Let R be resistance of material A then (x R) be resistance of material B.

The resistance of the series combination is,

$$R_{AB} = R_A + R_B$$

$$R_{AB} = R + xR = (1 + x)\ R\ \Omega$$

Let (α_{AB}) = resistance temperature coefficient of the series combination.

Let there be t °C change in temperature so,

$$R'_{AB} = (R_{AB})\ (\alpha_{AB})\ (t) = (1 + x)\ R\ (0.001)\ (t) \quad \ldots (1)$$

The resistance R'_{AB} is also $R'_A + R'_B$, where

R'_A is value of R_A due to change in temperature.

R'_B is value of R_B due to change in temperature.

$$R'_A = R_A \cdot (\alpha_A)\ (t) = R \cdot (0.004)\ (t)$$

and

$$R'_B = R_B \cdot (\alpha_B)\ (t) = xR \cdot (0.0004)\ (t)$$

$$R'_{AB} = R\ (0.004)\ (t) + x\ R\ (0.0004)\ (t)$$

$$R'_{AB} = R\ t(0.004 + 0.0004\ x) \quad \ldots (2)$$

Equating equation 1 and equation 2,

$\therefore R\ t\ (1 + x)\ (0.001) = R\ t\ (0.004 + 0.0004x)$

$$0.001 + 0.001\ x = 0.004 + 0.0004\ x \quad \text{i.e.} \quad 6 \times 10^{-4}\ x = 0.003$$

$\therefore \quad x = 5 \quad$ i.e. $R_B = 5\, R_A$

i.e. resistance R_A and R_B must be joined in the proportion 1 : 5.

Example 1.12 : *A resistor of 80 Ω resistance having a temperature coefficient of 0.0021 /°C at 0 °C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A the resistance is 80 Ω per 100 m and temperature coefficient is 0.003 /°C at 0 °C. For material B the corresponding figures are 60 Ω per 100 m and 0.0015 /°C at 0 °C. Calculate suitable lengths of the wires of materials A and B to be connected in series to get required resistor.*

Solution : R_{A0} = Resistance of A at 0 °C, R_{B0} = Resistance of B at 0°C

α_{A0} = R.T.C. of A at 0 °C = 0.003 /°C , α_{B0} = R.T.C. of B at 0 °C = 0.0015 /°C

R_{AB0} = Resistance of series combination of A and B at 0 °C = 80 Ω

α_{AB0} = R.T.C. of series combination at 0 °C = 0.0021 /°C

We know, $R_t = R_0\,(1 + \alpha_0\, t)$ i.e. $R_{At} = R_{A0}(1 + \alpha_{A0}\, t)$

$R_{Bt} = R_{B0}\,(1 + \alpha_{B0}\, t)$ and $R_{ABt} = R_{AB0}\,(1 + \alpha_{AB0}\, t)$

But $R_{ABt} = R_{At} + R_{Bt}$ ... series combination at t °C

Similarly $R_{AB0} = R_{A0} + R_{B0} = 80\ \Omega$ ... series combination at 0 °C

$\therefore\ R_{AB0}\,(1 + \alpha_{AB0}\, t) = R_{A0}\,(1 + \alpha_{A0}\, t) + R_{B0}\,(1 + \alpha_{B0}\, t)$

$\therefore\ 80\,(1 + 0.0021\, t) = R_{A0}\,(1 + 0.003\, t) + R_{B0}\,(1 + 0.0015\, t)$

$\therefore\ 80 + 0.168\, t = R_{A0} + 0.003\, R_{A0}\, t + R_{B0} + 0.0015\, R_{B0}\, t$

$\therefore\ 80 + 0.168\, t = (R_{A0} + R_{B0}) + 0.003\, R_{A0}\, t + 0.0015\, R_{B0}\, t$

Now $R_{A0} + R_{B0} = 80$ and $R_{B0} = 80 - R_{A0}$

$\therefore\ 80 + 0.168\, t = 80 + 0.003\, R_{A0}\, t + 0.0015\,(80 - R_{A0})\, t$

$\therefore\ 0.168\, t = [0.003\, R_{A0} + 0.0015\,(80 - R_{A0})]\, t$

$\therefore\ 0.168 = 0.003\, R_{A0} + 0.12 - 0.0015\, R_{A0}$

$\therefore\ R_{A0} = 32\ \Omega$

$\therefore\ R_{B0} = 80 - 32 = 48\ \Omega$

Now material A resistance is 80 Ω per 100 m so for 32 Ω the length required is,

$$\frac{32}{80} \times 100 = 40\ \text{m}$$

The material B has resistance of 60 Ω per 100 m so for 48 Ω the length required is ,

$$\frac{48}{60} \times 100 = 80\ \text{m}$$

1.12 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1.12.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.** Such a network is shown in the Fig. 1.11.

1.12.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element.**

Network elements can be either active elements or passive elements. Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. Passive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

1.12.3 Branch

A part of the network which connects the various points of the network with one another is called **a branch.** In the Fig. 1.11, AB, BC, CD, DA, DE, CF and EF are the various branches. A branch may consist more than one element.

1.12.4 Junction Point

A point where three or more branches meet is called **a junction point.** Point D and C are the junction points in the network shown in the Fig. 1.11.

1.12.5 Node

A point at which two or more elements are joined together is called **node.** The junction points are also the nodes of the network. In the network shown in the Fig. 1.11, A, B, C, D, E and F are the nodes of the network.

1.12.6 Mesh (or Loop)

Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any

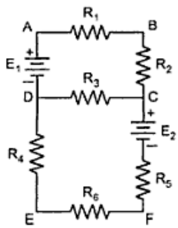


Fig. 1.11 An electrical network

node twice. In the Fig. 1.11 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

In this chapter, the analysis of d.c. circuits consisting of pure resistors and d.c. sources is included.

1.13 Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

i) Linear Network : A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network.** The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

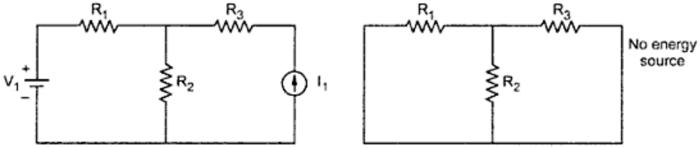
ii) Non linear Network : A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network .** The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

iii) Bilateral Network : A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

iv) Unilateral Network : A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network.** Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

v) Active Network : A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

vi) Passive Network : A circuit which contains no energy source is called **passive circuit**. This is shown in the Fig. 1.11.



(a) Active network

(b) Passive network

Fig. 1.12

vii) Lumped Network : A network in which all the network elements are physically separable is known as **lumped network**. Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

viii) Distributed Network : A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network**. The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

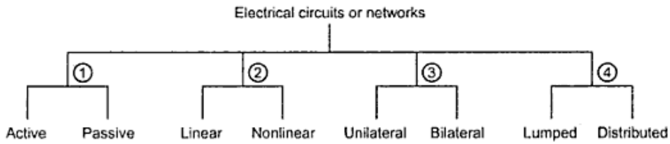


Fig. 1.13 Classification of networks

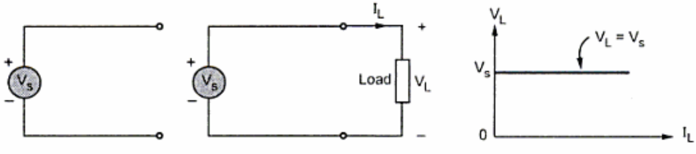
1.14 Energy Sources

There are basically two types of energy sources ; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

1.14.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 1.14 (a). This is connected to the load as shown in Fig. 1.14 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 1.14 (c).

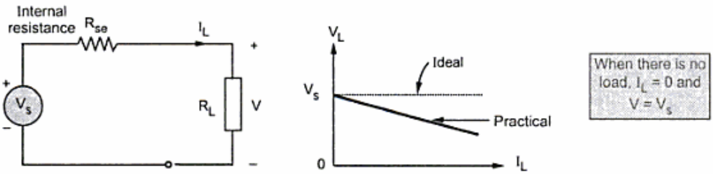


(a) Symbol (b) Circuit (c) Characteristics

Fig. 1.14 Ideal voltage source

Practical voltage source :

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.15.



(a) Circuit (b) Characteristics

Fig. 1.15 Practical voltage source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = -(R_{se})\, I_L + V_S = V_S - I_L R_{se}$$

Key Point: *For ideal voltage source, $R_{se} = 0$*

Voltage sources are further classified as follows,

i) Time Invariant Sources :

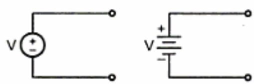


Fig. 1.16 (a) D.C. source

The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources.** These are denoted by capital letters. Such a source is represented in the Fig. 1.16 (a).

ii) Time Variant Sources :

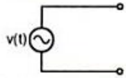
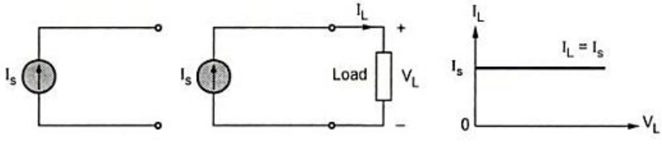


Fig. 1.16 (b) A.C. source

The sources in which voltage is varying with time are known as **time variant voltage sources** or **A.C. sources.** These are denoted by small letters. This is shown in the Fig. 1.16 (b).

1.14.2 Current Source

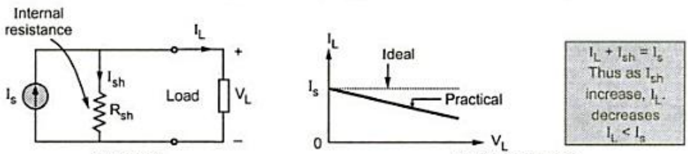
Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 1.17 (a). This is connected to the load as shown in the Fig. 1.17 (b). At any time, the value of the current flowing through load I_L is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 1.17 (c).



(a) Symbol (b) Circuit (c) Characteristics

Fig. 1.17 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.18.



(a) Circuit (b) Characteristic

Fig. 1.18 Practical current source

Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

Key Point: *For ideal current source,* $R_{sh} = \infty$.

Similar to voltage sources, current sources are classified as follows :

i) Time Invariant Sources :

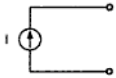


Fig. 1.19 (a) D.C. source

The sources in which current is not varying with time are known as **time invarient current sources** or **D.C. sources**. These are denoted by capital letters.

Such a current source is represented in the Fig. 1.19 (a).

ii) Time Variant Sources :

The sources in which current is varying with time are known as **time variant current sources** or **A.C. sources**. These are denoted by small letters.

Such a source is represented in the Fig. 1.19 (b).

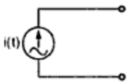


Fig. 1.19 (b) A.C. source

The sources which are discussed above are independent sources because these sources does not depend on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside.

1.14.3 Dependent Sources

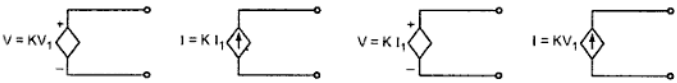
Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 1.20 and further classified as,

i) Voltage Dependent Voltage Source : It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. 1.20 (a).

ii) Current Dependent Current Source : It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 1.20 (b).

iii) Current Dependent Voltage Source : It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 1.20 (c).

iv) Voltage Dependent Current Source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 1.20 (d).



(a) (b) (c) (d)

Fig. 1.20

iv) Voltage Dependent Current Source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 1.20 (d).

K is constant and V_1 and I_1 are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as controlled sources.

In this chapter, d.c. circuits consisting of independent d.c. voltage and current sources are analysed.

1.15 Ohm's Law

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. **Dr. Ohm** in 1827 discovered a law called **Ohm's law.** It states,

Ohm's Law : *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

Mathematically, $I \propto \frac{V}{R}$

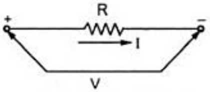


Fig. 1.21 Ohm's law

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 1.21.

Now $$I = \frac{V}{R}$$

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

Ohm's law is,
$$I = \frac{V}{R} \quad \text{amperes}$$
$$V = I\,R \quad \text{volts}$$
$$\frac{V}{I} = \text{constant} = R \quad \text{ohms}$$

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

Key Point : *Ohm's Law can be applied either to the entire circuit or to the part of a circuit. If it is applied to entire circuit, the voltage across the entire circuit and resistance of the entire circuit should be taken into account. If the Ohm's law is applied to the part of a circuit, then the resistance of that part and potential across that part should be used.*

1.15.1 Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = k\ I^m \quad \text{where k, m are constants.}$$

1.16 Series Circuit

A **series** circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is only one path for the flow of current.

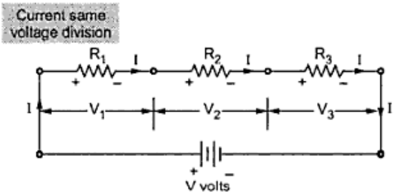


Fig. 1.22 A series circuit

Consider the resistances shown in the Fig. 1.22.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1,\ V_2 = I R_2,\ V_3 = I R_3$$

Current through all of them is same i.e. I

$$\therefore \quad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistences in series,	$R = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

1.16.1 Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots\ldots + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $\quad R > R_1 , R > R_2 , \ldots\ldots R > R_n$

1.17 Parallel Circuit

The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the Fig. 1.23.

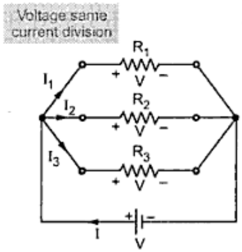


Fig. 1.23 A parallel circuit

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1 , \; V = I_2 R_2 , \; V = I_3 R_3$$

$$I_1 = \frac{V}{R_1} , \; I_2 = \frac{V}{R_2} , \; I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \quad \ldots (1)$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \text{... (2)}$$

where R_{eq} = Total or **equivalent resistance** of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

Conductance (G) :

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad G = G_1 + G_2 + G_3 + \ldots\ldots + G_n \quad \text{... For parallel circuit}$$

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

This formula is directly used hereafter, for two resistances in parallel.

1.17.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$

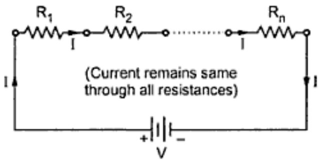
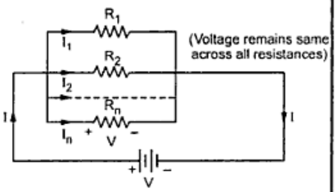
3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances.

$$R < R_1, \quad R<R_2, \ldots\ldots, R<R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

> **Key Point**: *The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.*

1.18 Comparison of Series and Parallel Circuits

Sr. No.	Series Circuit	Parallel Circuit
1.	The connection is as shown,  	The conncection is as shown, 
2.	The same current flows through each resistance.	The same voltage exists across all the resistances in parallel.
3.	The voltage across each resistance is different.	The current through each resistance is different.
4.	The sum of the voltages across all the resistances is the supply voltage. $V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$	The sum of the currents through all the resistances is the supply current. $I = I_1 + I_2 + \ldots\ldots I_n$
5.	The equivalent resistance is, $R_{eq} = R_1 + R_2 + \ldots\ldots + R_n$	The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$
6.	The equivalent resistance is the largest than each of the resistances in series. $R_{eq} > R_1, R_{eq} > R_2 \ldots\ldots R_{eq} > R_n$	The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

Example 1.13 : *Find the equivalent resistance between the two points A and B shown in the Fig. 1.24.*

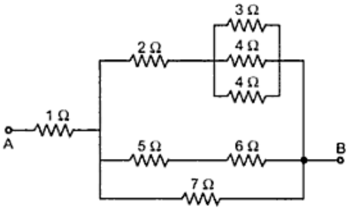


Fig. 1.24

Solution : Identify combinations of series and parallel resistances.

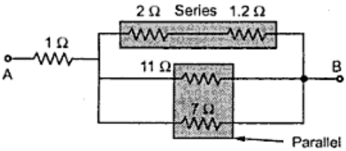
The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

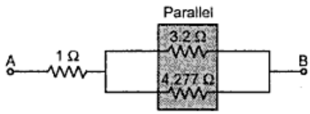
So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$





(a) **Fig. 1.24** (b)

Replacing these combinations redraw the figure as shown in the Fig. 1.24 (a).

Now again 1.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1 R_2}{R_1 + R_2}$ equivalent resistance is $\frac{11\times 7}{11+7} = \frac{77}{18} = 4.277\Omega$.

Replacing the respective combinations redraw the circuit as shown in the Fig. 1.24 (b).

Now 3.2 and 4.277 are in parallel.

∴ Replacing them by $\frac{3.2\times 4.277}{3.2+4.277} = 1.8304\ \Omega$

$$\therefore \quad R_{AB} = 1 + 1.8304 = \mathbf{2.8304\ \Omega}$$

1.19 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the nework plays an important role.

1.19.1 Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

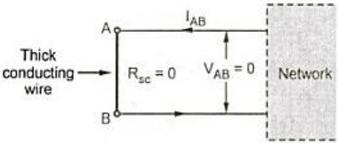


Fig. 1.25

The part of the network, which is short circuited is shown in the Fig. 1.25. The points A and B are short circuited. The resistance of the branch AB is $R_{sc} = 0\ \Omega$.

The curent I_{AB} is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V$$

Key Point : *Thus, voltage across short circuit is always zero though current flows through the short circuited path.*

1.19.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

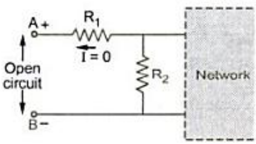


Fig. 1.26

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

The part of the network which is open circuited is shown in the Fig. 1.26. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{oc} = \infty\ \Omega$.

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{oc} = \infty\ \Omega$.

According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0\ A$$

Key Point : *Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.*

1.19.3 Redundant Branches and Combinations

The redundant means excessive and unwanted.

Key Point : *If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.*

The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of **redundancy** which may exist in practical circuits are,

Situation 1 : Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

If in a network, there exists a direct short circuit across a resistance or the combination of resistances then that resistance or the entire combination of resistances becomes **inactive** from the circuit point of view. Such a combination is redundant from circuit point of view.

To understand this, consider the combination of resistances and a short circuit as shown in the Fig. 1.27 (a) and (b).

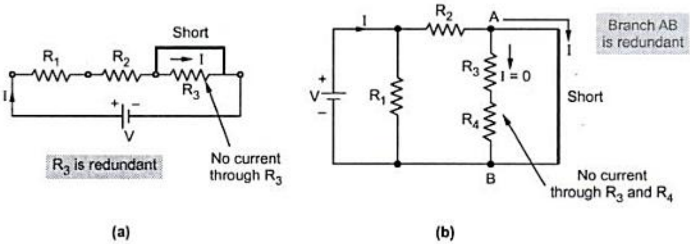


Fig. 1.27 Redundant branches

In Fig. 1.27 (a), there is short circuit across R_3. The current always prefers low resistance path hence entire current I passes through short circuit and hence resistance R_3 becomes redundant from the circuit point of view.

In Fig. 1.27 (b), there is short circuit across combination of R_3 and R_4. The entire current flows through short circuit across R_3 and R_4 and no current can flow through combination of R_3 and R_4. Thus that combination becomes meaningless from the circuit point of view. **Such combinations can be elliminated while analysing the circuit.**

Situation 2 : If there is open circuit in a branch or combination, it can not carry any current and becomes redundant.

In Fig. 1.28 as there exists open circuit in branch BC, the branch BC and CD cannot carry any current and are become redundant from circuit point of view.

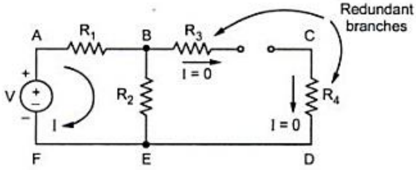


Fig. 1.28 Redundant branches due to open circuit

1.20 Voltage Division in Series Circuit of Resistors

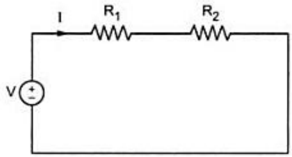


Fig. 1.29

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts.

As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

$$V = I R_1 + I R_2$$

$$\therefore \; I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \cdot R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $V_{R2} = I \cdot R_2$

$$\therefore \quad \boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So this circuit is a **voltage divider circuit.**

Key Point : *So in general, voltage drop across any resistor, or combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.*

Example 1.14 : *Find the voltage across the three resistances shown in the Fig. 1.30.*

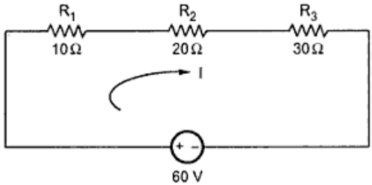


Fig. 1.30

Solution :

$$I = \frac{V}{R_1 + R_2 + R_3} \quad \text{... series circuit}$$

$$= \frac{60}{10+20+30} = 1 \text{ A}$$

$$\therefore \quad V_{R1} = I R_1 = \frac{V \times R_1}{R_1 + R_2 + R_3} = 1 \times 10 = \mathbf{10\ V}$$

$$\therefore \quad V_{R2} = I R_2 = \frac{V \times R_2}{R_1 + R_2 + R_3} = 1 \times 20 = \mathbf{20\ V}$$

and
$$V_{R3} = I R_3 = \frac{V \times R_3}{R_1 + R_2 + R_3} = 1 \times 30 = \mathbf{30\ V}$$

Key Point *: It can be seen that voltage across any resistance of series circuit is ratio of that resistance to the total resistance, multiplied by the source voltage.*

1.21 Current Division in Parallel Circuit of Resistors

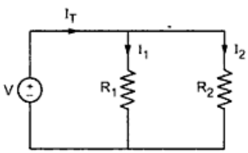


Fig. 1.31

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$$\therefore \quad I_T = I_1 + I_2$$

But $I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$

i.e. $V = I_1 R_1 = I_2 R_2$

$$\therefore \quad I_1 = I_2 \left(\frac{R_2}{R_1} \right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2\left(\frac{R_2}{R_1}\right) + I_2 = I_2\left[\frac{R_2}{R_1}+1\right] = I_2\left[\frac{R_1+R_2}{R_1}\right]$$

$$\therefore \quad \boxed{I_2 = \left[\frac{R_1}{R_1+R_2}\right] I_T}$$

Now

$$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad I_1 = \left[\frac{R_1+R_2-R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad \boxed{I_1 = \left[\frac{R_2}{R_1+R_2}\right] I_T}$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Example 1.15 : *Find the magnitudes of total current, current through R_1 and R_2 if, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.*

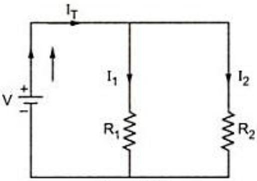


Fig. 1.32

Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1\, R_2}{R_1+R_2} = \frac{10\times 20}{10+20} = 6.67\ \Omega$$

$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5\ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1+R_2}\right) = 7.5\times\left(\frac{20}{10+20}\right)$$

$$= \mathbf{5\ A}$$

and $$I_2 = I_T\left(\frac{R_1}{R_1+R_2}\right) = 7.5\times\left(\frac{10}{10+20}\right)$$

$$= \mathbf{2.5 \ A}$$

It can be verified that $I_T = I_1 + I_2$

1.22 Source Transformation

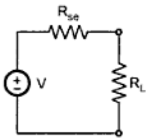


Fig. 1.33 (a) Voltage source

Consider a practical voltage source shown in the Fig. 1.33 (a) having internal resistance R_{se}, connected to the load having resistance R_L.

Now we can replace voltage source by equivalent current source.

> **Key Point** : *The two sources are said to be* ***equivalent****, if they supply equal load current to the load, with same load connected across its terminals*

The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se}+R_L)}, \qquad R_{se} \text{ and } R_L \text{ in series} \qquad \ldots(1)$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R_{se}+R_L)}$

Consider an equivalent current source shown in the Fig. 1.23 (b).

The total current is 'I'.

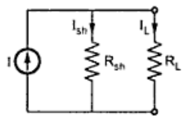


Fig. 1.33 (b) Current source

Both the resistances will take current proportional to their values.

From the current division in parallel circuit we can write,

$$I_L = I\times\frac{R_{sh}}{(R_{sh}+R_L)} \qquad \ldots(2)$$

Now this I_L and $\frac{V}{R_{se}+R_L}$ must be same, so equating (1) and equation (2),

$$\therefore \qquad \frac{V}{R_{se}+R_L} = \frac{I\times R_{sh}}{R_{sh}+R_L}$$

Let internal resistance be, $R_{se} = R_{sh} = R$ say.

Then, $V = I \times R_{sh} = I \times R$

or $I = \dfrac{V}{R_{sh}}$

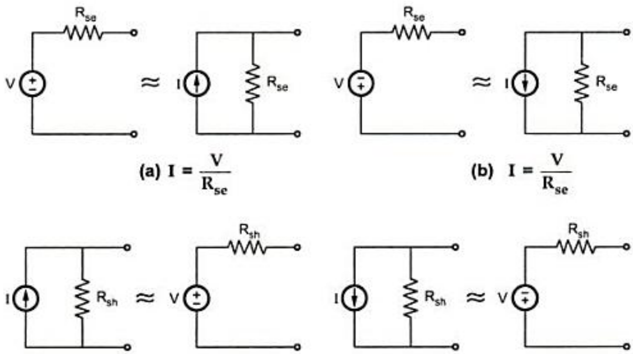
$\therefore$ $$\boxed{I = \frac{V}{R} = \frac{V}{R_{se}}}$$

Key Point: *If voltage source is converted to current source, then current source* $I = \dfrac{V}{R_{se}}$ *with parallel internal resistance equal to* R_{se}.

Key Point: *If current source is converted to voltage source, then voltage source* $V = I\,R_{sh}$ *with series internal resistance equal to* R_{sh}.

The direction of current of equivalent current source is always from **–ve to + ve, internal to the source.** While converting current source to voltage source, polarities of voltage is always as +ve terminal at top of arrow and –ve terminal at bottom of arrow, as direction of current is from –ve to +ve, internal to the source. **This ensures that current flows from positive to negative terminal in the external circuit.**

Note the directions of transformed sources, shown in the Fig. 1.23 (a), (b), (c) and (d).



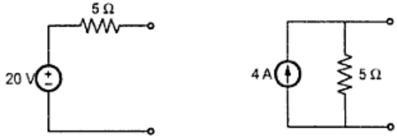
(a) $I = \dfrac{V}{R_{se}}$ **(b)** $I = \dfrac{V}{R_{se}}$

(c) $V = I \times R_{sh}$ **(d)** $V = I \times R_{sh}$

Fig. 1.34

Example 1.16 : *Transform a voltage source of 20 volts with an internal resistance of 5 Ω to a current source.*

Solution: Refer to the Fig. 1.35 (a).



(a) **Fig. 1.35** **(b)**

Then current of current source is, $I = \frac{V}{R_{se}} = \frac{20}{5} = 4$ A with internal parallel resistance same as R_{se}.

∴ Equivalent current source is as shown in the Fig. 1.34 (b).

Example 1.17 : *Convert the given current source of 50 A with internal resistance of 10 Ω to the equivalent voltage source.*

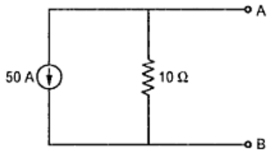


Fig. 1.36

Solution : The given values are, I = 50 A and R_{sh} = 10 Ω

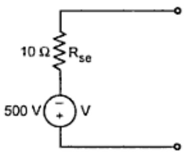


Fig. 1.36 (a)

For the equivalent voltage source,

$$V = I \times R_{sh} = 50 \times 10$$

$$= 500 \text{ V}$$

$$R_{se} = R_{sh} = 10\ \Omega \text{ in series.}$$

The equivalent voltage source is shown in the Fig. 1.36 (a).

Note the polarities of voltage source, which are such that + ve at top of arrow and – ve at bottom.

1.23 Kirchhoff's Laws

In 1847, a German Physicist, **Kirchhoff,** formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

1.23.1 Kirchhoff's Current Law (KCL)

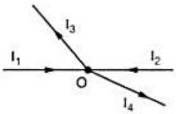


Fig. 1.37 Junction point

Consider a junction point in a complex network as shown in the Fig. 1.37.

At this junction point if $I_1 = 2$ A, $I_2 = 4$ A and $I_3 = 1$ A then to determine I_4 we write, total current entering is $2 + 4 = 6$ A while total current leaving is $1 + I_4$ A

And hence, $I_4 = 5$ A.

This analysis of currents entering and leaving is nothing but the application of Kirchhoff's Current Law. The law can be stated as,

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\sum I \text{ at junction point } = 0$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 1.37, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL, $\sum I$ at junction O $= 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

The law is very helpful in network simplification.

1.23.2 Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\text{Around a closed path } \sum V = 0$$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

This law is very useful in loop analysis of the network.

1.23.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

(a) Fig. 1.38 (b)

In the Fig. 1.38 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

In the Fig. 1.38 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise.**

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + IR while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop**.

For example, in the Fig. 1.38 (a) only, if the branch is traced from B to A then it should be taken as negative, as – IR while writing the equations.

Similarly in the Fig. 1.38 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as – IR while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as + IR while writing the equation.

Key Point:

1) ***Potential rise*** *i.e. travelling from negative to positively marked terminal, must be considered as* ***Positive.***
2) ***Potential drop*** *i.e. travelling from positive to negatively marked terminal, must be considered as* ***Negative.***
3) *While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.*

1.23.4 Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 1.39 (a).

As the loop is assumed to be a part of complex network, the branch currents are assumed to be different from each other.

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 1.39 (b).

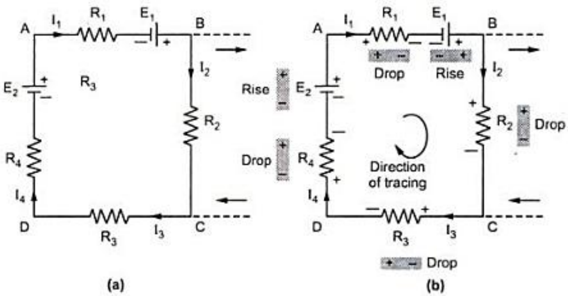


Fig. 1.39 Closed loop of a complex network

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (–).

Let us **trace** this closed path in clockwise direction i.e. **A-B-C-D-A.**

Across R_1 there is voltage drop $I_1 R_1$ and as getting traced from +ve to –ve, it is drop and must be taken as negative while applying KVL.

Battery E_1 is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Across R_2 there is a voltage drop $I_2 R_2$ and as getting traced from +ve to –ve, it is drop and must be taken negative.

Across R_3 there is a drop $I_3 R_3$ and as getting traced from +ve to –ve, it is drop and must be taken as negative.

Across R_4 there is drop $I_4 R_4$ and as getting traced from +ve to –ve, it is drop must be taken as negative.

Battery E_2 is getting traced from –ve to +ve, it is rise and must be taken as positive.

∴ We can write an equation by using KVL around this closed path as,

$$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0 \quad \text{... Required KVL equation}$$

i.e. $$E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$$

If we trace the closed loop in opposite direction i.e. along **A-D-C-B-A** and follow the same sign convention, the resulting equation will be same as what we have obtained above.

> **Key Point**: *So while applying KVL, direction in which loop is to be traced is not important but following the sign convention is most important.*

The same sign convention is followed in this book to solve the problems.

1.23.5 Steps to Apply Kirchhoff's Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1 : Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2 : Mark all the branch curretns with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

Assumed directions may be wrong, in such case answer of such current will be mathematically negative which indicates the correct direction of the current. A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3 : Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4 : Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equation.

> **Key Point**: *KVL must be applied to sufficient number of loops such that each element of the network is included at least once in any of the equations.*

Step 5 : Solve the simultaneous equations for the unknown currents. From these currents unknown voltages and power consumption in different resistances can be calculated.

What to do if current source exists ?

Key Point : *If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the* ***loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source.*** *This is because drop across current source is unknown.*

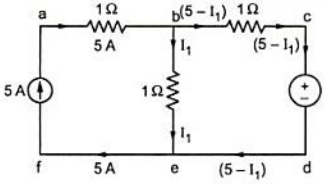


Fig. 1.40

For example, consider the circuit shown in the Fig. 1.40. The current distribution is completed interms of current source value. Then KVL must be applied to the loop bcdeb, which does not include current source. **The loop abefa should not be used for KVL application,** as it includes current source. Its effect is already considered at the time of current distribution.

1.24 Cramer's Rule

If the network is complex, the number of equations i.e. unknowns increases. In such case, the solution of simultaneous equations can be obtained by **Cramer's Rule** for determinants.

Let us assume that set of simultaneous equations obtained is, as follows :

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n &= C_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n &= C_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots + a_{nn}x_n &= C_n \end{aligned}$$

where $C_1, C_2, \ldots\ldots\ldots\ldots C_n$ are constants.

Then Cramer's rule says that form a system determinant Δ or D as,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D$$

Then obtain the subdeterminants D_j by replacing j^{th} column of Δ by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{vmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_2 & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad D_2 = \begin{vmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{vmatrix}$$

and
$$D_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ \vdots & & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & C_n \end{vmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D},\ X_2 = \frac{D_2}{D},\ \ldots\ldots\ldots\ldots,\ X_n = \frac{D_n}{D}$$

where $D_1, D_2, \ldots, D_n$ and D are values of the respective determinants.

Example 1.18 : *Apply Kirchhoff's current law and voltage law to the circuit shown in the Fig. 1.41.*

Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative ?

If yes, explain the significance of the negative sign.

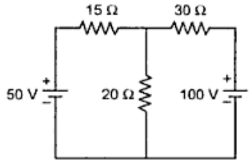


Fig. 1.41

Solution : Application of Kirchhoff's law :

Step 1 and 2 : Draw the circuit with all the values which are same as the given network. Mark all the branch currents starting from +ve of any of the source, say +ve of 50 V source.

Step 3 : Mark all the polarities for different voltages across the resistances. This is combined with step 2 shown in the network below in Fig. 1.41 (a).

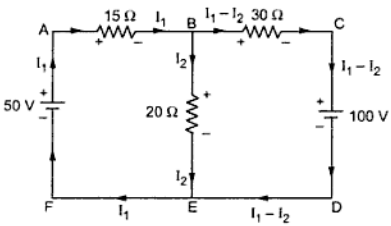


Fig. 1.41 (a)

Step 4 : Apply KVL to different loops.

Loop 1 : A-B-E-F-A., $-15\ I_1 - 20\ I_2 + 50 = 0$ ... (1)

Loop 2 : B-C-D-E-D, $-30\ (I_1 - I_2) - 100 + 20\ I_2 = 0$ ... (2)

Rewriting all the equations, taking constants on one side.

$15\ I_1 + 20\ I_2 = 50$ ...(1) and $-30\ I_1 + 50\ I_2 = 100$ ...(3)

Apply Cramer's rule, $$D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$$

Calculating D_1, $$D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = \mathbf{0.37\ A}$$

Calculating D_2, $$D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = \mathbf{2.22\ A}$$

For I_1 and I_2, as answer is positive, assumed direction is correct.

∴ For I_1 answer is 0.37 A. For I_2 answer is 2.22 A

$$I_1 - I_2 = 0.37 - 2.22 = -1.85\ A$$

Negative sign indicates assumed direction is wrong.

i.e. $I_1 - I_2 = 1.85$ A flowing in opposite direction to that of the assumed direction.

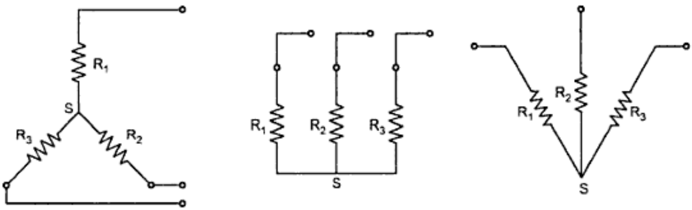
1.25 Star and Delta Connection of Resistances

In the complicated networks involving large number of resistances, Kirchhoff's laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analysed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

Let us see what is Star connection ?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.

The Fig. 1.42 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 1.42 (a) and (b) are exactly identical. The Fig. 1.42 (b) can be redrawn as Fig. 1.42 (a) or vice-versa, in the circuit from simplification point of view.



(a) (b) (c)

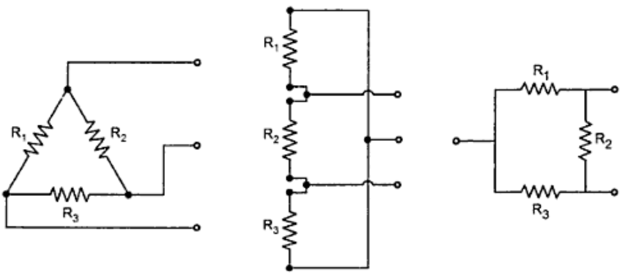
Fig. 1.42 Star connection of three resistances

Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.

Key Point: *Delta connection always forms a loop, closed path.*

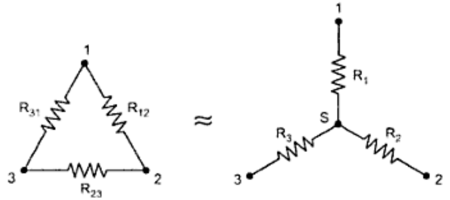
The Fig. 1.43 (a) and (b) show delta connection of three resistances. The Fig. 1.43 (a) and (b) are exactly identical.



(a) (b) (c)

Fig. 1.43 Delta connection of three resistances

1.25.1 Delta-Star Transformation



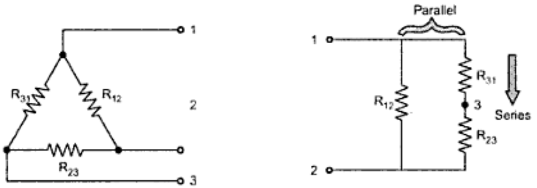
Given Delta Fig. 1.44 Equivalent Star

Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. 1.44. The terminals between which these are connected in Delta are named as 1, 2 and 3.

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 1.40 which is called **equivalent Star of Delta connected resistances.**

Key Point: *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 1.45 (a).



(a) Given Delta Fig. 1.45 (b) Equivalent between 1 and 2

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 1.45 (b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

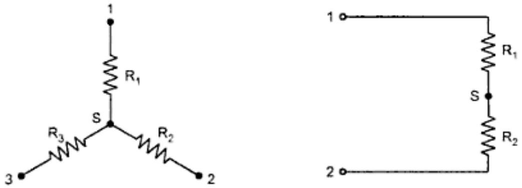
R_{12} parallel with $(R_{31} + R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31} + R_{23})}{R_{12} + (R_{31} + R_{23})} \qquad \text{...(a)}$$

[using $\frac{R_1 R_2}{R_1 + R_2}$ for parallel combination]

Now consider the same two terminals of equivalent Star connection shown in the Fig. 1.46.



(a) Star connection Fig. 1.46 (b) Equivalent between 1 and 2

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 1.46.

$\therefore$ Between (1) and (2) the resistance is $= R_1 + R_2$ ... (b)

This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (a) and (b),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \quad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \quad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \quad \text{...(e)}$$

Now we are interested in calculating what are the values of R_1, R_2, R_3 interms of known values R_{12}, R_{23}, and R_{31}.

Subtracting equation (d) from equation (c),

$$\frac{R_{12}(R_{31}+R_{23})-R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \quad R_1 - R_3 = \frac{R_{12}\,R_{31} - R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}} \quad \ldots(f)$$

Adding equation (f) and equation (e),

$$\frac{R_{12}\,R_{31} - R_{23}\,R_{31} + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\therefore \frac{R_{12}\,R_{31} - R_{23}\,R_{31} + R_{31}\,R_{12} + R_{31}\,R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1$$

$$2R_1 = \frac{2\,R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\boxed{R_1 = \frac{R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12}\,R_{23}}{R_{12} + R_{23} + R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}}}$$

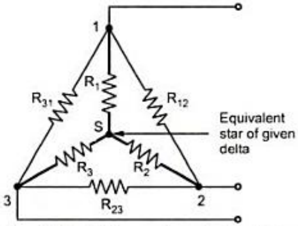


Fig. 1.47 Delta and equivalent Star

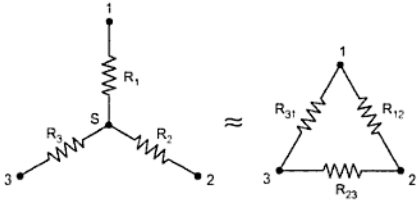
Easy way of remembering the result :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

So if we want equivalent resistance between terminal (2) and star point i.e. R_2 then it is the product of two resistances in delta which are connected to same terminal i.e. terminal (2) which are R_{12} and R_{23} divided by sum of all delta connected resistances i.e. R_{12}, R_{23} and R_{31}.

$$\therefore \quad R_2 = \frac{R_{12}\,R_{23}}{R_{12} + R_{23} + R_{31}}$$

1.25.2 Star-Delta Transformation



Given Star Equivalent Delta

Fig. 1.48

Consider the three resistances R_1, R_2 and R_3 connected in Star as shown in Fig. 1.48.

Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances R_{12}, R_{23} and R_{31}, between the same terminals. This is called **equivalent Delta of the given star.**

Now we are interested in finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12}+R_{23}+R_{31}} \qquad \text{...(g)}$$

$$R_2 = \frac{R_{12}\, R_{23}}{R_{12}+R_{23}+R_{31}} \qquad \text{...(h)}$$

$$R_3 = \frac{R_{23}\, R_{31}}{R_{12}+R_{23}+R_{31}} \qquad \text{...(i)}$$

Now multiply equations (g) and (h), equations (h) and (i), equations (i) and (g) to get following three equations.

$$R_1 R_2 = \frac{R_{12}^2\, R_{31}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \qquad \text{...(j)}$$

$$\therefore \quad R_2 R_3 = \frac{R_{23}^2\, R_{12}\, R_{31}}{(R_{12}+R_{23}+R_{31})^2} \qquad \text{...(k)}$$

$$R_3 R_1 = \frac{R_{31}^2\, R_{12}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \qquad \text{...}(l)$$

Now add equations (j) ,(k) and (*l*)

$$\therefore \quad R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{12}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{31} R_{23} (R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $$\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1 \quad \text{From equation (g)}$$

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = R_1 R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}}$$

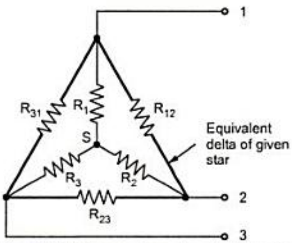


Fig. 1.49 Star and equivalent Delta

and $$\boxed{R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}}$$

Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

So if we want equivalent delta resistance between terminals (3) and (1), then take sum of the two resistances connected between same two terminals (3) and (1) and star point respectively i.e. terminal (3) to star point R_3 and terminal (1) to star point i.e. R_1. Then to this sum of R_1 and R_3, add the term which is the product of the same two resistances i.e. R_1 and R_3 divided by the third star resistance which is R_2.

$\therefore$ We can write, $R_{31} = R_1 + R_3 + \frac{R_1R_3}{R_2}$ which is same as derived above.

Result for equal resistances in star and delta :

If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = \frac{R}{3}$$

i.e. **equivalent Star contains three equal resistances, each of magnitude one third the magnitude of the resistances connected in Delta.**

If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of ,

$$R_{12} = R_{31} = R_{23} = R+R+\frac{R \times R}{R} = 3R$$

i.e. **equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.**

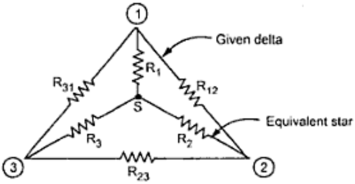
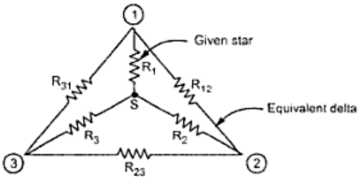
Delta-Star	Star-Delta
	
$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$
$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$	$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$
$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$

Table 1.4 Star-Delta and Delta-Star transformations

➡ **Example 1.19 :** *Convert the given Delta in the Fig. 1.50 into equivalent Star.*

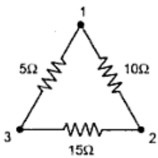


Fig. 1.50

Solution : Its equivalent star is as shown in the Fig. 1.50(a).

where

$$R_1 = \frac{10 \times 5}{5+10+15} = \mathbf{1.67\ \Omega}$$

$$R_2 = \frac{15 \times 10}{5+10+15} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{5 \times 15}{5+10+15} = \mathbf{2.5\ \Omega}$$

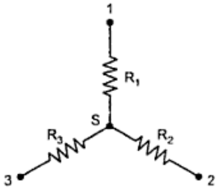


Fig. 1.50 (a)

Example 1.20 : *Convert the given star in the Fig. 1.51 into an equivalent delta.*

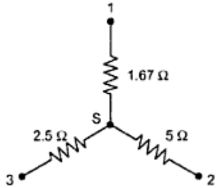


Fig. 1.51

Solution : Its equivalent delta is as shown in the Fig. 1.51 (a).

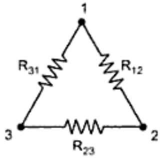


Fig. 1.51 (a)

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = \mathbf{10\ \Omega}$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = \mathbf{15\ \Omega}$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = \mathbf{5\ \Omega}$$

Example 1.21 : *Find equivalent resistance between points A-B.*

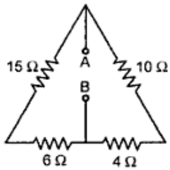


Fig. 1.52

Solution : Redrawing the circuit,

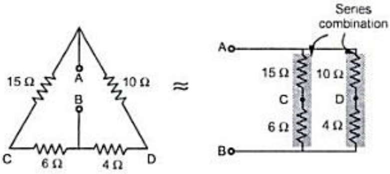
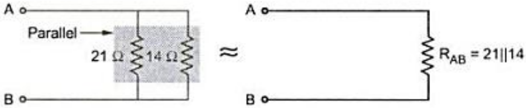


Fig. 1.52 (a)



$$R_{AB} = \frac{21 \times 14}{21 + 14} = \mathbf{8.4\ \Omega}$$

Fig. 1.52 (b)

Example 1.22 : *Find equivalent resistance between points A-B.*

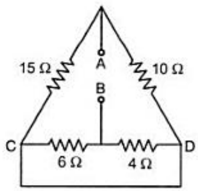


Fig. 1.53

Solution : Redraw the circuit,

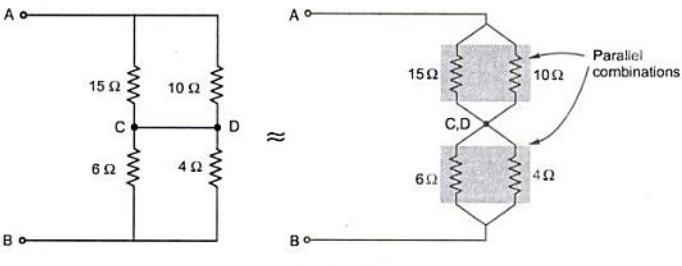


Fig. 1.53 (a)

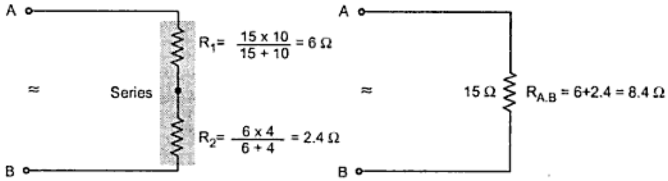


Fig. 1.53 (b)

$\therefore \quad R_{AB} = 8.4\ \Omega$

1.26 Concept of Loop Current

A loop current is that current which simultaneously links with all the branches, defining a particular loop.

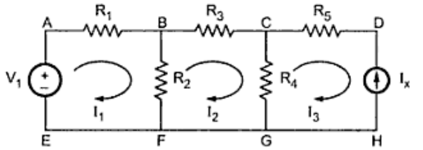


Fig. 1.54 Concept of loop current

The Fig. 1.54 shows a network. In this, I_1 is the loop current for the loop ABFEA and simultaneously links with the branches AB, BF, FE and EA. Similarly I_2 is the second loop current for the loop BCGFB and I_3 is the third loop current for the loop CDFGC.

Observe :

1. For the common branches of the various loops, multiple loop currents get associated with them. For example to the branch BF, both I_1 and I_2 are associated.
2. The branch current is always unique hence a branch current can be expressed interms of associated loop currents.

Key Point : *The total branch current is the algebraic sum of all the loop currents associated with that branches.*

$$I_{BF} = I_1 - I_2 \text{ from B to F}$$

$$I_{CG} = I_2 - I_3 \text{ from C to G}$$

3. The branches consisting current sources, directly decide the values of the loop currents flowing through them.

The branch DH consists current source of I_x amperes and only the loop current I_3 is associated with the branch DH in opposite direction. Hence $I_3 = -I_x$.

4. Assuming such loop currents and assigning the polarities for the drops across the various branches due to the assumed loop currents, the Kirchhoff's voltage law can be applied to the loops. Solving these equations, the various loop currents can be obtained. Once the loop currents are obtained, any branch current can be calculated.

Note : From the syllabus point of view, in this book, the branch current method is used to solve the problems. If loop currents are given in the problem, mark the branch currents interms of given loop currents and then use KVL, to solve the problem.

Examples with Solutions

Example 1.23 : *Prove that the length 'l' and diameter 'd' of a cylinder of copper are*

$$l = \left(\frac{rx}{\rho}\right)^{\frac{1}{2}} \text{ and } d = \left(\frac{16x\rho}{\pi^2 r}\right)^{\frac{1}{4}}$$

where x-volume, ρ-resistivity and r-resistance between opposite circular faces.

Solution : The resistance is given by,

$$r = \frac{\rho l}{a}$$

Now $\quad x = \text{volume} = a \times l$

Multiplying numerator and denominator by l,

$$r = \frac{\rho l \times l}{a \times l} = \frac{\rho l^2}{x}$$

$$\therefore \quad l = \left(\frac{rx}{\rho}\right)^{1/2} \quad \text{... proved}$$

Now $\quad a = \frac{\pi}{4} d^2$

$$r = \frac{\rho l \times a}{a \times a} = \frac{\rho x}{a^2} \quad \text{... multiplying and dividing by a}$$

$$\therefore \quad r = \frac{\rho x}{\left(\frac{\pi}{4} d^2\right)^2}$$

$$\therefore \quad d^4 = \frac{16 \rho x}{\pi^2 r}$$

$$\therefore \quad d = \left(\frac{16\rho x}{\pi^2 r}\right)^{1/4} \quad \text{... proved}$$

Example 1.24 : *At the instant of switching a 40 W lamp on a 230 V supply, the current is observed to be 2.5 A. The R.T.C. of filament is 0.0048 /°C at 0°C. The ambient temperature is 27 °C. Find the working temperature of the filament and current taken during normal operation.*

Solution : $P = 40$ W, $V = 230$ V, $I = 2.5$ A, $\alpha_0 = 0.0048$ /°C

At the time of switching, temperature is ambient i.e. t = 27 °C,

$$\therefore \quad R_{27} = \frac{V}{I} = \frac{230}{2.5} = 92\ \Omega$$

Under working condition, power consumption of lamp is 40 W.

$$\therefore \quad R_{t2} = \frac{V^2}{P} = \frac{(230)^2}{40} = 1322.5\ \Omega$$

Now $$R_{t2} = R_{27}\,[\,1 + \alpha_{27}\,(t_2 - 27)]$$

$$\alpha_{27} = \frac{\alpha_0}{1+\alpha_0 t} = \frac{0.0048}{1+0.0048\times 27} = 4.2492 \times 10^{-3}\ /°C$$

$$\therefore \quad 1322.5 = 92\,[1 + 4.2492 \times 10^{-3}\,(t_2 - 27)]$$

$$\therefore \quad 14.375 = 1 + 4.2492 \times 10^{-3}\,(t_2 - 27)$$

$$\therefore \quad t_2 - 27 = 3147.6513$$

$$\therefore \quad t_2 = \mathbf{3174.6513\ °C} \quad \text{... working temperature}$$

$$\therefore \quad I(\text{working}) = \frac{V}{R_{t2}} = \frac{230}{1322.5} = \mathbf{0.1739\ A} \quad \text{... working current}$$

Example 1.25 : *When a resistance of 2 Ω is placed across the terminals of battery, the current is 2 A. When the resistance is increased to 5 Ω, the current falls to 1 A. Find e.m.f. of battery and its internal resistance.*

Solution : The two cases are shown in the Fig. 1.65.

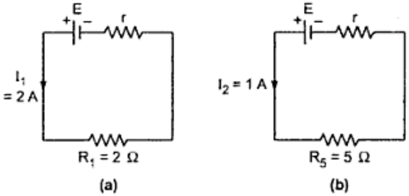


Fig. 1.55

Now $I_1 = \frac{E}{R_1 + r}$ i.e. $E = 2[2 + r] = 4 + 2r$ ...(1)

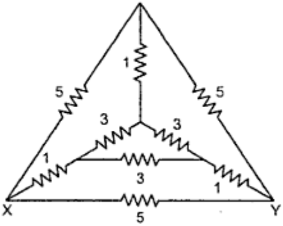
and $I_2 = \frac{E}{R_2 + r}$ i.e. $E = 1[5 + r] = 5 + r$ ...(2)

Subtracting equation (2) from equation (1),

$$0 = -1 + r \quad \text{i.e.} \quad r = \mathbf{1\ \Omega}$$

and $E = \mathbf{6\ V}$

Example 1.26 : *Determine the resistance between the terminals X and Y for the circuit shown in Fig. 1.56.*



(All resistances in ohm)

Fig. 1.56

Solution :

Converting inner delta to star.

$$\text{Each resistance} = \frac{3 \times 3}{3 + 3 + 3} = 1\,\Omega$$

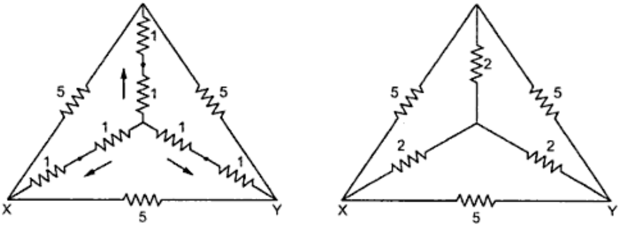
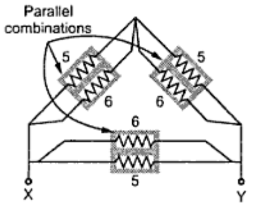


Fig. 1.56 (a)

Converting inner star to delta.

$$\text{Each resistance} = 2+2+\frac{2\times 2}{2} = 6\ \Omega$$



All three parallel combinations,

$$5||6 = \frac{5\times 6}{5+6}$$

$$= 2.7272\ \Omega$$

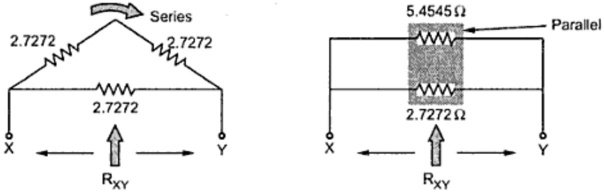


Fig. 1.56 (b)

$\therefore$ $R_{XY} = 5.4545||2.7272 = \mathbf{1.8181\ \Omega}$.

Example 1.27 : *Find the currents i_1, i_2, i_3 and powers delivered by the sources of the network in Fig. 1.57.*

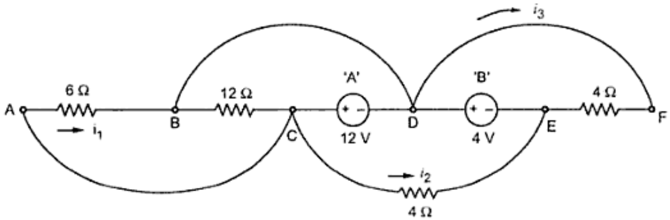


Fig. 1.57

Solution : The various branch currents are shown in the Fig. 1.57 (a), by applying KCL at various nodes interms of i_1, i_2, i_3 shown.

Loop ABCA, $-6\, i_1 - 12\,(i_1 + i_2 - i_4) = 0$ i.e. $-18\, i_1 - 12\, i_2 + 12\, i_4 = 0$ ...(1)

Loop CDEC, $-4\, i_2 + 4 + 12 = 0$ i.e. $i_2 = 4$ A ... (2)

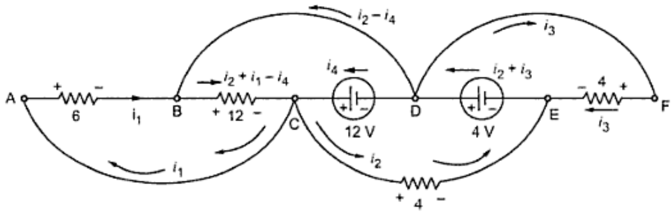


Fig. 1.57 (a)

Loop DEFD, $-4 + 4\,i_3 = 0$ i.e. $i_3 = 1$ A ... (3)

Loop BCDB, $-12\,(i_1 + i_2 - i_4) - 12 = 0$ i.e. $-i_1 - i_2 + i_4 = 1$ ... (4)

Substituting $i_2 = 4$ A in (1) and (4)

$$-3\,i_1 + 2\,i_4 = 8 \quad \text{... (5)}$$

$$-i_1 + i_4 = 5 \quad \text{... (6)}$$

$$-1.5\,i_1 + i_4 = 4 \quad \text{... (7)}$$

∴ $0.5\,i_1 = 1$

∴ $i_1 = $ **2 A** and $i_4 = $ **7 A**

∴ Power by 12 V source $= i_4 \times 12 = $ **84 W**

and Power by 4 V source $= 4 \times (i_2 + i_3) = $ **20 W**

➡ **Example 1.28 :** *Find the value of 'R' so that 1 A would flow in it, for the network in the Fig. 1.58.*

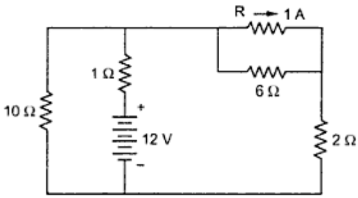


Fig. 1.58

Solution : The various branch currents are shown in the Fig. 1.58 (a).

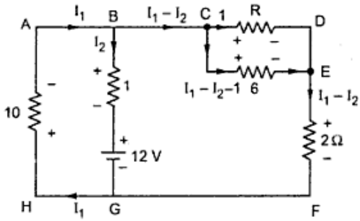


Fig. 1.58 (a)

Loop ABGH, $-I_2 - 12 - 10\,I_1 = 0$ i.e. $10\,I_1 + I_2 = -12$ ... (1)

Loop BCEFGB, $-6\,(I_1 - I_2 - 1) - 2\,(I_1 - I_2) + 12 + I_2 = 0$ i.e. $-8\,I_1 + 9\,I_2 = -18$ ... (2)

Multiplying equation (1) by 9 we get,

$\therefore \quad 90\,I_1 + 9\,I_2 = -108$ ... (3)

Subtracting equation (3) from (2) we get,

$\therefore \quad -98\,I_1 = +90$

$\therefore \quad I_1 = -0.9183\text{ A}$ and $I_2 = -2.8163\text{ A}$

$\therefore$ Current through 6 Ω = $I_1 - I_2 - 1 = -0.9183 + 2.8163 - 1$

= 0.898 A

$\therefore$ Drop across 6 Ω = 6 × current through 6 Ω = 6 × 0.898

= 5.388 V

Same is drop across R = R × 1 = 5.388

$\therefore$ R = **5.388 Ω**

Example 1.29 : *Using Kirchhoff's laws, find the current flowing in 2 ohm resistance for the circuit shown in Fig. 1.59.*

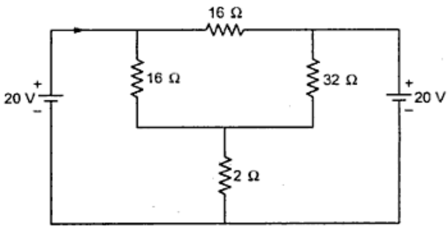


Fig. 1.59

Solution : The various branch currents are as shown.

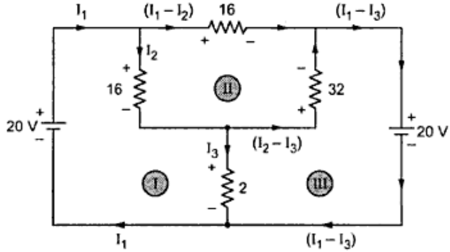


Fig. 1.59 (a)

Loop I : $-16\,I_2 - 2I_3 + 20 = 0$ i.e. $+16\,I_2 + 2I_3 = 20$ ...(1)

Loop II : $-16(I_1 - I_2) + 32(I_2 - I_3) + 16I_2 = 0$ i.e. $-16\,I_1 + 64I_2 - 32I_3 = 0$ ...(2)

Loop III : $-20 + 2I_3 - 32(I_2 - I_3) = 0$ i.e. $-32I_2 + 34I_3 = 20$ ...(3)

Solving for I_3, I_3 = **1.5789 A** ↓ ... Use Cramer's rule

➡ **Example 1.30 :** *Find the V_{CE} and V_{AG} for the circuit shown in Fig. 1.60.*

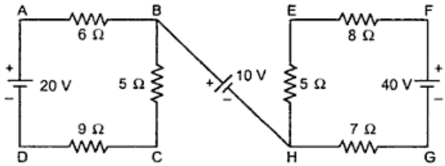


Fig. 1.60

Solution : Assume the two currents as shown in the Fig. 1.60 (a)

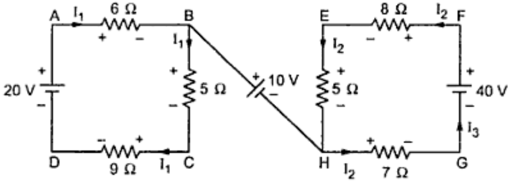


Fig. 1.60 (a)

Applying KVL to the two loops,

$$-6\,I_1 - 5\,I_1 - 9\,I_1 + 20 = 0 \quad \text{and} \quad -8\,I_2 - 5\,I_2 - 7\,I_2 + 40 = 0$$

$$\therefore \quad I_1 = 1\text{ A and } I_2 = 2\text{ A}$$

i) Trace the path C-E,

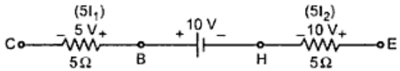


Fig. 1.60 (b)

$$\therefore \quad V_{CE} = -5\text{ V}$$

$$= \textbf{5 V with C negative}$$

ii) Trace the path A-G,

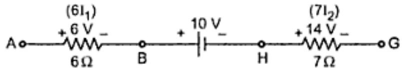


Fig. 1.60 (c)

$$\therefore \quad V_{AG} = \textbf{30 V with A positive}$$

Examples from G.U. and G.T.U. Papers

Example 1.31 : *A wire has a resistance of 2 Ω. It has been stretched to the length 3 times that of original. What will be the new resistance of wire?* **[GU : Aug.-2001]**

Solution : $R_1 = 2\ \Omega$, $l_2 = 3\ l_1$,

$$R = \frac{\rho l}{a}$$

Key Point: *When wire is stretched, its length changes and cross-sectional area changes as the total volume remains same.*

$$\text{Volume} = \text{area} \times \text{length} = a_1 \times l_1 = a_2 \times l_2$$

$$\frac{a_1}{a_2} = \frac{l_2}{l_1} = \frac{3l_1}{l_1} = 3 \quad \text{i.e. } a_2 = \frac{1}{3} a_1$$

Now,
$$\frac{R_1}{R_2} = \frac{\left(\frac{\rho l_1}{a_1}\right)}{\left(\frac{\rho l_2}{a_2}\right)} = \frac{l_1}{l_2} \times \frac{a_2}{a_1} = \frac{1}{3} \times \frac{1}{3}$$

$$R_2 = 9\ R_1 = 9 \times 2 = 18\ \Omega \quad \text{... New resistance}$$

Example 1.32 : *Calculate the resistance of 100 m length of wire having a uniform cross-sectional area of 0.1 mm² if the wire is made of Manganin having a resistivity of 50×10^{-8} Ω-m. If the wire to drawn three times its original length, find out new resistance.*
[GU. : Nov.-2005]

Solution : $a = 0.1\ mm^2$, $l = 100$ m, $\rho = 50 \times 10^{-8}$ Ω-m.

$$\therefore \quad R = \frac{\rho l}{a} = \frac{50 \times 10^{-8} \times 100}{0.1 \times 10^{-6}} = 500\ \Omega$$

Now, $l' = 3l$ but its volume remains same.

$\therefore$ Volume $= a \times l = a' \times l'$ where a' = new c/s

$$\therefore \quad \frac{a}{a'} = \frac{l'}{l} = 3 \quad \text{i.e.} \quad a' = \frac{1}{3}a$$

$$\therefore \quad R' = \frac{\rho l'}{a'} = \frac{\rho \times 3l}{\frac{1}{3}a} = 9\left(\frac{\rho l}{a}\right) = 9\,R = 4500\ \Omega$$

Example 1.33 : *Two wires of different conducting materials are connected in parallel. They share current in the ratio 5:6. If the wire of material A has 1.7 times length and double cross-section area that of material B, find the ratio of their specific resistances.*
[GU: July-2004]

Solution :

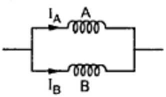


Fig. 1.61

$$\frac{I_A}{I_B} = \frac{5}{6},\ l_A = 1.7\ l_B,\ a_A = 2\ a_B$$

$$I \propto \frac{1}{R} \quad \text{...Ohm's law}$$

$$\therefore \quad \frac{I_A}{I_B} = \frac{R_B}{R_A} = \frac{5}{6}$$

$$\therefore \quad R_A = \frac{6}{5}\ R_B$$

$$\therefore \quad \frac{\rho_A l_A}{a_A} = \frac{6}{5}\ \frac{\rho_B l_B}{a_B} \quad \text{i.e.} \quad \frac{\rho_A}{\rho_B} = \frac{6}{5} \times \frac{l_B}{l_A} \times \frac{a_A}{a_B}$$

$$\therefore \quad \frac{\rho_A}{\rho_B} = \frac{6}{5} \times \frac{1}{1.7} \times 2 = 1.411 \quad \text{... Ratio of specific resistances}$$

Example 1.34 : *An electric radiator is required to dissipate 1 kW when connected to a 230 V supply. If the coils of the radiator are of wire 0.5 mm in diameter having resistivity of 60 μΩ-cm, calculate the necessary length of the wire.* **[GU : June 2005]**

Solution : P = 1 kW, V = 230 V, d = 0.5 mm, ρ = 60 μΩ-cm

$$P = \frac{V^2}{R} \quad \text{i.e.} \quad 1 \times 10^3 = \frac{(230)^2}{R}$$

$$\therefore \quad R = 52.9\ \Omega \quad \text{but} \quad R = \frac{\rho l}{a}$$

$$\therefore \quad 52.9 = \frac{60 \times 10^{-6} \times 10^{-2} \times l}{\frac{\pi}{4} \times (0.5)^2 \times 10^{-6}} \qquad \text{... } a = \frac{\pi}{4} d^2$$

$$\therefore \quad l = \mathbf{17.311\ m} \qquad \text{... Length required}$$

Example 1.35 : *The resistance of the field coil of a d.c. machine is 120 Ω at 15 °C. During its full load run, the resistance increases to 135 Ω. Find the average temperature of the field coil. Take resistance temperature coefficient to be 0.00401 / °C at 15 °C.* **[GU : Dec.-2001, June-2004]**

Solution : $R_1 = 120\ \Omega$, $t_1 = 15\ °C$, $R_2 = 135\ \Omega$, $\alpha_1 = 0.00401\ /°C$

$$R_2 = R_1 [1+\alpha_1(t_2-t_1)]$$

$$\therefore \quad 135 = 120 [1+0.00401 (t_2 - 15)] \quad \text{i.e.} \quad t_2 - 15 = 31.172$$

$$\therefore \quad t_2 = \mathbf{46.172\ °C} \qquad \text{... Temperature of field coil}$$

Example 1.36 : *The resistance of a given conductor is 60 Ω at 20 °C. Find its temperature coefficient and the resistance at 30 °C if its temperature coefficient at 0 °C is 0.004 /°C.* **[GU : June-2004]**

Solution : $R_1 = 60\ \Omega$, $t_1 = 20\ °C$, $R_2 = ?$, $t_2 = 30\ °C$, $\alpha_0 = 0.004\ /°C$

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{0.004}{1+0.004 \times 20} = 0.003703\ /°C \qquad \text{...at 20 °C}$$

$$\therefore \quad R_2 = R_1 [1+\alpha_1(t_2-t_1)] = 60 [1+0.003703 (30 - 20)]$$

$$\therefore \quad R_2 = \mathbf{62.22\ \Omega} \qquad \text{... Resistance at 30 °C}$$

$$\alpha_2 = \frac{\alpha_0}{1+\alpha_0 t_2} = \frac{0.004}{1+0.004 \times 30} = \mathbf{0.003571\ /\ °C} \qquad \text{...at 30 °C}$$

Example 1.37: *A wire 10 m long has cross-sectional area of 10 mm^2. At 0 °C, a current of 10 A passes through it, when it is connected to a d.c. supply of 200 V. Calculate :*
a) resistivity of the material, b) Current which will flow through the wire when the temperature rises to 50 °C. Given : α_0 = 0.0003 / °C. **[GU : June-2003]**

Solution : l = 10 m, a = 10 mm^2, I_0 = 10 A, V = 200 V

a) At t = 0 °C, $R_0 = \frac{V}{I_0} = \frac{200}{10} = 20\ \Omega$

But $R_0 = \frac{\rho l}{a}$ i.e. $20 = \frac{\rho \times 10}{10 \times 10^{-6}}$

$\therefore$ $\rho = \mathbf{2 \times 10^{-5}\ \Omega\text{-m}}$ ... Resistivity

b) Now $t_1 = 50\ °C$

$R_1 = R_0 (1 + \alpha_0 t_1) = 20\ (1 + 0.0003 \times 50)$

$= 20.3\ \Omega$

$\therefore$ $I_1 = \frac{V}{R_1} = \frac{200}{20.3} = \mathbf{9.8522\ A}$ ... Current at 50 °C

Example 1.38 : *Two conductors, one of copper and other of iron are connected in parallel and at 30 °C carry equal currents. What proportion of current will pass through each if the temperature is raised to 120 °C. The temperature coefficient of resistance for copper is 0.0043/ °C and for iron is 0.0062 / °C at 30 °C.* **[GU : June-95, 99]**

Solution : α_{1c} = 0.0043 °C , α_{1i} = 0.0062 /°C .

Let R_{1c} = Resistance of copper at t_1 = 30 °C .

R_{1i} = Resistance of iron at t_1 = 30 °C .

R_{2c} = Resistance of copper at t_2 = 120 °C.

R_{2i} = Resistance of iron at t_2 = 120 °C.

At t_1 = 30 °C, both carry equal current hence $R_{1c} = R_{1i}$.

$R_{2c} = R_{1c} [1 + \alpha_{1c} (t_2 - t_1)] = R_{1c} [1 + 0.0043 (120 - 30)]$

$= 1.387\ R_{1c}$

$R_{2i} = R_{1i} [1 + \alpha_{1i} (t_2 - t_1)] = R_{1i} [1 + 0.0062 (120 - 30)]$

$= 1.558\ R_{1i}$

Now $I \propto \frac{1}{R}$ ... Voltage constant as in parallel

$\therefore \quad \frac{I_{1c}}{I_{2c}} = \frac{R_{2c}}{R_{1c}}$ i.e. $I_{2c} = I_{1c} \times \frac{R_{1c}}{R_{2c}} = \frac{1}{0.387} I_{1c}$

And $\quad \frac{I_{1i}}{I_{2i}} = \frac{R_{2i}}{R_{1i}}$ i.e. $I_{2i} = I_{1i} \times \frac{R_{1i}}{R_{2i}} = \frac{1}{1.558} I_{1i}$

But $\quad I_{1c} = I_{1i} = I$

$\therefore \quad I_{2c} = 0.7209\ I$ and $I_{2i} = 0.6418\ I$

I_{T2} = Total current at (t_2 = 120 °C) = $I_{2c} + I_{2i}$

$= 0.7209\ I + 0.6418\ I = 1.627\ I$

$\therefore$ % Current through copper at 120 °C = $\frac{0.7209\ I}{1.3627\ I} \times 100$ = **52.9 %**

$\therefore$ % Current through iron at 120 °C = $\frac{0.6418\ I}{1.3627\ I}$ = **47.1%**

Example 1.39 : *A coil of copper wire has a resistance of 90 Ω at 20 °C and is connected to a 230 V supply. By how much the voltage is to be changed in order to maintain the current constant if the temperature of the coil rises to 60 °C. Assume α_0 = 0.00428 /°C.*

[GTU : Aug.-2001]

Solution : $R_1 = 90\ \Omega,\ t_1 = 20\ °C,\ t_2 = 60\ °C$

$R_2 = R_1 [1+\alpha_1 (t_2 - t_1)]$

While $\quad \alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{0.00428}{1+0.00428\times 20} = 0.003942$ /°C

$\therefore \quad R_2 = 90\ [1+0.003942 \times (60-20)] = 104.193\ \Omega$

$I_1 = \frac{V_1}{R_1} = \frac{230}{90} = 2.5555$ A

But now current has to remain same by changing voltage. The new voltage be V_2.

$\therefore \quad I_2 = \frac{V_2}{R_2}$ and $I_2 = I_1 = 2.5555$

$\therefore \quad 2.5555 = \frac{V_2}{104.193}$ i.e. $V_2 = 266.27$ V

$\therefore$ Increase in voltage = $V_2 - V_1$ = 266.27 − 230 = **36.27 V**

Example 1.40 : *A 100 W, 200 V filament lamp has operating temperature of 2000 °C. The temperature coefficient of filament material is 0.005 at 0 °C per °C. Calculate the current taken by the lamp at the instant of switching with 200 V supply, with the filament temperature at 20 °C* **[GU : June-2006]**

Solution : $\alpha_0 = 0.005\ /°C$, $t_2 = 2000\ °C$, $V = 200\ V$, $t_1 = 20\ °C$

The power 100 W is corresponding to operating temperature of $t_2 = 2000\ °C$. The voltage is constant at 200 V.

$$\therefore \quad R_2 = \frac{V^2}{P} = \frac{(200)^2}{100} = 400\ \Omega \quad \text{... Resistance at 2000 °C}$$

At the time of switching, $t_1 = 20\ °C$ and resistance be R_1.

$$\therefore \quad R_2 = R_1\,[1+\alpha_1\,(t_2-t_1)]$$

$$\text{Now,} \quad \alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{0.005}{1+0.005\times 20} = 0.004545/\ °C$$

$$\therefore \quad 400 = R_1[1+0.004545\ (2000-20)]$$

$$\therefore \quad R_1 = 40\,\Omega \quad \text{... Resistance at 20 °C}$$

$$\therefore \quad I_1 = \text{Current at the instant of switching} = \frac{V}{R_1}$$

$$= \frac{200}{40} = \mathbf{5\ A}$$

Example 1.41 : *The current flowing at the instant of switching on a 40 W lamp to a 240 V supply is 2 A. Resistance temperature coefficient of the filament material is 0.0055/°C at the room temperature of 20 °C. Calculate the working temperature of the filament and current taken during normal working.* **[GU : July-2007]**

Solution : At the instant of switching let the temperature of filament be $t_1\ °C = 20\ °C$

$$\alpha_1 = 0.0055\ /°C,\ I_1 = 2\ A,\ V = 240\ V,\ P = 40\ W$$

$$R_1 = \frac{V}{I_1} = \frac{240}{2} = 120\ \Omega \quad \text{... Resistance at 20 °C}$$

At the normal working , the power is 40 W and the temperature be $t_2\ °C$. The corresponding resistance is R_2.

$$\therefore \quad R_2 = \frac{V^2}{P} = \frac{(240)^2}{40} = 1440\ \Omega$$

$$\text{But} \quad R_2 = R_1\,[1+\alpha_1\,(t_2-t_1)] \quad \text{i.e.} \quad 1440 = 120\,[1+0.0055(t_2-20)]$$

$$\therefore \quad 12 = 1+0.0055\,(t_2-20) \quad \text{i.e} \quad t_2-20 = 2000$$

$\therefore \quad t_2 = 2020\ °C$ ...Working temperature

$$I_2 = \frac{V}{R_2} = \frac{240}{1440} = 0.1667\ A$$ ... Normal working current

Example 1.42 : *Find the equivalent resistance between A and B for the circuit shown in the Fig. 1.62.* **[GU : Nov.-2006]**

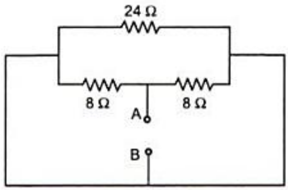
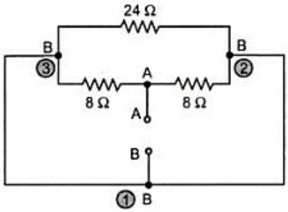


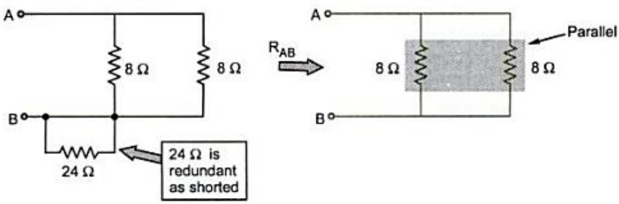
Fig. 1.62

Solution : Name the circuit as shown in the Fig. 1.62 (a).



(a)

The nodes 1,2 and 3 are directly joined to each other and hence are at same potential. Thus all three represent same node B. Showing them as a single node. The circuit can be redrawn as shown in the Fig. 1.62 (b)



(b) **(c)**

Fig. 1.62

$$\therefore \quad R_{AB} = 8 \parallel 8 = \frac{8 \times 8}{8+8} = 4\ \Omega$$

Example 1.43 : *Determine the values of R_a and R_b ($R_b \neq o$) for the circuit shown in the Fig. 1.63.*

If $V_2 = \frac{V_1}{2}$ and the equivalent resistance of the circuit across the terminals A-B is 100 Ω.

[GU : June-2001]

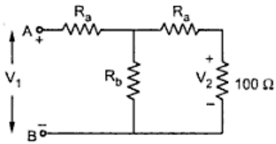
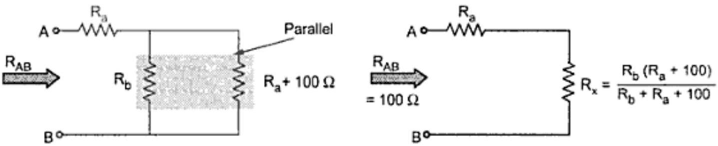


Fig. 1.63

Solution : R_{AB} = 100 Ω and R_a and 100 Ω are in series in second branch.



(a) (b)

Fig. 1.63

$$\therefore \quad 100 = R_a + \frac{R_b(R_a + 100)}{R_b + R_a + 100} = \frac{R_a(R_b + R_a + 100) + R_b(R_a + 100)}{(R_b + R_a + 100)} \quad \text{...(1)}$$

Now the currents are as shown in the Fig. 1.63 (c).

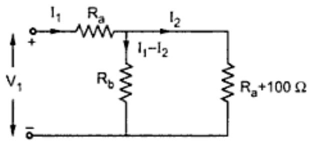


Fig. 1.63 (c)

From current distribution rule,

$$I_2 = I_1 \times \frac{R_b}{R_b + R_a + 100}$$

Same current I_2 passes through 100 Ω across which drop is V_2.

$$\therefore \quad V_2 = 100\, I_2 = 100 \times I_1 \times \frac{R_b}{R_b + R_a + 100} \quad \text{and} \quad V_2 = \frac{V_1}{2} \text{ (given)}$$

$$\therefore \quad \frac{V_1}{2} = 100 \times I_1 \times \frac{R_b}{R_b + R_a + 100} \quad \text{i.e.} \quad \frac{V_1}{I_1} = \frac{200\, R_b}{R_b + R_a + 100}$$

$$\text{But} \quad \frac{V_1}{I_1} = R_{AB} = 100 \quad \text{i.e.} \quad 100 = \frac{200\, R_b}{R_b + R_a + 100}$$

$\therefore \quad R_b+R_a+100 = 2R_b \quad$ i.e. $\quad R_b = R_a + 100$ ... (2)

Using in equation (1), $\quad 100 = \dfrac{R_a(R_a+100+R_a+100)+(R_a+100)(R_a+100)}{(R_a+100+R_a+100)}$

$\therefore \quad 100(2R_a+200) = R_a \times 2(R_a+100) + (R_a+100)^2$

$\therefore \quad 100 \times 2 = 2R_a + R_a + 100 \quad$ i.e. $\quad 3R_a = 100$

$\therefore \quad \mathbf{R_a = 33.333\ \Omega}$, $\mathbf{R_b = 133.333\ \Omega}$

Example 1.44 : *Find the resistance between the terminals A-B.* **[GU : July-2007]**

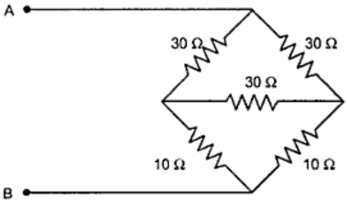
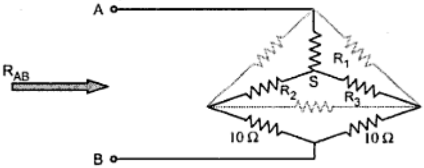


Fig. 1.64

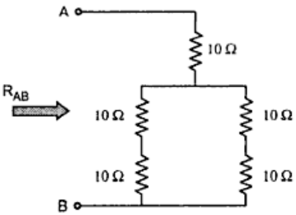
Solution : Convert delta of 30-30-30 Ω to star,



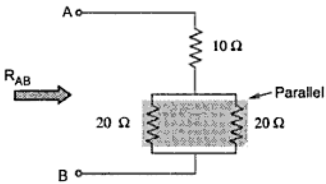
(a)

$R_1 = R_2 = R_3$ in star as all the resistance are equal in delta.

$\therefore \quad R_1 = R_2 = R_3 = \dfrac{30 \times 30}{30+30+30} = 10\ \Omega$



(b)



(c)

Fig. 1.64

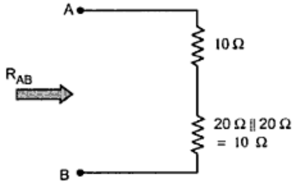


Fig. 1.64 (d)

$$\therefore \quad R_{AB} = 10 + 10 = 20 \ \Omega$$

Example 1.45 : *Using delta-star transformation, determine the resistance between the terminals A-B of the circuit shown in the Fig. 1.65.*
[GU : May-2001, July-2005]

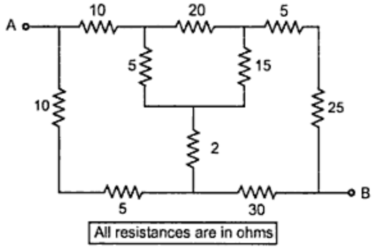
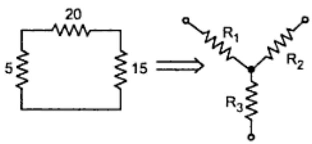


Fig. 1.65

Solution : Convert the delta of 20, 5 and 15 Ω to equivalent star.



(a)

$$R_1 = \frac{20 \times 5}{20+5+15} = 2.5 \ \Omega$$

$$R_2 = \frac{20 \times 15}{20+5+15} = 7.5 \ \Omega$$

$$R_3 = \frac{5 \times 15}{20+5+15} = 1.875 \ \Omega$$

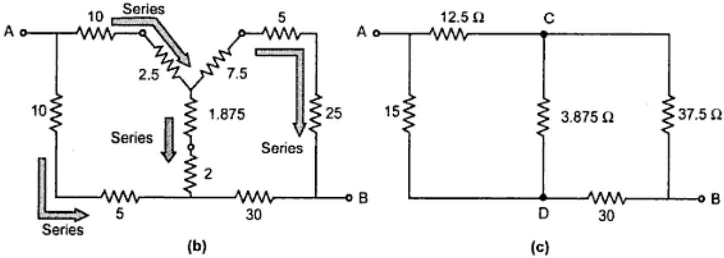
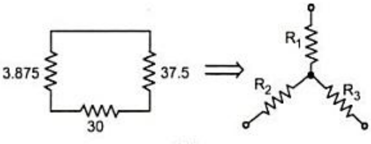


Fig. 1.65

Convert Δ CDB to equivalent star,



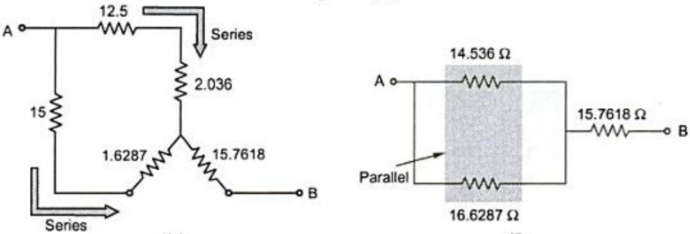
(d)

$$R_1 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = 2.036\,\Omega$$

$$R_2 = \frac{3.875 \times 30}{3.875 + 37.5 + 30} = 1.6287\,\Omega$$

$$R_3 = \frac{30 \times 37.5}{3.875 + 37.5 + 30} = 15.7618\,\Omega$$

The circuit reduces as shown in the Fig. 1.65 (e).



(e) (f)

Fig. 1.65

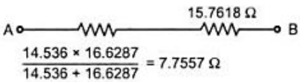


Fig. 1.65 (g)

$\therefore$

$$R_{AB} = 7.7557 + 15.7618$$

$$= \mathbf{23.5175\ \Omega}$$

➠ **Example 1.46 :** *Find out the current delivered by the battery in the circuit shown in the Fig. 1.66*

[GU : June-2006]

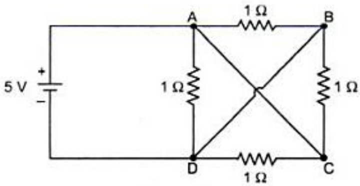


Fig. 1.66

Solution : Redraw the circuit by analyzing it.

Key Point: *Nodes A and C are directly joined hence are at same potential and can be shown as one node. Similarly nodes B and D can be shown as single node as directly joined.*

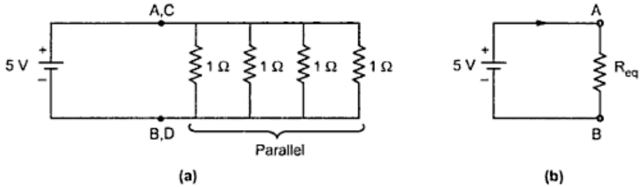


Fig. 1.66

$$\frac{1}{R_{eq}} = \frac{1}{1}+\frac{1}{1}+\frac{1}{1}+\frac{1}{1} \quad \text{i.e.} \quad R_{eq} = \frac{1}{4}\,\Omega$$

$$\therefore \qquad I = \frac{5}{R_{eq}} = \frac{5}{\left(\frac{1}{4}\right)} = 20\text{ A} \qquad \text{...Current delivered by battery}$$

Example 1.47 : *A lamp rated 75 W, 100 V is to be connected across 230 V supply. Find the value of resistance to be connected in series with the lamp. Also find the power loss occurring in the resistance.* **[GU : June-2005]**

Solution : The arrangement is shown in the Fig. 1.67

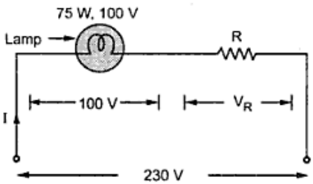


Fig. 1.67

The voltage across lamp must be 100 V.

P_L = Power in lamp = 75 W

V_L = Voltage across lamp = 100 V

$$I = \frac{P_L}{V_L} = \frac{75}{100} = 0.75\text{ A}$$

Now $\quad V_T = V_L + V_R \quad$ i.e. $\quad 230 = 100 + V_R$

$\therefore \quad V_R = 230 - 100 = 130$ V

But $\quad V_R = I\,R \quad$ i.e. $\quad 130 = 0.75 \times R$

$$\therefore \quad R = \frac{130}{0.75} = \mathbf{173.333\ \Omega}$$

$$P_R = V_R \times I \text{ or } I^2 \times R = 130 \times 0.75 = \mathbf{97.5\ W}$$

Example 1.48 : *A bulb rated 110 V, 60 W, is connected in series with another bulb rated 110 V, 100 W across a 220 V supply. Calculate the resistance which should be connected in parallel with the first bulb so that both the bulbs may get their rated power.*

[GU : June-98]

Solution : The bulbs are shown in the Fig. 1.68.

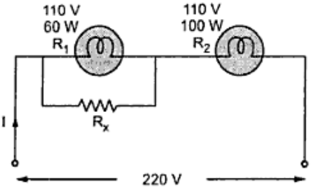


Fig. 1.68

$$R_1 = \text{Resistance of bulb 1} = \frac{V^2}{P} = \frac{110^2}{60}$$

$$= 201.667\ \Omega$$

$$R_2 = \text{Resistance of bulb 2} = \frac{(110)^2}{100} = 121\Omega$$

For rated power, the voltage across each bulb must be 110 V.

$$\therefore \quad I = \frac{V_{R2}}{R_2} = \frac{110}{121} = 0.90909\ A$$

The current remains same due to series circuit. And the voltage across bulb 1 must be 110 V.

$$\therefore \quad R_{eq} = \frac{\text{Voltage across bulb1}}{I} = \frac{110}{0.90909} = 121\ \Omega$$

$$\text{But} \quad R_{eq} = \frac{R_1 \times R_x}{R_1 + R_x} \quad \text{i.e.} \quad 121 = \frac{201.667\ R_x}{201.667 + R_x}$$

$$\therefore \quad 201.667 + R_x = 1.667\ R_x \quad \text{i.e.} \quad R_x = \mathbf{302.5\ \Omega}$$

This is the resistance required in parallel with bulb 1.

Example 1.49 : *Two coils are connected in parallel and a voltage of 200 V is applied between the terminals. The total current taken by the circuit is 25 A and power dissipated in one of the coils is 1500 W. Calculate the resistance of each coil.*

[GU : Nov.-2002, June-2003]

Solution : The coils are shown in the Fig.1.69.

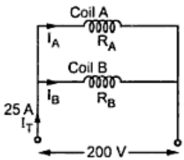


Fig. 1.69

The power dissipated in A = P_A = 1500 W

$$P_A = \frac{V^2}{R_A} = \frac{(200)^2}{R_A}$$

$\therefore \quad R_A = \frac{(200)^2}{1500} = \mathbf{26.667\ \Omega}$

$\therefore \quad I_A = \frac{V}{R_A} = \frac{200}{26.667} = 7.5\ A$

$\therefore \quad I_B = I_T - I_A = 25 - 7.5 = 17.5\ A$

$\therefore \quad R_B = \frac{V}{I_B} = \frac{200}{17.5} = \mathbf{11.4285\ \Omega}$

Example 1.50 : *Two batteries A and B are connected in parallel and combination is connected across 10 Ω resistance. Battery A has an internal resistance of 0.2 Ω and internal voltage is 110 V while the corresponding values of battery B are 0.25 Ω and 100 V. Calculate the current in each battery and also in 10 Ω resistor.* **[GU : Dec.-2001]**

Solution :

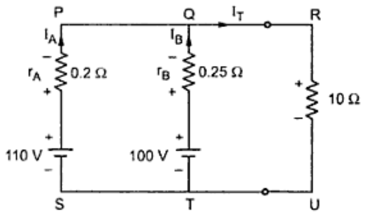


Fig. 1.70

Applying KVL to the two loops,

Loop PQTSP,

$- 0.2\ I_A + 0.25\ I_B - 100 + 110 = 0$

i.e. $- 0.2\ I_A + 0.25\ I_B = - 10$ ...(1)

Loop QRUTQ, $- 10\ I_T + 100 - 0.25\ I_B = 0$ but $I_T = I_A + I_B$

$\therefore \quad - 10\ (I_A + I_B) + 100 - 0.25\ I_B = 0$ i.e. $+ 10\ I_A + 10.25\ I_B = + 100$ ...(2)

Solving equation (1) and equation (2), I_A = **28.02 A ↑**, $I_B = - 17.58$ A i.e. **17.58 A ↓**

$\therefore \quad I_T = I_A + I_B = 28.02 - 17.58 = \mathbf{10.44\ A} \downarrow$

Example 1.51 : *In the network shown in the Fig. 1.71, calculate the value of resistance R in the branch BO and the current flowing through it if the current in the branch OC is zero.* **[GU : June-2001]**

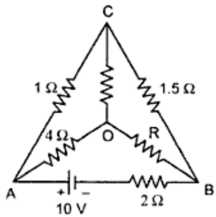


Fig. 1.71

Solution : As the current in the branch CO is zero, it is redundant from the circuit point of view :

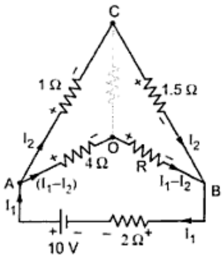


Fig. 1.71 (a)

Key Point: *As the current in branch CO is zero, the points C and O are equipotential i.e. at same voltage.*

$$\therefore \quad \text{Drop AC} = \text{Drop AO}$$

$$\therefore \quad I_2 \times 1 = (I_1 - I_2) \times 4$$

$$\therefore \quad I_2 = 0.8\, I_1 \quad \text{...(1)}$$

Apply KVL to two loops,

Loop AOBA, $-4(I_1-I_2) - R(I_1-I_2) - 2I_1 + 10 = 0$

i.e. $-4I_1 + 4I_2 - RI_1 + RI_2 - 2I_1 + 10 = 0$ and use equation (1),

i.e. $-4I_1 + 4 \times 0.8\, I_1 - RI_1 + R(0.8\, I_1) - 2I_1 + 10 = 0$

i.e. $I_1[-2.8 - 0.2\,R] = -10$ i.e. $I_1(2.8 + 0.2R) = 10$ ...(2)

Loop ACBA, $-I_2 - 1.5\, I_2 - 2\, I_1 + 10 = 0$ and use equation (1),

i.e. $-0.8\, I_1 - 1.5 \times 0.8\, I_1 - 2 \times I_1 + 10 = 0$

i.e. $4\, I_1 = 10$ i.e. $I_1 = 2.5$ A ...(3)

From equation (2), $2.5\,(2.8+0.2\,R) = 10$ i.e. $2.8 + 0.2\,R = 4$ $\therefore R = \mathbf{6\ \Omega}$

$$\therefore \quad \text{Current flowing in R} = I_1 - I_2 = I_1 - 0.8\, I_1$$

$$= 0.2\, I_1 = 0.2 \times 2.5 = \mathbf{0.5\ A}$$

Example 1.52 : *In the circuit shown in the Fig. 1.72, calculate the voltage across AB when*

a) The switch S is open

b) The switch S is closed.

[GU : July- 2005]

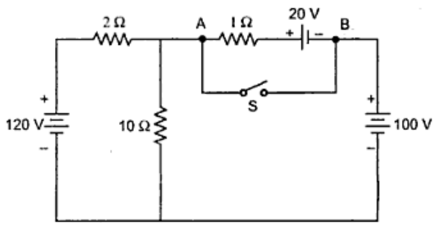


Fig. 1.72

Solution : a) The switch S is open.

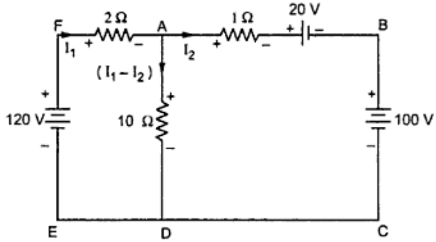


Fig. 1.72 (a)

Apply KVL to the two loops,

Loop FADEF, $-2 I_1 - 10 (I_1 - I_2) + 120 = 0$ i.e. $-12 I_1 + 10 I_2 = -120$ ...(1)

Loop ABCDA, $-I_2 - 20 - 100 + 10 (I_1 - I_2) = 0$ i.e. $-10 I_1 - 12 I_2 = 120$ ... (2)

Solving, $I_1 = 5.4545$ A, $I_2 = -5.4545$ A

$\therefore$ $V_{AB} = I_2 \times 1 + 20 = -5.4545 \times 1 + 20 =$ **14.5455 V** ...A +ve

b) The switch S is closed

When the switch S is closed, the point AB are shorted and voltage across short circuit is zero.

$\therefore$ V_{AB} = **0 V**

Example 1.53 : *Using delta-star transformation, determine the current drawn from the supply, in the circuit shown in the Fig. 1.73.*

[GU: June-2003, Nov.-2006, Dec.-2008]

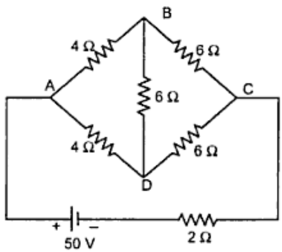
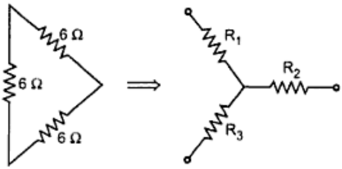


Fig. 1.73

Solution : Convert the delta of 6 - 6 - 6 Ω to equivalent star.



$$R_1 = R_2 = R_3 = \frac{6 \times 6}{6+6+6}$$

$$= 2\,\Omega$$

(a)

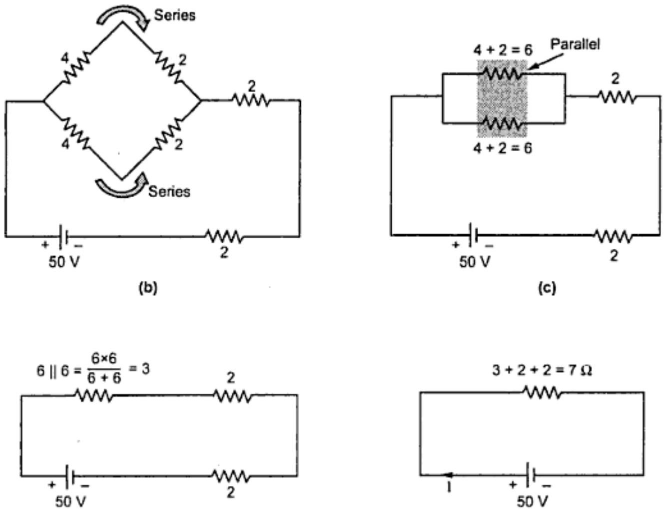


Fig. 1.73

$$\therefore \qquad I = \frac{50}{7} = 7.1428 \text{ A}$$

Example 1.54 : *Find the resistance between A and B by star delta transformation. Also find the current in 50 Ω connected between terminals C and D.*

[GU: Nov.-2006]

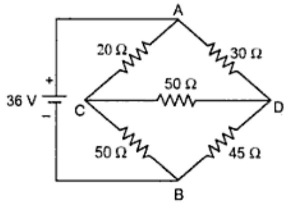
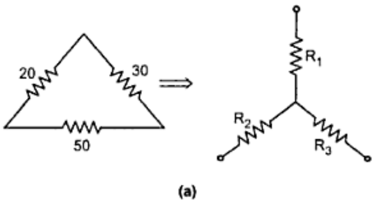


Fig. 1.74

Solution : Convert the delta of 20, 30, 50 to equivalent star.



$$R_1 = \frac{20\times 30}{20+30+50} = 6\ \Omega$$

$$R_2 = \frac{20\times 50}{20+30+50} = 10\ \Omega$$

$$R_3 = \frac{30\times 50}{20+30+50} = 15\ \Omega$$

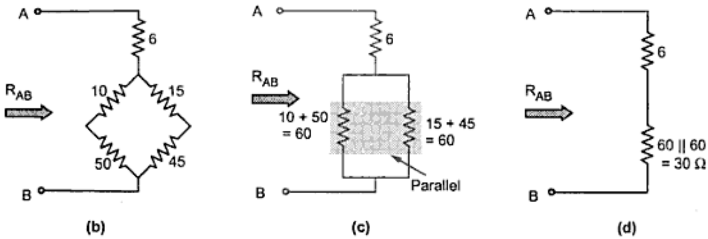


Fig. 1.74

$\therefore$ $R_{AB} = 6 + 30 = \mathbf{36}\ \Omega$ ...Resistance between A and B

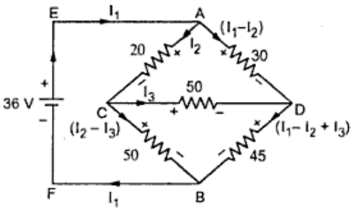


Fig. 1.74 (e)

To find the current through 50 Ω, show the current distribution as shown in the Fig. 1.74 (e). Apply KVL to the three loops,

$$-30\,(I_1-I_2) + 50\,I_3 + 20\,I_2 = 0$$

i.e. $-30\,I_1 + 50\,I_2 + 50\,I_3 = 0$ (Loop ADCP) ...(1)

Loop CDBC, $-50\,I_3 - 45\,(I_1-I_2+I_3) + 50\,(I_2-I_3) = 0$ ...(2)

i.e. $-45\,I_1 + 95\,I_2 - 145\,I_3 = 0$

Loop EACBFE, $-20\,I_2 - 50\,(I_2-I_3) + 36 = 0$

i.e. $70\,I_2 - 50\,I_3 = 36$ ... (3)

Use Cramer's rule for the equations (1), (2), (3) and we want current through 50 Ω i.e. I_3.

$$D = \begin{vmatrix} -30 & 50 & 50 \\ -45 & 95 & -145 \\ 0 & 70 & -50 \end{vmatrix} = -432000, \quad D_3 = \begin{vmatrix} -30 & 50 & 0 \\ -45 & 95 & 0 \\ 0 & 70 & 36 \end{vmatrix} = -21600$$

$$\therefore \quad I_3 = \frac{D_3}{D} = \frac{-21600}{-432000} = \mathbf{+0.05\ A\ \ from\ C\ to\ D}$$

Example 1.55 : *In the circuit shown in the Fig.1.75, find the value of resistance R, when V_{AB} = 5 V.* **[GU : Nov.-2006]**

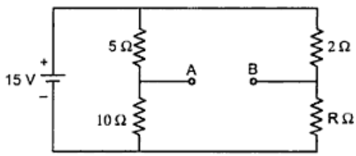


Fig. 1.75

Solution : The current distribution is shown in the Fig. 1.75 (a).

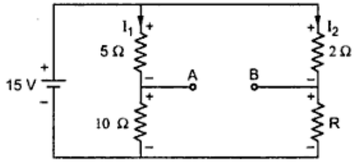


Fig. 1.75 (a)

I_1 flows from both 5 and 10 Ω.

$$I_1 = \frac{15}{5+10} = 1 \text{ A}$$

I_2 flows from both 2 and R Ω

$$I_2 = \frac{15}{2+R}$$

Trace the path from A to B as shown in the Fig. 1.75 (b).

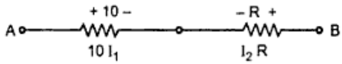


Fig. 1.75 (b)

$$V_{AB} = 10\, I_1 - I_2\, R$$

$$= 10 \times 1 - \frac{15}{2+R} \times R$$

But $V_{AB} = 5$ V (Given)

$$5 = 10 - \frac{15\,R}{2+R} \quad \text{i.e.} \quad \frac{15\,R}{2+R} = 5$$

i.e. 10 R = 10 i.e **R = 1 Ω**

Example 1.56 : *Determine the current in the 17 Ω resistor in the network shown in the Fig. 1.76.*

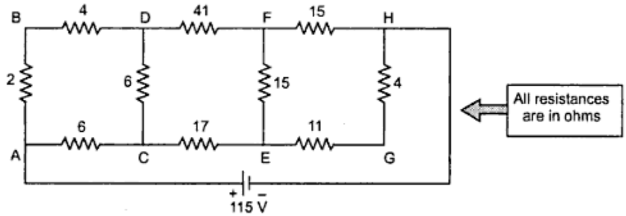
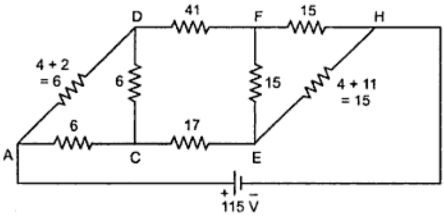


Fig. 1.76

[GU : June-2006]

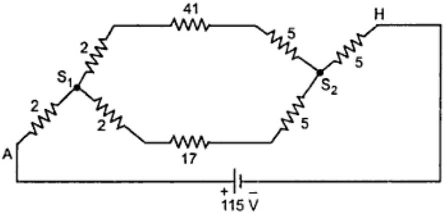
Solution :



(a)

4 Ω and 2 Ω are in series while 11 Ω and 4 Ω are in series, convert delta ADC to star giving each resistance as,

$$\frac{6\times 6}{6+6+6} = 2\ \Omega$$



(b)

Convert delta EFH to star giving each resistance as,

$$\frac{15\times 15}{15+15+15} = 5\ \Omega$$

The circuit reduces as shown in the Fig. 1.76 (b)

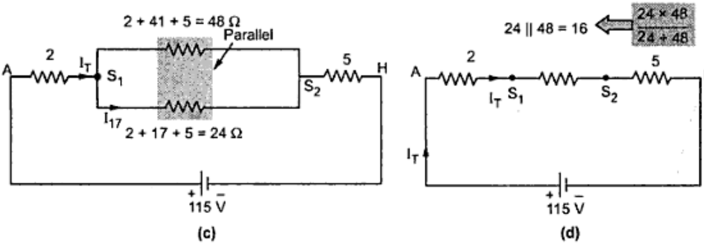


Fig. 1.76

$$I_T = \frac{115}{2+16+5} = 5\ A$$

Now the 17 Ω resistor is a part of one of the branches of parallel combination where total current entering the combination is 5 A. So using **Current division rule,**

$$I_{17} = I_T \times \frac{48}{48+24} = \frac{5\times 48}{72} = 3.333\ A$$

Example 1.57 : *Find R in the circuit shown in the Fig. 1.77.* [GU : July-2007]

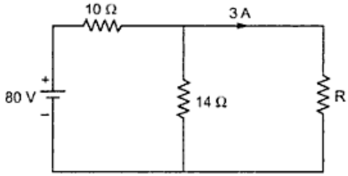


Fig. 1.77

Solution : The resistance R and 14 are in parallel.

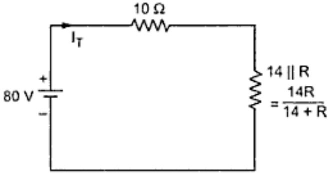


Fig. 1.77 (a)

$$\therefore \qquad I_T = \frac{80}{10+\frac{14R}{14+R}} = \frac{80(14+R)}{140+24R}$$

This I_T divides into two parallel branches and one of the two branch currents is given as 3 A. So by **Current division rule,**

$$3 = I_T \times \frac{14}{14+R} = \frac{80(14+R)}{(140+24R)} \times \frac{14}{14+R}$$

$$\therefore \qquad 3(140+24\ R) = 80 \times 14 \qquad \text{i.e.} \qquad 24\ R = 233.333$$

$$\therefore \qquad R = \mathbf{9.7222\ \Omega}$$

Example 1.58 : *The resistance of tungsten filament of a lamp is 20 Ω at the room temperature of 20 °C. What is the operating temperature of the filament if the resistance temperature coefficient of tungsten is 0.005/ °C at 20 °C? The base of the lamp is marked 120 V 50 W.* [GTU : March - 2009]

Solution : $R_1 = 20\ \Omega$ at $t_1 = 20\ °C$, $\alpha_1 = 0.005\ /°C$.

At working temperature of t_2 °C, P = 50 W, V = 120 V

$$\therefore \qquad R_2 = \frac{V^2}{P} = \frac{(120)^2}{50} = 288\ \Omega$$

Now, $R_2 = R_1\ [1+\alpha_1(t_2-t_1)]$ i.e. $288 = 20\ [1+0.005\ (t_2-20)]$

$$\therefore \qquad 13.4 = 0.005\ (t_2-20) \qquad \text{i.e.} \qquad t_2-20 = 2680$$

$$\therefore \qquad t_2 = \mathbf{2700\ °C}$$ ... Operating temperature

Example 1.59 : *The Wheatstone bridge network ABCD is as follows : Resistances between the terminals AB, BC, CD, DA and BD are 10, 30, 15, 20 and 40 Ω respectively. A 2 V battery of negligible resistance is connected between AC. Determine the value and direction of current in 40 Ω resistance.* **[GTU : March-2009]**

Solution : The Wheatstone bridge network is shown in the Fig. 1.78.

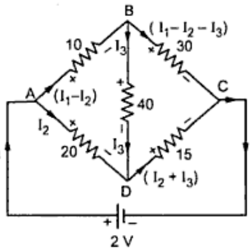


Fig. 1.78

The current distribution is shown in the Fig. 1.78.

Apply KVL to the various loops,

Loop ABDA, $-10\ (I_1 - I_2) - 40\ I_3 + 20\ I_2 = 0$ i.e. $-10\ I_1 + 30\ I_2 - 40\ I_3 = 0$ ...(1)

Loop BCDB, $-30\ (I_1 - I_2 - I_3) + 15\ (I_2 + I_3) + 40\ I_3 = 0$

i.e. $-30\ I_1 + 45\ I_2 + 85\ I_3 = 0$ ... (2)

Loop ADCA, $-20\ I_2 - 15\ (I_2 + I_3) + 2 = 0$ i.e. $+35\ I_2 + 15\ I_3 = +2$ ... (3)

Solving equations (1), (2) and (3) simultaneously,

$$I_3 = 0.01146 \text{ A from B to D}$$

Example 1.60 : *Determine the equivalent resistance between the terminals A and B of the network shown in the Fig. 1.79.* **[GTU : June-2009]**

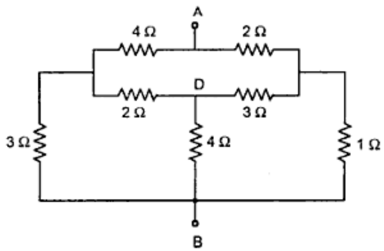
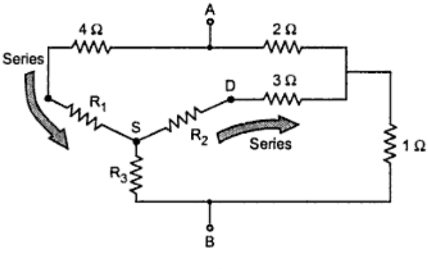


Fig. 1.79

Solution : Convert the delta of 2 Ω, 3 Ω and 4 Ω to star,



(a)

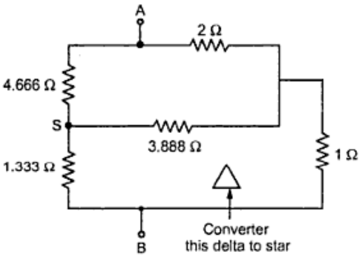
$$R_1 = \frac{3\times 2}{3+2+4} = 0.666\ \Omega$$

$$R_2 = \frac{2\times 4}{3+2+4} = 0.888\ \Omega$$

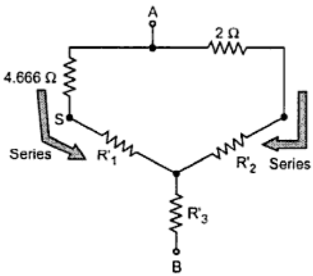
$$R_3 = \frac{3\times 4}{3+2+4} = 1.333\ \Omega$$

4 Ω and R_1 are in series.

3 Ω and R_2 are in series.



(b)



(c)

Fig. 1.79

$$R'_1 = \frac{1.333\times 3.888}{1.333+3.888+1} = 0.833\ \Omega,\quad R'_2 = \frac{1\times 3.888}{1.333+3.888+1} = 0.6248\ \Omega$$

$$R'_3 = \frac{1\times 1.333}{1.333+3.888+1} = 0.2142\ \Omega$$

4.666 Ω and R'_1 are in series giving 5.5 Ω.

2 Ω and R'_2 are in series giving 2.6248 Ω.

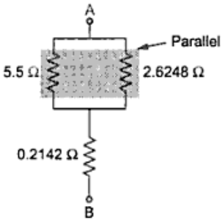


Fig. 1.79 (d)

$$\therefore \quad R_{AB} = [5.5 \| 2.6248] + 0.2142$$

$$= \left[\frac{5.5 \times 2.6248}{5.5 + 2.6248}\right] + 0.2142$$

$$= 1.7768 + 0.2142$$

$$= \mathbf{1.991\ \Omega}$$

Review Questions

1. *What is charge ? What is the unit of measurement of charge ?*
2. *Explain the relation between charge and current.*
3. *What is the difference between e.m.f. and potential difference ?*
4. *What is the resistance ? Which are the various factors affecting the resistance ?*
5. *Define the resistivity and conductivity of the material, stating their units.*
6. *Explain the effect of temperature on the resistance of i) Metals ii) Insulators and iii) Alloys.*
7. *Define resistance temperature coefficient. Derive its units.*
8. *Explain the use of R.T.C. in calculating resistance at t °C.*
9. *Explain the effect of temperature on R.T.C.*
10. *The field winding of a d.c. machine takes a current of 20 Amp. from 240 V d.c. supply at 25 °C. After a run of a 4 hours, the current drops to 15 Amp, supply voltage remaining constant. Determine its temperature rise.* **(Ans. : 86.5 °C)**
11. *A coil has resistance of 18 Ω at 20 °C and 20 Ω at 50 °C. Find its temperature rise when its resistance is 21 Ω and ambient temperature is 15 °C.* **(Ans. : 50 °C)**
12. *The current at the instant of switching a 40 W, 240 V lamp is 2 Amp. The resistance temperature co-efficient of the filament material is 0.0055 at the room temperature of 20 °C. Find the working temperature of lamp.* **(Ans. : 2020 °C)**
13. *The resistance of a copper wire is 50 Ω at a temperature of 35 °C. If the wire is heated to a temperature of 80 °C. find its resistance at that temperature. Assume the temperature co-efficient of resistance of copper at 0 °C to be 0.00427/ °C. Also find the temperature co-efficient at 35 °C.*

    (Ans. : 58.35 Ω, 3.71×10^{-3}/ °C)
14. *The field winding of a d.c. motor is connected across a 440 V supply. When the room temperature is 17 °C, winding current is 2.3 A. After the machine has been running for few*

hours, the current has fallen to 1.9 A, the voltage remaining unaltered. Calculate the average temperature throughout the winding, assuming α_0 of copper = 0.00426 /°C. **(Ans. : 70 °C)**

15. *Explain the various types of sources used in d.c. circuits.*
16. *Explains the concept of source transformation with suitable example.*
17. *Derive the relationship to express three star connected resistances into equivalent delta.*
18. *Derive the relationship to express three delta connected resistances into equivalent star.*
19. *Two resistances 15 Ω and 20 Ω are connected in parallel. A resistance of 12 Ω is connected in series with the combination . A voltage of 120 V is applied across the entire circuit. Find the current in each resistance, voltage across 12 Ω resistance and power consumed in all the resistances.* **(Ans. : 3.33 A, 2.5 A, 70 V)**
20. *A resistance R is connected in series with a parallel circuit comprising two resistances of 12 and 8 Ω. The total power dissipated in the circuit is 700 watts when the applied voltage is 200 V. Calculate the value of R..* **(Ans. : 52.3428 Ω)**
21. *In the series parallel circuit shown in the Fig. 1.80, find the*

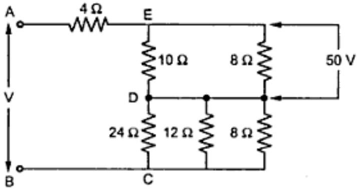


Fig. 1.80

i) Voltage drop across the 4 Ω resistance *ii) The supply voltage V* **(Ans. : 45 V, 140 V)**

22. *Find the current in all the branches of the network shown in the Fig. 1.81.*
(Ans. : 39 A, 21 A, 39 A, 81 A, 11 A, 41 A)

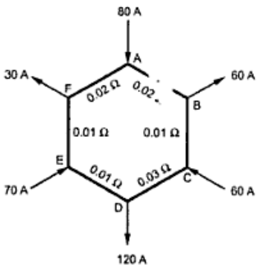


Fig. 1.81

23. *If the total power dissipated in the circuit shown in the Fig. 1.82 is 18 watts, find the value of R and current through it.* **(Ans.: 12 Ω, 0.6 A)**

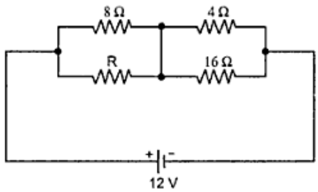


Fig. 1.82

24. *The current in the 6 Ω resista nce of the network shown in the Fig. 1.83 is 2 A. Determine the currents in all the other resistances and the supply voltage V.*

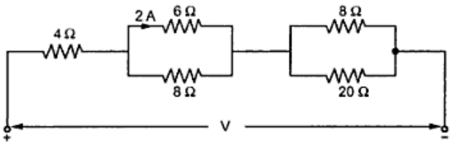


Fig. 1.83

(Ans.: 1.5 A, 2.5 A, 1A, 46 V)

25. *A particular battery when loaded by a resistance of 50 Ω gives the terminal voltage of 48.6 V. If the load resistance is increased to 100 Ω, the terminal voltage is observed to be 49.2 V.*

Determine, *i) E.M.F. of battery*

ii) Internal resistance of battery

Also calculate the load resistance required to be connected to get the terminal voltage of 49.5 V.

(Ans.: 49.815 V, 196.42 Ω)

26. *Determine the value of R shown in the Fig. 1.84, if the power dissipated in 10 Ω resistance is 90 W.* **(Ans.: 100 Ω)**

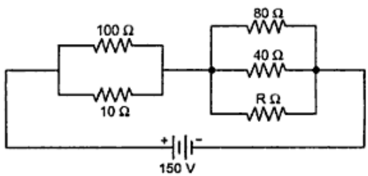


Fig. 1.84

27. *A resistance of 10 Ω is connected in series with the two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken will be 1.5 A from 20 V supply applied ?* **(Ans. : 6 Ω)**

28. *Two coils are connected in parallel and a voltage of 200 V is applied between the terminals. The total current taken is 25 A and power dissipated in one of the resistances is 1500 W. Calculate the resistances of two coils.* **(Ans. : 26.67 Ω, 11.43 Ω)**

29. *Two storage batteries A and B are connected in parallel to supply a load of 0.3 Ω. The open circuit e.m.f. of A is 11.7 V and that of B is 12.3 V. The internal resistances are 0.06 Ω and 0.05 Ω respectively. Find the current supplied to the load.* **(Ans. : 36.778 A)**

30. *Using Kirchhoff's laws, find the current flowing through the galvanometer G in the Wheatstone bridge network shown in the Fig. 1.85.* **(Ans. : 48.746 mA)**

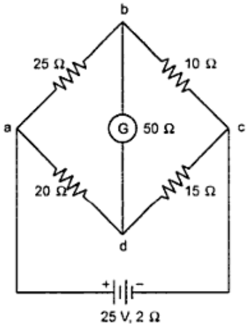


Fig. 1.85

31. *A network ABCD is made up as follows :*

AB has a cell of 2 V and negligible resistancs 98io, with the positive terminal connected to A; BC is a resistor of 25 Ω ; CD is a resistor of 100 Ω ; DA is a battery of 4 V and negligible resistance with positive terminal connected to D; AC is a milliammeter of resistance 10 Ω. Calculate the reading on the milliammeter. **(Ans. : 26.67 mA)**

University Questions

Q.1 *Define temperature co-efficient of resistance. How resistance of different materials vary with temperature? Prove that* $\alpha_1 = \alpha_0 / (1+\alpha_0 t)$. **[GTU : Dec.-2008, 8 Marks]**

Q.2 *Explain the method of transforming a star network of resistance into delta network and vice versa.* **[GTU : Dec.-2008, 9 Marks]**

Q.3 *State and explain Kirchhoff's voltage and current laws.* **[GTU : March-2009, 5 Marks]**

Q.4 *Show that* $R_t = R_0 (1 + \alpha t)$. *Notations have usual meaning.* **[GTU : March-2009, 4 Marks]**

Q.5 *Prove* $R_{t2} = R_{t1} [1 + \alpha_1 (t_2 - t_1)]$, *where notations have usual meanings.* **[GTU : June-2009, 5 Marks]**

Q.6 *Explain KCL and KVL.* **[GTU : June-2009, 4 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

❑❑❑

(1 - 100)

2

Electrostatics and Capacitance

2.1 Introduction

The branch of electrical engineering which deals with electricity at rest is called **electrostatics.** All the electric phenomena are produced due to the various types of charges. The moving charges produce current and magnetic effects. The accelerated charges produce radiation. Apart from moving and accelerated charges, there exists one more type of charges called **stationary charges** or **static charges.** Static charges are responsible for the generation of the forces on other charges which are called **electrostatic forces.** Electrostatics means the study of the static charges and the associated effects.

Such static charges may be situated at a point when they are called **point charges.** When the static charges are distributed along the telephone lines or power lines, they are called **line charges.** When distributed over the surfaces such as surfaces of plates of capacitor, they are called **surface charges**. Static charges may exist in the entire volume in the form of a charge cloud then they are called **volume charges.** In this chapter, we will discuss the behaviour of electricity due to the static charges, the laws governing such behaviour and concept of a capacitor.

2.2 Concept of an Electric Charge

The matter on the earth which occupies the space may be solid, liquid or gaseous. The matter is made up of one or more elements. Each element is made up of many atoms which are of similar nature. Now a days, scientists are successful in breaking atoms and studying the resulting products.

According to modern electron theory, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron,** the **proton** and the **electron**. The proton is positively charged while the electron is negatively charged. The neutron is electrically neutral i.e. possessing no charge. The mass of neutron and proton is same which is 1.675×10^{-27} kg while the mass of electron is 9.107×10^{-31} kg. The magnitude of positive charge on proton is same as the magnitude of negative charge on electron. Under normal conditions, number of protons is equal to number of electrons hence, the atom as a whole is electrically neutral. All the protons and neutrons are bound together into a compact **nucleus.** Nucleus may be thought as a central sun, about which, the electrons revolve in a particular fashion. The electrons are arranged in different **orbits** i.e. levels. The orbits are also called **shells.**

The orbits are more or less elliptical. The electrons revolving in various orbits are held by force of attraction exerted by nucleus. The orbit which is closest to the nucleus is under tremendous force of attraction while orbit which is farthest is under very weak force of attraction. Hence, electrons revolving in farthest orbit are loosely held to the nucleus. Such a shell is called **valence shell** and the electrons in this shell are **valence electrons.** In some atoms, at room temperature only, these valence electrons gain an additional energy and they escape from the shell. Such electrons exist in an atom as **free electrons.** Now, if such electrons are removed from an atom, it will loose negative charge and will become positively charged. Such positively charged atom is called **anion**. As against this, if excess electrons are added to an atom, it will become negatively charged. Such negatively charged atom is called **cation.**

Key Point : *This total deficiency or addition of excess electrons in an atom is called as its charge and the atom is said to be charged. The unit of charge is* ***coulomb.***

The deficiency or excess of electrons can be achieved by different methods. One of such methods is to rub two dissimilar materials against each other. When an ebonite rod is rubbed on a fur cloth, then the rod extracts electrons from fur cloth and behaves as negatively charged while fur cloth behaves as positively charged. This charged condition of rod cannot be sensed by eyes or by any sense organs. But, we can observe the effect of it by simple experiment. Such charged ebonite rod, when brought near light pieces of paper, attracts these pieces. This attraction is nothing but the effect of static charge present on the rod. This is the basic principle of the **static electricity.**

Such phenomena due to static charges are governed by some laws called **laws of electrostatics.** Let us study these laws.

2.3 Laws of Electrostatics

The two fundamental laws of electrostatics are as below :-

1) Like charges repel each other and unlike charges attract each other.

The law can be demonstrated by another simple experiment. The ebonite rod becomes negatively charged when rubbed against fur cloth. Now, if glass rod is rubbed against fur cloth, it gets positively charged. And if they are brought near each other, they try to attract each other. While two ebonite rods after rubbing against fur cloth, brought nearby, try to repel each other. This shows that like charges repel while unlike charges attract each other.

2) Coulomb's Inverse Square Law.

The law states that the mechanical force, attraction or repulsion, between the two small charged bodies is,

i) Directly proportional to the product of the charges present on the bodies.

ii) Inversely proportional to the square of the distance between the bodies and

iii) Depends upon the nature of the medium surrounding the bodies.

(a) Unlike charges attract (b) Like charges repel

Fig. 2.1 Force between charges

The Fig. 2.1 shows two point charges, separated by distance 'd' metres. The charges are Q_1 and Q_2 coulombs and K is the constant of proportionality.

According to Coulomb's law, force between the charges can be mathematically expressed as,

$$F \propto \frac{Q_1 Q_2}{d^2}$$

So,

$$\boxed{F = \frac{K Q_1 Q_2}{d^2} \quad \text{Newtons}}$$

The constant of proportionality, K depends on the surrounding medium and is given by,

$$\boxed{K = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_r \varepsilon_0}}$$

where ε = Absolute permittivity of the medium = $\varepsilon_0 \varepsilon_r$

ε_0 = Permittivity of free space and ε_r = Relative permittivity of the medium

And

$$\boxed{\varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} \quad F/m}$$

For air, $\varepsilon_r = 1$

The concept of permittivity is discussed later in this chapter.

If $Q_1 = Q_2 = 1\text{ C}$ and $d = 1\text{ m}$,

then,

$$F = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9 \text{ N}$$

Key Point: *Thus, one coulomb of charge may be defined as that charge, which, when placed in the air or vacuum at a distance of one metre away from an equal and similar charge, is repelled by a force of* 9×10^9 *N.*

Example 2.1 : *The two equal charges $Q_1 = 5\ \mu C$ and $Q_2 = 1\ \mu C$ are seperated by 50 cm, are kept in a vacuum. Find the force of repulsion.*

To have same force of repulsion, what should be the distance between them, if they are kept in a material having $\varepsilon_r = 5$?

Solution : **Case 1 :** $Q_1 = 5\ \mu C,\ Q_2 = 1\ \mu C,\ d = 50 \times 10^{-2}$ m

$$\varepsilon_r = 1,\ \varepsilon = \varepsilon_0 = 8.854 \times 10^{-12}$$

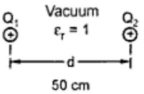


Fig. 2.2 (a)

$$\therefore\ F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} = \frac{5\times10^{-6}\times1\times10^{-6}}{4\pi\times8.854\times10^{-12}\times\left(50\times10^{-2}\right)^2}$$

$$= \mathbf{0.1797\ N}$$

Case 2 : The force must be same, 0.1797 N, but $\varepsilon_r = 5$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

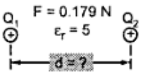


Fig. 2.2 (b)

$$\therefore\ 0.1797 = \frac{5\times10^{-6}\times1\times10^{-6}}{4\pi\times8.854\times10^{-12}\times5\times(d)^2}$$

$$\therefore\ d^2 = 0.05$$

$\therefore$ **d = 0.2236 m = 22.36 cm** .. New distance

2.4 Electrostatic Field

As we have seen in the previous section that unlike charges attract and like charges repel each other. Positively charged particle exerts a force of attraction on negatively charged while exerts a force of repulsion on positively charged particle. It must be kept in mind that the second charged particle also produces the electrostatic force on the first particle. So, it can be concluded that the space around the charge is always under the stress and exerts a force on another charge which is placed around it. The region or space around a charge or charged body in which the influence of electrostatic force or stress exists is called **electric field or dielectric field or electrostatic field.**

2.4.1 Electric Lines of Force

The electric field around a charge is imagined in terms of presence of lines of force around it. The imaginary lines, distributed around a charge, representing the stress of the charge around it are called as **electric or electrostatic lines of force.** The pattern of lines of force around isolated positive charge is shown in Fig. 2.3 (a) while the pattern of lines of force around isolated negative charge is shown in Fig. 2.3 (b). Such lines of force originate from the positive charge and terminate on the negative charge, when these charges are placed near each other. They exert the force of attraction on each other. This is shown in

Fig. 2.3 (c). While when two like charges are near each other, such lines will be in opposite direction as shown in Fig. 2.3 (d). There exists a force of repulsion between them.

Fig. 2.3 (a) Isolated positive charge

Fig. 2.3 (b) Isolated negative charge

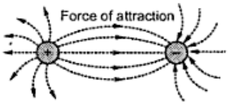


Fig. 2.3 (c) Two equal unlike charges

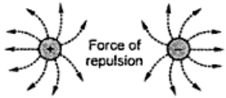


Fig. 2.3 (d) Two equal like charges

> **Key Point:** *In general, the directions of the lines of force at any point is the direction of movement of a unit positive charge placed at that point, if free to do so.*

2.4.2 Properties of Electric Lines of Force

The properties of electric lines of force are,

1) The lines of force always originate from a positive charge and terminate at negative charge.
2) They always enter or leave a conducting surface, normally.
3) They are always parallel and never cross each other.
4) The lines travelling in the same direction repel each other, while traveling in the opposite directions attract one another.
5) They behave like a stretched rubber band and always try to contract.
6) They pass only through the insulating medium between the charges and do not enter the charged bodies.

> **Key Point:** *Hence, they cannot form a closed loop as in case of the magnetic lines of force.*

2.5 Electric Flux

Theoretically, the lines of force emanating from a charge are infinite. Faraday suggested that the electric field should be assumed to be composed of very small bunches containing a fixed number of electric lines of force. Such a bunch or a closed area is called a **tube of flux.**

> **Key Point:** *The total number of lines of force or tubes of flux in any particular electric field is called as the* ***electric flux.***

This is represented by the symbol ψ. Similar to charge, unit of electric flux is also coulomb C.

One coulomb of electric flux is defined as that flux which emanates from a positive charge of one Coulomb.

In general, if the charge on a body is ± Q Coulombs, then the number of tubes of flux or total electric flux, starting or terminating on it is also Q.

So, for a charge of ± Q coulombs,

$$\boxed{\text{Electric Flux, } \psi = Q \text{ coulombs (numerically)}}$$

2.6 Electric Flux Density

This is defined as the flux passing at right angles through unit area of surface. It is represented by symbol D and measured in Coulomb per square metre.

If a flux of ψ Coulombs passes normally (at right angles) through an area of A m^2, then

$$D = \frac{\psi}{A} = \frac{Q}{A} \ \text{C/m}^2 \qquad \dots \text{As } \psi = Q$$

Let a point charge of Q coulombs placed at the centre of an imaginary sphere of radius 'r' metres.

$$\text{Total flux, } \psi = Q$$

This flux falls normally on a surface area of $4\pi r^2$ (metre)^2 of sphere. So, electric flux density,

$$\boxed{D = \frac{\psi}{A} = \frac{Q}{4\pi r^2} \ \text{C/m}^2}$$

The flux density is also called **displacement density.**

2.6.1 Surface Charge Density

If the charge is distributed over the surface, then the surface charge density is defined as the charge per unit area of the surface over which the charge is distributed. It is denoted as δ.

$$\therefore \qquad \boxed{\delta = \frac{Q}{A} \quad \text{C/m}^2}$$

2.7 Electric Field Strength or Field Intensity

It is defined as the force experienced by a unit positive charge placed at any point in the electric field. It is represented by symbol E and measured in newton per coulomb.

Suppose a charge of Q coulombs, placed at a point within an electric field, experiences a force of F newtons, then the intensity of the electric field at that point is given by,

$$E = \frac{F}{Q} \quad N/C$$

Higher the value of E, stronger is the electric field.

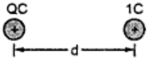


Fig. 2.4

Consider a positive charge of Q coulombs placed in a medium as shown in the Fig. 2.4.

Let a unit positive charge is placed at a distance of d metres from the charge Q.

The field intensity at the point where unit positive charge is placed can be obtained from force experienced by unit positive charge.

Now, $E = \frac{F}{Q}$ but here, $Q = 1$ C unit charge

$\therefore \quad E = F$

But, $F = \frac{Q \times 1}{4\pi\varepsilon d^2}$ ... By Coulomb's law

$\therefore \quad E = \frac{Q}{4\pi\varepsilon d^2}$ N/C

But, $\varepsilon = \varepsilon_0 \varepsilon_r$

$$\therefore \quad \boxed{E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d^2}}$$

The similar concept can be used to obtain the relation between electric field intensity and electric flux density.

2.7.1 Relation between D and E

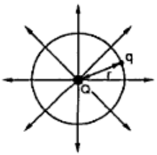


Fig. 2.5 Relation between D and E

Let there be a point charge of 'Q' coulombs placed at the centre of the sphere of radius 'r' metres. The small positive charge 'q' coulombs is placed at a distance 'r' from 'Q' on the surface of the sphere as shown in Fig. 2.5.

The force experienced on the charge 'q' due to 'Q' is given by

$$F = \frac{Q \cdot q}{4\pi\varepsilon_0 \varepsilon_r r^2} \quad \text{... By Coulomb's law}$$

Electric field strength is given by force per charge.

$$\therefore \quad \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2}$$

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2} \quad N/C$$

The flux density is, $D = \frac{\psi}{a}$

Now, $\psi = Q$

While a = Surface area of the sphere = $4 \pi r^2$ m^2

$$\therefore \quad D = \frac{Q}{4\pi r^2} \ C/m^2$$

Substituting in the equation for 'E' we get,

$$\therefore \quad E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

and

$$\boxed{D = E \varepsilon_0 \varepsilon_r \quad C/m^2}$$

2.8 Permittivity

From the relation derived above, we can say that electric flux density depends on electric field strength. Now the value of electric flux density depends on the value of electric field strength E along with the dielectric property of the medium which is known as **permittivity.**

Key Point : *Permittivity can be defined as the ease with which a dielectric medium permits an electric flux to be established in it.*

2.8.1 Absolute Permittivity

The ratio of the electric flux density D to electric field strength E at any point is defined as the **absolute permittivity.**

It is denoted by ε and measured in units farads/metre, (F/m).

$$\boxed{\varepsilon = \frac{D}{E} \quad F/m}$$

2.8.2 Permittivity and Free Space

It is also called as **electric space constant.**

Key Point : *The ratio of the electric flux density in a vacuum (or free space) to the corresponding electric field is defined as permittivity of the free space.*

It is denoted by ε_0 and measured in unit farads/m (F/m).

$$\boxed{\varepsilon_0 = \frac{D}{E} \quad F/m \quad \text{in vacuum}}$$

The value of ε_0 is less than the value of permittivity of any medium. Experimentally, its value has been derived as,

$$\varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} \quad F/m$$

2.8.3 Relative Permittivity

To define the permittivity of the dielectric medium, the vacuum or free space is considered to be a reference medium. So, **relative permittivity** of vacuum with respect to itself is unity.

The ratio of electric flux density in a dielectric medium to that produced in a vacuum by the same electric field strength under identical conditions is called **relative permittivity.**

It is denoted by ε_r and has no units.

$$\varepsilon_r = \frac{D}{D_0}$$

Now $D = \varepsilon E$ and $D_0 = \varepsilon_0 E$

$$\therefore \quad \varepsilon_r = \frac{\varepsilon E}{\varepsilon_0 E}$$

$$\therefore \quad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\therefore \quad \varepsilon = \varepsilon_r \varepsilon_0$$

It can also be defined as the ratio of the absolute permittivity of the dielectric medium to the permittivity of the free space. The relative permittivity of the air is also taken as unity, though its actual value is 1.0006. Most of the other materials have value of relative permittivity between 1 to 10.

Sr. No.	Material	Relative permittivity, ε_r
1	Free space	1
2	Air	1.0006 ≈ 1
3	Rubber	2 to 3.5
4	Paper	2 to 2.5
5	Mica	3 to 7
6	Porcelain	6 to 7
7	Bakelite	4.5 to 5.5
8	Glass	5 to 10

Table 2.1 Material and ε_r

The relative permittivity of air is assumed to be one for all practical purposes.

Key Point: *Higher the value of ε_r, easier is the flow of electric flux through the materials.*

In practice, paper and mica are extensively used for manufacturing of capacitors.

Key Point: *The relative permittivity is nothing but the dielectric constant of the material.*

2.9 Electric Potential and Potential Difference

When a mass is raised above the ground level, work is done against the force of gravity. This work done is stored in the mass as a potential energy (mgh). Hence, due to such potential energy, it is said that the mass, when raised above the ground level has a gravitational potential. Such potential of mass depends upon the position of the mass with respect to the ground.

An electric charge gives rise to an electric field around it, analogous to gravitational field around the earth. If any charge is introduced in this field, it gets attracted or repelled, depending on the nature of the charge. At the time of movement of this charge, work is done against or by the force acting on the charge due to the electric field. This depends on the position of the charge in the electric field and is analogous to the potential of mass due to gravitation field, when lifted upwards.

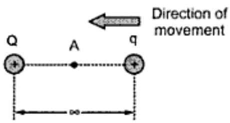


Fig. 2.6 Electric potential

Now, consider a small isolated positive charge 'q' placed at infinity with respect to another isolated positive charge 'Q' as shown in the Fig. 2.6. Theoretically, the electric field of charge 'Q' extends upto infinity but has a zero influence at infinity, where 'q' is placed. When charge 'q' is moved towards 'Q', work is done against the force of repulsion between these two like charges.

Due to this work done, when charge 'q' reaches position A, it acquires a potential energy. If charge 'q' is released, due to force of repulsion, it will go back to infinity i.e. position of zero potential. So, at point A, charge 'q' has some potential exactly equal to work done in bringing it from infinity to the point A, called **electric potential.**

It can be defined as the work done in joules, in moving a unit positive charge from infinity (position of zero potential) to the point against the electric field.

It is denoted by symbol V and is measured in joule per coulomb or volt.

Thus, $$\boxed{\text{Electric potential V} = \frac{\text{Workdone (W)}}{\text{Charge (Q)}}} \quad \text{... Volts}$$

Definition of 1 volt :

The electric potential at a point in an electric field is said to be **one volt** when the work done in bringing a unit positive charge from infinity to that point from infinity against the electric field is one joule.

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

2.9.1 Potential Difference

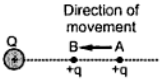


Fig. 2.7 Potential difference

Consider two points A and B in an electric field as shown in Fig. 2.7. The positive charge '+q' is moved from point A to B in an electric field. At point A, charge acquires certain electric potential say V_A. Some additional work is done in bringing it to point B. At point B, it has an electric potential say V_B.

Key Point: *The difference between these two potentials per unit positive charge is called potential difference.*

So, the potential difference between the two points in an electric field is defined as the work done in moving a unit positive charge from the point of lower potential to the higher potential.

i.e. $$V_{AB} = V_A - V_B = \frac{W_A - W_B}{q} \quad \text{volts}$$

2.9.2 Expressions for Potential and Potential Difference

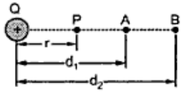


Fig. 2.8

Consider a positive charge Q placed in a medium of relative permittivity ε_r. Consider a point P at a distance r from the charge Q. Now, a unit positive charge of 1 C is placed at point P, there will exist a force of repulsion between the two charges. This is shown in the Fig. 5.8.

The force of repulsion is given by,

$$F = \frac{Q \times 1}{4\pi\varepsilon r^2} = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2} \quad N$$

Now, electric field intensity at point P is the ratio of force to charge at point P. But charge at P is unit charge,

$$\therefore \quad E = \frac{F}{1\,C} = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2} \quad N/m$$

Now, move the unit charge at point P towards charge Q against the force of repulsion.

Work done :

Let the distance moved by charge at P towards Q be dr and for this work is done against force of repulsion, given by,

$$dW = -E\,dr = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r^2}\cdot dr$$

The negative sign indicates that the work is done against the force of repulsion.

Now, to find electrical potential at point P, consider that the unit positive charge is moved from infinity to the point P. Hence, total work done in moving unit positive charge from infinity to point P can be obtained by integrating dW as,

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -\frac{Q}{4\,\pi\varepsilon_0\,\varepsilon_r\,r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\int_{\infty}^{r}\frac{1}{r^2}\,dr$$

Now, $\int \frac{1}{r^2}\,dr = -\frac{1}{r}$

$$\therefore \quad W = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$W = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r}$$

Key Point: *But this total work done is nothing but potential at point P.*

$$\therefore \quad \boxed{V_P = W = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r} \quad \text{volts}}$$

Thus, as r increases, potential decreases till it becomes zero at infinity.

Potential difference :

Consider point A at a distance d_1 from charge Q. Hence, potential of point A is given by,

$$V_A = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,d_1}$$

While the potential of point B which is at a distance d_2 from the charge Q is,

$$V_B = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,d_2}$$

Hence, the potential difference between the points A and B is given by,

$$V_{AB} = V_A - V_B$$

$$\therefore \quad \boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[\frac{1}{d_1}-\frac{1}{d_2}\right]}$$

We know that field intensity at a distance 'd' due to charge Q is given by,

$$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d^2}$$

While the potential of the same point is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d}$$

Substituting V in expression for E, we can write,

$$\boxed{E = \frac{V}{d}}$$

2.10 Potential Gradient

In practice, the electric field intensity is not uniform but varies from point to point in an electric field. Let the electric field strength at any point A in an electric field be E (N/C). Now, the unit positive charge at point A is displaced by distance dx metres in the direction of the field so that the electric field strength remains constant. Work done in moving this charge can be determined by force × displacement.

$$\text{Work done} = +E \times dx \text{ joules}$$

Let dV be the potential drop over this distance in the direction of the electric field. It is moved from point of **higher potential** to **lower potential.** So, by the definition of a potential difference,

$$dV = +E\,dx$$

i.e $$\boxed{E = +\frac{dV}{dx}}$$

The term $\frac{dV}{dx}$ in the above expression is called the **potential gradient.**

Key Point : *Potential gradient is defined as the drop in potential per metre in the direction of the electric field.*

It is measured in units volts/metre (V/m).

If the change in potential is from lower potential to higher potential, i.e. against the direction of the electric field then potential gradient is said to be negative i.e.

$$\boxed{E = -\frac{dV}{dx}}$$

Key Point: *From the above expression, it follows that numerically,*

Electric Field Strength = Potential Gradient

2.11 Capacitor

A capacitor is nothing but the two conducting surfaces, separated by an insulating medium called **dielectric.** These conducting surfaces could be in the form of rectangular, circular, spherical or cylindrical in shape.

A capacitor is also called **condenser**. The commonly used dielectrics in capacitors are paper, mica, air etc.

2.12 Capacitance

Capacitance is defined as the amount of charge required to create a unit potential difference between the plates.

Key Point: *The property of a capacitor to store an electric energy in the form of static charges is called its capacitance.*

2.13 Action of a Capacitor

Consider a capacitor formed by two flat metal plates X and Y, facing each other and separated by an air gap or other insulating material used as a dielectric medium. There is no electrical contact or connection between them. Such a capacitor is called **parallel plate capacitor.**

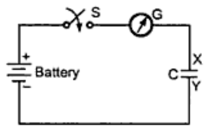


Fig. 2.9 A capacitor

Consider a circuit in which such a capacitor across a battery with the help of a switch 'S' and a galvanometer 'G' in series. The arrangement is shown in the Fig. 2.9.

Let us see what happens when the switch 'S' is closed. As soon as the switch 'S' is closed, the positive terminal of the battery attracts some of the free electrons from the plate 'X' of the capacitor. The electrons are then pumped from positive terminal of the battery to the negative terminal of the battery due to e.m.f. of the battery. Now, negative terminal and electrons are like charges and hence, electrons are repelled by the negative terminal to the plate 'Y' of the capacitor.

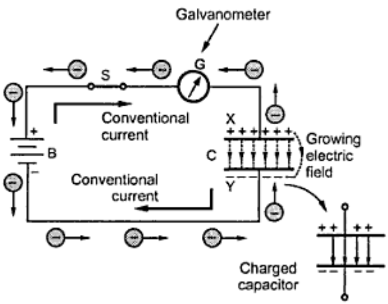


Fig. 2.10 Action of a capacitor

The action is shown in Fig. 2.10.

So, plate 'X' becomes positively charged while plate 'Y' becomes negatively charged. The flow of electrons constitutes a current, in the direction opposite to the flow of electrons. This is the conventional current called

charging current of the capacitor as shown in the Fig. 2.10. This can be experienced from the momentary deflection of the galvanometer 'G'. Because of this, there builds a potential difference across the plates 'X' and 'Y'. There builds an electric field between the two fields.

But this potential difference across the plates, acts as a counter e.m.f. and starts opposing the movement of the electrons. The magnitude of this potential difference is proportional to the charge that accumulates on the plates. When this potential difference becomes equal to the battery e.m.f., the flow of electrons ceases.

If under such condition, the battery is disconnected then the capacitor remains in the charged condition, for a long time. It stores an electrical energy and can be regarded as a reservoir of electricity. Now, if a conducting wire is connected across the two plates of capacitor, with the galvanometer in series, then galvanometer shows a momentary deflection again but in the opposite direction.

This is due to the fact that electrons rush back to plate X from plate Y through the wire. So, there is a rush of current through the wire. This is called **discharging current of a capacitor.** Thus, the energy stored in the capacitor is released and is dissipated in the form of the heat energy in the resistance of the wire connected.

The direction of the conventional current is always opposite to the flow of electrons. If the voltage of the battery is increased, the deflection of the galvanometer also increases at the time of charging and discharging.

Key Point: *So, charge on the capacitor is proportional to the voltage applied to it.*

2.14 Relation between Charge and Applied Voltage

As seen earlier, the charge on capacitor plates depends on the applied voltage. Let 'V' be the voltage applied to the capacitor and 'Q' be the charge accumulated on the capacitor plates, then mathematically, it can be written as,

$$Q \propto V$$

i.e. $$Q = C V$$

The constant of proportionality 'C' is called **capacitance** of the capacitor, defined earlier.

$\therefore$ $$C = \frac{Q}{V}$$

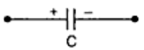


Fig. 2.11 Symbol of capacitance

From the above expression, the **capacitance** is defined as the ratio of charge acquired to attain the potential difference between the plates. It is the charge required per unit potential difference. It is measured in unit **farads**.

One farad capacitance is defined as the capacitance of a capacitor which requires a charge of one coulomb to establish a potential difference of one volt between its plates.

The capacitance is symbolically denoted as shown in the Fig. 2.11.

For practical use, the farad is too large unit and hence, micro farad (μF), nano farad (nF) and pico farad (pF) are commonly used.

$$1\ \mu F = 10^{-6}\ F$$
$$1\ nF = 10^{-9}\ F$$
$$1\ pF = 10^{-12}\ F$$

2.15 Capacitance of a Parallel Plate Capacitor

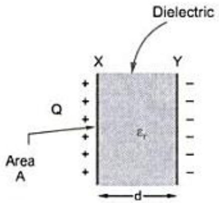


Fig. 2.12 Charged capacitor

Consider a parallel plate capacitor, fully charged, as shown in the Fig. 2.12.

The area of each plate X and Y is say A m^2 and plates are separated by distance 'd'.

The relative permittivity of the dielectric used in between is say ε_r.

Let Q be the charge accumulated on plate X, then the flux passing through the medium is $\psi = Q$.

The flux density, $D = \frac{\psi}{A} = \frac{Q}{A}$

The electric field intensity,

$$E = \frac{V}{d}$$

We know that $D = \varepsilon E$

$$\therefore \quad \frac{Q}{A} = \varepsilon \frac{V}{d}$$

$$\therefore \quad \frac{Q}{V} = \frac{\varepsilon A}{d}$$

But, $\frac{Q}{V} = C$

$$\therefore \quad C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad F$$

Key Point : *When the capacitor is fully charged, the potential difference across it is equal to the voltage applied to it.*

Example 2.2 : *A parallel plate capacitor has an area of 10 cm^2 and distance between the plates is 2 mm. The dielectric used between the plates has relative permittivity of 3. Determine the capacitance of the parallel plate capacitor.*

Solution : The capacitance can be calculated as,

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{3 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 1.328 \times 10^{-11} = \mathbf{13.28\ pF}$$

Example 2.3 : *The potential gradient between the plates in the above capacitor is 12 kV/cm, determine the voltage across the plates, charge, electric flux density and electricity flux between the plates.*

Solution : Electric intensity = Potential gradient

$$E = 12\ \text{k V/cm} = \frac{12 \times 10^3}{1 \times 10^{-2}}\ \text{V/m} = 1200 \times 10^3\ \text{V/m}$$

And $C = 13.28\ \text{pF}$ ... Calculated in Ex. 2.1.

Now, $E = \frac{V}{d}$

$\therefore$ $1200 \times 10^3 = \frac{V}{2 \times 10^{-3}}$ i.e. $V = 2400\ V$

This is the voltage across capacitor plates.

$$C = \frac{Q}{V}$$

$\therefore$ $Q = C V = 13.28 \times 10^{-12} \times 2400 = 31.87 \times 10^{-9}\ C$

$\therefore$ Charge = **31.87 nC**

Electric flux, $\psi = Q = 31.87\ \text{nC}$

Electric flux density, $D = \frac{Q}{A} = \frac{31.87 \times 10^{9}}{10 \times 10^{-4}} = 3.187 \times 10^{-5}\ C/m^2$

$= \mathbf{31.87\ \mu C / m^2}$

2.16 Dielectric Strength

We know that, $E = \frac{V}{d}$

So, as the voltage on the capacitor is increased with a given thickness (d) or the thickness (d) is reduced with a given voltage (V), the electric intensity E increases.

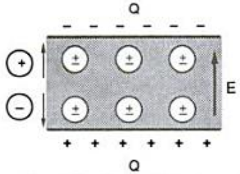


Fig. 2.13 Dielectric strength

This intensity represents the force exerted on the charges on the molecules or the dielectric material.

As E is increased, the centre of the positive charges is pushed in the direction of E and centre of the negative charges in the opposite direction.

Now, every dielectric medium has its capacity to withstand the increasing E. If the applied voltage and hence E is increased beyond a certain limit, then forces on the molecules become sufficiently large. The electrons break away from the molecules causing ionization and free charges.

The material then conducts due to ionization and the charge recombine, thereby vanish from the capacitor plates. The capacitor can no more hold the charge and is said to be breakdown. The dielectric medium is said to be punctured and becomes useless from using it as a dielectric.

The ability of an insulating medium to resist its breakdown when a voltage is increased across it, is called its **dielectric strength.**

This depends upon the temperature of the material and presence of air pockets and imperfections in the molecular arrangement of that material. It is generally expressed in kV/cm of kV/mm.

Key Point : *The voltage at which the dielectric medium of the capacitor breaksdown is known as* ***breakdown voltage of the capacitor.***

The factors affecting the dielectric strength are,

1. Temperature
2. Type of material
3. Size, thickness and shape of the plates.
4. Presence of air pockets in the material.
5. Moisture content of the material.
6. Molecular arrangement of the material.

Dielectric strength and dielectric constants of some materials are quoted below from published literature.

Sr. No.	Material	Dielectric constant	Dielectric strength in kV/mm
1	Air	1	3
2	Bakelite	5	15 to 25
3	Mica	6	46 to 200
4	Dry paper	2.2	5 to 10
5	Glass	6	6 to 26

Table 2.2 Dielectric strengths

The dielectric strength varies as thickness of dielectric material hence the range of values are given in the Table 2.2. The value indicates that if material is subjected to electric field more than specified dielectric strength then it will breakdown.

2.16.1 Dielectric Leakage and Losses

If there is no leakage of current in the dielectric and the insulation is perfect, then the charge on the capacitor plates can be held on for hours.

The fact however remains that the insulation resistance of most of the dielectric materials is only of the order of megaohms and hence charge on the capacitor leaks away through the insulating material in a few minutes.

Key Point : *In any case, it is dangerous to touch a charged capacitor even after it is disconnected from the supply.*

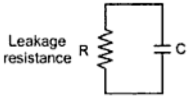


Fig. 2.14 Practical capacitor

In case of d.c. a practical capacitor is considered to be a charge storing device in parallel with a leakage resistance (R) as shown in the Fig. 2.14.

Further, when the voltage applied to the capacitor is alternating, due to molecular friction of dipoles created in the material, the value of R becomes frequency dependent. The loss due to such molecular friction is called **dielectric loss.**

2.17 Capacitors in Series

Consider the three capacitors in series connected across the applied voltage V as shown in the Fig. 2.15. Suppose this pushes charge Q on C_1 then the opposite plate of C_1 must have the same charge. This charge which is negative must have been obtained from the connecting leads by the charge separation which means that the charge on the upper plate of C_2 is also Q. In short, **all the three capacitors have the same charge Q.**

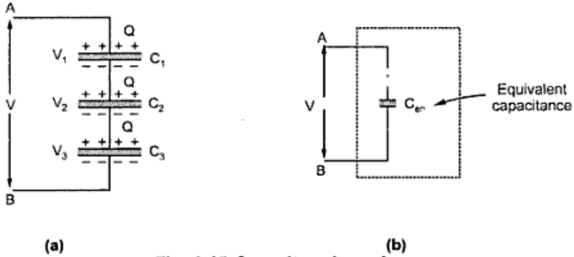


Fig. 2.15 Capacitors in series

$$\boxed{Q = C_1 V_1 = C_2 V_2 = C_3 V_3}$$

Giving , $V_1 = \frac{Q}{C_1}; \quad V_2 = \frac{Q}{C_2}; \quad V_3 = \frac{Q}{C_3}$

If an equivalent capacitor also stores the same charge, when applied with the same voltage, then it is obvious that,

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad V = \frac{Q}{C_{eq}}$$

But, $V = V_1 + V_2 + V_3$

$$\therefore \quad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

It is easy to find V_1, V_2 and V_3 if Q is known.

$$\boxed{\text{For 'n' capacitors in series, } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}}$$

Key Point: *For all the capacitors in series, the charge on all of them is always same, but the voltage across them is different.*

2.17.1 Voltage Distribution in Two Capacitors in Series

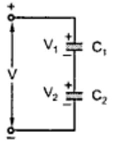


Fig. 2.16 Capacitors in series

Consider two capacitors C_1 and C_2 connected in series. The voltage across them is say, V volts. This is shown in Fig. 2.16.

As capacitors are in series, the charge on them is same, say Q.

$$\therefore \quad Q = C_1 V_1 = C_2 V_2$$

where V_1 is voltage across C_1 and V_2 is voltage across C_2

Now, $V = V_1 + V_2$

From the expression of Q, we can write,

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

Adding 1 to both sides, $\frac{V_1}{V_2} + 1 = \frac{C_2}{C_1} + 1$

$$\therefore \quad \frac{V_1 + V_2}{V_2} = \frac{C_2 + C_1}{C_1}$$

$$\therefore \quad \frac{V}{V_2} = \frac{C_2 + C_1}{C_1} \qquad \text{... As } V_1 + V_2 = V$$

$$\therefore \quad \boxed{V_2 = V \cdot \frac{C_1}{C_1 + C_2}}$$

Similarly,

$$\boxed{V_1 = V \cdot \frac{C_2}{C_1 + C_2}}$$

The result is exactly identical to the current distribution in two parallel resistances.

Example 2.4 : *Three capacitors of values 2 μF, 4 μF, and 6 μF have an applied voltage of 60 V across their series combination. Determine the voltage on each of the capacitors.*

Solution : This is the example of series connection.

$$\frac{1}{C_{eq}} = \frac{1}{2\times10^{-6}} + \frac{1}{4\times10^{-6}} + \frac{1}{6\times10^{-6}}$$

$$\frac{1}{C_{eq}} = 10^6\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right] = 10^6\left[\frac{6+3+2}{12}\right]$$

$$\therefore \quad C_{eq} = \frac{12}{11} \times 10^{-6}\text{ F} = \frac{12}{11}\ \mu\text{F}$$

$$Q = C_{eq} V = \left(\frac{12}{11}\times10^{-6}\right)\times 60 = \frac{720}{11}\times10^{-6} \text{ coulomb}$$

In series combination of capacitors, charge on each capacitor is same as Q calculated above.

$$\therefore \quad Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \left(\frac{\frac{720}{11}\times10^{-6}}{2\times10^{-6}}\right) = \frac{360}{11} = \mathbf{32.727\ volts}$$

$$\therefore \quad V_2 = \frac{Q}{C_2} = \left(\frac{\frac{720}{11}\times10^{-6}}{4\times10^{-6}}\right) = \frac{180}{11} = \mathbf{16.364\ volts}$$

$$\therefore \quad V_3 = \frac{Q}{C_3} = \left(\frac{\frac{720}{11}\times10^{-6}}{6\times10^{-6}}\right) = \frac{120}{11} = \mathbf{10.909\ volts}$$

Key Point: *Notice that the smallest capacitor has the largest of the three voltages across it and that C_{eq} is lesser than any of the capacitors in the series string.*

2.18 Capacitors in Parallel

Key Point : *When capacitors are in parallel, the same voltage exists across them, but charges are different.*

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V$$

The total charge stored by the parallel bank of capacitors Q is given by,

$$Q = Q_1 + Q_2 + Q_3$$

$$= C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3) V \quad \text{... (1)}$$

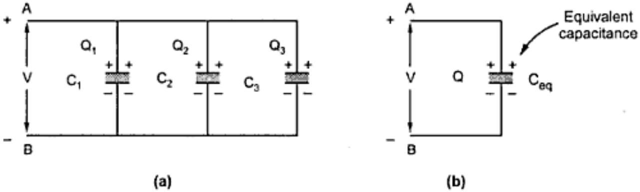


Fig. 2.17 Capacitors in parallel

An equivalent capacitor which stores the same charge Q at the same voltage V, will have

$$Q = C_{eq} V \quad \text{... (2)}$$

Comparing equations (1) and (2),

As $\quad C_{eq} = C_1 + C_2 + C_3$

$\therefore \quad Q = C_1 V + C_2 V + C_3 V$

It is easy to find Q1, Q_2 and Q_3 if V is known.

For 'n' capacitors in parallel, $C_{eq} = C_1 + C_2 + \ldots + C_n$

Example 2.5 : *Two capacitors are connected in parallel having equivalent capacitance of 10 μF while same capacitors when connected in series have equivalent capacitance of 2 μF. Find the values of two capacitors.*

Solution : Case 1 : Capacitors in parallel

$$\therefore \quad C_{eq} = C_1 + C_2 = 10\ \mu F \quad \text{... (1)}$$

Case 2 : Capacitors in series

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2\ \mu F \quad \text{... (2)}$$

From equation (1), $C_2 = 10 \times 10^{-6} - C_1$ and using in equation (2),

$$\frac{C_1\left[10\times10^{-6}-C_1\right]}{10\times10^{-6}} = 2\times10^{-6} \qquad \ldots C_1 + C_2 = 10\ \mu F$$

$$\therefore \quad 10\times10^{-6}C_1 - C_1^2 = 20\times10^{-12}$$

$$\therefore \quad C_1^2 - 10\times10^{-6}C_1 + 20\times10^{-12} = 0$$

$$\therefore \quad C_1 = \frac{10\times10^{-6} \pm\sqrt{\left(10\times10^{-6}\right)^2 - 4\times20\times10^{-12}}}{2}$$

$$= \frac{10\times10^{-6} \pm 4.4721\times10^{-6}}{2}$$

$$= 7.236\times10^{-6} \text{ or } 2.7639\times10^{-6}\ F$$

Thus C_1 = **7.236 μF**, C_2 = **2.7639 μF**

or C_1 = **2.7639 μF**, C_2 = **7.236 μF**

2.19 Parallel Plate Capacitor with Multiple Plates

Fig. 2.18 Parallel plate capacitor with multiple plates

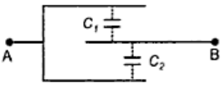


Fig. 2.18 (a) Three plate capacitor

In case of electronic apparatus, in many cases, a multiple plate capacitor is used. This is shown with terminals A and B in the Fig. 2.18. All the metal plates form capacitance with the adjacent plate.

In such capacitor, one set of plates is fixed while other is movable. The set of moving plates can be moved in and out of the space between the fixed plates, without touching them. The alternate plates are connected to the terminals A and B.

Consider such a capacitor with three plates as shown in the Fig. 2.18 (a).

It can be seen that there exists two capacitors C_1 and C_2 between the adjacent plates. Let A be the area of one side of each plate, d be the thickness of the medium between the plates and ε_r is the relative permittivity of the medium. Hence, according to the capacitance of a parallel plate capacitor,

We can write, $C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d}$

Now, as the parameters ε_r, A and d are same for any combination of plates,

$$C_1 = C_2$$

The two capacitors are in parallel,

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{2\varepsilon_0 \varepsilon_r A}{d}$$

Now, if the n plates are used, we can write that

$$C_{eq} = \frac{(n-1)\ \varepsilon_0\ \varepsilon_r\ A}{d}$$

Example 2.6 : *A multiple plate capacitor has 4 plates in all and the distance between the plates is 1 mm. If the area of the plates is effectively 5 cm² and the dielectric constant of material between the plates is 10, determine the capacitance.*

Solution :

$$C = \frac{(n-1)\,A\,\varepsilon_0\,\varepsilon_r}{d} = \frac{(4-1)\times(5\times10^{-4})}{1\times10^{-3}} \times \frac{1}{36\,\pi\times10^{9}} \times 10$$

$$= \frac{3\times5\times10\times10^{-4}}{36\,\pi\times10^{6}} = 1.32 \times 10^{-10}\ \text{F} = \mathbf{132\ pF}$$

2.20 Composite Dielectric Capacitors

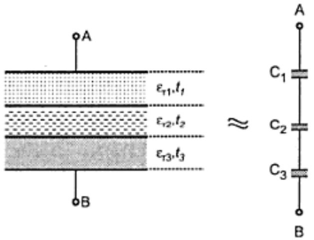


Fig. 2.19 Composite capacitor

When a parallel plate capacitor has two dielectrics or more between the plates, it is said to be **composite capacitor.** The various types of such composite capacitor exists in practice. Let us study few types of such composite capacitors.

Type 1 : In this type, number of dielectrics having different thicknesses and relative permittivities are filled in between the two parallel plates. The composite capacitor with three different dielectrics with permittivities ε_{r1}, ε_{r2} and ε_{r3} and thicknesses t_1, t_2 and t_3 is shown in Fig. 2.19.

Let V be the voltage applied across the capacitor.

It can be seen that there exists three capacitors in series. The values of three capacitors are different. Hence, the equivalent capacitance across the terminals A-B is,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

And $$C_1 = \frac{\varepsilon_0\,\varepsilon_{r1}\,A}{t_1},\quad C_2 = \frac{\varepsilon_0\,\varepsilon_{r1}\,A}{t_2},\quad C_3 = \frac{\varepsilon_0\,\varepsilon_{r1}\,A}{t_3}$$

$\therefore$ $$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0\,A}\left[\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right]$$

$\therefore$ $$C_{eq} = \frac{\varepsilon_0\,A}{\left[\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right]}$$

In general, for a composite capacitor with 'n' dielectrics,

$$C_{eq} = \frac{\varepsilon_0 A}{\sum_{k=1}^{n} \frac{t_k}{\varepsilon_{rk}}} = \frac{\varepsilon_0 A}{\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \ldots + \frac{t_n}{\varepsilon_{rn}}}$$

The voltage across each dielectric will be different,

$$\therefore \quad V = V_1 + V_2 + V_3$$

But, $\quad E = \frac{V}{t} \quad$ i.e. $\quad V = E\,t$

$$\therefore \quad V = E_1 t_1 + E_2 t_2 + E_3 t_3$$

where E_1, E_2 and E_3 are the values of electric intensities in the different dielectrics.

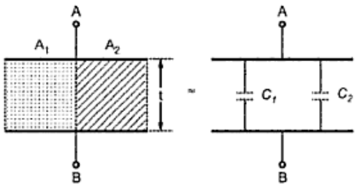


Fig. 2.20 Composite capacitor

Type 2 : In this type, in the same thickness, 't', the two dielectrics are arranged as shown in the Fig. 2.20.

Let the relative permittivity values for the two dielectrics be ε_{r1} and ε_{r2}. The thickness for both is same but the areas are different. It can be seen from the equivalent circuit that there exists two capacitors in parallel due to two different dielectrics.

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t} + \frac{\varepsilon_0 \varepsilon_{r2} A_2}{t} = \frac{\varepsilon_0}{t}(A_1 \varepsilon_{r1} + A_2 \varepsilon_{r2})$$

If one of the two dielectrics is air, then the corresponding relative permittivity is one, to be used in the above expression.

For 'n' dielectrics arranged in same thickness 't',

$$C_{eq} = \frac{\varepsilon_0}{t}[A_1\varepsilon_{r1} + A_2\varepsilon_{r2} + \ldots\ldots.. + A_n\varepsilon_{rn}]$$

Type 3 : In practice we can have the capacitor which is combination of above two types. One such capacitor is shown in the Fig. 2.21.

Basically it is Type 2 capacitor, consisting of Type 1 capacitor. So there are two capacitors in parallel. The C_1 is having thickness t_1, relative permitivity ε_{r1} and area A_1.

$$\therefore \quad C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t_1}$$

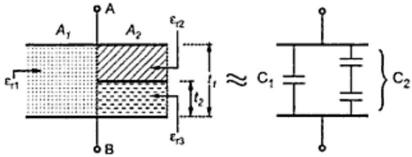


Fig. 2.21 Composite capacitor

Now the capacitor C_2 is again a composite capacitor of Type 1 which itself is made up of two capacitors in series. From the result of Type 1 we can write,

$$C_2 = \frac{\varepsilon_0 A_2}{\frac{(t_1 - t_2)}{\varepsilon_{r2}} + \frac{t_2}{\varepsilon_{r3}}}$$

Hence the total capacitance is the parallel combination of C_1 and C_2,

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t_1} + \frac{\varepsilon_0 A_2}{\frac{(t_1 - t_2)}{\varepsilon_{r2}} + \frac{t_2}{\varepsilon_{r3}}}$$

➡ **Example 2.7 :** *A parallel plate capacitor of area 50 cm^2 have two dielectrics of thicknesses 1 mm and 2 mm and dielectric constants 4 and 2 respectively. Find the value of the capacitance of the capacitor so formed.*

Solution :

$$C_{eq} = \frac{A\varepsilon_0}{\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}}} = \frac{\left(50 \times 10^{-4}\right) \times 8.854 \times 10^{-12}}{\frac{1 \times 10^{-3}}{4} + \frac{2 \times 10^{-3}}{2}}$$

$$= \frac{50 \times 10^{-4} \times 8.854 \times 10^{-12}}{10^{-3}\,(0.25 + 1)}$$

$$= 0.3536 \times 10^{-10}\ \text{F} = \mathbf{35.36\ pF}$$

2.21 Energy Stored in a Capacitor

When the capacitor is charged, energy is expended by the charging source. This is because charging the capacitor means the transfer of the charges from one plate to the another. This transfer is against the opposition due to potential difference across the plates. Due to this, there is expenditure of energy on the part of charging source. This energy is stored in the capacitor in terms of the electrostatic field set up in the dielectric medium. However, when the capacitor is discharged, this field collapses and energy stored in it is released.

Derivation of expression :

Let us determine the energy expended in charging a capacitor of capacitance C farads to a voltage V.

Let at any instant of charging, the potential difference across the plates be 'V' volts. As per definition, it is equal to the work done in shifting one coulomb of charge from one plate to another.

Now, if the charge of the capacitor is raised by a small amount 'dq' coulombs, the work done is,

$$dW = V \cdot dq$$

$$= \frac{q}{C} \cdot dq \text{ joules} \qquad \left(\text{As } V = \frac{q}{C}\right)$$

This work done is ultimately stored in the capacitor as a potential energy.

Therefore, the total energy stored when it is finally charged to 'Q' coulombs can be obtained as,

$$W = \frac{1}{C}\int_0^Q q \, . \, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C} \text{ joules}$$

But, $$V = \frac{Q}{C}$$

$\therefore$ $$W = \frac{C^2 V^2}{2C}$$

$\therefore$ $$\boxed{\text{Energy stored, } W = \frac{1}{2} C V^2 \text{ joules}}$$

2.22 Current in a Capacitor

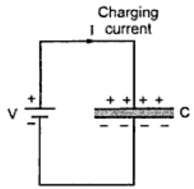


Fig. 2.22 Current in a capacitor

When voltage applied to the capacitor, more charges are supplied to the capacitor and there is a flow of current in the external circuit.

The Fig. 2.22 shows a capacitor C connected across a voltage 'V' and current 'I' is flowing through the external circuit.

Let charge dq be applied to the capacitor in time dt, to increase the voltage across capacitor by dV.

Now, $I = \frac{dq}{dt}$

And $C = \frac{dq}{dV}$

$\therefore \quad dq = C\,dV$

$\therefore \quad I = C\frac{dV}{dt}$

And $V = \frac{1}{C}\int_{-\infty}^{t} I\,dt$

The integral given here can also be interpreted as area under the current curve from $t = -\infty$ to the time t under consideration.

Now, capacitor is filled with good insulation, then how can current flow through it ?

The answer is that current flow inside the capacitor is not because of movement of electrons i.e. it is not a conduction current. In a capacitor, we can assume that an imaginary current flows through the capacitor whose value just equals the conduction current outside the capacitor. This current is called **'displacement current.'**

$$i = i\text{ displacement} = C\frac{dV}{dt} = \frac{d\psi}{dt}$$

Key Point : *Outside the capacitor, the current flows by conduction process and inside the capacitor by displacement process.*

2.23 Types of Capacitors

Mostly, the capacitors are classified based on the size and shape of the plates used. e.g. Parallel plate, cylindrical, concentric spherical etc. They may be classified based on the nature of the dielectric used as follows :-

i) Air capacitors : This type of capacitor consists of one set of fixed plates and another set of movable plates. Its capacitance can be changed by changing the position of the movable plates. This type is mainly used for radio work where the capacitance is required to be varied.

ii) Paper capacitors : This consists of metal foils interleaved with paper impregnated with wax or oil and it is rolled into a compact form. These are used in power supplies.

iii) Mica capacitors : It consists of alternate layers of mica and metal foil clamped together tightly. Use of mica makes its cost high. It is mainly used in high frequency circuits which requires greater accuracy, high voltages and less dielectric loss.

iv) Poly carbonate capacitors : This is a recent development where a film of polycarbonate, metallised with aluminium is wound to form the capacitor elements. It has a relative permittivity of 2.8 and has a high resistivity with very low dielectric loss.

v) Ceramic capacitors : It has a metallic coatings on the opposite faces of a thin disc of ceramic material like barium titanate, hydrous silicate of magnesia, etc. It is used in high frequency radio and electronic circuits.

vi) Electrolytic capacitors : These are most commonly used and consists of two aluminium foils, one with an oxide film and one without. The foils are interleaved with a material such as a paper saturated with a suitable electrolyte. The aluminium oxide film is formed on the one foil by passing it through an electrolytic bath. This oxide film acts as a dielectric. These are used where very large capacitance values are required so used in electronic and filter circuits. The main limitations of this type are the low insulation resistance and suitability only for those circuits where the voltage applied to the capacitor never reverses its direction.

2.24 Charging a Capacitor through Resistance

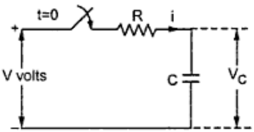


Fig. 2.23 Charging a capacitor

Consider a capacitor C in series with the resistance R. The capacitor has initially no charge and no voltage across it. When the switch is closed at the instant t = 0, the charge starts accumulating on capacitor and current starts flowing.

The rate of rise of charge at start is high and later becomes slow and behaves in exponential manner till it reaches equal to the source voltage V. The current at the instant of closing the switch is high and as the voltage across capacitor V_C at start is zero this initial current can be expressed as,

$$i = \frac{V - V_C}{R} = \frac{V}{R}\ A$$

This is maximum charging current. As capacitor starts charging, the capacitor voltage V_C increases and finally after a certain period achieves a value equal to V.

Then the charging current reduces to zero. Theoretically, the current becomes zero only after an infinite time. In actual practice the voltage across capacitor and current achieve their steady state values equal to V and zero respectively, in a relatively short time. The variation of charging current and capacitor voltage V_C against time is shown in Fig. 2.24 (a) and (b).

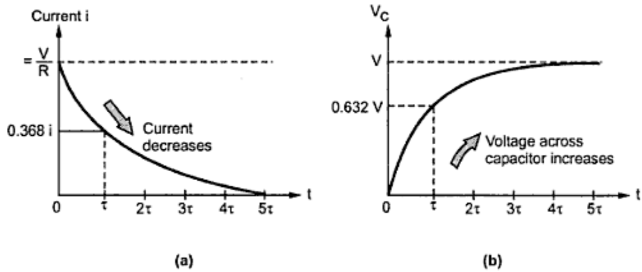


Fig. 2.24 Variation of charging current and voltage V_C

2.24.1 Mathematical Analysis

Let V_C = Voltage across capacitor at any instant.

q = Charge on capacitor in coulombs at any instant.

i = Charging current at any instant in amperes.

By Kirchhoff's voltage law,

$$V = V_R + V_C$$

$$= i R + V_C$$

but $$i = C \frac{dV_C}{dt}$$

$\therefore$ $$V = CR \frac{dV_C}{dt} + V_C$$

$\therefore$ $$V - V_C = RC \frac{dV_C}{dt}$$

$\therefore$ $$\frac{dt}{RC} = \frac{dV_C}{V - V_C}$$

Integrating both sides of the above equation,

$$\frac{t}{CR} = -\ln (V - V_C) + K$$

where K = Constant of integration.

At t = 0, V_C = 0, substituting in above,

$$0 = -\ln (V) + K$$

$\therefore$ $$K = \ln (V)$$

$$\therefore \quad \frac{t}{CR} = -ln\,(V - V_C) + ln\,(V)$$

$$\therefore \quad \frac{t}{CR} = ln\frac{V}{V - V_C}$$

$$\therefore \quad \frac{V}{V - V_C} = e^{t/CR}$$

$$\therefore \quad V - V_C = V\,e^{-t/CR}$$

$$\therefore \quad \boxed{V_C = V\,(1 - e^{-t/CR})}$$

When the steady state is achieved, the total charge on the capacitor is Q coulombs.

$$\therefore \quad V = \frac{Q}{C}$$

Similarly at any instant $V_C = \frac{q}{C}$

$$\therefore \quad \frac{q}{C} = \frac{Q}{C}(1 - e^{-t/CR})$$

$$\therefore \quad q = Q\,(1 - e^{-t/CR})$$

Now $V - V_C = i\,R$

$$\therefore \quad i = \frac{V - V_C}{R}$$

$$\therefore \quad i = \frac{V - e^{-t/CR}}{R}$$

$$\therefore \quad \boxed{i = \frac{V}{R}e^{-t/CR}}$$

So at $t = 0$, $i = \frac{V}{R}$ is maximum and in steady state it becomes zero.

Thus capacitor acts as short circuit at start and acts as open circuit in steady state.

2.24.2 Time Constant

The term CR in all the above equation is called the **Time Constant of the R-C** charging circuit and denoted by τ, measured in seconds.

When $t = CR = \tau$ then ,

$$V_C = V\,(1 - e^{-1})$$

$$V_C = 0.632\ V$$

So time constant of the R-C series circuit is defined as **time required by the capacitor voltage to rise from zero to 0.632 of its final steady state value during charging.**

Incidentally after t = 2τ, 3τ, 4τ the capacitor voltage attains the values as 0.863 V, 0.95 V, 0.982 V respectively and practically capacitor requires the time 4 to 5 times the time constant to charge fully.

When $t = CR$ then

$$i = \frac{V}{R} e^{-1} = 0.368 \left(\frac{V}{R}\right)$$

Now $\left(\frac{V}{R}\right)$ is starting charging current. From this time constant can be defined as below.

Key Point: *Time constant is the time required for the charging current of the capacitor to fall to 0.368 of its initial maximum value, starting from its maximum value.*

2.24.3 Initial Rate of Rise of Capacitor Voltage

The initial rate of rise of capacitor voltage is fast however when the capacitor charges this rate is reduced.

Let us find initial $\frac{dV_C}{dt}$, by differentiating the equation of V_C.

Since $\frac{dV_C}{dt} = V\left(+\frac{1}{CR}\right)e^{-t/CR}$

$$\text{At } t = 0, \quad \frac{dV_C}{dt} = \frac{V}{CR} = \frac{V}{\tau}$$

If the same rate is maintained through out after

$$t = \tau, V_C = V$$

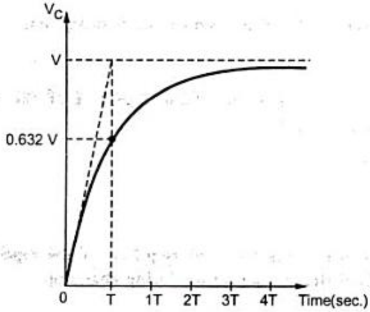


Fig. 2.25

Thus the tangent to the initial part of V_C joins $V_C = V$ at $t = \tau$ as shown in the graph.

From the above discussion, the time constant of R-C series circuit can be defined as the **time in seconds during which the voltage across the capacitor, starting from zero, would reach its final steady value if its rate of change was maintained constant at its initial value throughout the charging period.**

Example 2.8 : *A 80 μF capacitor is in series with, 10 kΩ resistance. The combination is connected suddenly across a 100 V supply. Find after 0.04 sec,*

i) Voltage across resistance. ii) Voltage across capacitor.

iii) Find the time at which voltage across resistance becomes 40 V.

iv) Find current at this time. v) What is charge on capacitor after 0.2 sec?

Solution : **Given :** $C = 80\ \mu F$ and $R = 10\ k\Omega = 10\times10^3\Omega$

$$\text{Initial current} = \frac{V}{R} = \frac{100}{10\times10^3} = 0.01\ A$$

$$\text{Time constant} \quad \tau = R\,C = 80\times10^{-6}\times10\times10^3 = 0.8\ sec$$

$$\therefore \quad i = \frac{V}{R}(e^{-t/RC})$$

$$\therefore \quad i = 0.01\ e^{-1.25\,t}$$

$$\text{After} \quad t = 0.4$$

$$i = 6.0653\times10^{-3}\ A$$

i) Voltage across resistance $= i\,R = 6.0653\times10\times10^{-3}\times10\times10^3$

$= \mathbf{60.653\ V}$

ii) Voltage across capacitor $= V - V_R = 100 - 60.653 = \mathbf{39.34\ V}$

iii) Voltage across resistance $= 40\ V$

$$\therefore \quad i\,R = 40\ V$$

$$\therefore \quad i = \frac{40}{10\times10^3} = 4\times10^{-3}\ A$$

$$\therefore \quad 4\times10^{-3} = 0.01\ e^{-1.25\,t}$$

$$\therefore \quad t = \mathbf{0.733\ sec}$$

iv) Current at this time $i = \mathbf{4\times10^{-3}\ A}$

v) After t = 0.2 sec, $V_C = V(1 - e^{-1.25\,t}) = 100\,(1 - e^{-1.25\times0.2})$

$= 22.119\ V$

$$\therefore \quad Q = C\times V_C = 80\times10^{-6}\times22.119$$

$= \mathbf{1.7695\times10^{-3}\ C}$

2.25 Discharging a Capacitor through a Resistance

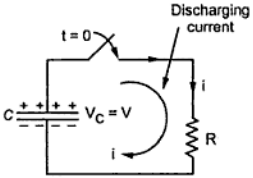


Fig. 2.26 Discharging of a capacitor

Now consider that a capacitor 'C' is being discharged through a resistor R by closing the switch at t = 0. At the time of closing the switch the capacitor 'C' is fully charged to V volts and it discharges through resistance 'R' and current through resistance flows in opposite direction to that at the time of charging.

As time passes, charge and hence the capacitor voltage V_C decreases gradually and hence discharge current also gradually decreases exponentially from maximum to zero.

The variation of capacitor voltage and discharging current as a function of time is shown in the Fig. 2.27.

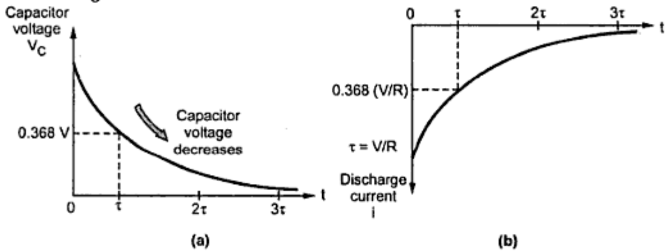


Fig. 2.27 Variation of discharge current and voltage

As direction of current is opposite to that of charging current, it is mathematically considered as negative. Hence graph of current against time is in fourth quadrant.

2.25.1 Mathematical Analysis

Applying Kirchhoff's voltage law, we had

$$V = V_C + i R$$

But V = 0,

$$\therefore \quad 0 = V_C + i R$$

$$\therefore \quad V_C = -i R$$

But

$$i = +C \frac{d V_C}{dt}$$

$\therefore \quad V_C = -CR \dfrac{dV_C}{dt}$

Hence $\quad +\dfrac{dt}{CR} = -\dfrac{dV_C}{V_C}$

Integrating both sides

$$+\frac{t}{CR} = -\ln V_C + K$$

At $t = 0$, $V_C = V$

$\therefore \quad 0 = \ln V + K$

$\therefore \quad K = \ln V$

$\therefore \quad \dfrac{t}{CR} = -\ln V_C + \ln V$

$\therefore \quad \dfrac{t}{CR} = -\ln \dfrac{V}{V_C}$

$\therefore \quad \dfrac{V}{V_C} = e^{-t/CR}$

$\therefore \quad \boxed{V_C = V e^{-t/CR} \text{ volts}}$

As $\quad V = \dfrac{Q}{C}$

And $\quad V_C = \dfrac{q}{C}$

$\therefore \quad \dfrac{q}{C} = \dfrac{Q}{C} e^{-t/CR}$

$\therefore \quad q = Q e^{-t/CR}$

Now $\quad i = -\dfrac{V_R}{R}$

But $\quad V_R = V_C$

$\therefore \quad i = -\dfrac{V_C}{R}$

$\therefore \quad \boxed{i = -\dfrac{V}{R} e^{-t/CR}}$

Key Point: *The negative sign indicates that the direction of the discharge current is the reverse of that of charging current.*

2.25.2 Time Constant

Similar to the previous definition, at the time of discharging also the term CR is called as **time constant** denoted by τ.

$$\tau = t = CR \text{ seconds}$$

When $t = CR$

$$V_C = V e^{-1} = 0.368 \text{ V}$$

and $i = -\frac{V}{R} e^{-1}$

∴ $$\boxed{i = -0.368 \left(\frac{V}{R}\right) \text{amps.}}$$

So **time constant** can be defined as

i) The time required for capacitor voltage to fall to 0.368 of its initial maximum value on discharge from its initial maximum value.

ii) The time required during which the capacitor discharge current falls to 0.368 of its initial maximum value.

2.25.3 Significance of Time Constant

The charging and discharging of a capacitor under the conditions discussed is said to be exponential. The 'time constant' (τ) of the circuit has following significance.

i) The whole charging or discharging process can be considered to be completed in a time equal to 4 times the time constant and the current falls to insignificant value (Theoretically the process takes infinite time).

ii) The charging current falls to 36.8% of its initial value in a time equal to time constant (τ) and to nearly 1.8% of initial value in $t = 4\tau$.

iii) The capacitor charges to nearly 63.2% of its final value in $t = \tau$ and nearly 98.2% of the final value in $t = 4\tau$ provided it is initially uncharged.

iv) The capacitor discharges to nearly 36.8% of its initial value in $t = \tau$ and nearly 1.8% of its initial value in $t = 4\tau$.

v) If the initial rate of rise of voltage is maintained then the capacitor charges to its final value in a time = τ.

➡ **Example 2.9 :** *A capacitor of 2 μF capacitance charged to p.d. of 200 V is discharged through a resistor of 2 MΩ.*

Calculate :

1) The initial value of discharged current

2) Its value 4 seconds later, and

3) Initial rate of decay of the capacitor voltage.

Solution : C = 2 μF, V = 200 V, R = 2 MΩ

i) The discharging current is given by

$$i = -\frac{V}{R}e^{-t/RC}$$

Initially t = 0, $i = -\frac{V}{R} = -\frac{200}{2\times10^{6}} = -100\ \mu A$

ii) At t = 4 sec,

$$i = -\frac{200}{2\times10^{6}}e^{-4/2\times10^{6}\times2\times10^{-6}} = \mathbf{-36.7879\ \mu A}$$

iii) $V_C = V\ e^{-t/RC}$

$$\therefore \quad \frac{dV_C}{dt} = V\left(-\frac{1}{RC}\right)e^{-t/RC}$$

Initial rate of decay is at t = 0,

$$\left.\frac{dV_C}{dt}\right|_{t=0} = -\frac{V}{RC} = -\frac{200}{2\times10^{6}\times2\times10^{-6}} = \mathbf{-50\ V/sec}$$

Negative sign indicates decay of voltage and current.

Examples with Solutions

Example 2.10 : *A parallel plate capacitor has plates 0.1 cm apart, a plate area of 100 cm² and a dielectric with relative permittivity of 4. Determine the electric flux, electric flux density, electric field intensity, voltage between the plates, value of capacitance and energy stored if the capacitor has a charge of 0.05 μC.*

Solution : Now electric flux ψ = charge on C i.e. Q

$$\therefore \quad \psi = \mathbf{0.05\ \mu C}$$

Electric flux density $D = \frac{Q}{A} = \frac{0.05\times10^{-6}}{100\times10^{-4}} = \mathbf{5\ \mu C/m^2}$

Now $D = \varepsilon E$

$$\therefore \quad E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0\varepsilon_r} = \frac{5\times10^{-6}}{8.854\times10^{-12}\times4} = \mathbf{141.179\ kV/m}$$

$$E = \frac{V}{d}$$

$$V = 141.179\times10^{3}\times0.1\times10^{-2} = \mathbf{141.179\ V}$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= \frac{8.854\times10^{-12}\times4\times100\times10^{-4}}{0.1\times10^{-2}} = \mathbf{354.16\ pF}$$

$$W = \frac{1}{2} C V^2 = \frac{1}{2}\, 354.16\times10^{-12}\times(141.179)^2$$

$$= \mathbf{3.53\ \mu C}$$

Example 2.11 : *A capacitor is composed of two plates separated by a sheet of insulating material 3 mm thick and of relative permittivity = 4. The distance between plates is increased to allow the insertion of a second sheet 5 mm thick and relative permittivity 'ε_r'. If the capacitance of the capacitor so formed is one half of the original capacitance, find the value of ε_r.*

Solution : Initially, t_1 = 3 mm and $\varepsilon_{r1} = 4$ thus $\varepsilon = \varepsilon_0 \varepsilon_{r1} = 4\varepsilon_0$

$$\therefore \quad C_1 = \frac{\varepsilon A}{t_1} = \frac{\varepsilon_0\, 4\, A}{3\times10^{-3}} = 1333.333\,\varepsilon_0\, A \qquad \text{... (1)}$$

With t_2 = 5 mm and $\varepsilon_r = \varepsilon_{r2}$, $C_2 = 0.5\, C_1$

$$\text{Now} \quad C_2 = \frac{A\varepsilon_0}{\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}}} = \frac{A\varepsilon_0}{\frac{3\times10^{-3}}{4} + \frac{5\times10^{-3}}{\varepsilon_{r2}}} \qquad \text{... (2)}$$

Take ratio of equations (1) and (2),

$$\therefore \quad \frac{C_1}{C_2} = \frac{1333.333\,\varepsilon_0\, A}{\left[\frac{A\varepsilon_0}{\frac{3\times10^{-3}}{4} + \frac{5\times10^{-3}}{\varepsilon_{r2}}}\right]}$$

$$\therefore \quad \frac{1}{0.5} = 1333.333\left[7.5\times10^{-4} + \frac{5\times10^{-3}}{\varepsilon_{r2}}\right]$$

$$\therefore \quad \frac{5\times10^{-3}}{\varepsilon_{r2}} = 7.5\times10^{-4}$$

$$\therefore \quad \varepsilon_{r2} = \mathbf{6.667}$$

Example 2.12 : *A capacitor is made of two parallel plates with an area of 11 cm^2 and are separated by mica sheet 2 mm thick. If for mica ε_r = 6, find its capacitance. If now, one plate of capacitor is moved further to give an air gap of 0.5 mm wide between the plates and mica. Find the new value of capacitance.*

Solution : Case 1 : d = 2 mm, A = 11 cm^2, $\varepsilon_r = 6$

$$\therefore \quad C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r}{d} = \frac{6 \times 8.854 \times 10^{-12} \times 11 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= \mathbf{29.2182\ pF}$$

Case 2 : An air gap of 0.5 mm between mica and plates

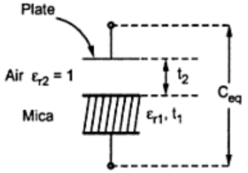


Fig. 2.28

It is a composite capacitor.

$\varepsilon_{r1} = 6$, $t_1 = 2$ mm for mica

$\varepsilon_{r2} = 1$, $t_2 = 0.5$ mm for air

$A = 11\ cm^2$ same for both

$C_{eq} = C_1$ series C_2

where $C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{t_1}$ and $C_2 = \frac{\varepsilon_0 A}{t_2}$

For series connection, $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore \quad C_{eq} = \frac{\frac{\varepsilon_0 \varepsilon_{r1} A}{t_1} \times \frac{\varepsilon_0 A}{t_2}}{\frac{\varepsilon_0 \varepsilon_{r1} A}{t_1} + \frac{\varepsilon_0 A}{t_2}} = \frac{\frac{\varepsilon_0^2 A^2}{t_1 t_2}[\varepsilon_{r1}]}{\frac{\varepsilon_0 A}{t_1 t_2}[t_2 \varepsilon_{r1} + t_1]}$$

$$= \frac{\varepsilon_0 A\, \varepsilon_{r1}}{\left[0.5 \times 10^{-3} \times 6 + 2 \times 10^{-3}\right]} = \frac{6 \times 8.854 \times 10^{-12} \times 11 \times 10^{-4}}{5 \times 10^{-3}}$$

$$= \mathbf{11.6872\ pF}$$

Examples from G.U. and G.T.U. Papers

Example 2.13 : *A parallel plate capacitor has plate of area 1500 cm^2 seperated by 5 mm with air as dielectric. If a layer of dielectric 2 mm thick with $\varepsilon_r = 3$ is now introduced between the plates, what must be the seperation between the plates to bring the capacitance to original value.* **[GU : June-2001]**

Solution : $A = 1500\ cm^2$, $d = 5$ mm, $\varepsilon_r = 1$

$$\therefore \quad C_1 = \frac{\varepsilon_0\, \varepsilon_{r1}\, A}{d} = \frac{8.854 \times 10^{-12} \times 1 \times 1500 \times 10^{-4}}{5 \times 10^{-3}} = 0.2656\ nF.$$

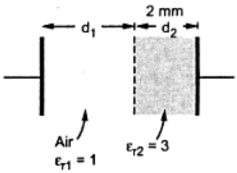


Fig. 2.29

Now the dielectric of $\varepsilon_{r2} = 3$ is introduced for $d_2 = 2$ mm.

The capacitance of a composite capacitor is,

$$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}}} = \frac{8.854 \times 10^{-12} \times 1500 \times 10^{-4}}{\frac{d_1}{1} + \frac{2 \times 10^{-3}}{3}}.$$

Now d_1 is to be selected such that the value of C is same as original C_1 = 0.2656 nF.

$$\therefore \quad 0.2656\times10^{-9} = \frac{8.854\times10^{-12}\times1500\times10^{-4}}{d_1+6.667\times10^{-4}}$$

$$\therefore \quad d_1 = 4.333\times10^{-3} \text{ m} = 4.333 \text{ mm}$$

So total seperation between the plates is d_1+d_2 = 4.333 + 2 i.e. **6.333 mm.**

Example 2.14 : *Two capacitors having capacitance of 6 μF and 10 μF are connected in parallel. A 16 μF capacitor is connected in series with this combination. The complete circuit is connected across 400 V. Find : a) Total capacitance of the circuit. b) Total charge in the circuit. c) Voltage across each capacitor d) The charge on each capacitor.*

[GU : Nov.-2002]

Solution : The arrangement is shown in the Fig. 2.30.

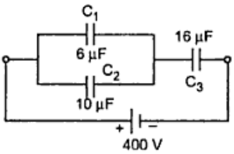


Fig. 2.30

a) C_1 and C_2 in parallel,

$$\therefore C' = C_1+C_2 = 6+10 = 16\ \mu F$$

Now C' and C_3 are in series,

$$\therefore C_{eq} = \frac{C'\times C_3}{C'+C_3} = \frac{16\times16}{16+16} = \mathbf{8\ \mu F}$$

b) $$Q = C_{eq}\times V = 8\times10^{-6}\times400 = \mathbf{3.2\times10^{-3}\ C} \quad \text{... Total charge}$$

c) For C' and C_3, the charge is same as total charge.

$$\therefore \quad Q = C'\times V' = C_3\times V_3 \quad \text{... V' = voltage across C'}$$

$$\therefore \quad V' = \frac{Q}{C'} = \frac{3.2\times10^{-3}}{16\times10^{-6}} = 200 \text{ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{3.2\times10^{-3}}{16\times10^{-6}} = \mathbf{200\ V} \quad \text{... Voltage across } C_3$$

C' is made up of C_1 and C_2 in parallel.

$$\therefore \quad V_1 = V_2 = \mathbf{200\ V.} \quad \text{... Voltage across } C_1 \text{ and } C_2$$

d) As C_1 and C_2 are in parallel, voltage across them is same.

$$\therefore \quad Q_1 = C_1V_1 = 6\times10^{-6}\times200 = \mathbf{1.2\times10^{-3}\ C} \quad \text{... Charge on } C_1$$

$$\therefore \quad Q_2 = C_2V_2 = 10\times10^{-6}\times200 = \mathbf{2\times10^{-3}\ C} \quad \text{... Charge on } C_2$$

$\therefore \quad Q_3 = C_3V_3 = 16\times10^{-6}\times200 = 3.2\times10^{-3}$ C ... Charge on C_3

Example 2.15 : *Two capacitors having capacitances of 20 μF and 30 μF are connected in series across a 600 V d.c. supply. Calculate the potential difference across each capacitor. If a third capacitor of unknown capacitance is now connected in parallel with the 20 μF capacitor such that the potential difference across 30 μF capacitor is 400 V. Calculate : 1) The value of unknown capacitance 2) Energy stored in the third capacitor.* **[GU : July-2005]**

Solution :

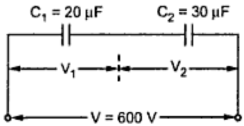


Fig. 2.31 (a)

As capacitors are in series, the charge across them remains same.

$$\therefore Q = C_1V_1 = C_2V_2 = C_{eq}V$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{20\times30}{20+30} = 12\ \mu F$$

$$\therefore \quad Q = C_{eq}\times V = 12\times10^{-6}\times600 = 7.2\times10^{-3}\ C$$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \frac{7.2\times10^{-3}}{20\times10^{-6}} = \mathbf{360\ V}, \quad V_2 = \frac{Q}{C_2} = \frac{7.2\times10^{-3}}{30\times10^{-6}} = \mathbf{240\ V}$$

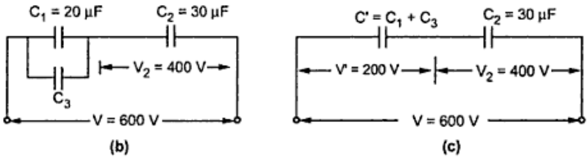


Fig. 2.31

The equivalent capacitor of C_1 and C_3 is C' . Now C' and C_2 are in series where charge remains same.

$$\therefore \quad Q = C'V' = C_2V_2$$

$$\therefore \quad (C_1+C_3)\times200 = 30\times10^{-6}\times400$$

$$\therefore \quad C_1+C_3 = 60\times10^{-6} \quad \text{i.e.} \quad C_3 = 60\times10^{-6}-20\times10^{-6}$$

$$\therefore \quad C_3 = \mathbf{40\ \mu F}$$ ... Unknown capacitor.

The voltage across C_3 is V' = 200 V.

$$\therefore \quad E = \frac{1}{2}C_3(V')^2 = \frac{1}{2}\times40\times10^{-6}\times200^2 = \mathbf{0.8\ J}$$

Example 2.16 : *A capacitor is made of two plates with an area of 11 cm^2 which are seperated by mica sheet 2 mm thick. If relative permittivity of mica is 6, find its capacitance. If now one plate is moved further to give an air gap of 0.5 mm wide between the plate and mica, find the change in capacitance.* **[GU : Nov.-2005]**

Solution :

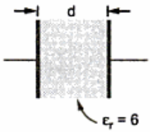


Fig. 2.32 (a)

Case 1 : $\varepsilon_r = 6$, d = 2 mm, A = 11 cm^2

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854\times10^{-12}\times6\times11\times10^{-4}}{2\times10^{-3}} = \mathbf{29.2182\ pF}$$

Case 2 : Air gap of 0.5 mm is added.

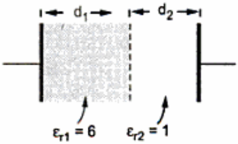


Fig. 2.32 (b)

d_1 = 2 mm, d_2 = 0.5 mm,

$\varepsilon_{r1} = 6$, $\varepsilon_{r2} = 1$

There are two capacitors in series.

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = 29.2182\ \text{pF} \quad \text{... Same as before}$$

$$C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{8.854\times10^{-12}\times11\times10^{-4}}{0.5\times10^{-3}} = 19.4788\ \text{pF} \quad \text{... } \varepsilon_{r2} = 1$$

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{29.2182\times10^{-12}\times19.4788\times10^{-12}}{29.2182\times10^{-12}+19.4788\times10^{-12}} = 11.6872\ \text{pF.}$$

$\therefore$ Change in capacitance = 29.2182 – 11.6872 = **17.531 pF.**

Example 2.17 : *The total capacitance of two capacitors is 0.03 μF when joined in series and 0.16 μF when connected in parallel. Calculate the capacitance of each capacitor.* **[GU : June-2003]**

Solution : Let the capacitors be C_1 and C_2

In series, $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 0.03\times10^{-6} \quad \text{... (1)}$$

In parallel, $$C_{eq} = C_1 + C_2 = 0.16\times10^{-6} \quad \text{... (2)}$$

Using equation (2) in equation (1),

$$C_1C_2 = 0.03\times10^{-6}\times0.16\times10^{-16} = 4.8\times10^{-15}$$

i.e.
$$C_2 = \frac{4.8\times10^{-15}}{C_1} \quad \text{... (3)}$$

Using equation (3) in equation (2), $C_1+\dfrac{4.8\times10^{-15}}{C_1} = 0.16\times10^{-6}$

$\therefore \quad C_1^2 - 0.16\times10^{-6}\,C_1 + 4.8\times10^{-15} = 0$

Solving, $C_1 = \mathbf{0.12\ \mu F}$ or $\mathbf{0.04\ \mu F}$

Hence, $C_2 = \mathbf{0.04\ \mu F}$ or $\mathbf{0.12\ \mu F}$.

Example 2.18 : *A capacitor of 50 μF in series with 100 Ω resistor is suddenly connected across 100 V d.c. supply. Find : a) Time constant of circuit b) Initial current c) Current equation as a function of time d) Voltage across R after 6 msec.*

[GU : Dec.- 2001, May-2003]

Solution : C = 50 μF, R = 100 Ω, V = 100 V

a) $\tau = CR = 50\times10^{-6}\times100 = 5\times10^{-3}$ sec $= \mathbf{5\ ms}$

b) $i_{initial} = \dfrac{V}{R} = \dfrac{100}{100} = \mathbf{1\ A}$

c) $i = \dfrac{V}{R}e^{-t/RC}$ i.e. $i = e^{-t/5\times10^{-3}}$

$\therefore \quad i = \mathbf{e^{-200t}\ A}$

d) After t = 6 ms, $i = e^{-200\times6\times10^{-3}} = 0.3012$ A

$\therefore \quad V_R = i\times R = 0.3012\times100 = \mathbf{30.12\ V.}$

Example 2.19 : *A 20 μF capacitor initially charged to a potential difference of 500 V, is discharged through an unknown resistance. After one minute the potential difference at the terminals of the capacitor is 200 V. What is the magnitude of the resistance?*

[GU : Dec.-2004]

Solution : V = 500 V, C = 20 μF, t = 1 min = 60 sec, V_C = 200 V

The equation for discharging capacitor voltage is,

$V_C = Ve^{-t/RC}$ i.e. $200 = 500\,e^{-60/R\times20\times10^{-6}}$

$\therefore \quad 0.4 = e^{-3\times10^6/R}$ i.e. $ln(0.4) = e^{-3\times10^6/R}$

$\therefore \quad -0.9163 = \dfrac{-3\times10^6}{R}$ i.e. $R = \dfrac{3\times10^6}{0.9163} = \mathbf{3.274\ M\Omega.}$

Example 2.20 : *A capacitor is charged through a resistance of 500 kΩ connected in series with it, across a d.c. supply. The potential difference across a capacitor is 80 % of its final value after 1 second during charging. Find the value of the capacitor.*

[GU : Dec.-2002, June-2006]

Solution : $R = 500\ k\Omega$, $V_C = 80\ \%$ of V for t = 1 sec.

$$V_C = V(1 - e^{-t/RC})$$

$$\therefore \quad 0.8\ V = V(1 - e^{-1/500\times10^3\ C}) \quad \text{... t = 1 sec.}$$

$$\therefore \quad 0.8 = (1 - e^{-1/500\times10^3\ C})$$

$$\therefore \quad +e^{-1/500\times10^3\ C} = 1 - 0.8 = 0.2$$

$$\therefore \quad -\frac{1}{500\times10^3\ C} = \ln(0.2) = -1.6094$$

$$\therefore \quad C = \frac{1}{1.6094\times500\times10^3} = \mathbf{1.2427\ \mu F.}$$

Example 2.21 : *Calculate the charge and voltage on each capacitor of the circuit shown in the Fig. 2.33.*

[GU : June-2006]

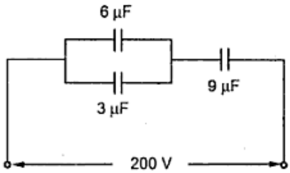


Fig. 2.33

Solution : $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$ are parallel hence their equivalent $C' = C_1 + C_2 = 9\ \mu F$. So circuit reduces as shown in the Fig. 2.33(a).

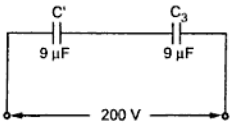


Fig. 2.33(a)

The C' and C_3 are in series hence,

$$C_{eq} = \frac{C' \times C_3}{C' + C_3} = \frac{9\times9}{9+9} = 4.5\ \mu F.$$

Key Point: *For series the charge remains same.*

$$\therefore \quad Q = C_{eq} \times V = 4.5\times10^{-6}\times200 = 9\times10^{-4}\text{ C}$$

$$\therefore \text{ Voltage across } C_3 = \frac{Q}{C_3} = \frac{9\times10^{-4}}{9\times10^{-6}} = \mathbf{100\ V}$$

$$\therefore \text{ Voltage across } C' = 200 - 100 = 100\text{ V}$$

$$\therefore \text{ Voltage across } 3\ \mu\text{F and } 6\ \mu\text{F} = \mathbf{100\ V}$$

$$\therefore \text{ Charge on } C_1\,(3\mu F) = C_1V = 3\times10^{-6}\times100 = 3\times10^{-4}\text{ C}$$

$$\therefore \text{ Charge on } C_2\,(6\mu F) = C_2V = 6\times10^{-6}\times100 = 6\times10^{-4}\text{ C}$$

$$\text{Charge on } C_3\,(9\mu F) = 9\times10^{-4}\text{ C}$$

Example 2.22 : *A capacitor is charged with 5000 μC. If the energy stored is 1 joule then find : a) Voltage and b) Capacitance.* **[GU : Nov.-2006]**

Solution : $Q = 5000\ \mu C,\ E = 1\ J$

a) $$E = \frac{1}{2}CV^2 \quad \text{and} \quad Q = CV \text{ i.e. } C = \frac{Q}{V}$$

$$\therefore \quad E = \frac{1}{2}\frac{Q}{V}\times V^2 \quad \text{i.e.} \quad 1 = \frac{1}{2}\times5000\times10^{-6}\times V$$

$$\therefore \quad V = \mathbf{400\ V} \quad \text{...Voltage}$$

b) $$C = \frac{Q}{V} = \frac{5000\times10^{-6}}{400} = \mathbf{12.5\ \mu F} \quad \text{... Capacitor}$$

Example 2.23 : *When two capacitors A and B are connected across 200 V d.c. supply, the potential difference across A is 120 V and that across B is 80 V. The potential difference across A rises to 140 V when B is shunted by 3 μF capacitor. Find the capacitances of A and B.* **[GU : July - 2007]**

Solution :

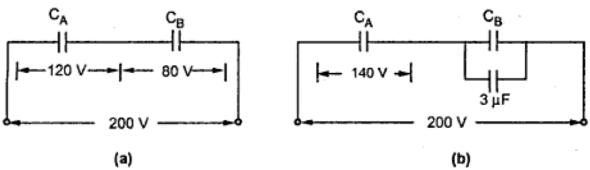


Fig. 2.34

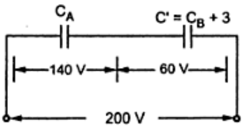


Fig. 2.34 (c)

C_B and 3 μF are in parallel hence
$C' = C_B + 3.$

For series capacitors, charge remains same.

$\therefore\ Q = C_A V_A = C'V'$

$\therefore\ C_A \times 140 = (C_B + 3) \times 60$

$\therefore\ C_A = 0.4285\ (C_B + 3)$ ... (1)

Similarly for the Fig. 2.34 (a), C_A and C_B are in series and for them charge is same.

$\therefore\quad Q = C_A V_A = C_B V_B \quad$ i.e. $\quad 120\ C_A = 80\ C_B$

$\therefore\quad C_A = 0.666\ C_B$ ... (2)

Use equation (2) in equation (1), $0.666\ C_B = 0.4285\ (C_B + 3)$

$$\therefore\quad C_B = \frac{3}{0.555} = \mathbf{5.399\ \mu F}, \quad C_A = \mathbf{3.599\ \mu F}$$

Example 2.24 : *A parallel plate capacitor has plate area of 4 cm². The plates are seperated by three slabs of different dielectric materials of thickness 0.3, 0.4 and 0.3 mm. Find the capacitance of each material and voltage across them if supply voltage is 1000 V. Take $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m. The relative permittivities of the materials are 3, 1.5 and 2.*

[GU : July-2004]

Solution : The capacitor is shown in the Fig. 2.35.

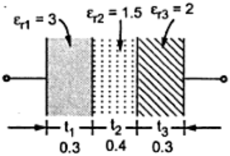


Fig. 2.35

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{t_1} = \frac{8.854 \times 10^{-12} \times 3 \times 4 \times 10^{-4}}{0.3 \times 10^{-3}}$$

$= \mathbf{35.416\ pF.}$

$$C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A}{t_2} = \frac{8.854 \times 10^{-12} \times 1.5 \times 4 \times 10^{-4}}{0.4 \times 10^{-3}} = \mathbf{13.281\ pF}$$

$$C_3 = \frac{\varepsilon_0 \varepsilon_{r3} A}{t_3} = \frac{8.854 \times 10^{-12} \times 2 \times 4 \times 10^{-4}}{0.3 \times 10^{-3}} = \mathbf{23.61\ pF}$$

All three are in series hence charge across them is same.

$$\therefore \quad Q = C_1V_1 = C_2V_2 = C_3V_3 = C_{eq}V \qquad \text{... V = 1000 V.}$$

Now, $$\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \quad \text{i.e.} \quad C_{eq} = 6.8547 \text{ pF}$$

$$\therefore \quad Q = 6.8547\times10^{-12}\times1000 = 6.8547\times10^{-9} \text{ C}$$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \mathbf{193.548\ V}, \quad V_2 = \frac{Q}{C_2} = \mathbf{516.1283\ V}, \quad V_3 = \frac{Q}{C_3} = \mathbf{290.323\ V}$$

Example 2.25 : *A capacitor of 10 μF is connected to a d.c. supply through a resistance of 1.1 MΩ. Calculate the time taken for the capacitor to reach 90 % of its final charge.*
[GU : Mar.-2009, July-2005, Nov.-2005]

Solution : C = 10 μF, R = 1.1 MΩ

The equation for capacitor voltage is,

$$V_C = V\left(1-e^{-t/RC}\right) = V\left(1-e^{-t/10\times10^{-6}\times1.1\times10^{6}}\right)$$

$$\therefore \quad V_C = V\left(1-e^{-0.0909t}\right)$$

Now $$V_C = 90\text{ \% of } V = 0.9 \text{ V}$$

$$\therefore \quad 0.9\text{ V} = V\left(1-e^{-0.0909t}\right) \quad \text{i.e.} \quad e^{-0.0909t} = 0.1$$

$$\therefore \quad -0.0909\text{ t} = \ln 0.1 = -2.30258$$

$$\therefore \quad t = \mathbf{25.328\ sec}$$

Example 2.26 : *Two capacitors 4 μF and 8 μF are connected in series and charged from a constant voltage of 210 V supply. Calculate : a) The voltage across each capacitor b) The charge on each capacitor.* **[GU : March-2009]**

Solution :

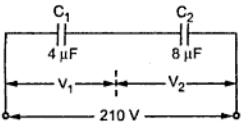


Fig. 2.36

For capacitors in series, the charge remains same.

$$\therefore Q = C_1V_1 = C_2V_2 = C_{eq}V$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{4\times8}{4+8} = 2.667\ \mu\text{F}$$

$$\therefore Q = 2.667\times10^{-6}\times210 = 5.6\times10^{-4} \text{ C}$$

a) $V_1 = \frac{Q}{C_1} = \frac{5.6\times10^{-6}}{4\times10^{-6}} = \mathbf{140\ V}$, $V_2 = \frac{Q}{C_2} = \frac{5.6\times10^{-4}}{8\times10^{-6}} = \mathbf{70\ V}$

b) The charge on each capacitor $= Q = \mathbf{5.6\times10^{-4}\ C}$

Example 2.27 : *A 10 μF capacitor is connected in series with a 1 MΩ resistor. This combination is connected across a 100 V d.c. supply. Determine i) Time constant of the circuit ii) The initial value of the charging current iii) The initial rate of rise of voltage across the capacitor iv) Time taken for the capacitor voltage to reach 60 V.*

[GTU : June-2009]

Solution : C = 10 μF, R = 1 MΩ, V = 100 V

i) $\tau = RC = 1\times10^{6}\times10\times10^{-6} = \mathbf{10\ sec}$ ... Time constant

ii) $i = \frac{V}{R} = \frac{100}{1\times10^{6}} = \mathbf{100\ \mu A}$ ... Initial current

iii) $\left.\frac{dV_C}{dt}\right|_{t=0} = \frac{V}{RC} = \frac{100}{10} = \mathbf{10\ V/sec}$

iv) $V_C = V\left(1-e^{-t/RC}\right) = 100\left(1-e^{-t/10}\right)$ and $V_C = 60\ V$

$\therefore$ $60 = 100\left(1-e^{-t/10}\right)$ i.e. $e^{-t/10} = 1-0.6 = 0.4$

$\therefore$ $ln\ 0.4 = -\frac{t}{10}$ i.e. $-\frac{t}{10} = -0.9163$

$\therefore$ $t = \mathbf{9.163\ sec}$

Review Questions

1. *Explain the concept of charge. What is its unit ?*
2. *Explain the laws of electrostatics.*
3. *What is electric field and electric lines of force ? State the properties of electric lines of force.*
4. *State and explain Coulomb's law.*
5. *Define the following terms stating their units.*
   i) Electric flux *ii) Electric field intensity*
   iii) Electric flux density *iv) Surface charge density*
   v) Absolute permittivity *vi) Relative permittivity*
6. *Derive the relation between electric field intensity and electric flux density.*
7. *What is electric potential ?What is its unit ? Define the unit.*
8. *Explain the concept of potential difference.*

9. *Derive the expressions for an electric potential and potential difference.*
10. *Explain what is potential gradient in an electric field.*
11. *What is capacitor question? Define its unit.*
12. *Obtain expressions for equivalent capacitance when the capacitors are connected in*
    i) Series and ii) Parallel
13. *With usual notation derive an expression for capacitance of a parallel plate capacitor.*
14. *Derive the equation for the capacitance of a multiple plate capacitor.*
15. *Derive the equation of the capacitance of a composite capacitor consisting of three different dielectric media with different thicknesses and relative permittivities.*
16. *Derive the expression for the energy stored in a capacitor.*
17. *Explain the action of a capacitor, when connected to a battery of V volts.*
18. *A parallel plate capacitor has plates, each of area 100 cm², separated by a distance of 3 cms. The dielectric between the plates has relative permittivity of 2.2. The potential difference between the plates is 10 kV.*
    Find (i) Capacitance of the capacitor ; (ii) Surface charge density; (iii) Field intensity; (iv) Energy stored. **(Ans. : 6.493 pF, 6.493×10^{-6} C/m^2, 3.3×10^5 V/m, 3.246×10^{-4} J)**
19. *Calculate the energy stored in a parallel plate capacitor which consists of two metal plates 60 cm², separated by a dielectric of 1.5 mm thick and of relative permittivity 3.5, if a potential difference of 1000 V is applied across it.* **(Ans. : 6.1978×10^{-5} J)**
20. *A potential difference of 400 V is maintained across a capacitor of 25 μF. Calculate (i) charge; (ii) electric field strength ; (iii) electric flux density. The distance between the plates of a capacitor is 0.5 mm and area of cross-section of plates is 1.2 cm². Find also the energy stored in the capacitor.* **(Ans. : .01 C, 8×10^5 V/m, 83.33 C/m^2, 2 J)**
21. *The capacity of a parallel plate capacitor is 0.0005 μF. The capacitor is made up of plates having area 200 cm², separated by a dielectric of 5 mm thickness. A p.d. of 10,000 V is applied across the condenser. Find (i) charge; (ii) potential gradient in the dielectric; (iii) relative permittivity and (i) electric flux density.*
    (Ans. : 5×10^{-6} C, 2×10^6 V/m, 14.12, 2.5×10^{-4} C/m^2)
22. *Three capacitors of 5, 10 and 15 μF are connected in series across a 100 V supply. Find the equivalent capacitance and the voltage across each.*
    If the capacitor, after being charged in series are disconnected and then connected in parallel, with plates of like polarity together, find the total charge of the parallel combination.
    (Ans. : 2.727 μF, 54.54 V, 27.27 V, 18.18 V, 8.181×10^{-4} C)
23. *Three capacitors A, B and C are charged as follows : A = 10 μF, 100 V; B = 15 μF, 150 V and C = 25 μF, 200 V. They are now connected in parallel with terminals of like polarity together. Find the voltage across the combination.* **(Ans. : 50 μF, 165 V)**

24. *A capacitor consists of two metallic plates, each 40 cm × 40 cm and placed 6 mm apart. The space between plates is filled with a glass plate 5 mm thick and a layer of paper 1 mm thick. The relative permittivities are 8 and 2 respectively. Calculate its capacitance.* **(Ans. : 1.26 nF)**

25. *A capacitor of 10 μF is charged to a p.d. of 200 V and then connected in parallel with an uncharged capacitor of 5 μF. Find the p.d. across the parallel combination and the energy stored in each capacitor.* **(Ans. : 133.33 V, 0.089 J, 0.044 J)**

26. *Determine the equivalent capacitance of the combination shown in Fig. 2.37.* **(Ans. : 2.85 F)**

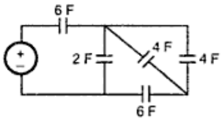


Fig. 2.37

27. *Three capacitors A, B and C are connected in series across 100 V d.c. supply. The p.d. across the capacitors are 20 V, 30 V and 50 V respectively. If the capacitance of A is 10 μF, calculate the capacitances of B and C.*

(Ans. : 6.67 μF, 4 μF)

University Questions

Q.1 *Derive the expression for the voltage across the capacitor at any instant after the application of dc voltage V to a circuit having a capacitance C in series with resistance R.* **[GTU. : Dec.-2008, 7 Marks]**

Q.2 *Derive an expression for the capacitance of a parallel plate capacitor with plate area 'A' and distance of separation between the plates 'd' in M.K.S.* **[GTU. : March-2009, 7 Marks]**

Q.3 *Derive an expression for the equivalent capacitance of parallel plate capacitors when they are connected in (i) series and (ii) Parallel.* **[GTU. : June-2009, 7 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

❑❑❑

3 Electromagnetics

3.1 Introduction

All of us are familiar with a magnet. It is a piece of solid body which possesses a property of attracting iron pieces and pieces of some other metals. This is called a **natural magnet**. While as per the discovery of Scientist **Oersted** we can have an electromagnet. Scientist Oersted stated that any current carrying conductor is always surrounded by a magnetic field. The property of such current is called **magnetic effect of an electric current**. Natural magnet or an electromagnet, both have close relation with electromotive force (e.m.f.), mechanical force experienced by conductor, electric current etc. To understand this relationship it is necessary to study the fundamental concepts of magnetic circuits.

3.2 Magnet and its Properties

As stated earlier, magnet is a piece of solid body which possesses property of attracting iron and some other metal pieces.

i) When such a magnet is rolled into iron pieces it will be observed that iron pieces cling to it as shown in Fig. 3.1.

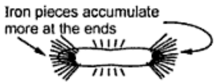


Fig. 3.1 Natural magnet

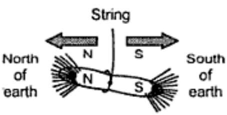


Fig. 3.2 Freely suspended

The maximum iron pieces accumulate at the two ends of the magnet while very few accumulate at the centre of the magnet.

The points at which the iron pieces accumulate maximum are called **Poles** of the magnet while imaginary line joining these poles is called **Axis** of the magnet.

ii) When such magnet is suspended freely by a piece of silk fibre, it turns and always adjusts itself in the direction of North and South of the earth.

The pole which adjusts itself in the direction of North is called North seeking or **North (N) pole,** while the pole which points in the direction of South is called South seeking or **South (S) pole**. Such freely suspended magnet is shown in the Fig. 3.2.

This is the property due to which it is used in the compass needle which is used by navigators to find the directions.

iii) When a magnet is placed near an iron or steel piece, its property of attraction gets transferred to iron or steel piece. Such transfer of property of attraction is also possible by actually rubbing the pole of magnet on an iron or steel piece. Such property is called **magnetic induction.**

Magnetic induction : The phenomenon due to which a magnet can induce magnetism in a (iron or steel) piece of magnetic material placed near it without actual physical contact is called magnetic induction.

iv) An ordinary piece of magnetic material when brought near to any pole N or S gets attracted towards the pole. But if another magnet is brought near the magnet such that two like poles ('N' and 'N' or 'S' and 'S'), it shows a repulsion in between them while if two unlike poles are brought near, it shows a force of attraction.

Key Point : *Like poles repel each other and the unlike poles attract each other. Repulsion is the sure test of magnetism as ordinary piece of magnetic material always shows attraction towards both the poles.*

Let us see the molecular theory behind this magnetism.

3.3 Molecular Theory of Magnetization

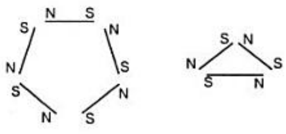


Fig. 3.3 Molecular magnets in unmagnetized material

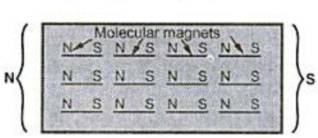


Fig. 3.4 Magnetized piece of material

Not only magnetized but materials like iron, steel are also complete magnets according to molecular theory. All materials consist of small magnets internally called **molecular magnets.** In unmagnetized materials such magnets arrange themselves in closed loops as shown in the Fig. 3.3.

So at any joint, effective strength at a point is zero, due to presence of two unlike poles. Such poles cancel each other's effect. But if magnetized material is considered or unmagnetized material subjected to magnetizing force is considered, then such small molecular magnets arrange themselves in the direction of magnetizing force, as shown in the Fig. 3.4.

Unlike poles of these small magnets in the middle are touching each other and hence neutralizing the effect.

But on one end 'N' poles of such magnets exist without neutralizing effect. Similarly on other end 'S' poles of such magnets exist. Thus one end behaves as 'N' pole while other as 'S' pole. So most of the iron particles get attracted towards end and not in the middle.

From this theory, we can note down the following points :

1) When magnetizing force is applied, immediately it is not possible to have alignment of all such small magnets, exactly horizontal as shown in the Fig. 3.4. There is always some limiting magnetizing force exists for which all such magnets align exactly in horizontal position.

Key Point : *Though magnetizing force is increased beyond certain value, there is no chance for further alignment of molecular magnets hence further magnetization is not possible. Such condition or phenomenon is called* ***saturation****.*

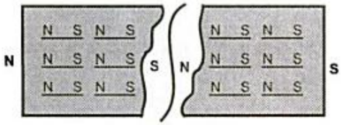


Fig. 3.5 Breaking of magnet

2) If the magnet is broken at any point, each piece behaves like an independent magnet with two poles to each, 'N' and 'S'.

3) The piece of soft iron gets magnetized more rapidly than hard steel. This is because alignment of molecular magnets in soft iron takes place quickly for less magnetizing force than in hard steel.

4) If unmagnetized piece is subjected to alternating magnetizing force i.e. changing magnetizing force, then heat is produced. This is because molecular magnets try to change themselves as per change in magnetizing force. So due to molecular friction heat is generated.

5) If a magnet is heated and allowed to cool, it demagnetizes. This is because heat sets molecular magnets into motion so that the molecules again form a closed loop, neutralizing the magnetism.

6) **Retentivity** : When a soft iron piece is magnetized by external magnetizing force due to magnetic induction, it loses its magnetism immediately if such force is removed. As against this hard steel continues to show magnetism though such force is removed. It retains magnetism for some time.

Key Point: *The power of retaining magnetism after the magnetizing force is removed is called* ***retentivity****. The time for which material retains such magnetism in absence of magnetizing force depends on its retentivity.*

3.4 Laws of Magnetism

There are two fundamental laws of magnetism which are as follows :

Law 1 : *It states that 'Like magnetic poles repel and unlike poles attract each other'.*

This is already mentioned in the properties of magnet.

Law 2 : *This law is experimentally proved by Scientist Coulomb and hence also known as Coulomb's law.*

The force (F) exerted by one pole on the other pole is,

a) directly proportional to the product of the pole strengths,

b) inversely proportional to the square of the distance between them and

c) nature of medium surrounding the poles.

Mathematically this law can be expressed as,

$$F \propto \frac{M_1 M_2}{d^2}$$

where M_1 and M_2 are pole strengths of the poles while d is distance between the poles.

$$\therefore \qquad \boxed{F = \frac{K M_1 M_2}{d^2}}$$

where K depends on the nature of the surroundings and called permeability.

3.5 Magnetic Field

We have seen that magnet has its influence on the surrounding medium. 'The region around a magnet within which the influence of the magnet can be experienced is called **magnetic field**'. Existence of such field can be experienced with the help of compass needle, iron or pieces of metals or by bringing another magnet in vicinity of a magnet.

3.5.1 Magnetic Lines of Force

The magnetic field of magnet is represented by imaginary lines around it which are called **magnetic lines of force.** Note that these lines have no physical existence, these are purely imaginary and were introduced by **Michael Faraday** to get the visualization of distribution of such lines of force.

3.5.2 Direction of Magnetic Field

The direction of magnetic field can be obtained by conducting small experiment.

Let us place a permanent magnet on table and cover it with a sheet of cardboard. Sprinkle steel or iron fillings uniformly over the sheet. Slight tapping of cardboard causes fillings to adjust themselves in a particular pattern as shown in the Fig. 3.6.

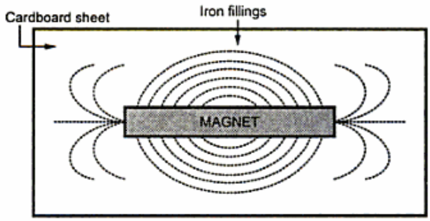


Fig. 3.6 Magnetic lines of force

The shape of this pattern projects a mental picture of the magnetic field present around a magnet.

A line of force can be defined as,

Consider the isolated N pole (we cannot separate the pole but imagine to explain line of force) and it is allowed to move freely, in a magnetic field. Then path along which it moves is called **line of force.** Its shape is as shown in the Fig. 3.6 and direction always from N-pole towards S-pole.

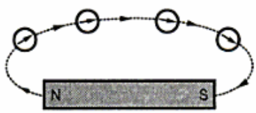


Fig. 3.7 Compass needle experiment

The direction of lines of force can be understood with the help of small compass needle. If magnet is placed with compass needles around it, then needles will take positions as shown in the Fig. 3.7. The tangent drawn at any point, of the dotted curve shown, gives direction of resultant force at that point. The N poles are all pointing along the dotted line shown, from N - pole to its S-pole.

The lines of force for a bar magnet and U-shaped magnet are shown in the Fig. 3.8.

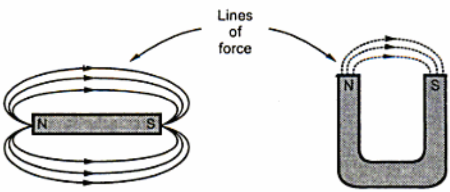


Fig. 3.8 (a) Bar magnet

Fig. 3.8 (b) U-shaped magnet

Attraction between the unlike poles and repulsion between the like poles of two magnets can be easily understood from the direction of magnetic lines of force. This is shown in the Fig. 3.9 (a) and (b).

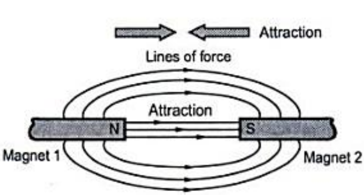


Fig. 3.9 (a) Force of attraction

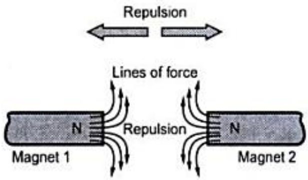


Fig. 3.9 (b) Force of repulsion

3.5.3 Properties of Lines of Force

Though the lines of force are imaginary, with the help of them various magnetic effects can be explained very conveniently. Let us see the various properties of these lines of force.

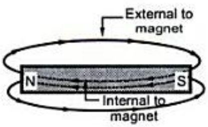


Fig. 3.10 Lines of force complete the closed path

1) Lines of force are always originating on a N-pole and terminating on a S-pole, external to the magnet.
2) Each line forms a **closed loop** as shown in the Fig. 3.10.

> **Key Point**: *This means that a line emerging from N-pole, continues upto S-pole* ***external to the magnet*** *while it is assumed to continue from S-pole to N-pole* ***internal to the magnet*** *completing a closed loop. Such lines internal to the magnet are called as* ***lines of induction.***

3) Lines of force never intersect each other.
4) The lines of force, are like stretched rubberbands and always try to contract in length.
5) The lines of force, which are parallel and travelling in the same direction repel each other.
6) Magnetic lines of force always prefer a path offering least opposition.

> **Key Point** : *The opposition by the material to the flow of lines of force is called reluctance. Air has more reluctance while magnetic materials like iron, steel etc. have low reluctance. Thus magnetic lines of force can easily pass through iron or steel but cannot pass easily through air.*

3.6 Magnetic Flux (ϕ)

The total number of lines of force existing in a particular magnetic field is called **magnetic flux**. Lines of force can be called **lines of magnetic flux**. The unit of flux is weber and flux is denoted by symbol (ϕ). The unit weber is denoted as Wb.

$$1 \text{ weber} = 10^8 \text{ lines of force}$$

3.7 Pole Strength

We have seen earlier that force between the poles depends on the pole strengths. As we are now familiar with flux, we can have idea of pole strength. Every pole has a capacity to radiate or accept certain number of magnetic lines of force i.e. magnetic flux which is called its **strength**. Pole strength is measurable quantity assigned to poles which depends on the force between the poles. If two poles are exerting equal force on one other, they are said to have equal pole strengths.

Unit of pole strength is **weber** as pole strength is directly related to flux i.e. lines of force.

Key Point: *A unit pole may be defined as that pole which when placed from an identical pole at a distance of 1 metre in free space experiences a force of* $\frac{10^7}{16\pi^2}$ *newtons.*

So when we say unit N-pole, it means a pole is having a pole strength of 1 weber.

3.8 Magnetic Flux Density (B)

It can be defined as 'The flux per unit area (a) in a plane at right angles to the flux is known as **flux density**'. Mathematically,

$$B = \frac{\phi}{a} \quad \frac{Wb}{m^2} \text{ or tesla}$$

It is shown in the Fig. 3.11.

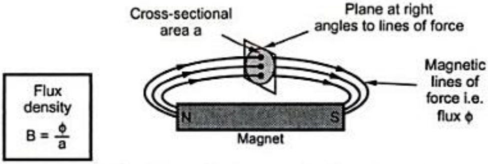


Fig. 3.11 Concept of magnetic flux density

Key Point: *The unit of flux density is Wb/m², also called* ***tesla*** *denoted as T.*

3.9 Magnetic Field Strength (H)

This gives quantitative measure of strongness or weakness of the magnetic field. Note that pole strength and magnetic field strength are different. This can be defined as 'The force experienced by a unit N-pole (i.e. N - pole with 1 Wb of pole strength) when placed at any point in a magnetic field is known as **magnetic field strength** at that point.

It is denoted by H and its unit is newtons per weber i.e. (N/Wb) or amperes per metre (A/m) or ampere turns per metre (AT/m). The mathematical expression for calculating magnetic field strength is,

$$H = \frac{\text{ampere turns}}{\text{length}}$$

$$\therefore \qquad H = \frac{NI}{l} \text{ AT/m}$$

Key Point : *More the value of 'H', more stronger is the magnetic field. This is also called magnetic field intensity.*

3.10 Magnetic Effect of an Electric Current (Electromagnets)

When a coil or a conductor carries a current, it produces the magnetic flux around it. Then it starts behaving as a magnet. Such a current carrying coil or conductor is called an **electromagnet.** This is due to magnetic effect of an electric current.

If such a coil is wound around a piece of magnetic material like iron or steel and carries current then piece of material around which the coil is wound, starts behaving as a magnet, which is called an electromagnet.

The flux produced and the flux density can be controlled by controlling the magnitude the current.

The direction and shape of the magnetic field around the coil or conductor depends on the direction of current and shape of the conductor through which it is passing. The magnetic field produced can be experienced with the help of iron fillings or compass needle.

Let us study two different types of electromagnets.

1) Electromagnet due to straight current carrying conductor
2) Electromagnet due to circular current carrying coil

3.10.1 Magnetic Field due to Straight Conductor

When a straight conductor carries a current, it produces a magnetic field all along its length. The lines of force are in the form of concentric circles in the planes right angles to the conductor. This can be demonstrated by a small experiment.

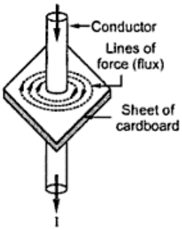


Fig. 3.12 Magnetic field due to a straight conductor

Consider a straight conductor carrying a current, passing through a sheet of cardboard as shown in the Fig. 3.12. Sprinkle iron fillings on the cardboard. Small tapping on the cardboard causes the iron filling to set themselves, in the concentric circular pattern. The direction of the magnetic flux can be determined by placing compass needle near the conductor. This direction depends on the direction of the current passing through the conductor. For the current direction shown in the Fig. 3.12 i.e. from top to bottom the direction of flux is clockwise around the conductor.

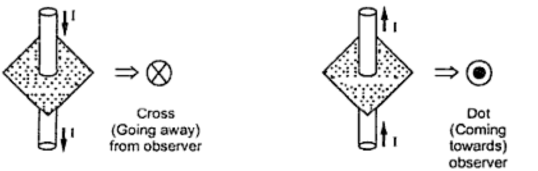
Conventionally such current carrying conductor is represented by small circle, (top view of conductor shown in the Fig. 3.12). Then current through such conductor will either come out of paper or will go into the plane of the paper.

Key Point: *When current is going into the plane of the paper, i.e. away from observer, it is represented by a 'cross', inside the circle indicating the conductors.*

The cross indicates rear view of feathered end of an arrow.

Key Point: *The current flowing towards the observer i.e. coming out of the plane of the paper is represented by a 'dot' inside the circle.*

The dot indicates front view i.e. tip of an arrow. This is shown in the Fig. 3.13.



(a) Current into the paper **(b) Current out of the paper**

Fig. 3.13 Cross and dot convention

3.10.1.1 Rules to Determine Direction of Flux Around Conductor

1) Right hand thumb rule : It states that, hold the current carrying conductor in the right hand such that the thumb pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field or flux around it. The Fig. 3.14 explains the rule.

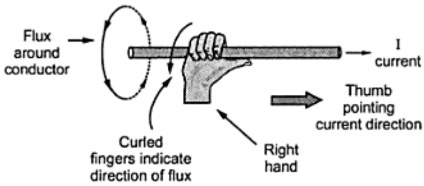


Fig. 3.14 Right hand thumb rule

Let us apply this rule to the conductor passing through card sheet considered earlier. This can be explained by the Fig. 3.15.

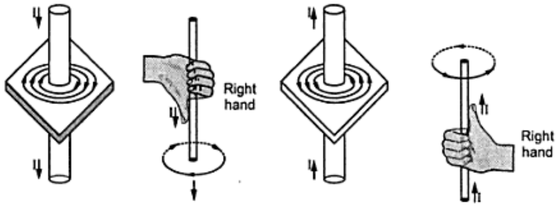


Fig. 3.15 Direction of magnetic lines by right hand thumb rule

Conventionally it is shown as in the Fig. 3.16.

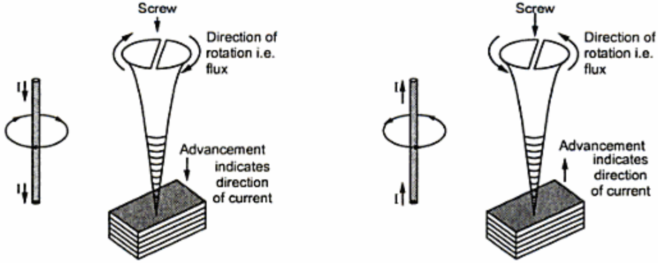
(a) Clockwise **(b) Anticlockwise**

Fig. 3.16 Representation of direction of flux

2) Corkscrew rule : Imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of the current flow.

Then the direction of the magnetic field is given by the direction in which the screw must be turned so as to advance in the direction of the current.

This is shown in the Fig. 3.17.



(a) Clockwise rotation (b) Anticlockwise rotation

Fig. 3.17 Corkscrew rule

3.10.2 Magnetic Field due to Circular Conductor i.e. Solenoid

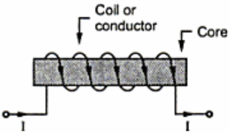


Fig. 3.18 (a) Solenoid

A **solenoid** is an arrangement in which long conductor is wound with number of turns close together to form a coil. The axial length of conductor is much more than the diameter of turns. The part or element around which the conductor is wound is called as **core** of the solenoid. Core may be air or may be some magnetic material. Solenoid with a steel or iron core in shown in Fig. 3.18 (a).

When such conductor is excited by the supply so that it carries a current then it produces a magnetic field which acts through the coil along its axis and also around the solenoid. Instead of using a straight core to wound the conductor, a circular core also can be used to wound the conductor. In such case the resulting solenoid is called **toroid**. Use of magnetic material for the core produces strong magnet. This is because current carrying conductor produces its own flux. In addition to this, the core behaves like a magnet due to magnetic induction, producing its own flux. The direction of two fluxes is same due to which resultant magnetic field becomes more strong.

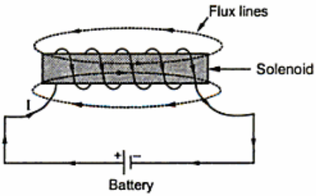


Fig. 3.18 (b) Flux around a solenoid

The pattern of the flux around the solenoid is shown in the Fig. 3.18 (b).

The rules to determine the direction of flux and poles of the magnet formed :

1) The right hand thumb rule :

Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid point to the **north pole** of the solenoid or point the direction of flux lines **inside the core.**

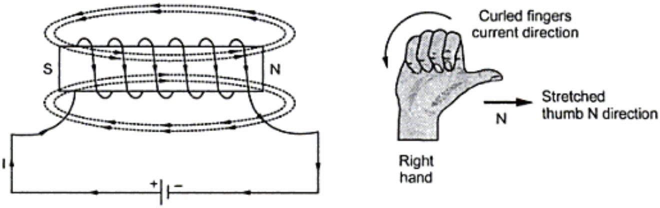


Fig. 3.19 (a) Direction of flux around a solenoid

This is shown in Fig. 3.19 (a) and (b).

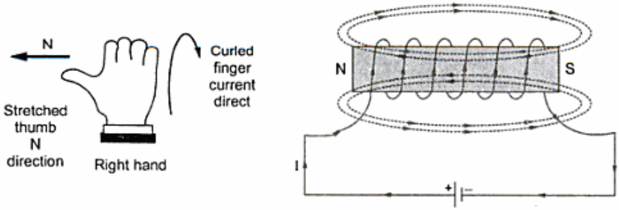


Fig. 3.19 (b) Direction of flux around a solenoid

In case of toroid, the core is circular and hence using right hand thumb rule, the direction of flux in the core, due to current carrying conductor can be determined. This is shown in the Fig. 3.20 (a) and (b). In the Fig. 3.20 (a), corresponding to direction of

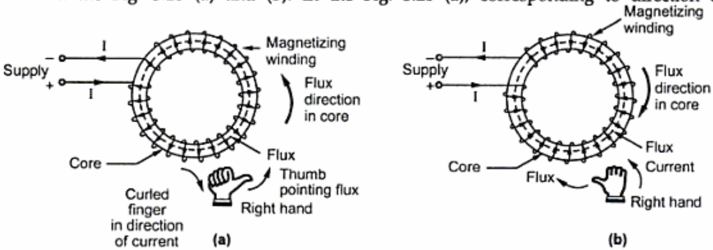


Fig. 3.20

winding, the flux set in the core is anticlockwise while in the Fig. 3.20 (b) due to direction of winding, the direction of flux set in the core is clockwise. The winding is also called **magnetizing winding** or **magnetizing coil** as it magnetizes the core.

2) Corkscrew rule : If axis of the screw is placed along the axis of the solenoid and if screw is turned in the direction of the current, then it travels towards the **N-pole** or in the direction of the magnetic field **inside** the solenoid.

3) End rule : If solenoid is observed from any one end then its polarity can be decided by noting direction of the current.

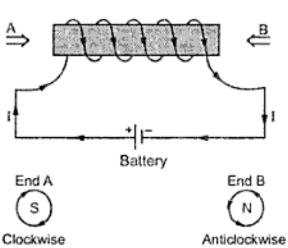


Fig. 3.21 End rule

Consider solenoid shown in the Fig. 3.21.

If it is seen from the end A, current will appear to flow in clockwise direction, so that end behaves as S-pole of the magnet. While as seen from the end B, current appears to flow in anticlockwise direction then that end which is B, behaves as N-pole of the magnet.

Generally right hand thumb rule is used to determine direction of flux and nature of the poles formed. Using such concept of an electromagnet, various magnetic circuits can be obtained.

3.11 Permeability

The flow of flux produced by the magnet not only depends on the magnetic field strength but also on one important property of the magnetic material called permeability. It is related to the medium in which magnet is placed. The force exerted by one magnetic pole on other depends on the medium in which magnets are placed.

Key Point : *The permeability is defined as the ability or ease with which the magnetic material forces the magnetic flux through a given medium.*

For any magnetic material, there are two permeabilities.

i) Absolute permeability ii) Relative permeability.

3.11.1 Absolute Permeability (μ)

The magnetic field strength (H) decides the flux density (B) to be produced by the magnet around it, in a given medium. The ratio of magnetic flux density B in a particular medium (other than vacuum or air) to the magnetic field strength H producing that flux density is called **absolute permeability** of that medium.

It is denoted by μ and mathematically can be expressed as,

$$\mu = \frac{B}{H}$$

i.e.

$$B = \mu H$$

The permeability is measured in units henries per metre denoted as H/m.

3.11.2 Permeability of Free Space or Vacuum (μ_0)

If the magnet is placed in a free space or vacuum or in air then the ratio of flux density B and magnetic field strength H is called **permeability of free space** or **vacuum** or **air**.

It is denoted as μ_0 and measured in H/m. It denotes the ease with which the magnetic flux permeates the free space or vacuum or air.

It is experimentally found that this μ_0 i.e. ratio of B and H in vacuum remains constant everywhere in the vacuum and its value is $4\pi\times10^{-7}$ H/m.

$$\therefore \quad \mu_0 = \frac{B}{H} \text{ in vacuum } = 4\pi\times10^{-7} \text{ H/m}$$

Key Point : *For a magnetic material, the absolute permeability μ is not constant. This is because B and H bears a nonlinear relation in case of magnetic materials. If magnetic field strength is increased, there is change in flux density B but not exactly proportional to the increase in H.*

The ratio B to H is constant only for free space, vacuum or air which is $\mu_0 = 4\pi\times10^{-7}$ H/m.

3.11.3 Relative Permeability (μ_r)

Generally the permeability of different magnetic materials is defined relative to the permeability of free space (μ_0). The **relative permeability** is defined as the ratio of flux density produced in a medium (other than free space) to the flux density produced in free space, under the influence of same magnetic field strength and under identical conditions.

Thus if the magnetic field strength is H which is producing flux density B in the medium while flux density B_0 in free space then the relative permeability is defined as,

$$\mu_r = \frac{B}{B_0} \quad \text{where H is same.}$$

It is dimensionless and has no units.

For free space, vacuum or air, $\mu_r = 1$

According to definition of absolute permeability we can write for given H,

$$\mu = \frac{B}{H} \quad \text{in medium} \qquad ...(1)$$

$$\mu_0 = \frac{B_0}{H} \quad \text{in free space} \qquad ...(2)$$

Dividing equation (1) and equation (2) , $\frac{\mu}{\mu_0} = \frac{B}{B_0}$

but $$\frac{B}{B_0} = \mu_r$$

$$\therefore \quad \frac{\mu}{\mu_0} = \mu_r$$

$$\therefore \quad \boxed{\mu = \mu_0 \mu_r \quad H/m}$$

The relative permeability of metals like iron, steel varies from 100 to 100,000.

Key Point: *If we require maximum flux production for the lesser magnetic field strength then the value of the relative permeability of the core material should be as high as possible.*

For example if relative permeability of the iron is 1000 means it is 1000 times more magnetic than the free space or air.

3.12 Magnetomotive Force (M.M.F. or F)

The flow of electrons is current which is basically due to electromotive force (e.m.f.). Similarly the force behind the flow of flux or production of flux in a magnetic circuit is called magnetomotive force (m.m.f.). The m.m.f. determines the magnetic field strength.

It is the driving force behind the magnetic circuit. It is given by the product of the number of turns of the magnetizing coil and the current passing through it.

Mathematically it can be expressed as,

$$\boxed{M.M.F. = N\,I \quad \text{ampere turns}}$$

where N = Number of turns of magnetizing coil

I = Current through coil

Its unit is ampere turns (AT) or amperes (A).

It is also defined as the work done in joules on a unit magnetic pole in taking it once round a closed magnetic circuit.

3.13 Reluctance (S)

In an electric circuit, current flow is opposed by the resistance of the material, similarly there is opposition by the material to the flow of flux which is called **reluctance.**

It is defined as the resistance offered by the material to the flow of magnetic flux through it. It is denoted by 'S'. It is directly proportional to the length of the magnetic circuit while inversely proportional to the area of cross-section.

$$S \propto \frac{l}{a} \quad \text{where } 'l' \text{ in 'm' while 'a' in } 'm^2'$$

$$\therefore \quad S = \frac{Kl}{a}$$

where K = Constant of proportionality

= Reciprocal of absolute permeability of material $= \frac{1}{\mu}$

$$\therefore \quad \boxed{S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a} \ \text{A/Wb}}$$

It is measured in amperes per weber (A/Wb).

The reluctance can be also expressed as the ratio of magnetomotive force to the flux produced.

i.e. $\text{Reluctance} = \frac{\text{M.M.F}}{\text{Flux}}$

$$\therefore \quad \boxed{S = \frac{NI}{\phi} \quad \text{AT/Wb or A/Wb}}$$

3.14 Permeance

The **permeance** of the magnetic circuit is defined as the reciprocal of the reluctance.

It is defined as the property of the magnetic circuit due to which it allows flow of the magnetic flux through it.

$$\therefore \quad \boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}}$$

It is measured in weber per amperes (Wb/A).

3.15 Magnetic Circuits

The **magnetic circuit** can be defined as, the closed path traced by the magnetic lines of force i.e. flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f., flux reluctance, permeability etc.

Consider simple magnetic circuit shown in the Fig. 3.22 (a). This circuit consists of an iron core with cross-sectional area of 'a' m^2 with a mean length of 'l' m. (This is mean length of the magnetic path which flux is going to trace.) A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil. This current carrying coil produces the flux (ϕ) which completes its path through the core as shown in the Fig. 3.22 (a).

This is analogous to simple electric circuit in which a supply i.e. e.m.f. of E volts drives a current I which completes its path through a closed conductor having resistance R. This analogous electrical circuit is shown in the Fig. 3.22 (b).

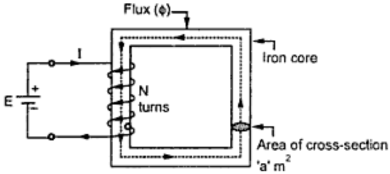


Fig. 3.22 (a) Magnetic circuit

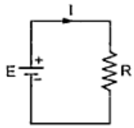


Fig. 3.22 (b) Electrical equivalent

Let us derive relationship between m.m.f., flux and reluctance.

I = Current flowing through the coil

N = Number of turns

ϕ = Flux in webers

B = Flux density in the core

μ = Absolute permeability of the magnetic material

μ_r = Relative permeability of the magnetic material

Magnetic field strength inside the solenoid is given by,

$$H = \frac{NI}{l} \quad \text{AT/m} \qquad \ldots(1)$$

Now flux density is,

$$B = \mu H$$

$$B = \frac{\mu_0 \mu_r NI}{l} \quad \text{Wb/m}^2 \qquad \ldots(2)$$

Now as area of cross-section is 'a ' m^2, total flux in core is,

$$\phi = B\,a = \frac{\mu_0 \mu_r NIa}{l} \quad \text{Wb} \qquad \ldots(3)$$

i.e.

$$\phi = \frac{NI}{\dfrac{l}{\mu_0 \mu_r a}}$$

$\therefore$

$$\boxed{\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{F}{S}}$$

where NI = Magnetomotive force m.m.f. in AT

$$S = \frac{l}{\mu_0 \mu_r a}$$

= Reluctance offered by the magnetic path

This expression of the flux is very much similar to expression for current in electric circuit.

$$I = \frac{\text{E.M.F.}}{\text{Resistance}}$$

Key Point : *So current is analogous to the flux, e.m.f. is analogous to the m.m.f. and resistance is analogous to the reluctance.*

Example 3.1 : *An iron ring of circular cross-sectional area of 3.0 cm^2 and mean diameter of 20 cm is wound with 500 turns of wire and carries a current of 2.09 A to produce the magnetic flux of 0.5 mWb in the ring. Determine the permeability of the material.*

Solution : The given values are :

$a = 3\ cm^2 = 3 \times 10^{-4}\ m^2$, $d = 20$ cm, $N = 500$, $I = 2$ A, $\phi = 0.5$ mWb

Now, $l = \pi \times d = \pi \times 20 = 62.8318 \text{ cm} = 0.628318 \text{ m}$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{0.628313}{4\pi \times 10^{-7} \times \mu_r \times 3 \times 10^{-4}} = \frac{1.6667 \times 10^9}{\mu_r} \quad \text{... (1)}$$

$$f = \frac{\text{M.M.F.}}{S} = \frac{NI}{S}$$

$$\therefore \quad S = \frac{NI}{\phi} = \frac{500 \times 2}{0.5 \times 10^{-3}} = 2 \times 10^6 \text{ AT/Wb} \quad \text{... (2)}$$

Equating equations (1) and (2),

$$\therefore \quad 2 \times 10^6 = \frac{1.6667 \times 10^9}{\mu_r}$$

$$\therefore \quad \mu_r = \mathbf{833.334}$$

3.15.1 Series Magnetic Circuits

In practice magnetic circuit may be composed of various materials of different permeabilities, of different lengths and of different cross-sectional areas. Such a circuit is called **composite** magnetic circuit. When such parts are connected one after the other the circuit is called **series magnetic circuit**.

Consider a circular ring made up of different materials of lengths l_1, l_2 and l_3 and with cross-sectional areas a_1, a_2 and a_3 with absolute permeabilities μ_1, μ_2 and μ_3 as shown in the Fig. 3.23.

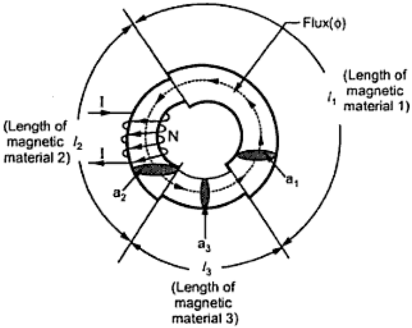


Fig. 3.23 A series magnetic circuit

Let coil wound on ring has N turns carrying a current of I amperes.

The total m.m.f. available is = NI AT.

This will set the flux 'ϕ' which is same through all the three elements of the circuit.

This is similar to three resistances connected in series in electrical circuit and connected to e.m.f. carrying same current 'I' through all of them.

Its analogous electric circuit can be shown as in the Fig. 3.24.

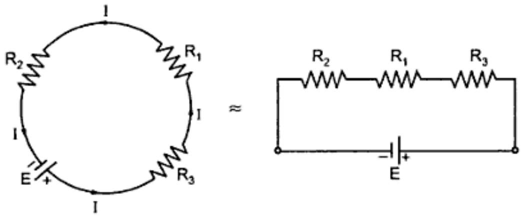


Fig. 3.24 Equivalent electrical circuit

The total resistance of the electric circuit is $R_1 + R_2 + R_3$. Similarly the total reluctance of the magnetic circuit is,

$$\text{Total } S_T = S_1 + S_2 + S_3 = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

$$\therefore \quad \text{Total } \phi = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{NI}{S_T} = \frac{NI}{(S_1 + S_2 + S_3)}$$

$$\therefore \quad N\,I = S_T\,\phi = (S_1 + S_2 + S_3)\,\phi$$

$$N\,I = S_1\,\phi + S_2\,\phi + S_3\,\phi$$

$$\therefore \quad (\text{m.m.f.})T = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 + (\text{m.m.f.})_3$$

The total m.m.f. also can be expressed as,

$$(\text{m.m.f.})T = H_1 l_1 + H_2 l_2 + H_3 l_3$$

where $H_1 = \frac{B_1}{\mu_1}, \quad H_2 = \frac{B_2}{\mu_2}, \quad H_3 = \frac{B_3}{\mu_3}$

So for a **series magnetic** circuit we can remember,

1) The magnetic flux through all the parts is same.
2) The equivalent reluctance is sum of the reluctance of different parts.
3) The resultant m.m.f. necessary is sum of the m.m.f.s in each individual part.

3.15.2 Series Circuit with Air Gap

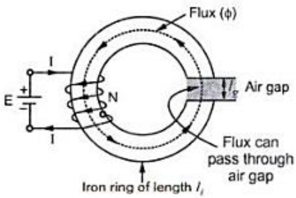


Fig. 3.25 A ring with an air gap

The series magnetic circuit can also have a short air gap.

Key Point : *This is possible because we have seen earlier that flux can pass through air also.*

Such air gap is not possible in case of electric circuit.

Consider a ring having mean length of iron part as 'l_i' as shown in the Fig. 3.25.

Total m.m.f = N I AT

Total reluctance $\boxed{S_T = S_i + S_g}$

where S_i = Reluctance of iron path

S_g = Reluctance of air gap

$$\therefore \quad S_i = \frac{l_i}{\mu a_i}$$

$$S_g = \frac{l_g}{\mu_0 a_i}$$

Key Point: *The absolute permeability of air* $\mu = \mu_0$.

The cross-sectional area of air gap is assumed to be equal to area of the iron ring.

$$\therefore \quad S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\therefore \quad \phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S_T}$$

or Total m.m.f. = m.m.f. for iron + m.m.f for air gap

$\therefore$ $$N\,I = S_i\phi + S_g\phi \quad \text{AT for ring.}$$

Example 3.2 : *An iron ring 8 cm mean diameter is made up of round iron of diameter 1 cm and permeability of 900, has an air gap of 2 mm wide. It consists of winding with 400 turns carrying a current of 3.5 A. Determine, i) m.m.f. ii) total reluctance iii) the flux iv) flux density in ring.*

Solution : The ring and the winding is shown in the Fig. 3.26.

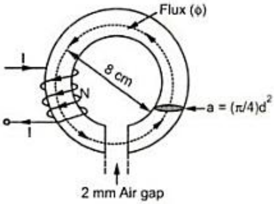


Fig. 3.26

Diameter of ring d = 8 cm,

$\therefore$ Length of iron = πd – Length of air gap

$$l_i = \pi\times\left(8\times10^{-2}\right)-2\times10^{-3}$$

$$= 0.2493 \text{ m.}$$

> **Key Point :** *While calculating iron length, do not forget to subtract length of air gap from total mean length.*

l_g = Length of air gap

$= 2 \text{ mm} = 2\times10^{-3}$ m

Diameter of iron = 1 cm

$\therefore$ Area of cross-section $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(1\times10^{-2}\right)^2$

$$a = 7.853\times10^{-5} \text{ m}^2$$

Area of cross-section of air gap and ring is to be assumed same.

i) Total m.m.f. produced = N I = 400×3.5

= **1400 AT** (ampere turns)

ii) Total reluctance $S_T = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} \quad \text{...Given } \mu_r = 900$$

$$= \frac{0.2493}{4\pi\times10^{-7}\times900\times7.853\times10^{-5}}$$

$$= 2806947.615 \text{ AT/ Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} \quad \text{as } \mu_r = 1 \text{ for air}$$

$$\therefore \quad S_g = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times7.853\times10^{-5}} = 20.2667\times10^{6} \quad \text{AT / Wb}$$

$$\therefore \quad S_T = 2806947.615+20.2667\times10^{6} = \mathbf{23.0737\times10^{6}} \quad \textbf{AT / Wb}$$

iii)
$$\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S_T} = \frac{1400}{23.0737\times10^{6}}$$

$$= \mathbf{6.067\times10^{-5}} \quad \textbf{Wb}$$

iv)
$$\text{Flux density} = \frac{\phi}{a} = \frac{6.067\times10^{-5}}{7.853\times10^{-5}} = \mathbf{0.7725} \quad \textbf{Wb / m}^2$$

3.15.3 Parallel Magnetic Circuits

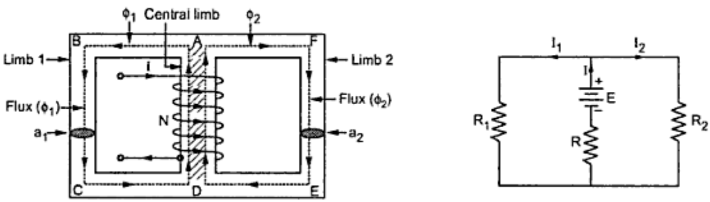
In case of electric circuits, resistances can be connected in parallel. Current through each of such resistances is different while voltage across all of them is same. Similarly different reluctances may be in parallel in case of magnetic circuits. A magnetic circuit which has more than one path for the flux is known as a **parallel magnetic circuit**.

Consider a magnetic circuit shown in the Fig. 3.27 (a). At point A the total flux ϕ, divides into two parts ϕ_1 and ϕ_2.

$$\therefore \quad \boxed{\phi = \phi_1 + \phi_2}$$

The fluxes ϕ_1 and ϕ_2 have their paths completed through ABCD and AFED respectively.

This is similar to division of current in case of parallel connection of two resistances in an electric circuit. The analogous electric circuit is shown in the Fig. 3.27 (b).



(a) Magnetic circuit (b) Equivalent electrical circuit

Fig. 3.27 A parallel magnetic circuit

The mean length of path ABCD = l_1 m

The mean length of the path AFED = l_2 m

The mean length of the path AD = l_c m

The reluctance of the path ABCD = S_1

The reluctance of path AFED = S_2

The reluctance of path AD = S_c

The total m.m.f. produced = N I AT

$$\text{Flux} = \frac{\text{M.M.F.}}{\text{Reluctance}}$$

$$\therefore \quad \text{M.M.F.} = \phi \times S$$

$$\therefore \quad \text{For path ABCDA, NI} = \phi_1 S_1 + \phi S_c$$

For path AFEDA,

$$\text{N I} = \phi_2 S_2 + \phi S_c$$

where

$$S_1 = \frac{l_1}{\mu a_1}, \quad S_2 = \frac{l_2}{\mu a_2} \quad \text{and} \quad S_c = \frac{l_c}{\mu a_c}$$

Generally

$$a_1 = a_2 = a_c = \text{Area of cross-section}$$

For parallel circuit,

$$\boxed{\text{Total m.m.f.} = \begin{matrix}\text{M.M.F. required}\\ \text{by central limb}\end{matrix} + \begin{matrix}\text{M.M.F. required by}\\ \text{any one of outer limbs}\end{matrix}}$$

$$\text{N I} = (\text{NI})_{AD} + (\text{NI})_{ABCD} \text{ or } (\text{NI})_{AFED}$$

$$\text{N I} = \phi S_c + [\phi_1 S_1 \text{ or } \phi_2 S_2]$$

As in the electric circuit e.m.f. across parallel branches is same, in the magnetic circuit the m.m.f. across parallel branches is same.

Thus same m.m.f. produces different fluxes in the two parallel branches. For such parallel branches,

$$\boxed{\phi_1 S_1 = \phi_2 S_2}$$

Hence while calculating total m.m.f., the m.m.f. of **only one** of the two parallel branches must be considered.

3.15.4 Parallel Magnetic Circuit with Air Gap

Consider a parallel magnetic circuit with air gap in the central limb as shown in the Fig. 3.28.

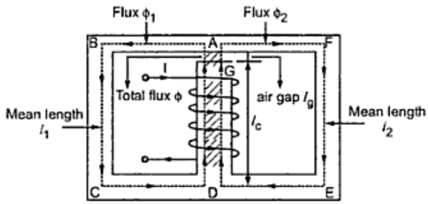


Fig. 3.28 Parallel circuit with air gap

The analysis of this circuit is exactly similar to the parallel circuit discussed above. The only change is the analysis of central limb. The central limb is series combination of iron path and air gap. The central limb is made up of,

$$\text{Path GD} = \text{Iron path} = l_c$$

$$\text{Path GA} = \text{Air gap} = l_g$$

The total flux produced is ϕ. It gets divided at A into ϕ_1 and ϕ_2.

$$\therefore \qquad \phi = \phi_1 + \phi_2$$

The reluctance of central limb is now,

$$S_c = S_i + S_g = \frac{l_c}{\mu a_c} + \frac{l_g}{\mu_0 a_c}$$

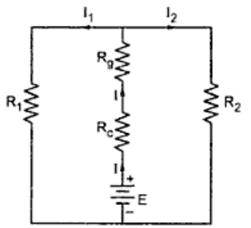


Fig. 3.29 Electrical equivalent circuit

Hence m.m.f. of central limb is now,

$$\boxed{(\text{m.m.f.})_{AD} = (\text{m.m.f.})_{GD} + (\text{m.m.f.})_{GA}}$$

Hence the total m.m.f. can be expressed as,

$$\boxed{(NI)_{total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD} \text{ or } (NI)_{AFED}}$$

Thus the electrical equivalent circuit for such case becomes as shown in the Fig. 3.29.

Similarly there may be air gaps in the side limbs but the method of analysis remains the same.

Example 3.3 : *A cast steel structure is made of a rod of square section 2.5 cm × 2.5 cm as shown in the Fig. 3.30. What is the current that should be passed in a 500 turn coil on the left limb so that a flux of 2.5 mWb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.*

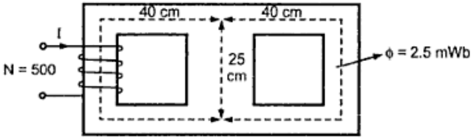


Fig. 3.30

Solution : This is parallel magnetic circuit. Its electrical equivalent is shown in the Fig. 3.30 (a).

The total flux produced gets distributed into two parts having reluctances S_1 and S_2.

S_1 = Reluctance of centre limb

S_2 = Reluctance of right side

$$S_1 = \frac{l_1}{\mu_0 \mu_r a_1} = \frac{25\times 10^{-2}}{4\pi\times 10^{-7}\times 750\times 2.5\times 2.5\times 10^{-4}}$$

$$= 424.413\times 10^3 \quad \text{AT/Wb}$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r a_1} = \frac{40\times 10^{-2}}{4\pi\times 10^{-7}\times 750\times 2.5\times 2.5\times 10^{-4}}$$

$$= 679.061\times 10^3 \quad \text{AT/Wb}$$

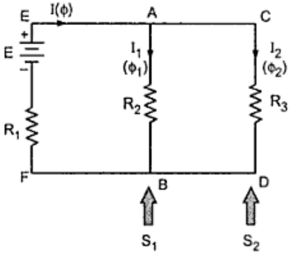


Fig. 3.30 (a)

Key Point: *For parallel branches, m.m.f. remains same.*

For branch AB and CD, m.m.f. is same.

$\therefore \quad \text{m.m.f.} = \phi_1 S_1 = \phi_2 S_2$

And $\quad \phi_2 = 2.5$ mWb ... Given

$$\therefore \quad \phi_1 = \frac{\phi_2 S_2}{S_1} = \frac{2.5\times 10^{-3}\times 679.061\times 10^3}{424.413\times 10^3} = 4 \text{ mWb}$$

$$\therefore \quad \phi = \phi_1 + \phi_2 = 2.5 + 4 = 6.5 \text{ mWb}$$

Total m.m.f. required is sum of the m.m.f. required for AEFB and that for either central or side limb.

$$S_{AEFB} = S_2 = 679.061\times 10^3 \quad \text{AT/Wb}$$

$$\therefore \quad \text{m.m.f. for AEFB} = S_{AEFB} \times \phi = 679.061 \times 10^3 \times 6.5 \times 10^{-3}$$

$$= 4413.8965 \text{ AT}$$

$$\therefore \quad \text{Total m.m.f.} = 4413.8965 + \phi_1 S_1$$

$$= 4413.8965 + 4 \times 10^{-3} \times 424.413 \times 10^3 = 6111.548 \text{ AT}$$

But $\quad NI = \text{Total m.m.f.}$

$$\therefore \quad I = \frac{6111.548}{500} = \mathbf{12.223\ A}$$

3.16 Kirchhoff's Laws for Magnetic Circuit

Similar to the electrical circuit Kirchhoff's laws can be used to analyze complex magnetic circuit. The laws can be stated as below.

3.16.1 Kirchhoff's Flux Law

The total magnetic flux arriving at any junction in a magnetic circuit is equal to the total magnetic flux leaving that junction.

At a junction, $\boxed{\sum \phi = 0}$

The law infact is used earlier to analyze parallel magnetic circuit at a junction A shown in the Fig. 3.27 (a), where

$$\phi = \phi_1 + \phi_2$$

3.16.2 Kirchhoff's M.M.F. Law

The resultant m.m.f. around a closed magnetic circuit is equal to the algebraic sum of the products of the flux and the reluctance of each part of the closed circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{M.M.F.} = \sum \phi S}$$

As $\quad \phi \times S = \text{flux} \times \text{reluctance} = \text{M.M.F.}$

M.M.F. also can be calculated as $H \times l$ where H is field strength and 'l' is mean length.

$$\therefore \quad \text{M.M.F.} = Hl$$

Alternatively the same law can be stated as :

The resultant m.m.f. around any closed loop of a magnetic circuit is equal to the algebraic sum of the products of the magnetic field strength and the length of each part of the circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{M.M.F.} = \sum H \cdot l}$$

3.17 Comparison of Magnetic and Electric Circuits

Similarities between electric and magnetic circuits are listed below :

Sr. No.	Electric circuit	Magnetic circuit
1.	Path traced by the current is called electric circuit.	Path traced by the magnetic flux is defined as magnetic circuit.
2.	E.M.F. is the driving force in electric circuit, the unit is **volts.**	M.M.F. is the driving force in the magnetic circuit, the unit of which is **ampere turns.**
3.	There is current I in the electric circuit measured in amperes.	There is flux ϕ in the magnetic circuit measured in webers.
4.	The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
5.	Resistance oppose the flow of the current. Unit is ohm.	Reluctance is opposed by magnetic path to the flux. Unit is ampere turn/weber.
6.	$R = \rho \frac{l}{a}$. Directly proportional to l. Inversely proportional to 'a'. Depends on nature of material.	$S = \frac{l}{\mu_0 \mu_r a}$. Directly proportional to l. Inversely proportional to $\mu = \mu_0 \mu_r$. Inversely proportional to area 'a'.
7.	The current $I = \frac{\text{E.M.F.}}{\text{resistance}}$	The flux $\phi = \frac{\text{M.M.F.}}{\text{Reluctance}}$
8.	The current density $\delta = \frac{I}{a}\ A/m^2$	The flux density $B = \frac{\phi}{a}\ Wb/m^2$
9.	Conductivity is reciprocal of the resistivity. Conductance $= \frac{1}{R}$	Permeance is reciprocal of the reluctance. Permeance $= \frac{1}{S}$
10.	Kirchhoff's current and voltage law is applicable to the electric circuit.	Kirchhoff's m.m.f. law and flux law is applicable to the magnetic circuit.

There are few dissimilarities between the two which are listed below :

Sr. No.	Electric circuit	Magnetic circuit
1.	In the electric circuit the current actually flows i.e. there is movement of electrons.	Due to m.m.f. flux gets established and does not flow in the sense in which current flows.
2.	There are many materials which can be used as insulators i.e. air, P.V.C., synthetic resin etc., from which current can not pass.	There is no magnetic insulator as flux can pass through all the materials, even through the air as well.
3.	Energy must be supplied to the electric circuit to maintain the flow of current.	Energy is required to create the magnetic flux, but is not required to maintain it.

4.	The resistance and the conductivity are independent of current density (δ) under constant temperature. But may change due to the temperature.	The reluctance, permeance and permeability are dependent on the flux density.
5.	Electric lines of flux are not closed. They start from positive charge and end on negative charge.	Magnetic lines of flux are closed lines. They flow from N pole to S pole externally while S pole to N pole internally.
6.	There is continuous consumption of electrical energy.	Energy is required to create the magnetic flux and not to maintain it.

3.18 Magnetic Leakage and Fringing

Most of the applications which are using magnetic effects of an electric current, are using flux in air gap for their operation. Such devices are generators, motors, measuring instruments like ammeter, voltmeter etc. Such devices consist of magnetic circuit with an air gap and flux in air gap is used to produce the required effect.

Such flux which is available in air gap and is utilized to produce the desired effect is called **useful flux** denoted by ϕ_u.

It is expected that whatever is the flux produced by the magnetizing coil, it should complete its path through the iron and air gap. So all the flux will be available in air gap. In actual practice it is not possible to have entire flux available in air gap. This is because, we have already seen that there is no perfect insulator for the flux. So part of the flux completes its path through the air or medium in which coil and magnetic circuit is placed.

Key Point : *Such flux which leaks and completes its path through surrounding air or medium instead of the desired path is called the leakage flux.*

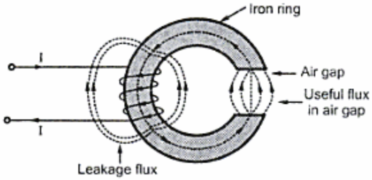


Fig. 3.31 Leakage and useful flux

The Fig. 3.31 shows the useful and leakage flux.

3.18.1 Leakage Coefficient or Hopkinson's Coefficient

The ratio of the total flux (ϕ_T) to the useful flux (ϕ_u) is defined as the **leakage coefficient** of **Hopkinson's coefficient** or **leakage factor** of that magnetic circuit.

It is denoted by λ.

$$\therefore \qquad \lambda = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_T}{\phi_u}$$

The value of 'λ' is always greater than 1 as ϕ_T is always more than ϕ_u. It generally varies between 1.1 and 1.25. Ideally its value should be 1.

3.18.2 Magnetic Fringing

When flux enters into the air gap, it passes through the air gap interms of parallel flux lines. There exists a force of repulsion between the magnetic lines of force which are parallel and having same direction. Due to this repulsive force there is tendency of the magnetic flux to bulge out (spread out) at the edge of the air gap. This tendency of flux to bulge out at the edges of the air gap is called **magnetic fringing**.

It has following two effects :

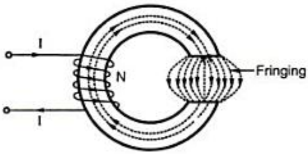


Fig. 3.32 Magnetic fringing

1) It increases the effective cross-sectional area of the air gap.
2) It reduces the flux density in the air gap.

So leakage, fringing and reluctance, in practice should be as small as possible.

> **Key Point** : *This is possible by choosing good magnetic material and making the air gap as narrow as possible.*

Example 3.4 : *A cast iron ring of 40 cm mean length and circular cross-section of 5 cm diameter is wound with a coil. The coil carries a current of 3 A and produces a flux of 3 mWb in the air gap. The length of the air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate number of turns of the coil.*

Solution : Given, $l_t = 40 \text{ cm} = 0.4 \text{ m}$, $l_g = 2\times10^{-3}$ m

$$l_i = l_t - l_{gap} = 0.4 - 2\times10^{-3} = 0.398 \text{ m}$$

$$I = 3 \text{ A}, \quad \phi_g = 2\times10^{-3} \text{ Wb}, \mu_r = 800, \quad \lambda = 1.2$$

$$\lambda = \frac{\phi_T}{\phi_g} \qquad \text{... Leakage coefficient}$$

$$\therefore \qquad 1.2 = \frac{\phi_T}{2\times10^{-3}}$$

$$\therefore \qquad \phi_T = 2.4\times10^{-3} \text{ Wb}$$

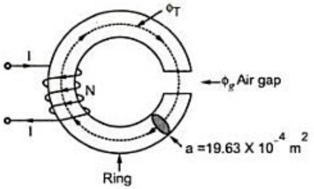


Fig. 3.33

Now reluctance of iron path $S_i = \dfrac{l_i}{\mu_0 \mu_r a}$

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times 5^2 = 19.6349 \text{ cm}^2 = 19.634 \times 10^{-4} \text{m}^2$$

$$\therefore \quad S_i = \frac{0.398}{4\pi\times 10^{-7}\times 800\times 19.63\times 10^{-4}} = 201629.16 \text{ AT/Wb}$$

$$\phi_T = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S_i}$$

$$\therefore \quad 2.4\times 10^{-3} = \frac{NI}{201629.16}$$

$$\therefore \quad \text{N I for iron path} = 483.909 \text{ AT}$$

Reluctance of air gap $S_g = \dfrac{l_g}{\mu_0 a}$

$$\therefore \quad S_g = \frac{2\times 10^{-3}}{4\pi\times 10^{-7}\times 19.634\times 10^{-4}}$$

$$= 810608.86 \text{ AT/Wb}$$

Now $\phi_g = \dfrac{\text{M.M.F.}}{S_g}$

$$\therefore \quad 2\times 10^{-3} = \frac{NI}{810608.86}$$

$$\therefore \quad \text{N I for air gap} = 1621.2177 \text{ AT}$$

$$\therefore \quad \text{Total m.m.f. required} = (NI)_{iron} + (NI)_{air\ gap} \quad NI = 483.909 + 1621.2177$$

$$\therefore \quad NI = 2105.1267 \quad \text{i.e. } N\times 3 = 2105.1267$$

$$\therefore \quad N = 701.7 \approx \textbf{702 turns.}$$

3.19 B-H Curve or Magnetization Curve

We have already seen that magnetic field strength H is $\dfrac{NI}{l}$. As current in coil changes, magnetic field strength also changes. Due to this flux produced and hence the flux density also changes. So there exists a particular relationship between B and H for a material which can be shown on the graph.

Key Point: *The graph between the flux density (B) and the magnetic field strength (H) for the magnetic material is called as its magnetization curve or B-H curve.*

Let us obtain the B-H curve experimentally for a magnetic material. The arrangement required is shown in the Fig. 3.34.

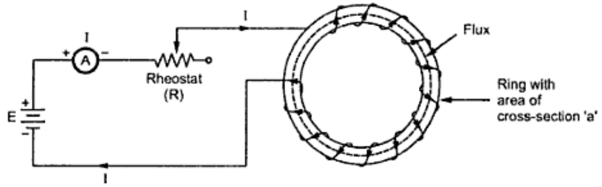


Fig. 3.34 Experimental set up to obtain B-H curve

The ring specimen as a mean length of 'l' metres with a cross-sectional area of 'a' square metres. Coil is wound for 'N' turns carrying a current 'I' which can be varied by changing the variable resistance 'R' connected in series. Ammeter is connected to measure the current. For measurement of flux produced, fluxmeter can be used which is not shown in the Fig. 3.34.

So H can be calculated as $\frac{NI}{l}$ while B can be calculated as $\frac{\phi}{a}$ for various values of current and plotted.

With the help of resistance R, I can be changed from zero to maximum possible value.

The B-H curve takes the following form, as shown in the Fig. 3.35.

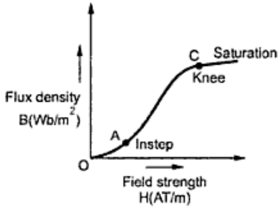


Fig. 3.35 B-H curve

The graph can be analyzed as below :

i) **Initial portion** : Near the origin for low values of 'H', the flux density does not increase rapidly. This is represented by curve OA. The point A is called as **instep.**

ii) **Middle portion** : In this portion as 'H' increases, the flux density B increases rapidly. This is almost straight line curve. At point 'C' it starts bending again. The point 'C' where this portion bends is called as **knee point.**

iii) **Saturation portion** : After the knee point, rate of increase in 'B' reduces drastically. Finally the curve becomes parallel to 'X' axis indicating that any increase in 'H' hereafter is not going to cause any change in 'B'. The ring is said to be **saturated** and region as **saturation region.**

We have seen already that according to molecular theory of magnetism, when all molecular magnets align themselves in the same direction due to application of H, saturation occurs.

Such curves are also called **saturation curves.**

3.19.1 B-H Curve and Permeability

From B-H curve, a curve of relative permeability μ_r and H can also be obtained.

As $$B = \mu H$$

$$= \mu_r \mu_0 H$$

$$\therefore \quad \mu_r = \frac{B}{\mu_0 H}$$

μ_0 is constant which is $4\pi \times 10^{-7}$. So B/H is nothing but slope of B-H curve.

Key Point: *So slope of B-H curve at various points decide the value of relative permeability at that point.*

Initially the slope is low so value of μ_r is also low. At knee point, the value of slope of B-H curve is maximum and hence μ_r is maximum. But in saturation region the value of μ_r maximum and hence μ_r is maximum. But in saturation region the value of μ_r falls down to very low value as slope of B-H curve in saturation region is almost zero.

The curve of μ_r against H is shown in the Fig. 3.36 for a magnetic material.

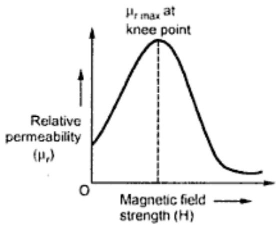


Fig. 3.36 μ_r-H curve for magnetic material

The value of μ_r is 1 which is constant in case of non-magnetic material. The slope of B-H curve is constant i.e. it is a straight line passing through the origin as shown in the Fig. 3.37.

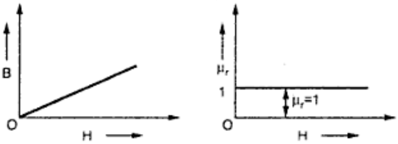


Fig. 3.37 μ_r - H curve for non magnetic material

3.19.2 Practical Use of B-H Curves

While designing the magnetic circuits, magnetization curves are useful to design the values of B corresponding to H. From this, proper material with required relative permeability can be selected.

The various materials like iron, steel are generally represented by the B-H curves and μ_r-H curves.

3.20 Modern Theory of Magnetism

We know that, in atoms, electrons are revolving around the nucleus with tremendous speeds, in different orbits. The moving electrons always constitute a current so such revolving electrons can be considered as flow of current. This current produces its own magnetic field depending upon the direction of rotation of electrons. The magnetic field is very small. When atoms combine to form molecules such magnetic fields balance each other and hence molecule becomes magnetically neutral in nature.

However in addition to such orbital motions, electrons, revolve about their own axes. A spinning electron also sets up its own magnetic field around it axis of spin. Direction of field depends on the direction of spin. In an atom all electrons are not rotating in same direction but they are revolving in such directions that the magnetic fields produced, due to spinning neutralize each other. In ferromagnetic materials like iron, nickel etc. have number of uncompensated electron spins and hence such materials show the magnetic properties.

After the applications of external m.m.f. all electrons start revolving in same direction and hence gives strong magnetic effect. The theory of electrons is exactly similar to the theory of molecular magnets present inside the materials, discussed earlier.

3.21 Magnetic Hysteresis

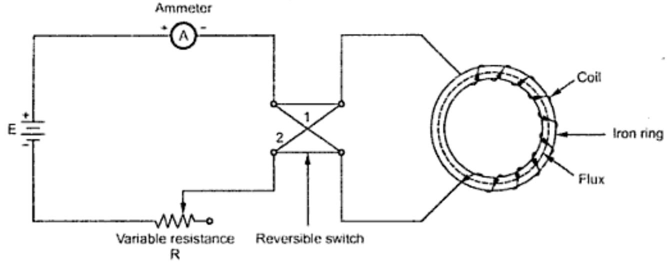


Fig. 3.38 Experimental set up to obtain hysteresis loop

Earlier we have studied B-H curve for a magnetic material. Magnetic hysteresis is extension to the same concept of B-H curve.

Instead of plotting B-H curve only for increase in current if plotted for one complete cycle of magnetization (increase in current) and demagnetization (decrease in current) then it is called **hysteresis curve or hysteresis loop**.

Consider a circuit consisting of a battery 'E', an ammeter, variable resistance R and reversible switch shown in the Fig. 3.38.

3.21.1 Steps in Obtaining Hysteresis Loop

i) Initially variable resistance is kept maximum so current through the circuit is very low. The field strength $H = \frac{NI}{l}$ is also very low. So as current is increased, for low values of field strengths, flux density do not increase rapidly. But after the knee point flux density increases rapidly upto certain point. This point is called **point of saturation**. Thereafter any change in current do not have an effect on the flux density. This curve is nothing but the magnetization curve (B-H curve). This is the initial part of hysteresis loop.

ii) After the saturation point, now current is again reduced to zero. Due to this field strength also reduces to zero. But it is observed that flux density do not trace the same curve back but falls back as compared to previous magnetization curve. This phenomenon of falling back of flux density while demagnetization cycle is called **hysteresis**. Hence due to this effect, when current becomes exactly zero, there remains some magnetism associated with a coil and hence the flux density. The core does not get completely demagnetized though current through coil becomes zero. This value of flux density when exciting current through the coil and magnetic field strength is reduced to zero is called **residual flux density or remanent flux density**. This is also called **residual magnetism** of the core. The magnitude of this residual flux or magnetism depends on the nature of the material of the core. And this property of the material is called **retentivity**.

iii) But now if it is required to demagnetize the core entirely then it is necessary to reverse the direction of the current through the coil. This is possible with the help of the intermediate switch.

Key Point : *The value of magnetic field strength required to wipe out the residual flux density is called the coercive force. It is measured interms of coercivity.*

iv) If now this reversed current is increased, core will get saturated but in opposite direction. At this point flux density is maximum but with opposite direction.

v) If this current is reduced to zero, again core shows a hysteresis property and does not get fully demagnetized. It shows same value of residual magnetism but with opposite direction.

vi) If current is reversed again, then for a certain magnitude of field strength, complete demagnetization of the core is possible.

vii) And if it is increased further, then saturation in the original direction is achieved completing one cycle of magnetization and demagnetization.

The curve plotted for such one cycle turns out to be a closed loop which is called hysteresis loop. Its nature is shown in the Fig. 3.39.

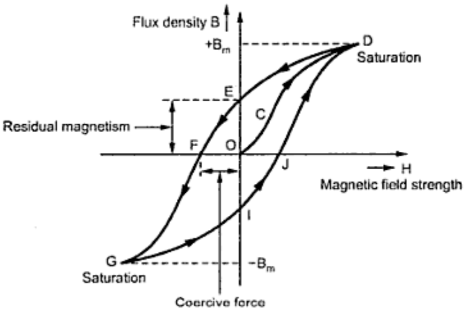


Fig. 3.39 Hysteresis loop

Part of curve **Represents what ?**

O-C-D : Region corresponding to normal magnetization curve increased form 'O' to 'I_{max} corresponding to 'B_m'. Maximum flux density is + B_m.

D-E : Current reduced to zero, but core cannot be completely demagnetized. O-E represents residual magnetism and residual flux density, denoted by + B_r.

E-F : Current is reversed and increased in reversed direction to get complete demagnetization of the core. O-F represent coercive force required to completely wipe out + B_r.

F-G : Current is increased in reversed direction till saturation in opposite direction is achieved. Maximum flux density same but with opposite direction i.e. – B_m.

G-I : Current is reduced to zero but again flux density lags and core cannot be completely demagnetized. O-I represents residual flux density in other direction i.e. – B_r.

I-J : Current is again reversed and increased till complete demagnetization is achieved.

J-D : Current is again increased in original direction till saturation is reached. Corresponding flux density is again + B_m.

3.21.2 Theory Behind Hysteresis Effect

As seen from the loop 'O-C-D-E-F-G-I-J-D' shown in the Fig. 3.39, the flux density B always lags behind the values of magnetic field strength H. When H is zero, corresponding flux density is + B_r. This effect is known as **hysteresis**.

This can be explained with the help of theory of molecular magnets inside a material. When a ferromagnetic material is subjected to a magnetic field strength, all the molecular magnets inside, align themselves in the direction of the applied m.m.f. If this applied force is removed or reduced some of the molecular magnets remain in the aligned state and magnetic neutralization of the material is not fully possible. So it continues to show some magnetic properties which is defined as the **residual magnetism**.

The value of the residual magnetism as said earlier depends on the quality of the magnetic material and the treatment it receives at the time of manufacturing. This property is called as **retentivity**.

Key Point: *Higher the value of retentivity, higher the value of the power of the magnetic material to retain its magnetism. For high retentivity, higher is the coercive force required.*

It can be measured interms coercivity of the material.

3.22 Hysteresis Loss

According to the molecular theory of magnetism groups of molecules acts like elementary magnets, which are magnetized to saturation. This magnetism is developed because of the magnetic effect of electron spins, which are known as '**domains**'.

When the material is unmagnetized, the axis of the different domains are in various direction. Thus the resultant magnetic effect is zero.

When the external magnetomotive force is applied the axes of the various domains are oriented. The axes coincide with the direction of the magnetomotive force. Hence the resultant of individual magnetic effects is a strong magnetic field.

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization, it results into disturbance in the alignment of the various domain. Now energy gets stored when magnetic field is established and energy is returned when field collapses. But due to hysteresis, all the energy is never returned though field completely collapses. This loss of energy appears as heat in the magnetic material. This is called as **hysteresis loss.** So disturbance in the alignment of the various domains causes hysteresis loss to take place. This hysteresis loss is undesirable and may cause undesirable high temperature rise due to heat produced. Due to such loss overall efficiency also reduces.

Such hysteresis loss depends on the following factors.

1. The hysteresis loss is directly proportional to the area under the hysteresis curve i.e. area of the hysteresis loop.
2. It is directly proportional to frequency i.e. number of cycles of magnetization per second.
3. It is directly proportional to volume of the material. It can be shown that quantitatively the hysteresis loss in joules per unit volume of the material in one cycle is equal to the area of the hysteresis loop.

3.22.1 Hysteresis Loss Per Unit Volume

Consider a ring shaped test piece of magnetic material having length of 'l' metres, cross-sectional area of 'a' m^2, wound with uniform 'N' turns of a coil.

The hysteresis loop for the piece is obtained as in the Fig. 3.40.

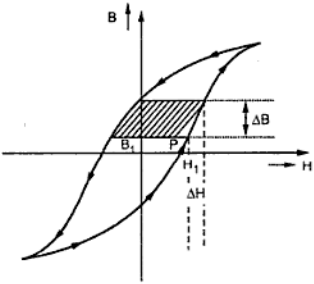


Fig. 3.40 Hysteresis loss per unit volume

Consider an instant when material is magnetized to a point P. The corresponding current be I_1.

$$\therefore \quad H_1 = \frac{N I_1}{l} \quad AT/m$$

Let current is increased by ΔI so there is increase in field strength by ΔH and flux density by ΔB. Due to ΔB change there is change in the flux $\Delta \phi$.

$$\therefore \quad \Delta\phi = \Delta B \times a \qquad \ldots \left(B = \frac{Flux}{Area} \right)$$

Now change in flux causes induced e.m.f. in coil according to Faraday's law.

$$\therefore \quad e = -N \frac{d\phi}{dt} = -N \frac{\Delta B \times a}{dt}$$

$\therefore$ Supply voltage has to overcome this

$$\therefore \quad V = -e$$

$$= N \frac{\Delta B \times a}{dt}$$

$\therefore$ Power supplied $P = V \times I$

$$\therefore \quad P = N \times a \times \frac{\Delta B}{dt} \times I$$

$\therefore$ Energy supplied in time dt = P × dt joules

$$\Delta E = N \times a \times \Delta B \times I$$

$$\Delta E = \frac{NI}{l} \times l \times a \times \Delta B$$

$$\Delta E = H \times a \times l \times \Delta B \quad \text{joules}$$

This is energy supplied within time dt.

$\therefore$ Energy supplied for one cycle can be obtained by integrating above expression.

$$\therefore \quad E = a \times l \times \oint H \Delta B \quad \text{joules}$$

Here $\oint$ is nothing but integration of the areas enclosed by strip H ΔB for one cycle i.e. the area enclosed by hysteresis loop for one cycle.

And $a \times l$ = Volume of the material

$\therefore$ Energy supplied during one cycle in joules = Volume × Area of the hysteresis loop

$\therefore$ Energy supplied per unit volume = Area of the hysteresis loop

When 'H' is increased from zero to maximum, energy is supplied while when 'H' is reduced energy is recovered. But all the energy is not recovered.

So net energy absorbed by material during one cycle appears as hysteresis loss.

∴ Energy per unit volume per cycle = Hysteresis loss per unit volume

∴ Hysteresis loss in joules/cycle/m^3 = Area of the hysteresis loop

If the hysteresis curve is drawn with scale as,

$$1 \text{ cm} = x \text{ ampere-turns/metre of H}$$

$$1 \text{ cm} = y \text{ teslas for B}$$

Then the hysteresis loss can be calculated as,

$$\text{Hysteresis loss/cycle/m}^3 = [x \times y \times \text{Area of hysteresis curve in cm}^2]$$

In practice the hysteresis loss is calculated with reasonable accuracy by experimentally determined mathematical expression devised by **Steinmetz**, which is as follows.

$$\boxed{\text{Hysteresis loss} = K_h (B_m)^{1.6} f \times \text{volume} \quad \text{watts}}$$

where K_h = Characteristic constant of the material

B_m = Maximum flux density

f = Frequency in cycles per second

3.22.2 Practical Use of Hysteresis Loop

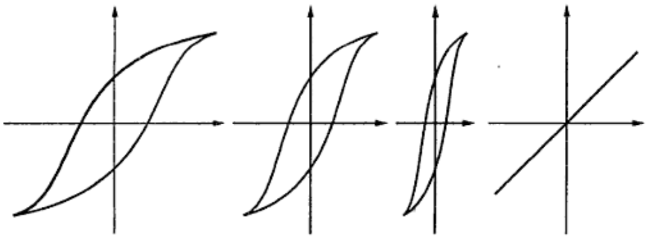
As we have seen that hysteresis loss is undesirable as it produces heat which increases temperature and also reduces the efficiency.

In machines where the frequency of the magnetization and demagnetization cycle is more, such hysteresis loss is bound to be more.

So selection of the magnetic material in such machines based on the hysteresis loss. Less the hysteresis loop area for the material, less is the hysteresis loss.

Key Point: *So generally material with less hysteresis loop area are preferred for different machines like transformer cores, alternating current machines, telephones.*

Shapes of hysteresis loops for different materials are shown in the Fig. 3.41.



(a) (b) (c) (d)

Fig. 3.41 Practical importance of hysteresis loop

The Fig. 3.41 (a) shows loop of hard steel, which is magnetic material.

The Fig. 3.41 (b) shows loop of cast steel.

The Fig. 3.41 (c) shows loop of permalloy (Alloy of nickel and iron) i.e. ferromagnetic materials.

The Fig. 3.41 (d) shows loop for air or non-magnetic material.

The materials iron, nickel, cobalt and some of their alloys and compounds show a strong tendency to move from weaker to stronger portion of a non-uniform magnetic field. Such substances are called **ferromagnetic materials.**

The hysteresis loss is proportional to the area of the hysteresis loop. For ferromagnetic materials the hysteresis loop area is less as shown in the Fig. 3.41(c) thus hysteresis loss is less in such materials.

In non-magnetic materials, the hysteresis loop is straight line having zero area hence hysteresis loss is also zero in such materials.

3.23 Eddy Current Loss

Consider a coil wound on a core. If this coil carries an alternating current i.e. current whose magnitude varies with respect to time, then flux produced by it is also of alternating nature. So core is under the influence of the changing flux and under such condition according to the Faraday's law of electromagnetic induction, e.m.f. gets induced in the core. Now if core is solid, then such induced e.m.f. circulates currents through the core. Such currents in the core which are due to induced e.m.f. in the core are called as eddy currents. Due to such currents there is power loss (I^2R) in the core. Such loss is called as **eddy current loss**. This loss, similar to hysteresis loss, reduces the efficiency. For solid core with less resistance, eddy currents are always very high.

The Fig. 3.42 shows a core carrying the eddy currents.

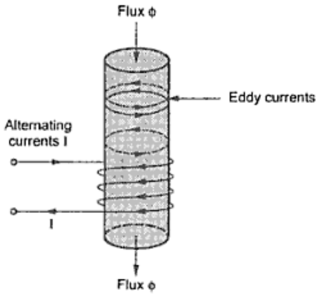


Fig. 3.42 Eddy currents

Eddy current loss depends on the various factors which are

i) Nature of the material

ii) Maximum flux density

iii) Frequency

iv) Thickness of laminations used to construct to core

v) Volume of magnetic material.

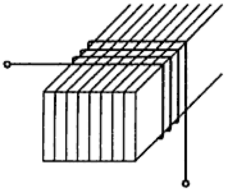


Fig. 3.43 Laminated core

It has been found that loss can be considerably reduced by selecting high resistivity magnetic material like silicon. Most popular method used to reduce eddy current loss is to use laminated construction to construct the core. Core is constructed by stacking thin pieces known as laminations as shown in the Fig. 3.43. The laminations are insulated from each other by thin layers of insulating material like varnish, paper, mica. This restricts the paths of eddy currents, to respective laminations only. So area through which currents flow decreases, increasing the resistance and magnitude of currents gets reduced considerably.

The loss may also be reduced by grinding the ferromagnetic material to a powder and mixing it with a binder that effectively insulates the particles one from other. This mixture is then formed under pressure into the desired shape and heat treated. Magnetic cores for use in communication equipment are frequently made by this process.

This loss is quantified by using the expression,

$$\text{Eddy current loss} = K_e (B_m)^2 f^2 t^2 \times \text{volume} \quad \text{watts}$$

where

K_e = A characteristic constant of material

B_m = Maximum flux density

f = Frequency

t = Thickness of the lamination

3.24 Magnetic Loss [Core Loss or Iron Loss]

The magnetic losses can be classified as

i) Hysteresis loss ii) Eddy current loss

The magnetic loss occurs in the core hence they are known as **core losses**. Since core material is generally iron or its alloy this loss is also referred as iron loss. The magnetic loss will result in the following effects.

i) It reduces the efficiency of the electrical equipment.

ii) It increases the temperature because of heating of the core.

Key Point: *The hysteresis loss can be reduced by selecting good quality magnetic material.*

The area of hysteresis loop should be narrow. Silicon steel is employed for the core material so that hysteresis loss can be minimized.

Key Point: *The eddy current loss can be reduced by using thin laminations for the core.*

3.25 Force on a Current Carrying Conductor in a Magnetic Field

We have already mentioned that magnetic effects of electric current are very useful in analyzing various practical applications like generators, motors etc. One of such important effects is force experienced by a current carrying conductor in a magnetic field.

Let a straight conductor, carrying a current is placed in a magnetic field as shown in the Fig. 3.44 (a).

The magnetic field in which it is placed has a flux pattern as shown in the Fig. 3.44 (a).

(a) Flux due to magnet **(b) Flux due to current carrying conductor**

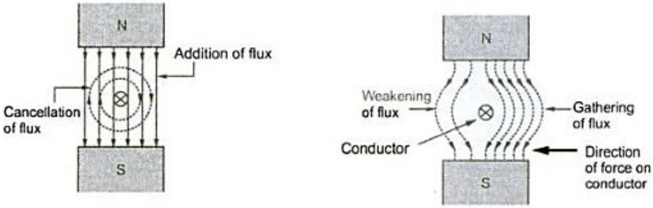
Fig. 3.44 Current carrying conductor in a magnetic field

Now current carrying conductor also produces its own magnetic field around it. Assuming current direction away from observer i.e. into the paper, the direction of its flux can be determined by right hand thumb rule. This is clockwise as shown in the Fig. 3.44 (b). [For simplicity, flux only due to current carrying conductor is shown in the Fig. 3.44 (b)].

Now there is presence of two magnetic fields namely due to permanent magnet and due to current carrying conductor. These two fluxes interact with each other. Such interaction is shown in the Fig. 3.45 (a).

This interaction as seen is in such a way that on one side of the conductor the two lines help each other, while on other side the two try to cancel each other. This means on left hand side of the conductor shown in the Fig. 3.45 the two fluxes are in the same direction and hence assisting each other. As against this, on the right hand side of the conductor the two fluxes are in opposite direction hence trying to cancel each other. Due to such interaction on one side of the conductor, there is **accumulation** of flux lines (gathering of the flux lines) while on the other side there is **weakening** of the flux lines.

The resultant flux pattern around the conductor is shown in the Fig. 3.45 (b).



(a) Presence of the two fluxes (b) Resultant flux pattern

Fig. 3.45 Interaction of the two flux lines

According to properties of the flux lines, these flux lines will try to shorten themselves. While doing so, flux lines which are gathered will exert force on the conductor. So conductor experiences a mechanical force from high flux lines area towards low flux lines area i.e. from left to right for a conductor shown in the Fig. 3.45.

Key Point : *Thus we can conclude that current carrying conductor placed in the magnetic field, experiences a mechanical force, due to interaction of two fluxes.*

This is the basic principle on which d.c. electric motors work and hence also called **motoring action.**

3.25.1 Fleming's Left Hand Rule

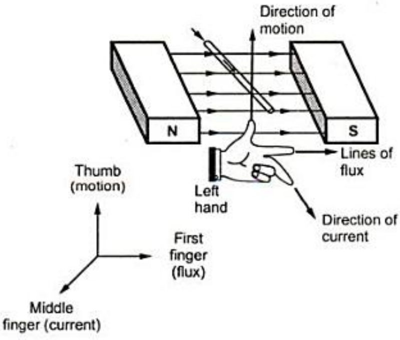


Fig. 3.46 Fleming's left hand rule

The direction of the force experienced by the current carrying conductor placed in magnetic field can be determined by a rule called **'fleming's left hand rule'**. The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.

The rule is explained in the diagrammatic form in the Fig. 3.46.

Apply the rule to crosscheck the direction of force experienced by a single conductor, placed in the magnetic field, shown in the Fig. 3.47 (a), (b), (c) and (d).

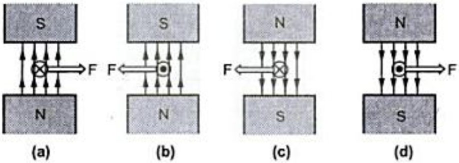


Fig. 3.47 Direction of force experienced by conductor

3.25.2 Magnitude of Force Experienced by the Conductor

The magnitude of the force experienced by the conductor depends on the following factors.

1) Flux density (B) of the magnetic field in which the conductor is placed measured in Wb/m^2 i.e. tesla.
2) Magnitude of the current I passing through the conductor in amperes.
3) Active length 'l' of the conductor in metres.

The **active length** of the conductor is that part of the conductor which is actually under the influence of magnetic field.

If the conductor is at right angles to the magnetic field as shown in Fig. 3.48 (a) then force F is given by,

$$F = B I l \text{ newtons}$$

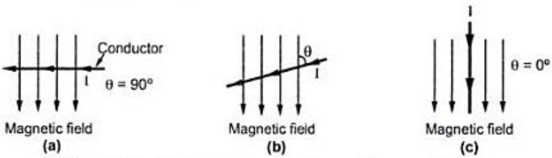


Fig. 3.48 Force on a current carrying conductor

But if the conductor is not exactly at right angles, but inclined at angle θ degrees with respect to axis of magnetic field as shown in the Fig. 3.48 (b) then force F is given by,

$$F = B I l \sin \theta \text{ newtons}$$

As shown in the Fig. 3.48 (c), if conductor is kept along the lines of magnetic field then $\theta = 0°$ and as $\sin 0° = 0$, the force experienced by the conductor is also zero.

Key Point: *The direction of such force can be reversed either by changing the direction of current or by changing the direction of the flux lines in which it is kept. If both are reversed, the direction of force remains same.*

➠ **Example 3.5 :** *Calculate the force experienced by the conductor of 20 cm long, carrying 50 amperes, placed at right angles to the lines of force of flux density 10×10^{-3} Wb/m².*

Solution : The force experienced is given by,

$$F = B I l \sin\theta \qquad \text{where} \quad \sin(\theta) = 1 \quad \text{as } \theta = 90 \text{ degrees}$$

$$B = \text{Flux density} = 10 \times 10^{-3} \ \text{Wb/m}^2$$

$$l = \text{Active length} = 20 \text{ cm} = 0.2 \text{ m}$$

$$I = \text{Current} = 50 \text{ A}$$

$$F = 10 \times 10^{-3} \times 50 \times 0.2 = \mathbf{0.1 \ N}$$

3.26 Introduction to Electromagnetic Induction

Uptill now we have discussed the basic properties, concepts of magnetism and magnetic circuits. Similarly we have studied, the magnetic effects of an electric current. But we have not seen the generation of e.m.f. with the help of magnetism. The e.m.f. can be generated by different ways, by chemical action, by heating thermocouples etc. But the most popular and extensively used method of generating an e.m.f. is based on electromagnetism.

After the magnetic effects of an electric current, attempts were made to produce electric current with the help of magnetism rather than getting magnetism due to current carrying conductor. In 1831, an English Physicist, **Michael Faraday** succeeded in getting e.m.f. from magnetic flux. The phenomenon by which e.m.f. is obtained from flux is called **electromagnetic induction**. Let us discuss, what is electromagnetic induction and its effect on the electrical engineering branch, in brief.

3.27 Faraday's Experiment

Let us study first the experiment conducted by **Faraday** to get understanding of electromagnetic induction.

Consider a coil having 'N' turns connected to a galvanometer as shown in the Fig. 3.49. Galvanometer indicates flow of current in the circuit, if any. A permanent magnet is moved relative to coil, such that magnetic lines of force associated with coil get changed.

Whenever, there is motion of permanent magnet, galvanometer deflects indicating flow of current through the circuit.

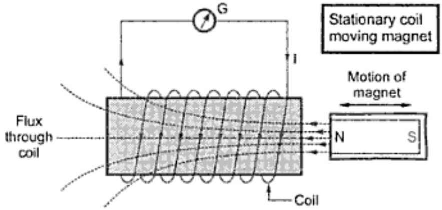


Fig. 3.49 Faraday's experiment

Key Point : *The galvanometer deflects in one direction, when magnet is moved towards a coil. It deflects in other direction, when moved away from the coil.*

The deflection continues as long as motion of magnet exists. More quickly the magnet is moved, the greater is the deflection. Now deflection of galvanometer indicates flow of current. But to exist flow of current there must be presence of e.m.f. Hence such movement of flux lines with respect to coil generates an e.m.f. which drives current through the coil. This is the situation where coil in which e.m.f. is generated is fixed and magnet is moved to create relative motion of flux with respect to coil.

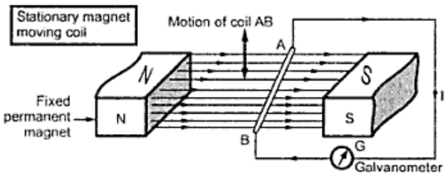


Fig. 3.50 Another form of Faraday's experiment

Similar observations can be made by moving a coil in the magnetic field of fixed permanent magnet, creating relative motion between flux and coil. This arrangement is shown in the Fig. 3.50. The coil AB is moved by some external means in the magnetic field of fixed permanent magnet. Coil is connected to galvanometer.

Whenever conductor AB is moved in the direction shown in the Fig. 3.50 the galvanometer deflects indicating flow of current through coil AB.

Key Point: *The deflection is on one side when conductor is moved up. While it is in other direction, when it is moved down.*

Similarly greater is the deflection if conductor is moved quickly in magnetic field.

In both cases, basically there is **change of flux lines with respect to conductor** i.e. there is cutting of the flux lines by the conductor in which e.m.f. induced.

With this experiment Faraday stated laws called faraday's laws of electromagnetic induction.

*This phenomenon of cutting of flux lines by the conductor to get the induced e.m.f. in the conductor or coil is called **electromagnetic induction**.*

Thus, to have induced e.m.f. there must exist,

1) A coil or conductor.
2) A magnetic field (permanent magnet or electromagnet).
3) Relative motion between conductor and magnetic flux (achieved by moving conductor with respect to flux or moving with respect to conductor).

Key Point: *The e.m.f. exists as long as relative motion persists.*

3.28 Faraday's Laws of Electromagnetic Induction

From the experiment discussed above, Michael Faraday a British scientist stated two laws of electromagnetic induction.

3.28.1 First Law

Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f. gets induced in that coil or circuit.

3.28.2 Second Law

The magnitude of the induced e.m.f. is directly proportional to the rate of change of flux linkages (flux × turns of coil).

Flux linkages = Flux × Number of turns of coil

The law can be explained as below.

Consider a coil having N turns. The initial flux linking with a coil is ϕ_1.

$$\therefore \quad \text{Initial flux linkages} = N\,\phi_1$$

In time interval t, the flux linking with the coil changes from ϕ_1 to ϕ_2.

$$\therefore \quad \text{Final flux linkages} = N\,\phi_2$$

$$\therefore \quad \text{Rate of change of flux linkages} = \frac{N\phi_2 - N\phi_1}{t}$$

Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$\therefore \quad e \propto \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \qquad e = K \times \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \qquad e = N\frac{d\phi}{dt}$$

With K as unity to get units of e as volts, $d\phi$ is change in flux, dt is change in time hence $(d\phi / dt)$ is rate of change of flux.

Now as per Lenz's law (discussed later), the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.

Thus such an induced e.m.f. is mathematically expressed along with its sign as,

$$\therefore \qquad \boxed{e = -N\frac{d\phi}{dt} \quad \text{volts}}$$

3.29 Nature of the Induced E.M.F.

E.M.F. gets induced in a conductor, whenever there exists change in flux with that conductor, according to Faraday's law. Such change in flux can be brought about by different methods.

Depending upon the nature of methods, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f. and
2) Statically induced e.m.f.

3.30 Dynamically Induced E.M.F.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux. Both these methods are discussed earlier in discussion of Faraday's experiment.

Key Point: *Such an induced e.m.f. which is due to physical movement of coil, conductor with respect to flux or movement of magnet with respect to stationary coil, conductor is called dynamically induced e.m.f. or motional induced e.m.f.*

3.30.1 Magnitude of Dynamically Induced E.M.F.

Consider a conductor of length l metres moving in the air gap between the poles of the magnet.

If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines and there cannot be any induced e.m.f. in the conductor such condition is shown in the Fig. 3.51 (a).

Key Point : *When plane of the flux is parallel to the plane of the motion of conductors then there is no cutting of flux, hence no induced e.m.f.*

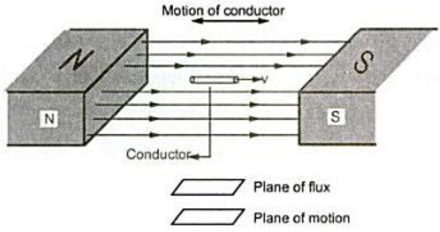


Fig. 3.51 (a) No cutting of flux

In second case as shown in the Fig. 3.51 (b), the velocity direction i.e. motion of conductor is perpendicular to the flux. Hence whole length of conductor is cutting the flux line hence there is maximum possible induced e.m.f. in the conductor. **Under such condition plane of flux and plane of motion are perpendicular to each other.**

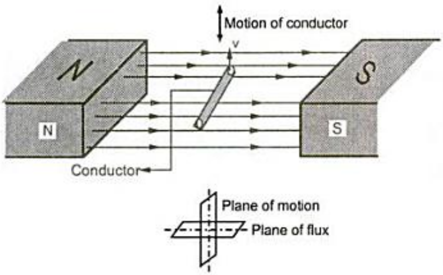
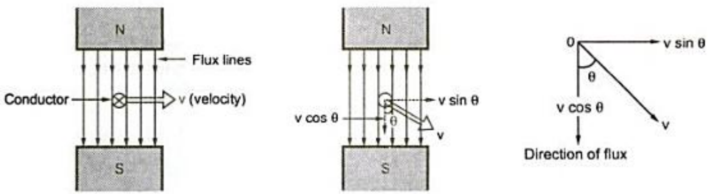


Fig. 3.51 (b) Maximum cutting of flux

Key Point : *When plane of the flux is perpendicular to the plane of the motion of the conductors then the cutting of flux is maximum and hence induced e.m.f. is also maximum.*

Consider a conductor moving with velocity v m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown in Fig. 3.52 (a).



(a) **(b)** **(c)**

Fig. 3.52

B = Flux density in Wb/m^2

l = Active length of conductor in metres

(This is the length of conductor which is actually responsible for cutting of flux lines.)

v = Velocity in m/sec

Let this conductor is moved through distance dx in a small time interval dt, then

$$\text{Area swept by conductor} = l \times dx \quad m^2$$

$$\therefore \quad \text{Flux cut by conductor} = \text{Flux density} \times \text{Area swept}$$

$$d\phi = B \times l \times dx \quad Wb$$

According to Faraday's law, magnitude of induced e.m.f. is proportional to the rate of change of flux.

$$\therefore \quad e = \frac{\text{Flux cut}}{\text{Time}}$$

$$= \frac{d\phi}{dt} \qquad \text{[Here N = 1 as single conductor]}$$

$$= \frac{B\,l\,dx}{dt}$$

But $\frac{dx}{dt}$ = Rate of change of displacement

= Velocity of the conductor

= v

$$\therefore \quad e = B\,l\,v \quad \text{volts}$$

This is the induced e.m.f. when plane of motion is exactly perpendicular to the plane of flux. This is maximum possible e.m.f. as plane of motion is at right angles to plane of the flux.

But if conductor is moving with a velocity v but at a certain angle θ measured with respect to direction of the field (plane of the flux) as shown in the Fig. 3.52 (b) then component of velocity which is $v \sin\theta$ is perpendicular to the direction of flux and hence responsible for the induced e.m.f. The other component $v \cos\theta$ is parallel to the plane of the flux and hence will not contribute to the dynamically induced e.m.f.

Under this condition magnitude of induced e.m.f. is given by,

$$e = B l v \sin\theta \quad \text{volts}$$

where θ is measured with respect to plane of the flux.

Example 3.6 : *A conductor of 2 m length moves with a uniform velocity of 1.27 m/sec under a magnetic field having a flux density of 1.2 Wb/m² (tesla). Calculate the magnitude of induced e.m.f. if conductor moves,*

i) at right angles to axis of field.

ii) at an angle of 60° to the direction of field.

Solution : i) The magnitude of induced e.m.f.

$$e = B l v \quad \text{for } \theta = 90°$$

$$\therefore \quad e = 1.2\times2\times1.27 = \mathbf{3.048\ volts}$$

ii)

$$e = B l v \sin\theta \quad \text{where } \theta = 60°$$

$$e = 1.2\times2\times1.27\times\sin 60 = \mathbf{2.6397\ volts}$$

Example 3.7 : *A coil carries 200 turns gives rise a flux of 500 μWb when carrying a certain current. If this current is reversed in $\frac{1}{10}$ th of a second. Find the average e.m.f. induced in the coil.*

Solution : The magnitude of induced e.m.f. is,

$$= N\frac{d\phi}{dt}$$

where $d\phi$ is change in flux linkages i.e. change in $N\phi$. Now in this problem flux is 500×10^{-6} for given current. After reversing this current, flux will reverse its direction. So flux becomes $\left(-500\times10^{-6}\right)$.

$$\therefore \quad d\phi = \phi_2 - \phi_1 = -500\times10^{-6} - \left(+500\times10^{-6}\right)$$

This happens in time dt = 0.1 sec.

$$\therefore \quad \text{Average e.m.f.} = -N\frac{d\phi}{dt} = -200\times\frac{\left(-1\times10^{3}\right)}{0.1} = \mathbf{2\ volts}$$

3.30.2 Direction of Dynamically Induced E.M.F.

The direction of induced e.m.f. can be decided by using two rules.

1) Fleming's right hand rule

As discussed earlier, the Fleming's left hand rule is used to get direction of force experienced by conductor carrying current, placed in a magnetic field while Fleming's right hand rule is to be used to get direction of induced e.m.f. when conductor is moving in a magnetic field.

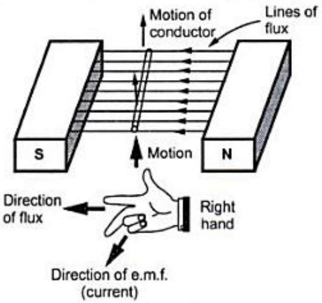


Fig. 3.53 (a)

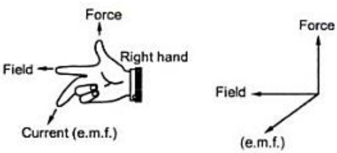


Fig. 3.53 (b)

According to Fleming's right hand rule, outstretch the three fingers of right hand namely the thumb, fore finger and the middle finger, perpendicular to each other. Arrange the right hand so that first finger point in the direction of flux lines (from N to S) and thumb in the direction of motion of conductor with respect to the flux then the middle finger will point in the direction of the induced e.m.f. (or current).

Consider the conductor moving in a magnetic field as shown in the Fig. 3.53 (a). It can be verified using Fleming's right hand rule that the direction of the current due to the induced e.m.f. is coming out. Symbolically this is shown in the Fig. 3.53 (b).

Key Point: *In practice though magnet is moved keeping the conductor stationary, while application of rule, thumb should point in the direction of relative motion of* ***conductor with respect to*** *flux, assuming the flux stationary.*

This rule mainly gives direction of current which induced e.m.f. in conductor will set up when closed path is provided to it.

Verify the direction of the current through conductor in the four cases shown in the Fig. 3.54 by the use of Fleming's right hand rule.

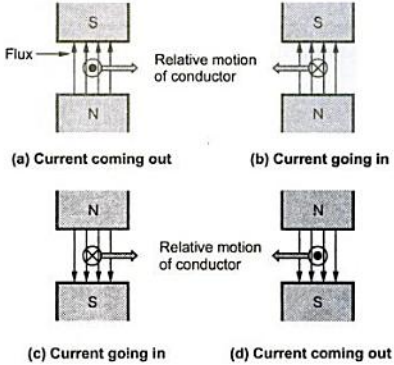


Fig. 3.54 Verifying Fleming's right hand rule

2) Lenz's law

This rule is based on the principles derived by German Physicist Heinrich Lenz.

The Lenz's law states that, 'The **direction of an induced e.m.f. produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.**'

In short the induced e.m.f. always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression.

$$\therefore \qquad e = -N\frac{d\phi}{dt}$$

The explanation can be given as below.

Consider a solenoid as shown in the Fig. 3.55. Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil which will circulate the current through the coil.

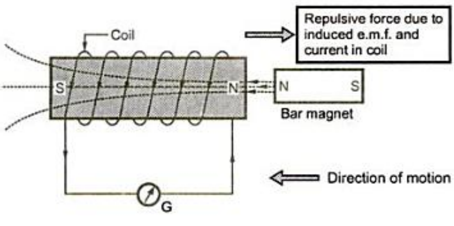


Fig. 3.55 Lenz's law

According to Lenz's law, the direction of current due to induced e.m.f. is so as to oppose the cause. The cause is motion of bar magnet towards coil. So e.m.f. will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole. Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the Fig. 3.55.

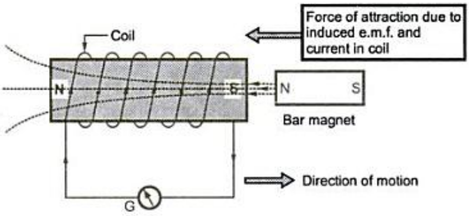


Fig. 3.56 Lenz's law

If the same bar magnet is moved away from the coil, then induced e.m.f. will set up a current in the direction which will cause, the end of solenoid facing bar magnet to behave as S-pole. Because of this two unlike poles face each other and there will be force of attraction which is direction of magnet, away from the coil. The galvanometer shows deflection in other direction as shown in the Fig. 3.56.

The Lenz's law can be summarized as,

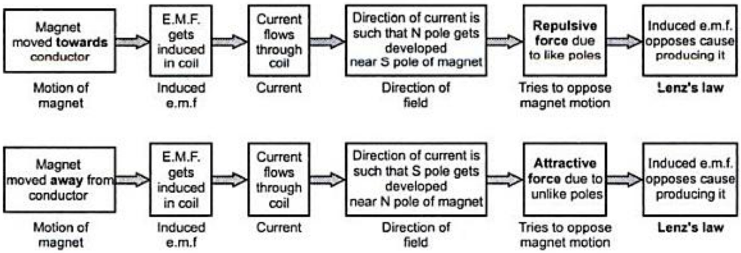


Fig. 3.57 Concept of Lenz's law

3.31 Statically Induced E.M.F.

Key Point : *The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called* ***statically induced e.m.f.***

Explanation : To have an induced e.m.f. there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

Consider an electromagnet which is producing the necessary flux for producing e.m.f. Now let current through the coil of an electromagnet be an alternating one. Such alternating current means **it changes its magnitude periodically with time**. This produces the flux which is also alternating i.e. changing with time. Thus there exists $d\phi/dt$ associated with coil placed in the vicinity of an electromagnet. This is responsible for producing an e.m.f. in the coil. This is called statically induced e.m.f.

Key Point: *It can be noted that there is no physical movement of magnet or conductor, it is the alternating supply which is responsible for such an induced e.m.f.*

The concept of statically induced e.m.f. is shown in the Fig. 3.58.

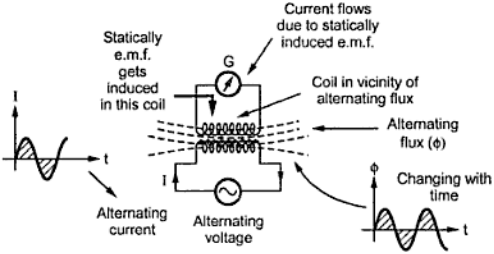


Fig. 3.58 Concept of statically induced e.m.f.

Such an induced e.m.f. can be observed in case of a device known as transformer.

Note : Due to alternating flux linking with the coil itself, the e.m.f. gets induced in that coil itself which carries an alternating current.

The statically induced e.m.f. is further classified as,

1) Self induced e.m.f. and 2) Mutually induced e.m.f.

We shall study now these two types of statically induced e.m.f.s.

3.32 Self Induced E.M.F.

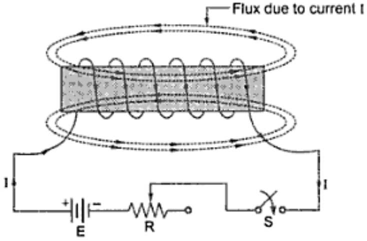


Fig. 3.59

Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position. The current magnitude can be varied with the help of variable resistance connected in series with battery, coil and switch as shown in the Fig. 3.59.

The flux produced by the coil links with the coil itself. The total flux linkages of coil will be N ϕ Wb-turns. Now if the current 'I' is changed with the help of variable resistance, then flux produced will also change, due to which flux linkages will also change.

Hence according to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f. in the coil. So without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called **self induction**.

The e.m.f. induced in a coil due to the change of its own flux linked with it is called **self induced e.m.f.**

Key Point : *The self induced e.m.f. lasts till the current in the coil is changing. The direction of such induced e.m.f. can be obtained by Lenz's law.*

3.32.1 Self Inductance

According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence the self induced e.m.f. will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep it to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. **So any change in current through coil is opposed by the coil.**

This property of the coil which opposes any change in the current passing through it is called **self inductance or only inductance.**

It is analogous to electrical inertia or electromagnetic inertia.

3.32.2 Magnitude of Self Induced E.M.F.

From the Faraday's law of electromagnetic induction, self induced e.m.f. can be expressed as

$$e = -N\frac{d\phi}{dt}$$

Negative sign indicates that direction of this e.m.f. is opposing change in current due to which it exists.

The flux can be expressed as,

$$\phi = (\text{Flux / Ampere}) \times \text{Ampere} = \frac{\phi}{I} \times I$$

Now for a circuit, as long as permeability 'μ' is constant, ratio of flux to current (i.e. B/H) remains constant.

$$\therefore \text{Rate of change of flux} = \frac{\phi}{I} \times \text{Rate of change of current}$$

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -N \cdot \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

The constant $\frac{N\phi}{I}$ in this expression is nothing but the quantitative measure of the property due to which coil opposes any change in current.

So this constant $\frac{N\phi}{I}$ is called **coefficient of self inductance and denoted by 'L'.**

$$\therefore \quad \boxed{L = \frac{N\phi}{I}}$$

It can be defined as flux linkages per ampere current in it. Its unit is **henry (H).**

A circuit possesses **a self inductance of 1 H** when a current of 1 A through it produces flux linkages of 1 Wb-turn in it.

$$\therefore \quad \boxed{e = -L\frac{dI}{dt} \quad \text{volts}}$$

From this equation, the unit henry of self inductance can be defined as below.

Key Point : *A circuit possesses an inductance of 1 H when a current through coil is changing uniformly at the rate of one ampere per second inducing an opposing e.m.f. 1 volt in it.*

The coefficient of self inductance is also defined as the e.m.f. induced in volts when the current in the circuit changes uniformly at the rate of one ampere per second.

3.32.3 Expressions for Coefficient of Self Inductance (L)

$$L = \frac{N\phi}{I} \quad \ldots (1)$$

But $$\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S}$$

$$\therefore \quad L = \frac{N \cdot N \cdot I}{I \cdot S}$$

$$\therefore \quad \boxed{L = \frac{N^2}{S} \quad \text{henries}} \quad \ldots (2)$$

Now $$S = \frac{l}{\mu a}$$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$$\therefore \quad \boxed{L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \quad \text{henries}} \quad \ldots (3)$$

where l = Length of magnetic circuit

a = Area of cross-section of magnetic circuit through which flux is passing.

3.32.4 Factors Affecting Self Inductance of a Coil

Now as defined in last section,

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

We can define factors on which self inductance of a coil depends as,

1) It is directly proportional to the square of number of turns of a coil. This means for same length, if number of turns are more then self inductance of coil will be more.
2) It is directly proportional to the cross-sectional area of the magnetic circuit.
3) It is inversely proportional to the length of the magnetic circuit.
4) It is directly proportional to the relative permeability of the core. So for iron and other magnetic materials inductance is high as their relative permeabilities are high.
5) For air cored or non-magnetic cored magnetic circuits, μ_r=1 and constant, hence self inductance coefficient is also small and always constant.

   As against this for magnetic materials, as current i.e. magnetic field strength H (N I/l) is changed, μ_r also changes. Due to this change in current, cause change in value of self inductance. So for magnetic materials it is not constant but varies with current.

Key Point: *For magnetic materials, L changes as the current I.*

6) Since the relative permeability of iron varies with respect to flux density, the coefficient of self inductance varies with respect to flux density.
7) If the conductor is bent back on itself, then magnetic fields produced by current through it will be opposite to each other and hence will neutralize each other. Hence **inductance will be zero** under such condition.

Example 3.8 : *If a coil has 500 turns is linked with a flux of 50 mWb, when carrying a current of 125 A. Calculate the inductance of the coil. If this current is reduced to zero uniformly in 0.1 sec, calculate the self induced e.m.f. in the coil.*

Solution : The inductance is given by,

$$L = \frac{N\phi}{I}$$

where $N = 500, \quad \phi = 50 \text{ mWb} = 50\times10^{-3} \text{ Wb}, \quad I = 25 \text{ A}$

$$\therefore \quad L = \frac{500\times50\times10^{-3}}{125} = 0.2 \text{ H}$$

$$e = -L\frac{dI}{dt} = -L\left[\frac{\text{Final value of I} - \text{Initial value of I}}{\text{Time}}\right]$$

$$= -0.2\times\left(\frac{0-125}{0.1}\right) = \mathbf{250 \text{ volts}}$$

This is positive because current is decreased. So this 'e' will try to oppose this decrease, means will try to increase current and will help the growth of the current.

Example 3.9 : *A coil is wound uniformly on an iron core. The relative permeability of the iron is 1400. The length of the magnetic circuit is 70 cm. The cross-sectional area of the core is 5 cm^2. The coil has 1000 turns. Calculate,*

i) Reluctance of magnetic circuit ii) Inductance of coil in henries.

iii) E.M.F. induced in coil if a current of 10 A is uniformly reversed in 0.2 seconds.

Solution : $\mu_r = 1400, \; L = 70 \text{ cm} = 0.7 \text{ m}, \; N = 1000$

$$A = 5 \text{ cm}^2 = 5\times10^{-4} \text{ m}^2, \; \mu_0 = 4\pi\times10^{-7}$$

i) $$S = \frac{l}{\mu_0\mu_r a} = \frac{0.7}{4\pi\times10^{-7}\times1400\times5\times10^{-4}} = \mathbf{7.957\times10^5 \text{ AT/Wb}}$$

ii) $$L = \frac{N^2}{S} = \frac{(1000)^2}{7.957\times10^5} = \mathbf{1.2566 \text{ H}}$$

iii) A current of + 10 A is made − 10 A in 0.2 sec.

$$\therefore \quad \frac{dI}{dt} = \frac{-10-10}{0.2} = -100$$

$$e = -L\frac{dI}{dt} = -1.2566\times(-100) = \mathbf{125.66 \text{ volts}}$$

Again it is positive indicating that this e.m.f. opposes the reversal i.e. decrease of current from +10 towards −10 A.

3.33 Mutually Induced E.M.F.

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called **mutually induced e.m.f.**

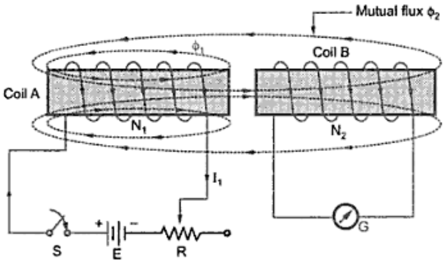


Fig. 3.60 Mutually induced e.m.f.

Consider two coils which are placed adjacent to each other as shown in the Fig. 3.60. The coil A has N_1 turns while coil B has N_2 number of turns. The coil A has switch S, variable resistance R and battery of 'E' volts in series with it. A galvanometer is connected across coil B to sense induced e.m.f. and current because of it.

Current through coil A is I_1 producing flux ϕ_1. Part of this flux will link with coil B i.e. will complete its path through coil B as shown in the Fig. 3.60. This is the mutual flux ϕ_2.

Now if current through coil A is changed by means of variable resistance R, then flux ϕ_1 changes. Due to this, flux associated with coil B, which is mutual flux ϕ_2 also changes. Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

Key Point : *Any change in current through coil A produces e.m.f. in coil B, this phenomenon is called mutual induction and e.m.f. is called mutually induced e.m.f.*

3.33.1 Magnitude of Mutually Induced E.M.F.

Let

N_1 = Number of turns of coil A

N_2 = Number of turns of coil B

I_1 = Current flowing through coil A

ϕ_1 = Flux produced due to current I_1 in webers

ϕ_2 = Flux linking with coil B

According to Faraday's law, the induced e.m.f. in coil B is,

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

Negative sign indicates that this e.m.f. will set up a current which will oppose the change of flux linking with it.

Now $$\phi_2 = \frac{\phi_2}{I_1} \times I_1$$

If permeability of the surroundings is assumed constant then $\phi_2 \propto I_1$ and hence ϕ_2 / I_1 is constant.

$\therefore$ Rate of change of $\phi_2 = \frac{\phi_2}{I_1} \times$ Rate of change of current I_1

$$\therefore \quad \frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -N_2 \cdot \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_1}{dt}$$

Here $\left(\frac{N_2 \phi_2}{I_1}\right)$ **is called coefficient of mutual inductance denoted by M.**

$$\therefore \quad \boxed{e_2 = -M \frac{dI_1}{dt} \quad \text{volts}}$$

Coefficient of mutual inductance is defined as the property by which e.m.f. gets induced in the second coil because of change in current through first coil.

Coefficient of mutual inductance is also called mutual inductance. It is measured in henries.

3.33.2 Definitions of Mutual Inductance and its Unit

1) The coefficient of mutual inductance is defined as the flux linkages of the coil per ampere current in other coil.
2) It can also be defined as equal to e.m.f. induced in volts in one coil when current in other coil changes uniformly at a rate of one ampere per second.

Similarly its unit can be defined as follows :

1) Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current of one ampere flowing through one coil produces a flux linkage of one weber turn in the other coil.

2) Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f. of one volt in the other coil.

3.33.3 Expressions of the Mutual Inductance (M)

1)
$$M = \frac{N_2 \phi_2}{I_1}$$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linking with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$\therefore \quad M = \frac{N_2 K_1 \phi_1}{I_1}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

$$\therefore \quad M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$M = \frac{K_1 N_1 N_2}{S}$$

If all the flux produced by coil A links with coil B then $K_1 = 1$.

$$M = \frac{N_1 N_2}{S}$$

4) Now $S = \frac{l}{\mu a}$ and $K_1 = 1$

Then
$$M = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l}$$

$$\therefore \quad M = \frac{N_1 N_2 a \mu_0 \mu_r}{l}$$

5) If second coil carries current I_2, producing flux ϕ_2, the part of which links with coil A i.e. ϕ_1 then,

$$\phi_1 = K_2 \phi_2 \quad \text{and} \quad M = \frac{N_1 \phi_1}{I_2}$$

$$M = \frac{N_1 K_2 \phi_2}{I_2}$$

Now $$\phi_2 = \frac{N_2 I_2}{S}$$

$$\therefore \quad M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$\therefore \quad \boxed{M = \frac{K_2 N_1 N_2}{S}}$$

If entire flux produced by coil B_2 links with coil 1, $K_2 = 1$ hence,

$$M = \frac{N_1 N_2}{S}$$

3.33.4 Coefficient of Coupling or Magnetic Coupling Coefficient

We know that, $M = \frac{N_2 K_1 \phi_1}{I_1}$ and $M = \frac{N_1 K_2 \phi_2}{I_2}$

Multiplying the two expressions of M,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

$$\therefore \quad M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

But $$\frac{N_1 \phi_1}{I_1} = \text{Self inductance of coil 1} = L_1$$

$$\frac{N_2 \phi_2}{I_2} = \text{Self inductance of coil 2} = L_2$$

$$\therefore \quad M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 L_2} = K\sqrt{L_1 L_2}$$

where $$\boxed{K = \sqrt{K_1 K_2}}$$

The K is called **coefficient of coupling.**

If entire flux produced by one coil links with other then $K = K_1 = K_2 = 1$ and maximum mutual inductance existing between the coil is $M = K\sqrt{L_1 L_2}$.

This gives an idea about magnetic coupling between the two coils. When entire flux produced by one coil links with other, this coefficient is maximum i.e. unity.

It can be defined as the ratio of the actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance.

The expression for K is,

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Key Point: *When K = 1 coils are said to be* ***tightly coupled*** *and if K is a fraction the coils are said to be* ***loosely coupled.***

3.34 Effective Inductance of Series Connection

Similar to the resistances, the two inductances can be coupled in series. The inductances can be connected in series either in series aiding mode called **cumulatively coupled connection** or series opposition mode called **differentially coupled connection.**

3.34.1 Series Aiding or Cumulatively Coupled Connection

Two coils are said to be cumulatively coupled if their fluxes are always in the same direction at any instant.

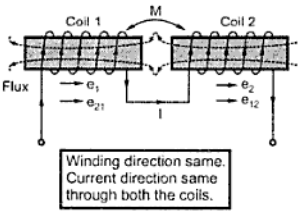


Fig. 3.61 Series aiding

For this, winding direction of the two coils on the core must be the same so that both will carry current in same direction. The Fig. 3.61 shows cumulatively coupled connection.

Coil 1 has self inductance L_1 and Coil 2 has self inductance L_2.

While both have a mutual inductance of M.

3.34.2 Equivalent Inductance of Series Aiding Connection

Refer to Fig. 3.61 which shows two coil of self inductances L_1 and L_2 connected in series aiding mode. The mutual inductance between the two is M.

If current flow through the circuit is changing at the rate of $\frac{di}{dt}$ then total e.m.f. induced will be due to self induced e.m.f.s and due to mutually induced e.m.f.s.

Due to flux linking with coil 1 itself, there is self induced e.m.f.,

$$e_1 = -L_1 \frac{di}{dt}$$

Due to flux produced by coil 2 linking with coil 1 there is mutually induced e.m.f.,

$$e_{21} = -M\frac{di}{dt}$$

Due to flux produced by coil 1 linking with coil 2 there is mutually induced e.m.f.,

$$e_{12} = -M\frac{di}{dt}$$

Due to flux produced by coil 2 linking with itself there is self induced e.m.f.

$$e_2 = -L_2\frac{di}{dt}$$

The total induced e.m.f. is addition of these e.m.f.s as all are in the same direction.

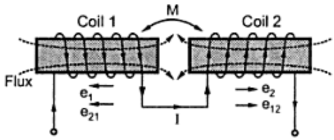
$$e = e_1 + e_{21} + e_{12} + e_2 = -L_1\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} - L_2\frac{di}{dt}$$

$$= -[L_1 + L_2 + 2M]\frac{di}{dt} = -L_{eq}\frac{di}{dt}$$

Where L_{eq} = Equivalent inductance

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2 + 2M}$$

3.34.3 Series Opposition or Differentially Coupled Connection



Winding direction opposite.
Hence current direction opposite

Fig. 3.62 Series opposition

Two coils are said to be differently coupled if their fluxes are always in the opposite direction at any instant.

Such a connection is shown in the Fig. 3.62.

Coil 1 has self inductance L_1.

Coil 2 has self inductance L_2.

and the mutual inductance between the two is M.

3.34.4 Equivalent Inductance of Series Opposition Connection

In series opposition, flux produced by coil 2 is in opposite direction to the flux produced by coil 1.

If current in the circuit is changed at a rate $\frac{di}{dt}$ then their self induced e.m.f.s will oppose the applied voltage but mutually induced e.m.f. will assist the applied voltage.

Similar to the cumulative connection there will exist four e.m.f.s which are,

$$e_1 = -L_1 \frac{di}{dt}, \qquad e_{21} = +M\frac{di}{dt}$$

$$e_{12} = +M\frac{di}{dt} \quad \text{and} \quad e_2 = -L_2 \frac{di}{dt}$$

Hence the total e.m.f. is the addition of these four e.m.f.s,

$$\therefore \quad e = e_1 + e_{21} + e_{12} + e_2$$

$$= -L_1 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} - L_2 \frac{di}{dt}$$

$$= -\left[L_1 + L_2 - 2M\right]\frac{di}{dt} = -L_{eq} \frac{di}{dt}$$

where L_{eq} = Equivalent inductance of the differentially coupled connection.

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Example 3.10 : *Two coils A and B are kept in parallel planes, such that 70 % of the flux produced by coil A links with coil B. Coil A has 10,000 turns. Coil B has 12,000 turns. A current of 4 A in coil A produces a flux of 0.04 mWb while a current of 4 A in coil B produces a flux of 0.08 mWb. Calculate,*
i) Self inductances L_A and L_B ii) Mutual inductance M iii) Coupling coefficient.

Solution : The given values are,

$$N_A = 10{,}000, \quad N_B = 12{,}000, \quad \phi_B = 0.7\,\phi_A$$

$$\therefore \quad K_A = \frac{\phi_B}{\phi_A} = 0.7$$

$$\phi_A = 0.04\times10^{-3}\ \text{Wb} \quad \text{for } I_A = 4\ \text{A}$$

$$\phi_B = 0.08\times10^{-3}\ \text{Wb} \quad \text{for } I_B = 4\ \text{A}$$

i) Self inductance $$L_A = \frac{N_A\phi_A}{I_A} = \frac{10{,}000\times0.04\times10^{-3}}{4} = \mathbf{0.1\ H}$$

and $$L_B = \frac{N_B\phi_B}{I_B} = \frac{12000\times0.08\times10^{-3}}{4} = \mathbf{0.24\ \ H}$$

ii) Mutual inductance $$M = \frac{N_B\phi_B}{I_A} = \frac{N_B K_A \phi_A}{I_A}$$

$$= \frac{12000\times0.7\times0.04\times10^{-3}}{4} = \mathbf{0.084\ H}$$

iii) Coupling coefficient $K = \frac{M}{\sqrt{L_A L_B}} = \frac{0.084}{\sqrt{0.1 \times 0.24}} = \mathbf{0.5422}$

3.35 Energy Stored in the Magnetic Field

We know that energy is required to establish flux i.e. magnetic field but it is not required to maintain it. This is similar to the fact that the energy is required to raise the water through a certain height (h) which is 'mgh' joules. But energy is not required to maintain the water at height 'h'. This energy 'mgh' gets stored in it as its potential energy and can be utilized for many purposes.

Key Point : *The energy required to establish magnetic field then gets stored into it as a potential energy. This energy can be recovered when magnetic field established, collapses.*

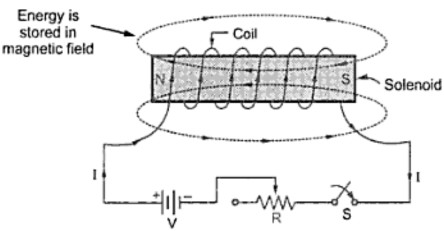


Fig. 3.63 Energy stored in magnetic field

This can be explained as below.

Consider a solenoid, the current through which can be controlled with the help of switch S, resistance R shown in the Fig. 3.63.

Initially switch 'S' is open, so current through coil, I is zero. When switch is closed, current will try to built its value equal to I. Neglect the resistance of coil.

It will take sometime to increase the current from 'zero' to 'I' say 'dt' seconds.

In the mean time, flux linkages associated with the coil will change, due to which there will be self induced e.m.f. in the coil whose value is given by,

$$e = -L\frac{dI}{dt}$$

So at every instant, coil will try to oppose the increase in the current. To overcome this opposition, supply has to provide the energy to the circuit. This is nothing but the energy required to establish the current i.e. magnetic field or flux around the coil.

Once current achieves its maximum value 'I' then change in current stops. Hence there cannot be any induced e.m.f. in the coil and no energy will be drawn from the supply. So no energy is required to maintain the established flux. This is because, induced e.m.f. lasts as long as there is change in flux lines associated with the coil, according to Faraday's law.

Key Point : *Now the energy which supply has provided, gets stored in the coil which is energy stored in the magnetic field, as its potential energy.*

When current is again reduced to zero by opening the switch then current through the coil starts decreasing and flux starts decreasing. So there is induced e.m.f. in the coil according to Faraday's law. But as per Lenz's law it will try to oppose cause producing it which is decrease in current. So this induced e.m.f. now will try to maintain current to its original value. So instantaneously this induced e.m.f. acts as a source and supplies the energy to the source. This is nothing but the same energy which is stored in the magnetic field which gets recovered while field collapses. So energy stored while increase in the current is returned back to the supply when current decreases i.e. when field collapses.

Key Point : *The energy which is stored in the coil earlier is returned back to the supply. No additional energy can exist as coil cannot generate any energy.*

The expression for this energy stored is derived below.

3.35.1 Expression for Energy Stored in the Magnetic Field

Let the induced e.m.f. in a coil be,

$$e = -L\frac{dI}{dt}$$

This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

$$\therefore \quad V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$$

$$\therefore \quad \text{Power supplied} = V \times I = L\frac{dI}{dt} \times I$$

$\therefore$ Energy supplied in time dt is,

$$E = \text{Power} \times \text{Time} = L\frac{dI}{dt} \times I \times dt$$

$$= L\, di \times I \quad \text{joules}$$

This is energy supplied for change in current of dI but actually current changes from zero to I.

$\therefore$ Integrating above total energy stored is,

$$E = \int_0^I L\, dI\, I = L \int_0^I dI\, I$$

$$= L\left[\frac{I^2}{2}\right]_0^I = L\left[\frac{I^2}{2} - 0\right]$$

$$\therefore \quad \boxed{E = \frac{1}{2} L I^2 \quad \text{joules}}$$

3.35.2 Energy Stored Per Unit Volume

The above expression for the energy stored can be expressed in the different form as,

$$E = \frac{1}{2} L I^2 \text{ joules}$$

Now $$L = \frac{N\phi}{I}$$

$$\therefore \quad E = \frac{1}{2}\frac{N\phi}{I} I^2 \text{ joules } = \frac{1}{2} N\phi I \text{ joules}$$

Now $$N I = H l \quad \text{ampere-turns}$$

$$\phi = B a$$

$$\therefore \quad E = \frac{1}{2} B a H l$$

But $$a \times l = \text{Area} \times \text{Length} = \text{Volume of magnetic circuit}$$

$\therefore$ Energy stored per unit volume is,

$$= \frac{1}{2} B H$$

But $$B = \mu H$$

$\therefore$ Energy per unit volume,

$$= \frac{1}{2} \mu H^2 \quad \text{joules / m}^3$$

$$\boxed{E / \text{unit volume} = \frac{1}{2}\frac{B^2}{\mu} \quad \text{joules / m}^3}$$

where $$\mu = \mu_0 \mu_r$$

In case of inductive circuit when circuit is opened with the help of switch then current decays and finally becomes zero. In such case energy stored is recovered and if there is resistance in the circuit, appears in the form of heat across the resistance.

While if the resistance is not present then this energy appears in the form of an arc across the switch, when switch is opened.

If the medium is air, $\mu_r = 1$ hence $\mu = \mu_0$ must be used in the above expressions of energy.

Example 3.11 : *A coil is wound on an iron core to form a solenoid. A certain current is passed through the coil which is producing a flux of 40 μWb. The length of magnetic circuit is 75 cm while its cross-sectional area is 3 cm². Calculate the energy stored in the circuit. Assume relative permeability of iron as 1500.*

Solution :

$$l = 75 \text{ cm} = 0.75 \text{ m}, \quad a = 3 \text{ cm}^2 = 3\times10^{-4} \text{ m}^2$$

$$\phi = 40 \ \mu\text{Wb} = 40\times10^{-6} \text{ Wb}, \quad \mu_r = 1500$$

$$\therefore \quad B = \frac{\phi}{a} = \frac{40\times10^{-6}}{3\times10^{-4}} = 0.133 \text{ Wb/m}^2$$

∴ Energy stored per unit volume,

$$\frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r} = \frac{1}{2}\frac{(0.133)^2}{4\pi\times10^{-7}\times1500} = 4.7157 \quad \text{J/m}^3$$

$$\therefore \text{ Total energy stored} = \text{Energy per unit volume} \times \text{Volume} = E\times(a\times l)$$

$$= 4.7157\times(3\times10^{-4}\times0.75) = \mathbf{0.00106 \ joules}$$

3.36 Lifting Power of Electromagnets

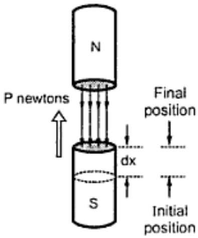


Fig. 3.64

Force of attraction between the two magnetized surfaces forms the basis of operation of devices like lifting magnets, solenoid valves, magnetically operated contactors, clutches etc.

Consider two poles of two magnetized surface N and S having an air gap of length 'l' m between them and a cross-sectional area of 'a' sq.m. Let P newtons be the force of attraction between them. This is shown in the Fig. 3.64.

The energy stored in a magnetic field in air per unit volume is,

$$E = \frac{1}{2}\frac{B^2}{\mu_0} \ \text{J/m}^3 \qquad \ldots \mu_r = 1$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}\frac{B^2}{\mu_0} a\times l \quad \text{J}$$

If south pole is moved further by distance dx then energy stored will further increase by,

$$= \frac{B^2}{2\mu_0} a \times dx \quad \text{joules}$$

This increased energy must be equal to the mechanical work done to move pole by distance dx which is,

$$P \times dx = (\text{Force} \times \text{Displacement})$$

$$\therefore \quad P\,dx = \frac{B^2}{2\mu_0} a \times dx$$

$$\boxed{P = \frac{B^2 a}{2\mu_0} \quad \text{newtons}}$$

This is the force in newtons existing between two magnetized surfaces.

3.37 Effective Inductance of Parallel Connection

The two inductances can be coupled in parallel. In such connection also they can be connected as cumulatively coupled (parallel aiding) or differentially coupled (parallel opposing). Let us obtain the effective inductance for these two cases.

3.37.1 Parallel Aiding or Cumulatively Coupled

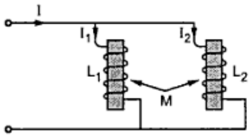


Fig. 3.65 Parallel aiding

Let the coil 1 has a self inductance L_1 while the coil 2 has self inductance L_2. The coils are connected in parallel aiding as shown in the Fig. 3.65

The currents through the coils 1 and 2 are I_1 and I_2 respectively.

Key Point: *Due to parallel aiding, the mutually induced e.m.f. helps the self induced e.m.f., in each coil.*

As coils are in parallel, the e.m.f.s induced in the two coils must be same.

$$\therefore \quad L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} = L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt} \qquad \text{...(1)}$$

$$\therefore \quad \frac{dI_1}{dt}[L_1 - M] = \frac{dI_2}{dt}[L_2 - M]$$

$$\therefore \quad \frac{dI_1}{dt} = \left[\frac{L_2 - M}{L_1 - M}\right]\frac{dI_2}{dt} \qquad \text{...(2)}$$

Now $\quad I = I_1 + I_2 \quad$ i.e. $\quad \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_2}{dt}$

$$\therefore \quad \frac{dI}{dt} = \left[\frac{L_2 - M}{L_1 - M}\right]\frac{dI_2}{dt} + \frac{dI_2}{dt} = \left[1 + \frac{L_2 - M}{L_1 - M}\right]\frac{dI_2}{dt} \qquad ...(3)$$

Now let L_{eq} be the equivalent inductance of the parallel connection then the induced e.m.f. can be written as,

$$e = L_{eq}\frac{dI}{dt} \qquad ...(4)$$

But due to parallel connection, this must be equal to the e.m.f. induced in any one coil.

$$\therefore \quad L_{eq}\frac{dI}{dt} = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt} \qquad ...(5)$$

Using equation (2) and (3) in the equation (5),

$$\therefore \quad L_{eq}\left[1 + \frac{L_2 - M}{L_1 - M}\right]\frac{dI_2}{dt} = L_1\left[\frac{L_2 - M}{L_1 - M}\right]\frac{dI_2}{dt} + M\frac{dI_2}{dt}$$

$$\therefore \quad L_{eq}\left[\frac{L_1 + L_2 - 2M}{L_1 - M}\right] = \frac{L_1(L_2 - M)}{(L_1 - M)} + M$$

$$\therefore \quad L_{eq}\left[\frac{L_1 + L_2 - 2M}{L_1 - M}\right] = \frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 - M}$$

$$\therefore \quad \boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}} \qquad ...(6)$$

3.37.2 Parallel Opposing or Differentially Coupled

The connection of coils is shown in the Fig. 3.66

Key Point: *Due to parallel opposing, the mutually induced e.m.f. opposes the self induced e.m.f., in each coil. As coils are in parallel, the e.m.f.s induced in the two coils must be same.*

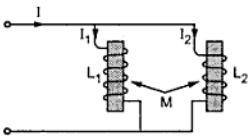


Fig. 3.66 Parallel opposing

$$\therefore \quad L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

$$\therefore \quad \frac{dI_1}{dt} = \left[\frac{L_2 + M}{L_1 + M}\right]\frac{dI_2}{dt} \qquad ...(7)$$

Now $$I = I_1 + I_2 \quad \text{i.e.} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$\therefore \quad \frac{dI}{dt} = \left[1 + \frac{L_2 + M}{L_1 + M}\right] \frac{dI_2}{dt} \quad \text{(Using equation (7))} \qquad ...(8)$$

Now let L_{eq} be the equivalent inductance of the parallel connection then the induced e.m.f. can be written as,

$$e = L_{eq} \frac{dI}{dt} \qquad ...(9)$$

But due to parallel connection, this total induced e.m.f. must be equal to the e.m.f. induced in any one coil.

$$\therefore \quad L_{eq} \frac{dI}{dt} = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \qquad ...(10)$$

Using equation (7) and (8) in the equation (10),

$$\therefore \quad L_{eq} \left[1 + \frac{L_2 + M}{L_1 + M}\right] \frac{dI_2}{dt} = L_1 \left[\frac{L_2 + M}{L_1 + M}\right] \frac{dI_2}{dt} - M \frac{dI_2}{dt}$$

$$\therefore \quad L_{eq} \left[\frac{L_1 + L_2 + 2M}{L_1 + M}\right] = \frac{L_1 L_2 + L_1 M - L_1 M - M^2}{L_1 + M}$$

$$\therefore \quad \boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}} \qquad ...(11)$$

3.38 Current Rise in Inductive Circuit

If a pure resistance R is connected to the supply, the current in the circuit increases to its maximum value $\frac{V}{R}$ instantaneously. Similarly if the resistive circuit is opened by means of a switch, the current reduces to zero value instantaneously. But in case of an inductive circuit, the rise and decay of the current is not instantaneous but gradual.

Consider an inductor L in series with the resistance R, connected across the voltage V, through a switch, as shown in the Fig. 3.67.

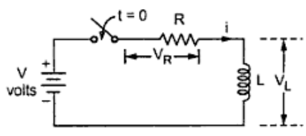


Fig. 3.67 Current rise in an inductor

The inductor has initially no current flowing through it. At the instant t = 0, the switch is closed as the d.c. voltage of V volts gets applied to the circuit.

Due to inductance L, the current increases gradually to its final value. At any instant during the rise of the current, let the current flowing is i amperes and it is increasing at a rate (di/dt) amperes/sec. This (di/dt) induces e.m.f. in the inductor, which opposes the cause producing it as per Lenz's law, which is rise in current. Thus the inductor opposes the rise in current due to which the current cannot increase to its maximum value instantaneously.

3.38.1 Mathematical Analysis

Applying KVL to the circuit, at any instant t,

$$V = V_R + V_L \quad \text{i.e.} \quad V = iR + L\frac{di}{dt}$$

$$\therefore \quad V - iR = L\frac{di}{dt} \quad \text{i.e.} \quad \frac{V}{R} - i = \frac{L}{R}\frac{di}{dt}$$

$$\therefore \quad \frac{R}{L}dt = \frac{di}{\frac{V}{R} - i}$$

Integrating both sides,

$$\frac{R}{L}t = -ln\left[\frac{V}{R} - i\right] + K \qquad \text{...(1)}$$

As initial inductor current is zero, i = 0 at t = 0.

$$\therefore \quad 0 = -ln\left[\frac{V}{R}\right] + K \quad \text{i.e.} \quad K = ln\left[\frac{V}{R}\right]$$

$$\therefore \quad \frac{R}{L}t = -ln\left[\frac{V}{R} - i\right] + ln\left[\frac{V}{R}\right]$$

$$\therefore \quad -\frac{R}{L}t = ln\left[\frac{V}{R} - i\right] - ln\left[\frac{V}{R}\right] = ln\left[\frac{\frac{V}{R} - i}{\left(\frac{V}{R}\right)}\right]$$

Taking antilog,

$$e^{-\frac{R}{L}t} = \frac{\frac{V}{R} - i}{\frac{V}{R}}$$

$$\therefore \quad \frac{V}{R} - i = \frac{V}{R}e^{-\frac{R}{L}t} \quad \text{i.e.} \quad i = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\therefore \quad \boxed{i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)} \qquad \text{... (2)}$$

As $t \to \infty$, $e^{-\frac{R}{L}t} \to 0$ and hence $i = I_m = \frac{V}{R}$ is the maximum value of current.

$$\therefore \quad \boxed{i = I_m \left(1 - e^{-\frac{R}{L}t}\right)} \quad \ldots (3)$$

3.38.2 Time Constant

The time constant of R-L series circuit is defined as the time required by the inductor current to rise from 0 to 0.632 of its final value, during its rise.

Thus at $t = \frac{L}{R}$, $i = I_m\left(1 - e^{-1}\right) = 0.632\ I_m$ hence the time constant of R-L series circuit is $\frac{L}{R}$ and is denoted as τ.

$$\therefore \quad \boxed{\tau = \frac{L}{R} \text{ seconds}}$$

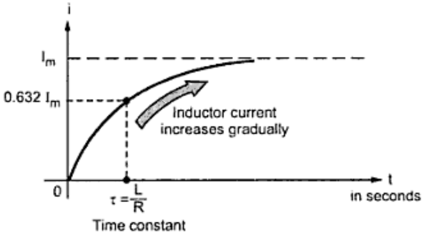


Fig. 3.68 Rise in inductor current

The Fig. 3.68 shows the exponential behaviour of the inductor current.

The initial rate of rise of current is large upto first time constant. At later stage the rate of rise of current reduces. Ideally the current reaches to its maximum value after infinite time but practically it reaches to its maximum value within 4 to 5 time constants.

3.39 Current Decay in Inductive Circuit

Consider a series R-L circuit as shown in the Fig. 3.69.

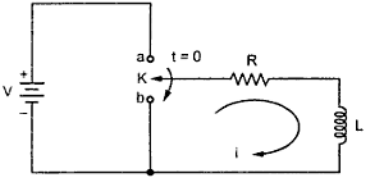


Fig. 3.69 Decay in inductive circuit

The switch K is in position 'a', long enough so that the current reaches to its maximum value of V/R. At the instant t = 0, the switch position is changed from 'a' to 'b'. Due to this, the current starts decreasing. But current cannot become zero instantaneously but there is gradual decay in the current due to the inductor.

3.39.1 Mathematical Analysis

Applying KVL to the circuit, when switch is in position 'b',

$$V_R + V_L = 0 \quad \text{i.e.} \quad iR + L\frac{di}{dt} = 0 \qquad ...(1)$$

And initially at t = 0, $i = I_m = \frac{V}{R}$

$$\therefore \quad L\frac{di}{dt} = -iR \quad \text{i.e.} \quad \frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides,

$$ln\,[i] = -\frac{R}{L}t + K'$$

Using initial condition,

$$ln\left[\frac{V}{R}\right] = 0 + K' \quad \text{i.e.} \quad K' = ln\left[\frac{V}{R}\right] \qquad ...(2)$$

$$\therefore \quad ln\,[i] = -\frac{R}{L}t + ln\left[\frac{V}{R}\right]$$

$$\therefore \quad ln\,[i] - ln\left[\frac{V}{R}\right] = -\frac{R}{L}t$$

$$\therefore \quad ln\left[\frac{i}{\frac{V}{R}}\right] = -\frac{R}{L}t \quad \text{i.e.} \quad \frac{i}{\left(\frac{V}{R}\right)} = e^{-\frac{R}{L}t}$$

$$\therefore \quad \boxed{i = \frac{V}{R}e^{-\frac{R}{L}t} = I_m\, e^{-\frac{R}{L}t}} \qquad ...(3)$$

Key Point: *The equation (3) shows that the current is exponentially decaying.*

The Fig. 3.70 shows the variation of current with respect to time.

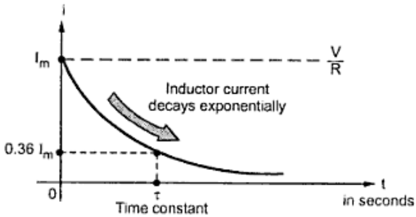


Fig. 3.70 Decay of inductor current

3.39.2 Time Constant

The time required by the inductor current to decay to 0.368 times its maximum value is called the time constant. This is shown in the Fig. 3.70.

Thus when $t = \tau = \frac{L}{R}$,

$$i = I_m \; e^{-\frac{R}{L} \times \frac{L}{R}} = I_m \; e^{-1} = 0.368 I_m$$

Hence the time constant τ is given by $\frac{L}{R}$.

$$\therefore \qquad \boxed{\tau = \frac{L}{R} \text{ seconds}}$$

Ideally current becomes zero after infinite time but practically it becomes almost zero within 4 to 5 time constants.

Examples with Solutions

Example 3.12 : *A magnetic circuit is excited by three coils as shown in the Fig. 3.71. Calculate the flux produced in the air gap. The material used for core is iron having relative permeability of 800. The length of the magnetic circuit is 100 cm with an air gap of 2 mm in it. The core has uniform cross-section of 6 cm².*

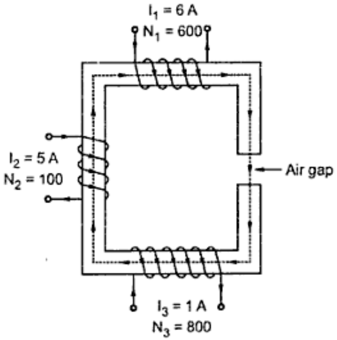


Fig. 3.71

Solution : Given, $N_1 = 600$, $I_1 = 6$ A, $N_2 = 100$, $I_2 = 5$ A

$$N_3 = 800, \quad I_3 = 1 \text{ A}, \quad l_T = 100 \text{ cm} = 1 \text{ m}$$

$$l_i = l_T - l_g = 1 \text{ m} - 2 \times 10^{-3} = 0.998 \text{ m}$$

$$l_g = 2\times10^{-3} \text{ m},\ \mu_r = 800,\ a = 6 \text{ cm}^2 = 6 \times 10^{-4} \text{ m}^2$$

Now total reluctance $S = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.998}{4\pi\times10^{-7}\times800\times6\times10^{-4}} = 1654548.263 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times6\times10^{-4}} = 2652582.385 \text{ AT/Wb}$$

$\therefore$ $S = 4307130.648$ AT/Wb

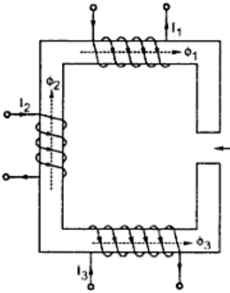


Fig. 3.71 (a)

Let us find the direction of flux due to various coils using **right hand thumb rule.**

As shown in the Fig. 3.71 (a) m.m.f of coil (1) and (2) are in same direction while m.m.f. of coil (3) is in opposite direction.

$\therefore$ Net m.m.f.= $(N_1 I_1) + (N_2 I_2) - (N_3 I_3)$

$$= (600 \times 6) + (100 \times 5) - (1 \times 800)$$

$$N I = 3300 \text{ AT}$$

$$\therefore \quad \phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S} = \frac{3300}{4307130.648}$$

$\therefore$ Flux in air gap ϕ = **0.7661 mWb**

Example 3.13 : *The Fig. 3.72 shows a magnetic circuit with two similar branches and an exciting coil of 1500 turns on central limb. The flux density in the air gap is 1 Wb/m² and leakage coefficient 1.2. Determine exciting current through the coil. Assume relative permeability of the iron constant equal to 600.*

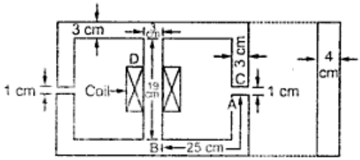


Fig. 3.72

Solution : Flux density in air gap $B_g = 1 \text{ Wb/m}^2$

$$a = 3 \times 4 = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad \phi_g = B_g \times a = 1 \times 12 \times 10^{-4} = 12 \times 10^{-4} \text{ Wb}$$

$$\lambda = \frac{\phi_T}{\phi_g} = \frac{\phi_T}{12 \times 10^{-4}} = 1.2$$

$$\phi_T = \phi_{side} = 1.2 \times 12 \times 10^{-4}$$

$$\phi_{side} = 1.44 \times 10^{-3} \text{ Wb}$$

As the circuit is parallel magnetic circuit,

$$\phi_{coil} = \phi_{side\ 1} + \phi_{side\ 2} = 2 \times 1.44 \times 10^{-3}$$

$$= 2.88 \times 10^{-3} \text{ (as sides are similar)}$$

Section I] Central limb

$$l_c = 19 \text{ cm} = 0.19 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_c = \frac{l_c}{\mu_0 \mu_r a} = \frac{0.19}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 209996.11 \text{ AT/Wb}$$

Section II] One side limb

$$l_i = 25 + 25 = 50 \text{ cm} = 0.5 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.5}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 552621.33 \text{ AT/Wb}$$

Section III] Air gap

$$l_g = 1 \text{ cm} = 0.01 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{0.01}{4\pi \times 10^{-7} \times 12 \times 10^{-4}}$$

$$= 6631456 \text{ AT/Wb}$$

$$\therefore \quad \text{M.M.F. for central limb} = \phi_{coil} \times S_c = 2.88 \times 10^{-3} \times 209996.11$$

$$= 604.78 \text{ AT}$$

$\therefore$ M.M.F. for one side limb $= \phi_{side} \times S_i = 1.44 \times 10^{-3} \times 552621.33$

$= 795.774$ AT

$\therefore$ M.M.F. for air gap $= \phi_g \times S_g = 12 \times 10^{-4} \times 6631456$

$= 7957.7472$ AT

$\therefore$ Total m.m.f. = M.M.F. for central limb + M.M.F. for one side + M.M.F. for air gap

$$N I = 604.78 + 795.774 + 7957.7452$$

$$\therefore \quad 1500 I = 9358.3107$$

$$\therefore \quad \mathbf{I = 6.2388\ A}$$

Example 3.14 : *A conductor of length 10 cm carrying 5 A is placed in a uniform magnetic field of flux density 1.25 tesla. Find the force acting on the conductor, if it is placed (i) along the lines of magnetic flux, (ii) perpendicular to the lines of flux and (ii) at 30° to the flux.*

Solution : $l = 10$ cm $= 10 \times 10^{-2}$ m, I = 5 A, B = 1.25 T

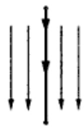


Fig. 3.73

Case 1 : Along lines of magnetic flux

θ = Angle between conductor and axis of magnetic field

$= 0°$

$\therefore F = B \cdot I \cdot l \sin\theta = 1.25 \times 5 \times 10 \times 10^{-2} \sin 0° = \mathbf{0\ N}$

Case 2 : Perpendicular to lines of flux i.e. $\theta = 90°$

$$\therefore \quad F = B \cdot I \cdot l \sin 90 = B \cdot I \cdot l = 1.25 \times 5 \times 10 \times 10^{-2} = \mathbf{0.625\ N}$$

Case 3 : At 30° to the flux i.e. $\theta = 30°$

$$\therefore \quad F = B \cdot I \cdot l \sin 30° = 1.25 \times 5 \times 10 \times 10^{-2} \times \frac{1}{2} = \mathbf{0.3125\ N}$$

Example 3.15 : *A square coil of 20 cm side is rotated about its axis at a speed of 200 revolutions per minute in a magnetic field of density 0.8 Wb/m². If the number of turns of coil is 25, determine maximum e.m.f. induced in the coil.*

Solution : The arrangement is shown in the Fig. 3.74.

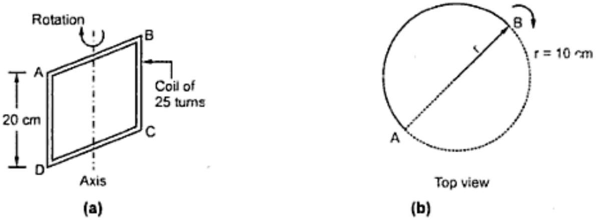


Fig. 3.74

As shown in the Fig. 3.74 (b) the active length responsible for cutting flux lines becomes $l = 20$ cm = 0.2 m.

Now $N = 200$ r.p.m.

We want the velocity of m/sec

$$v = r\,\omega$$

$$= r \times \frac{2\pi N}{60} \text{ [where N is in r.p.m. and r = 10 cm = 0.1 m]}$$

$$\therefore \quad v = 0.1 \times \frac{2\pi \times 200}{60} = 2.094 \text{ m/sec}$$

$$B = 0.8 \text{ Wb/m}^2 \quad \text{and} \quad \text{Active length} = 0.2 \text{ m}$$

The maximum e.m.f. induced in conductor AB, shown in Fig. 3.74 (b) will be,

$$e = B\,l\,v \sin\theta$$

For e_{max}, $\theta = 90°$

$$\therefore \quad e = 0.8 \times 0.2 \times 2.094 = 0.335 \text{ volts}$$

The e.m.f. induced in sides BC and AD is almost zero as their plane of rotation becomes parallel to plane of field.

And maximum e.m.f. induced in conductor CD will be same as AB = 0.335 volts.

∴ E.M.F. induced in one turn of the coil [AB + CD]

$$= 2 \times 0.335 = 0.67 \text{ volts}$$

In all, there are 25 turns in that coil.

∴ Total e.m.f. induced in a coil is

$$= 25 \times 0.67 = \mathbf{16.75 \text{ volts}}$$

Example 3.16 : *A conductor has 50 cm length is mounted on the periphery of a rotating part of d.c. machine. The diameter of a rotating drum is 75 cm. The drum is rotated at a speed of 1500 r.p.m. The flux density through which conductor passes at right angles is 1.1 T. Calculate the induced e.m.f. in the conductor.*

Solution : The active length l = 50 cm = 0.5 m, N = 1500 r.p.m, B = 1.1 T, $\theta = 90°$.

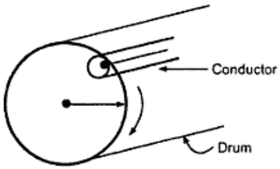


Fig. 3.75

The rotating drum on which conductor is mounted is called **armature** of a d.c. machine.

The arrangement is as shown in Fig. 3.75.

The linear velocity

$$v = r\,\omega = r \times \frac{2\pi N}{60}$$

$$= 0.375 \times \frac{2\pi \times 1500}{60} = 58.9 \text{ m/sec}$$

$$r = \frac{75}{2} = 37.5 \text{ cm}$$

$\therefore$ Induced e.m.f. in a conductor $= B\,lv = 1.1 \times 0.5 \times 58.9$

$= $ **32.397 volts**

Example 3.17 : *A coil of 200 turns having a mean diameter of 6 cm is placed co-axially at the centre of a solenoid of 50 cm long with 1500 turns and carrying current of 2.5 A. Calculate the mutual inductance between the two coils.*

Solution : Given values are,

N_1 = 1500 (solenoid), N_2 = 200, l_1 = 50 cm = 0.5 m, I_1 = 2.5 A

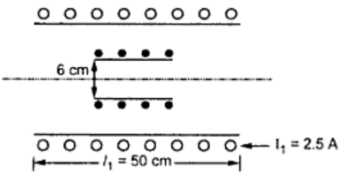


Fig. 3.76

Now magnetic field strength H at the centre of coil due to solenoid current is

$$H = \frac{N_1 I_1}{l_1}$$

$$= \frac{1500 \times 2.5}{0.5} = 7500 \text{ AT/m}$$

$\therefore$ Flux density at centre is $\quad B = \mu_0 H \quad (\mu_r = 1)$

$\therefore \quad B = 4\pi \times 10^{-7} \times 7500 = 9.424 \times 10^{-3} \text{ Wb/m}^2$

∴ Flux linking with second coil is,

$$\phi_2 = B \times a_2 = 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (d_2)^2$$

$$= 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (6 \times 10^{-2})^2 = 2.664 \times 10^{-5} \text{ Wb}$$

∴ Mutual inductance between the coils is,

$$M = \frac{N_2 \phi_2}{I_1} = \frac{200 \times 2.664 \times 10^{-5}}{2.5} = \mathbf{2.1318 \times 10^{-3} \text{ H}}$$

Example 3.18 : *Two coils with a coefficient of coupling of 0.5 between them are connected in series so as to magnetize a) in the same direction (series aiding), b) in the opposite direction (series opposition). The corresponding values of equivalent inductance for a) is 1.9 H and b) 0.7 H. Find the self inductance of each coil, mutual inductance between the coil.*

Solution : Given values are, K = 0.5

Now for series aiding, $L_{eq} = L_1 + L_2 - 2M = 1.9 \text{ H}$ ...(1)

For series opposition, $L_{eq} = L_1 + L_2 - 2M = 0.7 \text{ H}$ ...(2)

and $M = K\sqrt{L_1 L_2} = 0.5\sqrt{L_1 L_2}$ ... (3)

Subtracting equation (2) from equation (1), 4M = 1.2 i.e. **M = 0.3 H**

Substituting in equation (3), $0.3 = 0.5\sqrt{L_1 L_2}$ i.e. $L_1 L_2 = 0.36$

$$L_2 = \frac{0.36}{L_1}$$

Substituting in equation (1), $L_1 + \frac{0.36}{L_1} + 2 \times 0.3 = 1.9$

$$\therefore L_1^2 + 0.36 - 1.3 L_1 = 0$$

$$\therefore \quad L_1 = \frac{1.3 \pm \sqrt{(1.3)^2 - 4 \times 0.36}}{2}$$

$$L_1 = \mathbf{0.9 \text{ H} \text{ or } L_1 = 0.4 \text{ H}}$$

$$L_2 = \mathbf{0.4 \text{ H} \text{ or } L_2 = 0.9 \text{ H}}$$

Example 3.19 : *A coil of 800 turns of copper wire those diameter is of 0.375 mm. The length of the core is 90 cm. The diameter of core is 2.5 cm. Find the resistance and inductance of the coil. Assume specific resistance of copper as 1.73×10^{-6} Ω-cm.*

Solution :

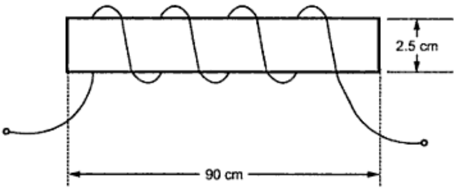


Fig. 3.77

Length of the coil $= (\pi \times d) \times \text{Number of turns}$

As $\pi \times d = \text{Circumference of 1 turn}$

And $d = \text{Diameter of the core}$

∴ Length of the coil $= (\pi \times 2.5 \times 10^{-2}) \times 800 = 62.83 \text{ m}$

$$\rho = 1.73 \times 10^{-6}\ \Omega\text{-cm} = 1.73 \times 10^{-8}\ \Omega\text{-m}$$

∴ $$R = \frac{\rho l}{a}$$

where $a = \frac{\pi}{4} d^2$ where d = Diameter of coil

∴ $d = 0.375 \text{ mm} = 0.375 \times 10^{-3} \text{ m}$

∴ $a = \frac{\pi}{4} \times (0.375 \times 10^{-3})^2 = 1.104 \times 10^{-7} \text{ m}^2$

∴ $$R = \frac{\rho l}{a} = \frac{1.73 \times 10^{-8} \times 62.68}{1.1044 \times 10^{-7}} = \mathbf{9.84\ \Omega}$$

While $$L = \frac{N^2}{S}$$

Reluctance $S = \frac{l}{\mu_0 \mu_r a}$ where l = Length of core = 0.9 m

$$a = \text{Cross-section area of core} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\times(2.5\times10^{-2})^2$$

$$= 4.908\times10^{-4} \ \ m^2$$

Assume $\mu_r = 1$

$$\therefore \quad S = \frac{0.9}{4\pi\times10^{-7}\times1\times4.908\times10^{-4}} = 1.45902\times10^9 \ \text{AT/Wb}$$

$$L = \frac{N^2}{S} = \frac{(800)^2}{1.45902\times10^9} = 4.386\times10^{-4}\ \text{H}$$

$$= \mathbf{0.4386\ mH}$$

Example 3.20 : *Two coils A and B are placed such that 40 % of flux produced by coil A links with coil B coils A and B have 2000 and 1000 turns respectively. A current of 2.5 A in coil A produces a flux of 0.035 mWb in coil B. For the above coil combination, find out (i) M, the mutual inductance and (ii) the coefficient of coupling K_A, K_B and K (iii) Self inductances L_A and L_B.*

Solution : $N_A = 2000$, $N_B = 1000$, $K_A = 0.4$, $\phi_B = 0.4\ \phi_A$

$I_A = 2.5$ A and $\phi_B = 0.035$ mWb

(i) Mutual inductance, $$M = \frac{N_B\,\phi_B}{I_A} = \frac{1000\times0.035\times10^{-3}}{2.5} = \mathbf{0.014\ H}$$

(ii) $\phi_B = 0.035$ mWb and $\phi_B = 0.4\ \phi_A$

$$\therefore \quad \phi_A = \frac{\phi_B}{0.4} = \frac{0.035}{0.4} = 0.0875\ \text{mWb}$$

$$\therefore \quad L_A = \frac{N_A\,\phi_A}{I_A} = \frac{2000\times0.0875\times10^{-3}}{2.5}$$

$$\therefore \quad L_A = \mathbf{0.07\ H}$$

Assuming that same current in coil B produces 0.035 mWb in coil B.

$$\therefore \quad L_B = \frac{N_B\,\phi_B}{I_B} = \frac{1000\times0.035\times10^{-3}}{2.5} = \mathbf{0.014\ H}$$

(iii) $$M = \frac{N_A\,\phi_A}{I_B} \qquad M = \frac{N_A\,K_B\,\phi_B}{I_B}$$

$$\therefore \quad 0.014 = \frac{2000\times K_B\times0.035\times10^{-3}}{2.5}$$

$\therefore \quad K_B = \mathbf{0.5}$

$\phi_B = K_A \phi_A$ and it is given that 40 % of ϕ_A links with coil B.

$\therefore \quad K_A = \mathbf{0.4}$

$$K = \sqrt{K_A K_B} = \sqrt{0.4 \times 0.5} = \mathbf{0.4472}$$

Example 3.21 : *Two long, single layered solenoids 'x' and 'y' have the same length and the same number of turns. The cross-sectional areas of the two are 'a_x' and 'a_y' respectively, with 'a_y' < 'a_x'. They are placed co-axially, with solenoid 'y' placed within the solenoid 'x'. Show that the coefficient of coupling between them is equal to $\sqrt{a_y / a_x}$.*

Solution : The arrangement is shown in the Fig. 3.78.

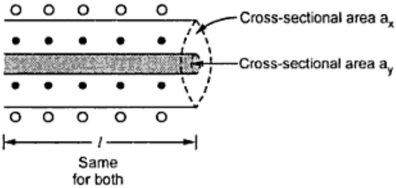


Fig. 3.78

It is known that

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

For coil x,

$$L_x = \frac{N^2 \mu_0 \mu_r a_x}{l}$$

and $$L_y = \frac{N^2 \mu_0 \mu_r a_y}{l}$$

The number of turns N and μ_r is same for both.

Considering coil y,

$$M = \frac{N_1 N_2 a_y \mu_0 \mu_r}{l} \quad \text{where } N_1 = N_2 = N$$

$$\therefore \quad M = \frac{N^2 a_y \mu_0 \mu_r}{l}$$

The coefficient of coupling is given by,

$$K = \frac{M}{\sqrt{L_x L_y}} = \frac{\frac{N^2 a_y \mu_0 \mu_r}{l}}{\sqrt{\frac{N^2 \mu_0 \mu_r a_x}{l} \times \frac{N^2 \mu_0 \mu_r a_y}{l}}}$$

$$= \frac{\left(\frac{N^2\mu_0\mu_r}{l}\right)a_y}{\left(\frac{N^2\mu_0\mu_r}{l}\right)\sqrt{a_x\,a_y}} = \sqrt{\frac{a_y}{a_x}}$$

$$\therefore \quad K = \sqrt{\frac{a_y}{a_x}} \qquad \text{...Proved}$$

Examples from G.U. and G.T.U. Papers

Example 3.22 : *A series magnetic circuit has an iron path of length 50 cm and an air gap of length 1 mm. The cross-sectional area of the iron is 6.66 cm² and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the air gap. The following points are taken from the magnetization curve for the iron :*

Flux density (T)	1.2	1.35	1.45	1.55
Magnetizing force (AT/m)	500	1000	2000	4000

[GU: Dec.-98, July-2004]

Solution : l_i = 50 cm, l_g = 1 mm, a = 6.66 cm², N = 400, ϕ = 0.9 mWb

$$B = \frac{\phi}{a} = \frac{0.9\times10^{-3}}{6.66\times10^{-4}} = 1.35 \text{ Wb/m}^2$$

From the given table, $H_i = 1000\,\text{AT/m}$ for $B = 1.35\,\text{T}$.

$$\therefore \quad (\text{M.M.F.})_{\text{iron}} = H_i\times l_i = 1000\times50\times10^{-2} = 500\,\text{AT}$$

For air gap, $B = \mu_0 H_g$

$$\therefore \quad H_g = \frac{B}{\mu_0} = \frac{1.35}{4\pi\times10^{-7}} = 1.07429\times10^6 \text{ AT/m}$$

$$\therefore \quad (\text{M.M.F.})_{\text{air gap}} = H_g\times l_g = 1.07429\times10^6\times1\times10^{-3} = 1074.2958\,\text{AT}$$

$$\therefore \quad \text{Total m.m.f.} = 500+1074.2958 = 1574.2958 = NI$$

$$\therefore \quad 1574.2958 = 400\times I$$

$$\therefore \quad I = \mathbf{3.9357\ A}$$

Example 3.23 : *Two coils have self inductances of 3 H and 2 H and mutual inductance is 2 H. They are connected in series and a current of 5 A is passed through them. Calculate the energy of the magnetic field when coils are connected i) Cumulatively and ii) Differentially* **[GU: Dec.-98]**

Solution : $L_A = 3\,H,\ L_B = 2\,H,\ M = 2\ H,\ I = 5\ A$

i) Cumulative connection

$$\therefore \quad L_{eq} = L_A + L_B + 2M = 3+2+(2\times2) = 9\ H$$

$$\therefore \quad E = \frac{1}{2}\ L_{eq}\ I^2 = \frac{1}{2}\times9\times5^2 = \mathbf{112.5\ J}$$

i) Differential connection

$$\therefore \quad L_{eq} = L_A + L_B - 2M = 3+2-(2\times2) = 1\,H$$

$$\therefore \quad E = \frac{1}{2}\ L_{eq}\ I^2 = \frac{1}{2}\times1\times5^2 = \mathbf{12.5\ J}$$

Example 3.24 : *Two long single layer solenoids have the same length and same number of turns, but are placed co-axially one within the other. The diameter of inner coil is 8 cm and that of outer coil is 10 cm. Calculate the co-efficient of coupling.* **[GU: May-2001]**

Solution : The arrangement is shown in the Fig. 3.79

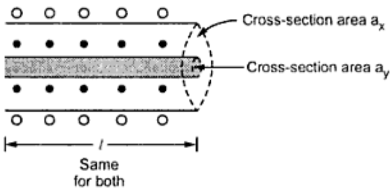


Fig. 3.79

It is known that

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

For coil x,

$$L_x = \frac{N^2 \mu_0 \mu_r a_x}{l}$$

and $$L_y = \frac{N^2 \mu_0 \mu_r a_y}{l}$$

The number of turns N and μ_r is same for both.

Considering coil y,

$$M = \frac{N_1 N_2 a_y \mu_0 \mu_r}{l} \quad \text{where } N_1 = N_2 = N$$

$$\therefore \quad M = \frac{N^2 a_y \mu_0 \mu_r}{l}$$

The coefficient of coupling is given by,

$$K = \frac{M}{\sqrt{L_x L_y}} = \frac{\frac{N^2 a_y \mu_0 \mu_r}{l}}{\sqrt{\frac{N^2 \mu_0 \mu_r a_x}{l} \times \frac{N^2 \mu_0 \mu_r a_y}{l}}}$$

$$= \frac{\left(\frac{N^2 \mu_0 \mu_r}{l}\right) a_y}{\left(\frac{N^2 \mu_0 \mu_r}{l}\right)\sqrt{a_x a_y}} = \sqrt{\frac{a_y}{a_x}}$$

$$\therefore \quad K = \sqrt{\frac{a_y}{a_x}}$$

In this example , $a_x = \frac{\pi}{4} d_{out}^2$, $a_y = \frac{\pi}{4} d_{in}^2$

where $d_{out} = 10\,\text{cm}$, $d_{in} = 8$ cm

$$\therefore \quad K = \sqrt{\frac{\frac{\pi}{4} d_{in}^2}{\frac{\pi}{4} d_{out}^2}} = \sqrt{\frac{8^2}{10^2}} = 0.8$$

Example 3.25 : *A direct current of 2 A is passed through a coil of 2000 turns and produces a flux of 0.2 mWb. Assuming that whole of flux links with all turns, then what is inductance of coil ? Find the voltage developed across the coil if current is interrupted in 1 ms.* **[GU : June-2000]**

Solution : I = 2 A, N = 2000, ϕ = 0.2 mWb

$$\therefore \quad L = \frac{N\phi}{I} = \frac{2000 \times 0.2 \times 10^{-3}}{2} = 0.2\text{ H}$$

$$e = -L\frac{di}{dt} = -L\frac{[i_{final} - i_{initial}]}{\text{Time}} = \frac{-0.2[0-2]}{1 \times 10^{-3}}$$

$$\therefore \quad e = 400\text{ V}$$ ...Current interrupted hence $i_{final} = 0$ A

Example 3.26 : *The coils A and B with 50 and 600 turns respectively are wound side by side on the same closed iron circuit of section 50 cm^2 and mean length of 1.2 m. Calculate :*

a) Self inductance of each coil b) Mutual inductance between the coils.

Assume $\mu_r = 1000$. Also find the e.m.f. induced in coil A if current in coil B grows steadily from 0 to 5 A in 0.01 sec. **[GU : Nov.-2001, 2004]**

Solution : $N_A = 50$, $N_B = 600$, $l_i = 1.2$ m, $a = 50\times10^{-4}\ m^2$, $\mu_r = 1000$

$$S = \frac{l_i}{\mu_0\mu_r a} = \frac{1.2}{4\pi\times10^{-7}\times1000\times50\times10^{-4}} = 190.986\times10^3\ \text{AT/Wb}$$

a) $$L_A = \frac{N_A^2}{S} = \frac{(50)^2}{190.986\times10^3} = \mathbf{13.089\ mH}$$

b) $$L_B = \frac{N_B^2}{S} = \frac{(600)^2}{190.986\times10^3} = \mathbf{1.885\ H}$$

c) $$M = \frac{N_A N_B}{S} = \frac{50\times600}{190.986\times10^3} = \mathbf{0.157\ H}$$

d) $$e_A = M\frac{di_B}{dt} = \frac{0.157(5-0)}{0.01} = \mathbf{78.5398\ V}$$

Example 3.27 : *A circular ring of mean circumference of 63 cm and cross-sectional area of 6 cm^2 is uniformly wound with a coil of 500 turns. Calculate : a) The current required to produce a flux of 0.45 mWb in the steel ring b) The current required for the same amount of flux when a saw cut of 0.1 cm width is made in the ring.*

The magnetization characteristics is given below :

B Wb/m^2	0.6	0.72	0.785	0.815
H AT/m	600	650	700	750

[GU : July-2002]

Solution : $l_i = 63$ cm, $a = 6\times10^{-4}\ m^2$, $N = 500$

Case 1 : No air gap, $\phi = 0.45\times10^{-3}$ Wb

$\therefore$ $$B = \frac{\phi}{a} = \frac{0.45\times10^{-3}}{6\times10^{-4}} = 0.75\ \text{Wb/m}^2$$

The B-H curve is shown in the Fig. 3.80 from which H = 680 for B = 0.75 Wb/m^2.

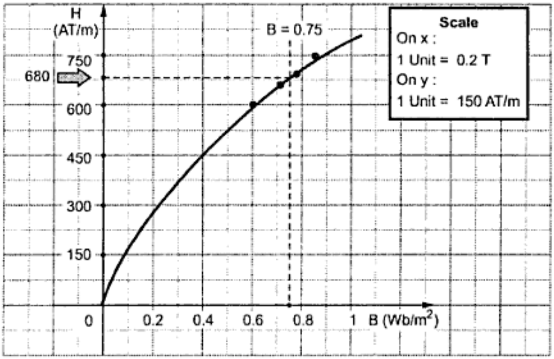


Fig. 3.80

$$\therefore \quad H = \frac{AT}{l_i}$$

$$\therefore \quad 680 = \frac{AT}{63\times10^{-2}}$$

$$\therefore \quad AT = 428.4 = N\,I$$

$$\therefore \quad I = \frac{AT}{N} = \frac{428.4}{500}$$

$$= \mathbf{0.857\ A}$$

Case 2 : Air gap of $l_g = 0.1\ \text{cm}$, $l_i = 63 - 0.1 = 62.9\ \text{cm}$

$B = \frac{\phi}{a} = 0.75\ Wb/m^2$ remains same as ϕ is same.

$\therefore \quad H_i = 680$ AT/m for iron path as obtained above.

$$\therefore \quad (AT)_i = H_i \times l_i = 680\times62.9\times10^{-2} = 427.72\ AT$$

For air gap, $B = \mu_0 H_g$ i.e. $H_g = \frac{0.75}{4\pi\times10^{-7}} = 596.831\times10^3$ **AT/m**

$$\therefore \quad (AT)_g = H_g \times l_g = 596.831\times10^3\times0.1\times10^{-2} = 596.831\ AT$$

$$\therefore \quad \text{Total AT} = (AT)_i + (AT)_g = 1024.551\ AT = NI$$

$$I = \frac{AT}{N} = \frac{1024.551}{500} = \mathbf{2.049\ A}$$

Example 3.28 : *A steel ring of 25 cm mean diameter has circular cross-section of 3 cm diameter has a gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 A. Calculate : a) M.M.F. b) Flux density c) Magnetic flux d) Reluctance e) μ_r for iron. Assume that iron path takes 30 % of total m.m.f.* **[GU : June-2002, 2003]**

Solution : $D_{mean} = 25$ cm, $l_g = 1.5$ mm, d = 3 cm, N = 700, I = 2 A

$$l_i = \pi D_{mean} - l_g = \pi \times 25 \times 10^{-2} - 1.5 \times 10^{-3} = 0.7839 \text{ m}$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times \left(3 \times 10^{-2}\right)^2 = 7.068 \times 10^{-4} \text{ m}^2$$

$$\phi = \text{Constant} = \frac{(\text{M.M.F.})_{iron}}{S_i} = \frac{(\text{M.M.F.})_{air\ gap}}{S_g}$$

a) $$\text{Total M.M.F.} = \text{N I} = 700 \times 2 = 1400 \text{ AT}$$

$$(\text{M.M.F.})_{iron} = 30\ \% \text{ of total M.M.F.} = 420$$

$$(\text{M.M.F.})_{air\ gap} = 70\ \% \text{ of total M.M.F.} = 980$$

b) $$S_g = \frac{l_g}{\mu_0 a} = \frac{1.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 7.068 \times 10^{-4}} = 1.6888 \times 10^6 \text{ AT/Wb}$$

$\therefore$ $$\phi = \frac{(\text{M.M.F.})_{air\ gap}}{S_g} = \frac{980}{1.6888 \times 10^6} = \mathbf{0.5802\ mWb}$$

c) $$B = \frac{\phi}{a} = \frac{0.5802 \times 10^{-3}}{7.068 \times 10^{-4}} = \mathbf{0.821\ Wb/m^2}$$

d) $$S_i = \frac{(\text{M.M.F.})_{iron}}{\phi} = \frac{420}{0.5802 \times 10^{-3}} = \mathbf{723.888 \times 10^3\ AT/Wb}$$

e) $$S_i = \frac{l_i}{\mu_0 \mu_r a} \quad \text{i.e.} \quad 723.888 \times 10^3 = \frac{0.7839}{4\pi \times 10^{-7} \times \mu_r \times 7.068 \times 10^{-4}}$$

$\therefore$ $$\mu_r = \mathbf{1224.085}$$

Example 3.29 : *A magnetic circuit is made of mild steel arranged as shown in the Fig. 3.81.*

The central limb is wound with 500 turns and has a cross-section of 800 mm². Each of the outer limb has a cross-section of 500 mm². The air gap has a length of 1 mm. Calculate the current required to set up a flux of 1.3 mWb in the central limb assuming no magnetic leakage

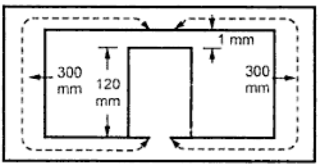


Fig. 3.81

and fringing. The mean lengths of various paths are shown in the Fig. 3.81. The B-H curve for mild steel is as follows :

B Wb/m^2	1	1.1	1.2	1.3	1.4	1.5	1.625
H AT/m	400	500	650	850	1250	2000	3800

[GU : Aug.-2001]

Solution : $N = 500$, $a_c = 800\ mm^2$, $a_s = 500\ mm^2$, $l_g = 1$ mm, $\phi_c = 1.3$ mWb

i) Air gap : ϕ_g = Air gap flux = ϕ_c = Flux in central limb = 1.3 mWb

$$\therefore \quad B_g = \frac{\phi_g}{a_c} = \frac{1.3\times10^{-3}}{800\times10^{-6}} = 1.625\ Wb/m^2$$

$$B_g = \mu_0 H_g \qquad \text{i.e.} \qquad H_g = \frac{1.625}{4\pi\times10^{-7}} = 1.2931\times10^6\ AT/m$$

$$\therefore \quad (AT)_g = H_g \times l_g = 1.2931\times10^6\times1\times10^{-3} = 1293.1339\ AT$$

ii) Central limb : For central limb, flux density is same as air gap hence

$$B_c = B_g = 1.625\ Wb/m^2$$

From the given table, $H_c = 3800$ for $B_c = 1.625\ Wb/m^2$.

$$\therefore \quad (AT)_c = H_c \times l_c = 3800\times120\times10^{-3} = 456\ AT$$

iii) Side limb : For one side limb, $l_i = 300$ mm

$$B_s = \frac{\phi_s}{a_s}$$

and $\phi_s = \frac{\phi_g}{2} = \frac{1.3}{2}$

$= 0.65$ mWb

$$\therefore \quad B_s = \frac{0.65\times10^{-3}}{500\times10^{-6}} = 1.3\ Wb/m^2$$

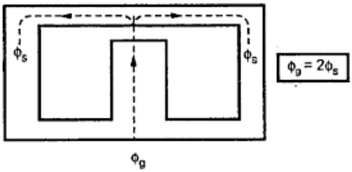


Fig. 3.81 (a)

From the given table, $H_s = 850$ AT/m for $B_s = 1.3\ Wb/m^2$.

$$\therefore \quad (AT)_s = H_s \times l_i = 850\times300\times10^{-3} = 255\ AT$$

$$\therefore \quad \text{Total M.M.F.} = (AT)_c + (AT)_g + (AT)_s$$

Key Point: *Due to parallel magnetic circuit, AT for any one side limb is required to be considered as M.M.F. remains same for the parallel paths.*

$\therefore$ Total M.M.F. $= 456 + 1293.1339 + 255 = 2004.1339$ AT

But Total M.M.F. $=$ N I i.e. $2004.1339 = 500$ I

$$\therefore \quad I = \frac{2004.1339}{500} = 4 \text{ A}$$

Example 3.30 : *A ring has a mean diameter of 21 cm and cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel with each joint having reluctance equal to an air gap of 0.2 mm. Find the ampere-turns required to produce a flux of 0.8 mWb. Relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.* **[GU : Nov.-2005]**

Solution : The ring is shown in the Fig. 3.82.

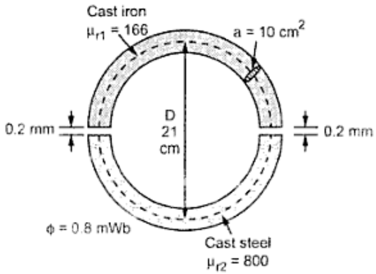


Fig. 3.82

$$l = \pi D = \pi \times 21 \times 10^{-2} = 0.6597 \text{ m}$$

$$l_g = 0.2 + 0.2 = 0.4 \text{ mm}$$

$$l_1 = l_2 = \frac{l - l_g}{2} = \frac{0.6597 - 0.4 \times 10^{-3}}{2}$$

$$= 0.3296 \text{ m}$$

Case 1 : Cast iron

$$S_1 = \frac{l_1}{\mu_0 \mu_{r1} a} = \frac{0.3296}{4\pi \times 10^{-7} \times 166 \times 10 \times 10^{-4}}$$

$$= 1.58036 \times 10^6 \text{ AT/Wb}$$

$$\therefore \quad (AT)_1 = (M.M.F.)_1 = S_1 \times \phi = 1.58036 \times 10^6 \times 0.8 \times 10^{-3} = \mathbf{1264.293 \text{ AT}}$$

Case 2 : Cast steel

$$S_2 = \frac{l_2}{\mu_0 \mu_{r2} a} = \frac{0.3296}{4\pi \times 10^{-7} \times 800 \times 10 \times 10^{-4}} = 327.8591 \times 10^3 \text{ AT/Wb}$$

$$\therefore \quad (AT)_2 = (M.M.F.)_2 = S_2 \times \phi = 327.8591 \times 10^3 \times 0.8 \times 10^{-3} = \mathbf{262.287 \text{ AT}}$$

Case 3 : Total air gap

$$S_3 = \frac{l_g}{\mu_0 a} = \frac{0.4 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 318.3098 \times 10^3 \text{ AT/Wb} \qquad \ldots \mu_r = 1 \text{ for air}$$

$$\therefore \quad (AT)_3 = (M.M.F.)_3 = S_3 \times \phi = 318.3098 \times 10^3 \times 0.8 \times 10^{-3} = \mathbf{254.648\ AT}$$

$$\therefore \quad \text{Total AT} = (AT)_1 + (AT)_2 + (AT)_3 = \mathbf{1781.228\ AT}$$

Example 3.31 : *A flux density of 1.2 Wb/m² is required in the 2 mm air gap of an electromagnet having an iron path of 1.5 m. Calculate the ampere-turns required, assuming a relative permeability of 1000 for the iron.* **[GU: May-2006]**

Solution : $B = 1.2\ Wb/m^2,\ l_g = 2\ mm,\ l_i = 1.5\ m,\ \mu_r = 1000$

For the air gap, $B = \mu_0 H_g$ i.e. $1.2 = 4\pi \times 10^{-7} \times H_g$

$$\therefore \quad H_g = 954.9296 \times 10^3\ AT/m$$

$$\therefore \quad (AT)_g = H_g \times l_g = 954.9296 \times 10^3 \times 2 \times 10^{-3} = 1909.8593\ AT$$

For the iron path, $B = \mu_0 \mu_r H_i$ ...Assuming no leakage

$$\therefore \quad H_i = \frac{B}{\mu_0 \mu_r} = \frac{1.2}{4\pi \times 10^{-7} \times 1000} = 954.9296\ AT/m$$

$$\therefore \quad (AT)_i = H_i \times l_i = 954.9296 \times 1.5 = 1432.3944\ AT$$

$$\therefore \quad \text{Total AT} = (AT)_g + (AT)_i = \mathbf{3342.2537\ AT}$$

Example 3.32 : *A coil is uniformly wound with 300 turns over a steel ring of relative permeability 900 and mean diameter of 20 cm. The steel ring is made up of bar having cross-section of diameter 2 cm. If a coil has resistance of 50 Ω and connected to 250 V d.c. supply, calculate : a) M.M.F. b) Field intensity c) Reluctance d) Total flux.* **[GU : Nov.-2002]**

Solution :

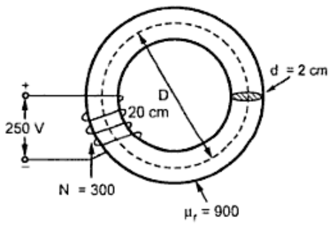


Fig. 3.83

$$l_i = \pi \times D = \pi \times 20 \times 10^{-2} = 0.6283\ m$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(2 \times 10^{-2}\right)^2 = 3.1416 \times 10^{-4}\ m^2$$

$$I = \frac{V}{R} = \frac{250}{50} = 5\ A$$

a) $M.M.F. = N\ I = 300 \times 5 = \mathbf{1500\ AT}$

b) $H = \frac{NI}{l_i} = \frac{1500}{0.6283} = \mathbf{2387.3945\ AT/m}$

c) $$S = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.6283}{4\pi\times10^{-7}\times900\times3.1416\times10^{-4}} = \mathbf{1.76833\times10^{6}\ AT/Wb}$$

d) $$\phi = \frac{M.M.F.}{S} = \frac{1500}{1.76833\times10^{6}} = \mathbf{0.8482\ mWb}$$

Example 3.33 : *An iron ring has 15 cm diameter and 10 cm^2 cross-sectional area, wound with 200 turns of wire. For flux density of 1 Wb/m^2 and permeability of 500, find the exciting current.* **[GU : Nov.-2006]**

Solution : D = 15 cm, a = 10 cm^2, N = 200, B = 1 Wb/m^2, μ_r = 500,

$$l_i = \pi D = \pi\times15\times10^{-2} = 0.4712\ m$$

$$\therefore \quad S = \frac{l_i}{\mu_0\mu_r a} = \frac{0.47123}{4\pi\times10^{-7}\times500\times10\times10^{-4}} = 750\times10^{3}\ AT/Wb$$

$$\phi = B\times a = 1\times10\times10^{-4} = 10\times10^{-4}\ Wb$$

$$\phi = \frac{M.M.F.}{S} = \frac{NI}{S} \quad \text{i.e.} \quad 10\times10^{-4} = \frac{200\times I}{750\times10^{3}}$$

$$\therefore \quad I = \frac{10\times10^{-4}\times750\times10^{3}}{200} = \mathbf{3.75\ A}$$

Example 3.34 : *Find the ampere turns required to produce flux of 0.4 mWb in the air gap of a magnetic circuit which has an air gap of 0.5 mm. The iron ring has cross-section of 4 cm^2 and 63 cm mean length. Take μ_r = 1800 and leakage coefficient of 1.15.* **[GU : July-2005]**

Solution : The ring is shown in the Fig. 3.84.

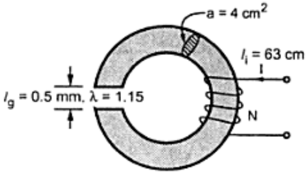


Fig. 3.84

$$\phi_g = 0.4\ mWb$$

$$\lambda = \frac{\phi_T}{\phi_g} \quad \text{i.e.} \quad 1.15 = \frac{\phi_T}{0.4\times10^{-3}}$$

$$\therefore \quad \phi_T = 4.6\times10^{-4}\ Wb$$

$$B_i = \frac{\phi_T}{a} = \frac{4.6\times10^{-4}}{4\times10^{-4}} = 1.15\ Wb/m^2$$

$$\therefore \quad H_i = \frac{B_i}{\mu_0\mu_r} = \frac{1.15}{4\pi\times10^{-7}\times1800} = 508.4116\ AT/m$$

$$\therefore \quad (AT)_i = H_i\times l_i = 508.4116\times63\times10^{-2} = \mathbf{320.3\ AT}$$

$$B_g = \frac{\phi_g}{a} = \frac{0.4\times10^{-3}}{4\times10^{-4}} = 1\ Wb/m^2$$

$$\therefore \quad H_g = \frac{B_g}{\mu_0} = \frac{1}{4\pi\times10^{-7}} = 795.7747\times10^3 \text{ AT/m} \qquad \ldots \mu_r = 1 \text{ for air}$$

$$\therefore \quad (AT)_g = H_g \times l_g = 795.7747\times10^3\times0.5\times10^{-3} = \mathbf{397.8873\ AT}$$

$$\therefore \quad \text{Total AT} = (AT)_i + (AT)_g = \mathbf{718.1873\ AT}$$

Example 3.35 : *An electromagnet has a cross-sectional area of 12 cm² and 50 cm length of iron path. It is excited by two coils with helping polarity, each having 400 turns. When the current in the coils is 1 A, the resultant flux density gives relative permeability of 1300. The magnet has an air gap of 0.4 cm. Calculate : a) Reluctance of iron path b) Reluctance of air gap c) Total reluctance d) Total flux e) Flux density in the air gap. Neglect leakage and fringing.* **[GU : July-2007]**

Solution : An electromagnet is shown in the Fig. 3.85.

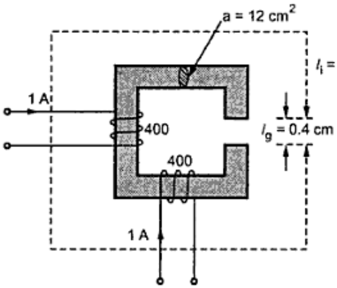


Fig. 3.85

a) For iron path

$$\therefore \quad S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{50\times10^{-2}}{4\pi\times10^{-7}\times1300\times12\times10^{-4}} = \mathbf{255.056\times10^3\ AT/Wb}$$

b) For air gap, $\mu_r = 1$

$$\therefore \quad S_g = \frac{l_g}{\mu_0 a} = \frac{0.4\times10^{-2}}{4\pi\times10^{-7}\times12\times10^{-4}} = \mathbf{2.6526\times10^6\ AT/Wb}$$

c) $$S_T = S_i + S_g = \mathbf{2.90764\times10^6\ AT/Wb}$$

d) $$\text{Total AT} = NI + NI = 400 + 400 = 800 \text{ AT} \qquad \ldots \text{2 coils helping}$$

$$\phi = \frac{\text{M.M.F.}}{S} = \frac{\text{Total AT}}{S_T} = \frac{800}{2.90764\times10^6} = \mathbf{0.2751\ mWb}$$

e) $$B_g = \frac{\phi}{a} = \frac{0.2751\times10^{-3}}{12\times10^{-4}} = \mathbf{0.2293\ Wb/m^2}$$

Example 3.36 : *Two coils A and B when connected in series cumulatively has total inductance of 0.5 H. When they are connected in series differentially, the resultant inductance is 0.2 H. If coil B has a self inductance of 0.15 H, calculate the self inductance of* *coil A and the induced e.m.f. in coil B when the rate of decrease of current in series combination is 1000 A/sec. Also find coefficient of coupling.*

[GU : June-2006, Nov.-2006, July-2007]

Solution : $L_{eq} = 0.5$ H when cumulative, $= 0.2$ H when differential.

$\therefore \quad 0.5 = L_A + L_B + 2M \quad$ and $\quad 0.2 = L_A + L_B - 2M$

Subtracting the two, $\quad (0.5-0.2) = 4M$

$$\therefore \quad M = \frac{0.3}{4} = 0.075 \text{ H}$$

Now $\quad L_B = 0.15$ H $\quad$ hence, $\quad 0.5 = L_A + 0.15 + (2 \times 0.075)$

$\therefore \quad L_A =$ **0.2 H**

Now $\quad \frac{di}{dt} = -1000$ A/sec $\quad$ ... Negative as decrease.

$$e_B = -L_B \frac{di}{dt} \pm M\frac{di}{dt} \quad \text{... + for cumulative, – for differential.}$$

$$= -0.15(-1000) \pm 0.075\,(-1000) = 150 \pm (-75)$$

$$= 150 \mp 75 = \mathbf{75\ V} \quad \textbf{or} \quad \mathbf{225\ V}$$

$$K = \frac{M}{\sqrt{L_A L_B}} = \frac{0.075}{\sqrt{0.2 \times 0.15}} = \mathbf{0.433}$$

Example 3.37 : *An iron ring having a cross-sectional area of 5 cm × 4 cm and a mean diameter of 18 cm has a coil of 270 turns uniformly wound over it. A current of 1.27 A flows through the coil which produces a flux of 1.13 mWb in the ring. Find the reluctance of the circuit, the absolute and relative permeabilities of the iron.* **[GTU : Dec.-2008]**

Solution : The ring is shown in the Fig. 3.86.

$$l_i = \pi \times D = \pi \times 18 \times 10^{-2} = 0.5655 \text{ m}$$

$$NI = \text{M.M.F.} = 1.27 \times 270$$

$$= 342.9 \text{ AT}$$

$$\therefore \quad S = \frac{\text{M.M.F.}}{\phi} = \frac{342.9}{1.13 \times 10^{-3}}$$

$$= \mathbf{303.4513 \times 10^3\ AT/Wb}$$

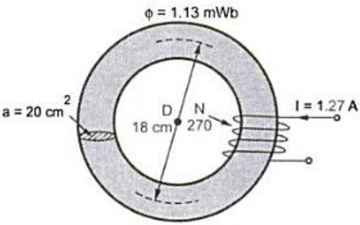


Fig. 3.86

Now, $S = \frac{l_i}{\mu_0 \mu_r a}$ i.e. $303.4513 \times 10^3 = \frac{0.5655}{4\pi \times 10^{-7} \times \mu_r \times 20 \times 10^{-4}}$

$\therefore \quad \mu_r = \mathbf{741.4872}$ ...relative permeability

$\therefore \quad \mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 741.4872 = \mathbf{9.3178 \times 10^{-4}}$

Example 3.38 : *Two coils having 100 and 1000 turns have a common magnetic circuit of 25 cm diameter and 62.5 cm² cross-section with a constant relative permeability of 2000. Calculate : a) The self inductance of both the coils. b) The mutual inductance between them if the coefficient of coupling is 0.5.* **[GTU : March-2009]**

Solution : $N_1 = 100$, $N_2 = 1000$, $D = 25$ cm, $a = 62.5\ cm^2$, $\mu_r = 2000$

$$l = \pi D = \pi \times 25 \times 10^{-2} = 0.7854 \text{ m}$$

$$\therefore \quad S = \frac{l}{\mu_0 \mu_r a} = \frac{0.7854}{4\pi \times 10^{-7} \times 2000 \times 62.5 \times 10^{-4}} = 50000 \text{ AT/Wb}$$

a) $$L_1 = \frac{N_1^2}{S} = \frac{(100)^2}{50000} = \mathbf{0.2 \ H}$$

$$L_2 = \frac{N_2^2}{S} = \frac{(1000)^2}{50000} = \mathbf{20 \ H}$$

b) $$K = 0.5 \quad \text{and} \quad K = \frac{M}{\sqrt{L_1 L_2}}$$

$$\therefore \quad M = 0.5 \times \sqrt{0.2 \times 20} = \mathbf{1 \ H}$$

Example 3.39 : *A circular ring of mild steel has diameter of 20 cm and 2 mm side air gap. The cross-sectional area is 3.2 cm². Estimate the m.m.f. required to establish 0.6 mWb flux. Assume μ_r for mild steel as 900.* **[GTU : June-2009]**

Solution : The ring is shown in the Fig. 3.87.

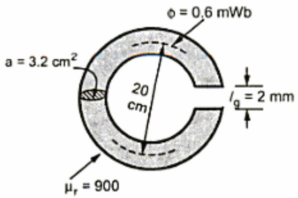


Fig. 3.87

$$l = \pi \times D = \pi \times 20 \times 10^{-2} = 0.6283 \text{ m}$$

$$\therefore \quad l_i = l - l_g = 0.6283 - 2 \times 10^{-3}$$

$$= 0.6263 \text{ m}$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.6263}{4\pi \times 10^{-7} \times 900 \times 3.2 \times 10^{-4}}$$

$$= 1.73058 \times 10^6 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times3.2\times10^{-4}}$$

$$= 4.9736\times10^{6} \text{ AT/Wb}$$

$$\therefore \quad S_T = S_i + S_g = 6.70417\times10^{6} \text{ AT/Wb}$$

$$\phi = \frac{\text{M.M.F.}}{S_T} \quad \text{i.e.} \quad 0.6\times10^{-3} = \frac{\text{M.M.F.}}{6.70417\times10^{6}}$$

$$\therefore \quad \text{M.M.F.} = \mathbf{4022.5031 \text{ AT}}$$

Review Questions

1. *State and explain the laws of magnetism.*
2. *What is magnetic field and magnetic lines of force? State the properties of lines of force.*
3. *Define and state the units of following parameters :*
   i) magnetic flux *ii) magnetic pole strength*
   iii) magnetic flux density *iv) magnetic field strength*
   v) absolute permeability *vi) relative permeability*
   vii) m.m.f. *viii) reluctance*
4. *Derive the relation between m.m.f., reluctance and the flux.*
5. *State and explain the following rules :*
   i) Right hand thumb rule
   ii) Fleming's left hand rule
   iii) Fleming's right hand rule
   iv) Lenz's law
   v) Kirchhoff's laws for magnetic circuits
6. *Explain the proceduce to analyze following circuit, with suitable example :*
   i) Series magnetic circuit
   ii) Series magnetic circuit with air gap
   iii) Parallel magnetic circuit
7. *What is an electromagnet ? What is solenoid?*
8. *Point out the analogy between electric and magnetic circuits.*
9. *Explain the magnetic leakage and magnetic fringing.*
10. *Define leakage coefficient.*
11. *Explain how current carrying conductor when placed in a magnetic field experiences a force.*

12. *A steel ring of 180 cm mean diameter has a cross-sectional area of 250 mm^2. Flux developed in the ring is 500 μWb when a 4000 turns coil carries certain current. Find*
 i) m.m.f. required ii) reluctance iii) current in the coil
 Given that the relative permeability of the steel is 1100.
 (Ans. : 8181.72 AT, 1.6363×10^7, 2.045 A)
13. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 A is passed through the coil, determine*
 i) m.m.f. ii) reluctance of ring and iii) flux **(Ans. : 7500 AT, 707355.3 AT/Wb, 0.0106 Wb)**
14. *Find the number of ampere turns required to produce a flux of 0.44 milli-weber in an iron ring of 100 cm mean circumference and 4 cm^2 in cross-section. B Vs μ_r test for the iron gives the following result :*

B in Wb/m^2	0.8	1.0	1.1	1.2	1.4
μ_r	2300	2000	1800	1600	1000

 If a saw cut of 2 mm wide is made in the above ring, how many extra ampere turns are required to maintain same flux ? **(Ans. : 486.307 AT, 1744 AT)**
15. *An iron ring of 20 cm mean diameter and 10 cm^2 cross-section is magnetized by a coil of 500 turns. The current through the coil is 8 A. The relative permeability of iron is 500. Find the flux density inside the ring.* **(Ans. : 4 Wb/m^2)**
16. *An iron ring of 100 cm mean circumference is made from round iron of cross-section $10 cm^2$, it relative permeability is 800. If it is wound with 300 turns, what current is required to produce a flux of 1.1×10^{-3} Wb ?* **(Ans. 3.647 A)**
17. *A coil of 300 turns and of resistance 10 Ω is wound uniformly over a steel ring of mean circumference 30 cm and cross-sectional area 9 cm^2. It is connected to a supply at 20 V d.c. If the relative permeability of the ring is 1500, find : (i) the magnetizing force ; (ii) the reluctance ; (iii) the m.m.f. ; and (iv) the flux.*
 (Ans. : 600 AT, 176838.82 AT/Wb, 2000 AT/m, 3.3929 mWb)
18. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean circumference of 400 mm and cross-sectional area of 500 mm^2. If a current of 25 A is passed through the coil find i) m.m.f. ii) reluctance and iii) flux.*
 (Ans. : 7500 AT, 707355.3 AT/Wb, 10.6 mWb)
19. *State the Faraday's laws of electromagnetism.*
20. *What is the difference between dynamically induced e.m.f. and statically induced e.m.f. ?*
21. *Derive the expression for the magnitude of the dynamically induced e.m.f.*

22. *Explain clearly the difference between self inductance and mutual inductance. State their units.*
23. *Derive the various expressions for the self inductance.*
24. *Explain the factors on which self inductances depends.*
25. *Derive the various expressions for the mutual inductance.*
26. *Derive the expression for coefficient of coupling.*
27. *Derive the expression for the equivalent inductance when two inductances are connected in*
    i) Series aiding (cumulatively coupled) ii) Series opposition (differentially coupled).
28. *How energy gets stored in the magnetic field ?*
29. *Derive the expression for energy stored in the magnetic field.*
30. *Write a note on lifting power of an electromagnet.*
31. *Two identical 1000 turn coils X and Y lie in parallel planes such that 60 % of the flux produced by one coil links with the other. A current of 5 A in X produces a flux of* 5×10^{-6} *Wb in itself. If the current in X changes from + 6 A to – 6 A in 0.01 sec, what will be the magnitude of the e.m.f. induced in Y ? Calculate the self inductance of each coil.*
    (Ans. : 0.72 V, 0.001 H)
32. *Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius. Also calculate energy stored in it if current rises from zero to 5 A.*
    (Ans. : 1.579 mH, 0.01973 J)
33. *Two 200 turns, air cored solenoids, 25 cm long have a cross-sectional area of 3 cm² each. The mutual inductance between them is 0.5 μH. Find the self inductance of the coils and the coefficient of coupling.* **(Ans. : 60.31 μH, 0.00828)**
34. *Two coils A and B having 5000 and 2500 turns respectively are wound on a magnetic ring. 60 % of the flux produced by coil A links with coil B. A current of 1 A produces a flux of 0.25 mWb in coil A while same current produces a flux of 0.15 mWb in coil B. Find the mutual inductance and coefficient of coupling.* **(Ans. : 1.25 H, 0.375 H, 0.5477)**
35. *A conductor has 1.9 m length. It moves at right angles to a uniform magnetic field. The flux density of the magnetic field is 0.9 tesla. The velocity of the conductor is 65 m/sec. Calculate the e.m.f. induced in the conductor.* **(Ans. : 111.15 volts)**
36. *An air cored coil has 800 turns. Length of the coil is 6 cm while its diameter is 4 cm. Find the current required to establish flux density of 0.01 T in core and self inductance of the coil.*
    (Ans. : 0.5968 A, 16.844 mH)
37. *A flux of 0.25 mWb is produced when a current of 2.5 a passes through a coil of 1000 turns. Calculate*
    i) Self inductance
    ii) E.M.F. induced in the coil if the current of 2.5 A is reduced to zero in 1 milliseconds.

iii) If second coil of 100 turns is placed near to the first on the same iron ring, calculate the mutual inductance between the coils. **(Ans. : 0.1 H, 250 V, 0.01 H)**

38. *Two coils A and B having 180 and 275 number of turns respectively are closely wound on an iron magnetic circuit, which has a mean length of 1.5 m and cross-sectional area of 150 cm. The relative permeability of iron is 1500. Determine mutual inductance between the coils. When will be the e.m.f. induced in a coil B if the current in coil A changes uniformly from 0 to 2.5 a in 0.03 seconds?* **(Ans. : 0.933 H, – 77.75 volts)**
39. *What is B-H curve ? Draw and explain the experimental setup to obtain B-H curve of a specimen.*
40. *Draw a typical B-H curve for a magnetic material and explain its various regions.*
41. *Draw and compare the B-H curves for magnetic and non-magnetic materials.*
42. *What is magnetic hysteresis ? Draw and explain a hysteresis loop.*
43. *Explain the theory behind hysteresis effect.*
44. *What is hysteresis loss ? On which factors it depends?*
45. *Derive the expression for the hysteresis loss per unit volume.*
46. *Explain the practical use of hysteresis loop.*
47. *Explain the eddy current loss and the factors on which it depends.*

University Questions

Q.1 *Explain the following terms : i) Magnetomotive force ii) Magnetic field intensity iii) Reluctance.* **[GTU : Dec.-2008, 6 Marks]**

Q.2 *Two inductive coils are connected in parallel. Derive the expression for total inductance when i) Coils are in parallel aiding connection ii) Coils are in parallel opposing connection.* **[GTU : Dec.-2008, 7 Marks]**

Q.3 *Give the comparison between electric and magnetic circuit.* **[GTU : March-2009, 5 Marks]**

Q.4 *State and explain Faraday's laws of electromagnetic induction.* **[GTU : March-2009, 4 Marks]**

Q.5 *Compare electric and magnetic circuit.* **[GTU : June-2009, 5 Marks]**

Q.6 *Distinguish statically induced and dynamically induced E.M.F. Derive expression for dynamically induced e.m.f.* **[GTU : June-2009, 5 Marks]**

Q.7 *i) Explain magnetic hysteresis. ii) What do you understand by coefficient of coupling between two magnetic coils.* **[GTU : June-2009, 4 Marks]**

Q.8 *Derive the expressions of equivalent inductance, when two magnetically coupled coils are connected in series in two different ways.* **[GTU : June-2009, 5 Marks]**

Q.9 *State and explain Faraday's laws of electromagnetic induction.* **[GTU : June-2009, 5 Marks]**

Q.10 *Explain the term i) Reluctance ii) Permeability* **[GTU : June-2009, 4 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

❑❑❑

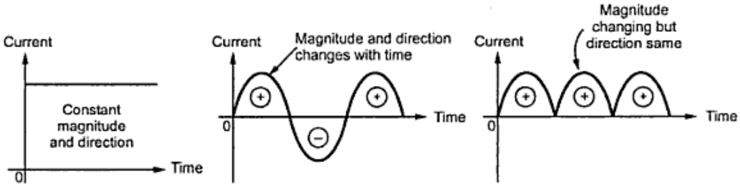
4 Fundamentals of A.C. Circuits

4.1 Introduction

Uptill now, we have discussed about D.C. supply and D.C. circuits. But 90 % of electrical energy used now a days is a.c. in nature. Electrical supply used for commercial purposes is alternating. The d.c. supply has constant magnitude with respect to time. The Fig. 4.1(a) shows a graph of such current with respect to time.

> **Key Point**: *An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.*

Such change in magnitude and direction is measured in terms of cycles. Each cycle of a.c. consists of two half cycles namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly. The Fig. 4.1 (b) shows a graph of alternating current against time.



(a) Direct current **(b) Alternating current** **(c) Pulsating d.c.**

Fig. 4.1

In practice some waveforms are available in which magnitude changes but its direction remains same as positive or negative. This is shown in the Fig. 4.1(c). Such waveform is called **pulsating d.c.** The waveform obtained as output of full wave rectifier is an example of pulsating d.c.

Let us see, why in practice, there is generation of a.c. Use of a.c. definitely offers certain advantages.

4.2 Advantages of A.C.

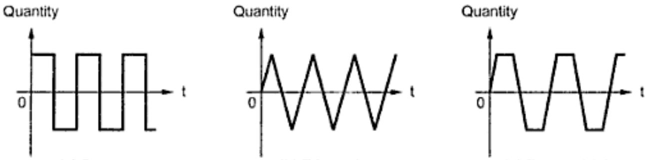
The various advantages of a.c. are,

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high a.c. voltage; high speed a.c. generators of large capacities. The construction and cost of such generators are very low. High a.c. voltages of about 11 kV can be generated and can be raised upto 220 kV for transmission purpose at sending end, while can be lowered down at 400 V at receiving end. This is not possible in case of d.c.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, a.c. supply can be easily converted to obtain d.c. supply. This is required as d.c. is very much essential for the applications like cranes, printing process, battery charging, telephone system, etc. But, such requirement of d.c. is very small compared to a.c.

Due to these advantages, a.c. is used extensively in practice and hence, it is necessary to study a.c. fundamentals.

4.3 Types of A.C. Waveforms

The waveform of alternating voltage or current is shown purely sinusoidal in the Fig. 4.1 (b). But, in practice, a quantity which undergoes variations in its instantaneous values, in magnitude as well as direction with respect to some zero reference is called an **alternating quantity.** The graph of such quantity against time is called its **waveform.** Various types of alternating waveforms other than sinusoidal are shown in the Fig. 4.2 (a), (b) and (c).



(a) Square (b) Triangular (c) Trapezoidal

Fig. 4.2

Out of all these types of alternating waveforms, purely sinusoidal waveform is preferred for a.c. system. There are few advantages of selecting purely sinusoidal as the standard waveform.

4.3.1 Advantages of Purely Sinusoidal Waveform

1) Mathematically, it is very easy to write the equations for purely sinusoidal waveform.
2) Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original wave form. Hence, it is always better to have sinusoidal waveform as the standard waveform.
3) The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through a linear circuit.
4) The integration and derivative of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical networks with sinusoidal inputs, very easy.

4.4 Generation of A.C. Voltage

The machines which are used to generate electrical voltages are called **generators.** The generators which generate purely sinusoidal a.c. voltages are called **alternators.**

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to **Faraday's law of electromagnetic induction**. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called **single turn** or **single loop alternator.**

4.4.1 Single Turn Alternator

Construction : It consists of a permanent magnet of two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil is made up of same conducting material like copper or aluminium. The coil is made up of two conductors namely a-b and c-d. Such two conductors are connected at one end to form a coil. This is shown in the Fig. 4.3.

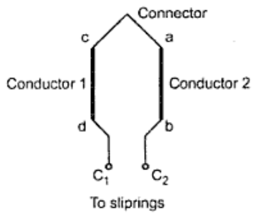


Fig. 4.3 Single turn coil

The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The remaining two ends C1 and C2 of the coil are connected to the rings mounted on the shaft called **slip rings.** Slip rings are also rotating members of the alternator. The two brushes P and Q are resting on the slip rings. The **brushes are stationary** and just making contact with the slip rings. The slip rings and brush assembly is necessary to collect the current induced in the rotating coil and make it available to the stationary external resistance. The overall construction is shown in the Fig. 4.4.

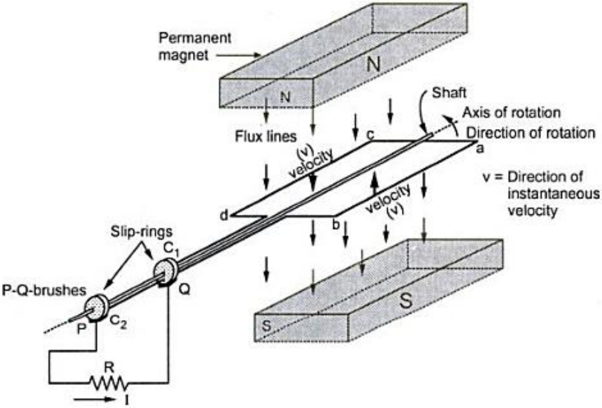


Fig. 4.4 Single turn alternator

Working : The coil is rotated in anticlockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, an e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected across the brushes P and Q. The magnitude of the induced e.m.f. depends on the position of the coil in the magnetic field. Let us see the relation between magnitude of the induced e.m.f. and the positions of the coil. Consider different instants and the different positions of the coil.

Instant 1 : Let the initial position of the coil be as shown in the Fig. 4.4. The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductors ab and cd, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence, no e.m.f. will be generated in the conductors ab and cd and no current will flow through the external resistance R. This position can be represented by considering the front view of the Fig. 4.4 as shown in the Fig. 4.5 (a).

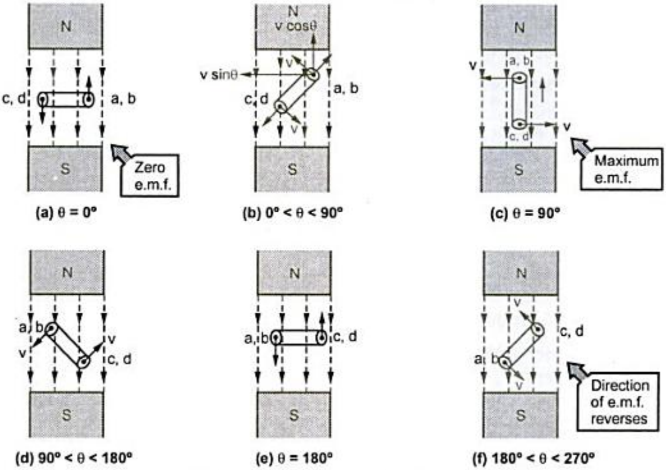


Fig. 4.5 The different instants of induced e.m.f.

The angle θ is measured from plane of the magnetic flux

Instant 2 : When the coil is rotated in anticlockwise direction through some angle θ, then the velocity will have two components v sin θ perpendicular to flux lines and v cos θ parallel to the flux lines. Due to v sin θ component, there will be cutting of the flux and proportionally, there will be induced e.m.f. in the conductors ab and cd. This e.m.f. will drive a current through the external resistance R. This is shown in the Fig. 4.5 (b).

Instant 3 : As angle 'θ' increases, the component of velocity acting perpendicular to flux lines increases, hence induced e.m.f. also increases. At θ = 90°, the plane of the coil is parallel to the plane of the magnetic field while the component of velocity cutting the lines of flux is at its maximum. **So, induced e.m.f. in this position, is at its maximum value.** This is shown in the Fig. 4.5 (c).

So, as θ increases from 0° to 90°, e.m.f. induced in the conductors increases gradually from 0 to maximum value. The current through external resistance R also varies according to the induced e.m.f.

Instant 4 : As the coil continues to rotate further from θ = 90° to 180°, the component of velocity, perpendicular to magnetic field starts decreasing, hence, gradually decreasing the magnitude of the induced e.m.f. This is shown in the Fig. 4.5 (d).

Instant 5 : In this position, the velocity component is fully parallel to the lines of flux similar to the instant 1.

Hence, there is no cutting of flux and hence, no induced e.m.f. in both the conductors. Hence, current through external circuit is also zero.

Instant 6 : As the coil rotates beyond $\theta = 180°$, the conductor ab uptil now cutting flux lines in one particular direction reverses the direction of cutting the flux lines. Similar is the behaviour of conductor cd. So, direction of induced e.m.f. in conductor ab is opposite to the direction of induced e.m.f. in it for the rotation of $\theta = 0°$ to $180°$. Similarly, the direction of induced e.m.f. in conductor cd also reverses. This change in direction of induced e.m.f. occurs because the direction of rotation of conductors ab and cd reverses with respect to the field as θ varies from 180° to 360°. This process continues as coil rotates further. At $\theta = 270°$ again, the induced e.m.f. achieves its maximum value but the direction of this e.m.f. in both the conductors is opposite to the previous maximum position i.e. at $\theta = 90°$. From $\theta = 270°$ to 360°, induced e.m.f. decreases without change in direction and at $\theta = 360°$, coil achieves the starting position with zero induced e.m.f.

So, as θ varies from 0° to 360°, the e.m.f. in a conductor ab or cd varies in an alternating manner i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductors rotate in a circular motion with a certain speed.

This variation of e.m.f. in a conductor can be graphically represented.

4.4.2 Graphical Representation of the Induced E.M.F.

The instantaneous values of the induced e.m.f. in any conductor, as it is rotated from $\theta = 0°$ to 360°, i.e. through one complete revolution can be represented as shown in the Fig. 4.6.

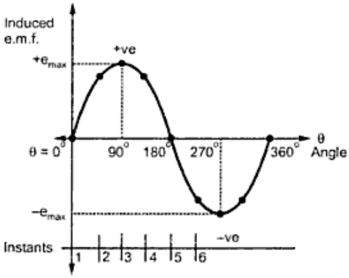


Fig. 4.6 Graphical representation of the induced e.m.f.

From the Fig. 4.6, it is clear that the waveform generated by the instantaneous values of the induced e.m.f. in any conductor (ab or cd) is **purely sinusoidal** in nature.

4.5 Standard Terminology Related to Alternating Quantity

Before further analysis of alternating quantity, it is necessary to be familiar with the different terms which are very frequently used related to the alternating quantities.

4.5.1 Instantaneous Value

The value of an alternating quantity at a particular instant is known as its **instantaneous value.**

e.g. e_1 and e_2 are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 4.7.

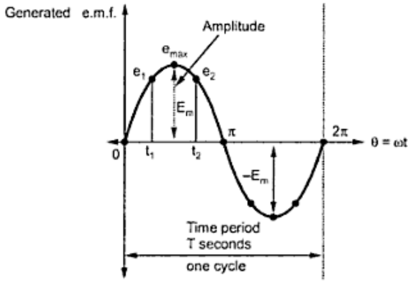


Fig. 4.7 Waveform of an alternating e.m.f.

4.5.2 Waveform

The graph of instantaneous values of an alternating quantity plotted against time in called its **waveform.**

4.5.3 Cycle

Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**

A **cycle** can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops). One such cycle of the alternating quantity is shown in the Fig. 4.7.

Key Point: *One cycle corresponds to 2π radians or 360°.*

4.5.4 Time Period (T)

The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 4.7.

4.5.5 Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by **f** and it is measured in **cycles / second** which is known as

Hertz, denoted as **Hz.** As time period T is time for one cycle i.e. seconds / cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$f = \frac{1}{T} \quad \text{Hz}$$

As time period increases, frequency decreases while as time period decreases, frequency increases. This is shown in the Fig. 4.8.

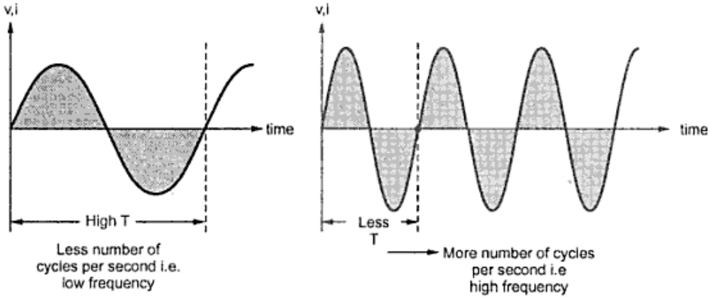


Fig. 4.8 Relation between T and f

In our nation, standard frequency of alternating voltages and currents is 50 Hz.

4.5.6 Amplitude

The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

Thus E_m is called peak value of the voltage while I_m is called peak value of the current.

4.5.7 Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2 π radians, the angular frequency can be expressed as (2 π × cycles/sec.) It is denoted by 'ω' and its unit is radians/second. Now, cycles/ sec. means frequency. Hence the relation between frequency 'f' and angular frequency 'ω' is,

$$\omega = 2\pi f \quad \text{radians/sec.} \quad \text{or} \quad \omega = \frac{2\pi}{T} \quad \text{radians/sec.}$$

4.6 Equation of an Alternating Quantity

For the derivation of the equation of an alternating quantity, consider single turn, 2 pole alternator discussed earlier. The coil is rotated with constant angular velocity in the magnetic field. An alternating e.m.f. induced is purely sinusoidal in nature.

Let B = Flux density of the magnetic field in Wb/m^2

l = Active length of each conductor in metres

r = Radius of circular path traced by conductors in metres.

ω = Angular velocity of coil in radians / second

v = Linear velocity of each conductor in m / sec.

Now, $v = r\,\omega$

Consider an instant where coil has rotated through angle θ from the position corresponding to $\theta = 0°$ i.e. from the instant when induced e.m.f. is zero. It requires time 't' to rotate through θ. So, θ in radians can be expressed as,

$$\boxed{\theta = \omega t \quad \text{radians}}$$

The position of the coil is shown in the Fig. 4.9 (a). The instantaneous peripheral velocity of any conductor can be resolved into two components as shown in the Fig. 4.9 (b).

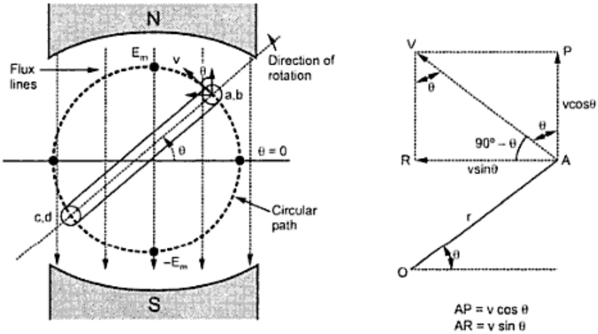


Fig. 4.9 Instantaneous value of an induced e.m.f.

The components of velocity, v are,

1) Parallel to the magnetic flux lines, $(AP) = v\cos\theta$
2) Perpendicular to the magnetic flux lines, $(AR) = v\sin\theta$

Out of the two, due to the component parallel to the flux, there cannot be the generation of the e.m.f. as there cannot be the cutting of the flux lines. Hence, the component which is acting perpendicular to the magnetic flux lines i.e. $v \sin \theta$ is solely responsible for the generation of the e.m.f.

According to the Faraday's law of electromagnetic induction, the expression for the generated e.m.f. in each conductor is,

$$e = B\, l\, v \sin \theta \quad \text{volts}$$

The active length 'l' means the length of the conductor which is under the influence of the magnetic field.

Now, $E_m = B\, l\, v$

= Maximum value of induced e.m.f. in conductor

This is achieved at $\theta = 90^\circ$ and is the peak value or amplitude of the sinusoidal induced e.m.f.

Hence, equation giving instantaneous value of the generated e.m.f. can be expressed as,

$$\boxed{e = E_m \sin \theta \ \text{volts}}$$

4.6.1 Different Forms of E.M.F. Equation

Now, $\theta = \omega t$ radians

$$\therefore \quad \boxed{e = E_m \sin(\omega t)} \quad \dots (1)$$

But, $\omega = 2\pi f$ rad / sec.

$$\therefore \quad \boxed{e = E_m \sin(2\pi f t)} \quad \dots (2)$$

But, $f = \frac{1}{T}$ seconds

$$\therefore \quad \boxed{e = E_m \sin\left(\frac{2\pi}{T} t\right)} \quad \dots (3)$$

Important Note : *In all the above equations, the angle θ is expressed in radians. Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.*

> **Key Point** : *Mode of the calculator should be converted to radians, to calculate the sine of the angle expressed in radians, before substituting in any of the above equations.*

This alternating e.m.f. drives a current through the electrical load which also varies in similar manner.

Its frequency is the same as the frequency of the generated e.m.f. Hence, it can be expressed as,

$$i = I_m \sin \theta$$

where I_m = Maximum or peak value of the current. This maximum value depends on the resistance of the electrical circuit to which an e.m.f. is applied. The instantaneous value of this sinusoidal current set up by the e.m.f. can be expressed as,

$$i = I_m \sin \omega t$$

or

$$i = I_m \sin 2 \pi f t$$

or

$$i = I_m \sin\left(\frac{2\pi t}{T}\right)$$

Example 4.1 : *Write the 4 ways of representing an a.c. voltage given by a magnitude of 5 V and frequency of 50 Hz.*

Solution : Given values are, $E_m = 5$ V and $f = 50$ Hz

So $\omega = 2\pi f = 100\pi$ rad/sec and $T = 1/f = 1/50$ sec

The voltage can be represented as,

1) $e = E_m \sin(\omega t) = 5 \sin(100\pi t)$ V

2) $e = E_m \sin\theta = 5 \sin\theta$ V

3) $e = E_m \sin(2\pi f t) = 5 \sin(100\pi t)$ V

4) $e = E_m \sin\left(\frac{2\pi t}{T}\right) = 5 \sin\left(\frac{2\pi t}{1/50}\right)$

Note that, after substituting the values of E_m, f, ω and T the resultant equation obtained remains same by all four ways.

Example 4.2 : *An alternating current of frequency 60 Hz has a maximum value of 12 A :*

i) Write down the equation for instantaneous values. ii) Find the value of the current after 1/360 second. iii) Time taken to reach 9.6 A for the first time.

In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction.

Solution : $f = 60$ Hz and $I_m = 12$ A

$$\omega = 2\pi f = 2\pi \times 60 = 377 \text{ rad/sec}$$

i) Equation of instantaneous value is

$$i = I_m \sin \omega t$$

$$\therefore \quad i = 12 \sin 377t$$

ii) $$t = \frac{1}{360} \text{ sec}$$

$$i = 12 \sin 377 \frac{1}{360} = 12 \sin 1.0472 = \mathbf{10.3924\ A}$$

Note : sin of 1.0472 must be calculated in **radian** mode.

iii) $$i = 9.6 \text{ A}$$

$$\therefore \quad 9.6 = 12 \sin 377t$$

$$\therefore \quad \sin 377t = 0.8$$

$$\therefore \quad 377t = 0.9272$$

Note : find inverse of sin in **radian** mode.

$$t = \mathbf{2.459 \times 10^{-3}\ sec.}$$

Example 4.3 : *A sinusoidal voltage of 50 Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be equal to 141.4 volts ?*

Solution : $f = 50$ Hz, $V_m = 200\sqrt{2}$ V, $v_1 = 141.4$ V

The equation of the voltage is,

$$v = V_m \sin(2\pi f t) = 200\sqrt{2} \sin(2\pi \times 50\, t)\ \text{V}$$

For $v = v_1$

$$141.4 = 200\sqrt{2} \sin(2\pi \times 50 \times t_1)$$

$$\therefore \quad t_1 = 1.666 \times 10^{-3} \text{ sec}$$ ... Use **radian mode** for sin

But this time is measured from t = 0. At positive maximum, time is $\frac{T}{4} = \frac{1}{4f} = 5 \times 10^{-3}$ sec so $t = t_1 = 1.666 \times 10^{-3}$ sec is before positive maximum.

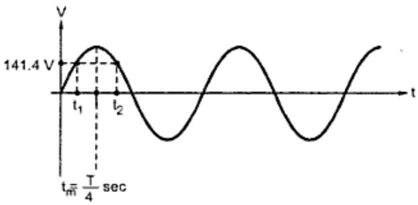


Fig. 4.10

From Fig. 4.10.

$$t_m - t_1 = 5 \times 10^{-3} - 1.666 \times 10^{-3}$$

$$= \mathbf{3.314 \times 10^{-3}\ sec}$$

As the waveform is symmetrical, at the time of **3.314×10^{-3} sec measured after positive maximum** value, the instantaneous voltage will be again 141.4 V.

4.7 Effective Value or R.M.S. Value

An alternating current varies from instant to instant, while the direct current is constant, with respect to time. So, for the comparison of the two, there must be some common platform. Such platform can be the effect produced by the two currents. One of the such effects is heating of the resistance, due to current passing through it. The heating effect can be used to compare the alternating and direct current. From this, r.m.s. value of an alternating current can be defined as,

> **Key Point**: *The effective or r.m.s. value of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

The following simple experiment gives the clear understanding of the r.m.s. value of an alternating current. The arrangement is shown in the Fig. 4.11.

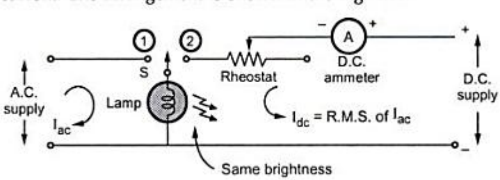


Fig. 4.11 Experiment to demonstrate r.m.s. value

A lamp is provided with double throw switch S. On position 1, it gets connected to an a.c. supply. The brightness of filament is observed.

Then, switch is thrown in position 2 and using the rheostats, the d.c. current is adjusted so as to achieve the same brightness of the filament.

The reading on the ammeter on d.c. side gives the value of the direct current that produces the same heating effect as that produced by the alternating current. This ammeter reading is nothing but the r.m.s. value of the alternating current.

R.M.S. value can be determined by two methods :

1) **Graphical Method** : This can be used for an alternating current having any wave form i.e. sinusoidal, triangular, square, etc.

2) **Analytical Method** : This is to be used for purely sinusoidally varying alternating current.

4.7.1 Graphical Method

Consider sinusoidally varying current.The r.m.s. value is to be obtained by comparing heat produced. Heat produced is proportional to square of current ($i^2 R$) so heat produced in both positive and negative half cycles will be the same. Hence, consider only positive half cycle, which is divided into 'n' intervals as shown in the Fig. 4.12. The width of each interval is 't/n' seconds and average height of each interval is assumed to be the average instantaneous values of current i.e. $i_1, i_2, ..., i_n$.

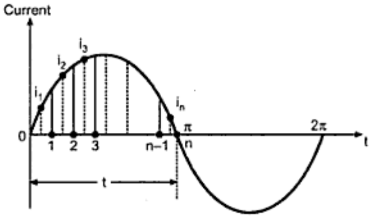


Fig. 4.12 Determining r.m.s. value

Let this current be passing through resistance 'R' ohms. Hence, heat produced can be calculated as,

joules Heat Produced $= i^2 R t$ joules

$\therefore$ Heat produced due to 1st interval $= i_1^2 R \frac{t}{n}$ joules

$\therefore$ Heat produced due to 2nd interval $= i_2^2 R \frac{t}{n}$ joules

$\therefore$ Heat produced due to nth interval $= i_n^2 R \frac{t}{n}$ joules

$\therefore$ Total heat produced in 't' seconds $= R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$

Now, heat produced by direct current I amperes passing through same resistance 'R' for the same time 't' is $= I^2 R t$ joules

For I to be the r.m.s. value of an alternating current, these two heats must be equal.

$$\therefore \quad I^2 R t = R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad I^2 = \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad \boxed{I = \sqrt{\frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}} = I_{r.m.s.}}$$

$I_{r.m.s.}$ = square root of the mean of the squares of ordinates of the current.

This is called **Effective value** of an alternating current or **Virtual value** of an alternating current. This expression is equally applicable to sinusoidally varying alternating voltage as,

$$V_{r.m.s.} = \sqrt{\frac{[V_1^2 + V_2^2 + ... + V_n^2]}{n}}$$

4.7.2 Analytical Method

Consider sinusoidally varying alternating current and square of this current as shown in the Fig. 4.13.

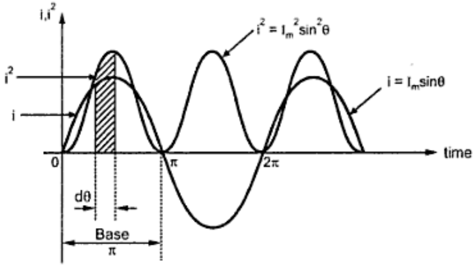


Fig. 4.13 Waveform of current and square of the current

The current $i = I_m \sin\theta$ while

Square of current $i^2 = I_m^2 \sin^2\theta$

Area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle = $\int_0^{\pi} i^2\, d\theta$ and Length of the base is π.

∴ Average value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta = \frac{I_m^2}{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\,\theta$$

$$= \frac{I_m^2}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_m^2}{2\pi}[\,\pi\,]$$

$$= \frac{I_m^2}{2}$$

Hence, root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{I_{r.m.s.} = 0.707\ I_m}$$

The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.

> **Key Point** : *The instantaneous values are denoted by small letters like i, e etc. while r.m.s. values are represented by capital letters like I, E etc.*

The above result is also applicable to **sinusoidal** alternating voltages.

So, we can write,

$$\therefore \quad \boxed{V_{r.m.s.} = 0.707\ V_m}$$

4.7.3 Importance of R.M.S. Value

1. In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quatntities unless and otherwise specicied to beother than r.m.s.

> **Key Point** : *In practice, everywhere, r.m.s. values are used to analyze alternating quantities.*

2. The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3. The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

Steps to find **r.m.s. value** of an a.c. quantity :

1. Write the equation of an a.c. quantity. Observe its behaviour during various time intervals.
2. Find square of the a.c. quantity from its equation.
3. Find average value of square of an alternating quantity as,

$$\text{Average} = \frac{\text{Area of curve over one cycle of squared waveform}}{\text{Length of the cycle}}$$

4. Find square root of average value which gives r.m.s. value of an alternating quantity.

4.8 Average Value

The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

> **Key Point:** *For a symmetrical a.c., the average value over a complete cycle is zero as both positive and negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.*

Average value can also be expressed by that steady current which transfers across any circuit, the same amount of charge as is transferred by that alternating current during the same time. The average value for sinusoidally varying alternating current can be obtained by,

1) Graphical Method and 2) Analytical Method

4.8.1 Graphical Method

Consider 'n' equal intervals of half cycle as shown in the Fig. 4.12. For r.m.s. value, we have calculated average value of the heat produced by the average currents during each of the 'n' intervals. In this case, it is necessary to determine the average value of current over half cycle.

$$\text{Average value of current over half cycle} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

$$\therefore \quad I_{av} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

While

$$V_{av} = \frac{V_1 + V_2 + \ldots\ldots + V_n}{n}$$

4.8.2 Analytical Method

For an unsymmetrical a.c., the average value must be obtained for one complete cycle but for symmetrical a.c. like sinusoidal, it is to be obtained for half cycle.

Consider sinusoidally varying current, $I = I_m \sin\theta$

Consider the elementary interval of instant 'dθ' as shown in the Fig. 4.14. The average instantaneous value of current in this interval is say, 'i' as shown.

The average value can be obtained by taking ratio of area under curve over half cycle to length of the base for half cycle.

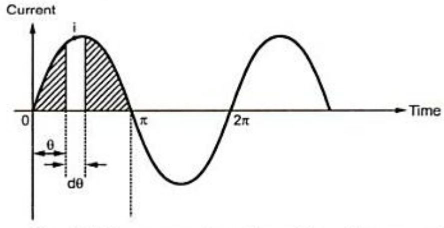


Fig. 4.14 Average value of an alternating current

$\therefore$ $$I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i\ d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{\pi}[-\cos\pi + \cos 0]$$

$$= \frac{I_m}{\pi}[2] = \frac{2 I_m}{\pi}$$

$\therefore$ $$I_{av} = \frac{2}{\pi} I_m$$

i.e. $$I_{av} = 0.637\ I_m$$

Similarly, $$V_{av} = 0.637\ V_m$$

4.8.3 Importance of Average Value

1. The average value is used for applications like battery charging.
2. The charge transferred in capacitor circuits is measured using average values.
3. The average values of voltages and currents play an important role in analysis of the rectifier circuits.
4. The average value is indicated by d.c. ammeters and voltmeters.
5. The average value of purely sinusoidal waveform is always zero.

4.9 Form Factor (K_f)

The form factor of an alternating quantities defined as the ratio of r.m.s. value to the average value,

Form factor, $$K_f = \frac{\text{r.m.s. value}}{\text{average value}}$$

The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\ I_m}{0.637\ I_m}$$

$\therefore$ $K_f = 1.11$ for sinusoidally varying quantity

4.10 Crest or Peak Factor (K_p)

The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

Peak factor $K_p = \frac{\text{maximum value}}{\text{r.m.s. value}}$

The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{I_m}{0.707\, I_m} = 1.414 \quad \text{for sinusoidal waveform}$$

The r.m.s. value is always greater than the average, except for a rectangular wave, in which case, the heating effect remains constant so that average and r.m.s. values are same. Hence, **in practice, for all the waveforms, except rectangular, form factor is greater than one.**

➡ **Example 4.4 :** *Calculate the r.m.s. value, average value, form factor, peak factor of a periodic current having following values for equal time intervals changing suddenly from one value to next as 0, 2, 4, 6, 8, 10, 8, 6, 4, 2, 0, –2, –4, –6, –8, –10, –8, ...*

Solution : The waveform can be represented as shown in the Fig. 4.16.

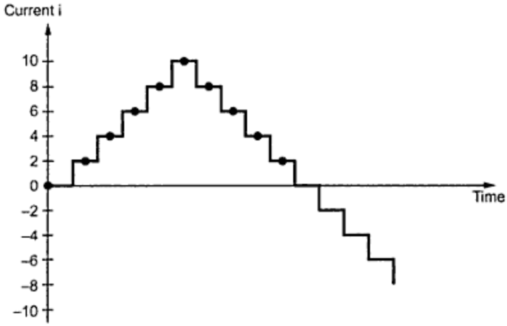


Fig. 4.15

The average value of the current is given by,

$$\text{Average value} = \frac{0+2+4+6+8+10+8+6+4+2}{10} = \mathbf{5\ A}$$

$$\text{The r.m.s. value of the current} = \sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+8^2+6^2+4^2+2^2}{10}}$$

$$= \mathbf{5.8309\ A}$$

Form factor $K_f = \frac{\text{r.m.s.}}{\text{average}} = \frac{5.8309}{5} = \mathbf{1.1661}$

Peak factor $K_p = \frac{\text{maximum}}{\text{r.m.s.}} = \frac{10}{5.8309} = \mathbf{1.715}$

4.11 R.M.S. Value of Combined Waveform

Consider a wire carrying simultaneously more than one alternating current of different magnitudes and frequencies alongwith certain d.c. current. It is required to calculate resultant **r.m.s. value** i.e. **effective value** of the current.

Let the wire carries three different currents as shown in the Fig. 4.16. It is required to obtain resultant I_{rms} through the wire.

Method : It is based on heating effect of various currents.

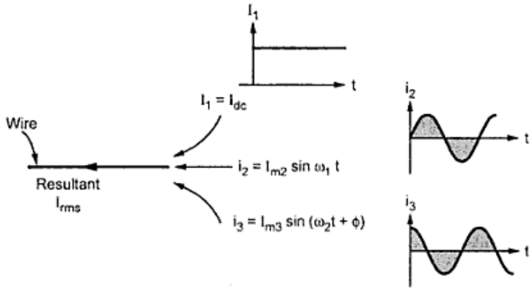


Fig. 4.16 Wire carrying 3 different currents simultaneously

Let I_{rms} = Resultant r.m.s. value of current

R = Resistance of wire

t = Time for which current is flowing

∴ H = Heat produced by resultant $= I_{rms}^2 \times R \times t$ ...(1)

This heat produced is sum of the heats produced by the individual current components flowing for the same time t.

H_1 = Heat produced by d.c. component $= I_{dc}^2 \times R \times t$

H_2 = Heat produced by first a.c. component $= I_{rms2}^2 \times R \times t$

$$= \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t$$

$$H_3 = \text{Heat produced by second a.c. component} = I^2_{rms3} \times R \times t$$

$$= \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

Note that for alternating currents $I_{rms} = I_m / \sqrt{2}$.

Thus equating the total heat produced to sum of the individual heats produced,

$$H = H_1 + H_2 + H_3$$

$$\therefore \quad I^2_{rms} Rt = I^2_{dc} Rt + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 Rt + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 Rt$$

$$\therefore \quad \boxed{I_{rms} = \sqrt{I^2_{dc} + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2}}$$

Key Point: *The result can be extended to n number of current components flowing through the wire.*

If average value is to be calculated, it must be noted that average value of purely sinusoidal quantity over a cycle is zero. Hence average value of the resultant is the only d.c. component flowing through the wire.

$$\therefore \quad \boxed{I_{av} = I_{dc}}$$

Example 4.5 : *Find the effective value of a resultant current in a wire which carries simultaneously a direct current of 10 A and alternating current given by,*
$i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin(5\omega t + \pi/3)$.

Solution : The **effective** value means **r.m.s.** value. It is based on the heating effect of the currents.

$$I_{dc} = 10 \text{ A}, \; I_{m1} = 12 \text{ A}, \; I_{m2} = 6 \text{ A}, \; I_{m3} = 4 \text{ A},$$

Let, I_{rms} = Resultant r.m.s. value, R = Resistance of wire.

Equating heat produced in time t due to resultant to the sum of individual heats produced by various components.

$$\therefore \quad I^2_{rms} \times R \times t = I^2_{dc} \times R \times t + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

Note that heat produced = $(\text{rms value})^2 \times R \times t$

and r.m.s. of a.c. = $\frac{I_m}{\sqrt{2}}$

$$\therefore \quad I_{rms}^2 = 10^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 198$$

$$\therefore \quad I_{rms} = \mathbf{14.0712\ A}$$ ...Effective value of the resultant

4.12 Phasor Representation of an Alternating Quantity

In the analysis of a.c. circuits, it is very difficult to deal with alternating quantities in terms of their waveforms and mathematical equations. The job of adding, subtracting, etc. of the two alternating quantities is tedious and time consuming in terms of their mathematical equations. Hence, it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called **phasor representation** of an alternating quantity.

The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in this method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

Key Point: *The phasors are assumed to be rotated in anticlockwise direction.*

One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 4.17. If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as $\theta = \omega t$), we get a sine waveform.

Consider the various positions shown in the Fig. 4.17.

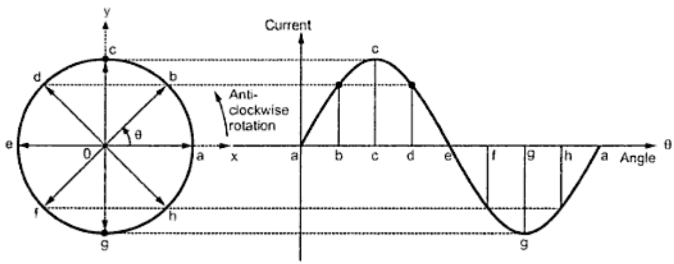


Fig. 4.17 Phasor representation of an alternating quantity

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [l (ob) sin θ)]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is $I = I_m \sin \theta$, represented in the waveform.
3. At point 'c', the Y-axis projection 'oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes $I_m \sin \theta$ which is the instantaneous value of the current at that instant.
5. At point 'e', the Y-axis projection is zero and instantaneous value of the current is zero at this instant.
6. Similarly, at points f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants and when plotted, give us negative half cycle of the alternating quantity.

Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y–axis gives the instantaneous value of the alternating quantity at that particular instant. The angular velocity 'ω' in an anticlockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle i.e. $\theta = \omega t$,

where $\omega = 2 \pi f$ rad/sec

Points to Remember :

In practice, the alternating quantities are represented by their r.m.s. values. Hence, the length of the phasor represents r.m.s. value of the alternating quantity. In such case, projection on Y-axis does not give directly the instantaneous value but as $I_m = \sqrt{2}\, I_{r.m.s.}$, the projection on Y-axis must be multiplied by $\sqrt{2}$ to get an instantaneous value of that alternating quantity.

Phasors are always assumed to be rotated in **anticlockwise** direction.

Two alternating quantities of same frequencies can be represented on same phasor diagram.

Key Point : *If frequencies of the two quantities are different, then such quantities cannot be represented on the same phasor diagram.*

4.13 Concept of Phase of an Alternating Quantity

In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant. It is represented in terms of angle θ in radians or degrees, measured from certain reference. Thus, phase can be defined as,

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

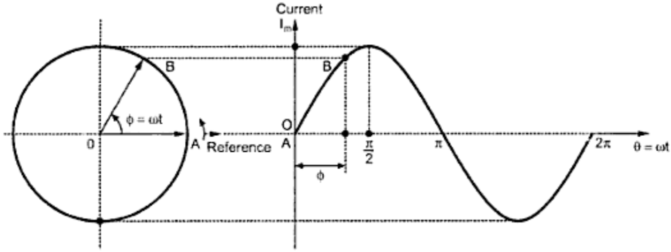


Fig. 4.18 Concept of phase

Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 4.19 at the instant A is $\phi = 0°$. While the phase of the current at the instant B is the angle ϕ through which the phasor has travelled, measured from the reference axis i.e. X-axis.

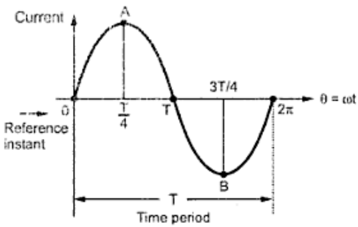


Fig. 4.19

In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.

Another way of defining the phase is in terms of a time period T. The phase of an alternating quantity at any particular instant is the fraction of the time period (T) through which the quantity is advanced from the reference instant.

Consider alternating quantity represented in the Fig. 4.19. As per above definition, the phase of quantity at instant A is $\frac{T}{4}$, while phase at instant B is $\frac{3T}{4}$. Generally, the phase is expressed in terms of angle ϕ which varies from 0 to 2π radians and measured with respect to positive x-axis direction.

In terms of phase the equation of alternating quantity can be modified as,

$$e = E_m \sin(\omega t \pm \phi)$$

where ϕ = Phase of the alternating quantity.

The phasor representation and waveforms of both the quantities are shown in the Fig. 4.21.

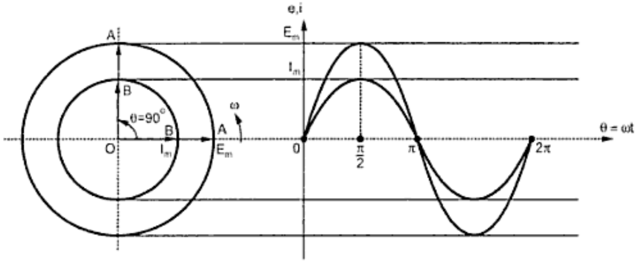


Fig. 4.21

The phasors OA = E_m

and OB = I_m

After $\theta = \frac{\pi}{2}$ radians, the OA phasor achieves its maximum E_m while at the same instant, the OB phasor achieves its maximum I_m. As the frequency of both is same, the angular velocity ω of both is also the same. So, they rotate together in synchronism.

So, at any instant, we can say that the phase of voltage e will be same as phase of i. Thus, the angle travelled by both within a particular time is always the same. So, the difference between the phases of the two quantities is zero at any instant. **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

Key Point : *When such phase difference between the two alternating quantities is zero, the two quantities are said to be in phase.*

The two alternating quantities having same frequency, reaching maximum positive and negative values and zero values at the same time are said to be in phase. **Their amplitudes may be different.**

In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value, at the same instant, the other is having some negative value or positive value.

Such two quantities are said to have phase difference between them. If there is difference between the phases (angles) of the two quantities, expressed in degrees or radians at any particular instant, then as both rotate with same speed, this difference remains same at all the instants.

Consider an e.m.f. having maximum value E_m and current having maximum value I_m. Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig. 4.22.

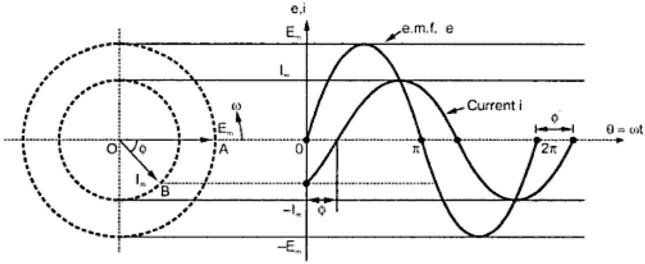


Fig. 4.22 Concept of phase difference (Lag)

Thus, there exists a phase difference ϕ between the two phasors. Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the times by angle ϕ. This is called **lagging phase difference.** The current i is said to lag the voltage e by angle ϕ. The current i achieves its maxima, zero values ϕ angle later than the corresponding maximum, zero values of voltage.

The equations of the two quantities are written as,

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin (\omega t - \phi)$$

'i' is said to lag 'e' by angle ϕ.

It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig. 4.23.

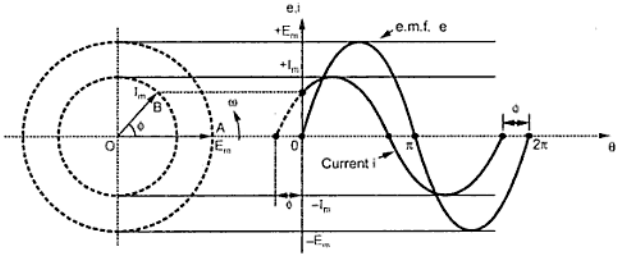


Fig. 4.23 Concept of phase difference (Lead)

It can be seen that, there exists a phase difference of ϕ angle between the two. But in this case, current 'i' is ahead of voltage 'e', as both are rotating in anticlockwise direction with same speed. Thus, current is said to be leading with respect to voltage and the phase difference is called **leading phase difference.** The current i achieves its maximum, zero values ϕ angle before than the corresponding maximum, zero values of the voltage. At all instants, current i is going to remain ahead of voltage 'e' by angle 'ϕ'.

The equations of such two quantities are written as

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin (\omega t + \phi)$$

'i' is said to lead 'e' by angle ϕ.

Key Point : *Thus, related to the phase difference, it can be remembered that a plus (+) sign of angle indicates lead where as a minus (–) sign of angle indicates lag with respect to the reference.*

4.13.2 Phasor Diagram

The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**

The phasors are rotating in anticlockwise direction with an angular velocity of $\omega = 2\pi f$ rad/sec. Hence, all phasors have a particular fixed position with respect to each other.

Key Point : *Hence, phasor diagram can be considered as a still picture of these phasors at a particular instant.*

To clear this point, consider two alternating quantities in phase with each other.

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin \omega t$$

At any instant, phase difference between them is zero i.e. angle difference between the two phasors is zero. Hence, the phasor diagram for such case drawn at different instants will be alike giving us the same information that two quantities are in phase. The phasor diagram drawn at different instants are shown in the Fig. 4.24.

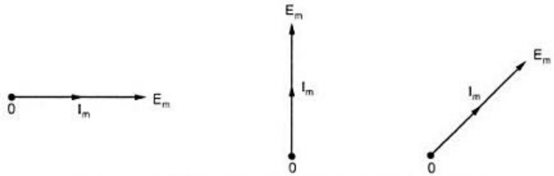


Fig. 4.24 Same phasor diagram at different instants

Consider another example where current i is lagging voltage e by angle ϕ So, difference between the angles of the phasors representing the two quantities is angle ϕ

$$e = E_m \sin \omega t$$

and $$i = I_m \sin (\omega t - \phi)$$

The phasor diagram for such case, at various instants will be same, as shown in the Fig. 4.25 (a), (b) and (c).

The phasor diagram drawn at any instant gives the same information.

Key Point : ***Remember that the lagging and leading word is relative to the reference.*** *In the above case, if we take current as reference, we have to say that the voltage leads current by angle ϕ. The direction of rotation of phasors is always anticlockwise.*

Fig. 4.25

Important Points Regarding Phasor Diagram :

1) As phasor diagram can be drawn at any instant, X and Y axis are not included in it. But, generally, the reference phasor chosen is shown along the positive X axis direction and at that instant other phasors are shown. This is just from convenience point of view. **The individual phase of an alternating quantity is always referred with respect to the positive x-axis direction.**

2) There may be more than two quantities represented in phasor diagram. Some of them may be current and some may be voltages or any other alternating quantities like flux, etc. The frequency of all of them must be the same.

3) Generally, length of phasor is drawn equal to r.m.s. value of an alternating quantity, rather than maximum value.

4) The phasors which are ahead, in anticlockwise direction, with respect to reference phasor are said to be leading with respect to reference and phasors behind are said to be lagging.

5) Different arrow heads may be used to differentiate phasors drawn for different alternating quantities like current, voltage, flux, etc.

Example 4.6 : *Two sinusoidal currents are given by,*

$i_1 = 10 \sin (\omega t + \pi/3)$ *and*

$i_2 = 15 \sin (\omega t - \pi/4)$

Calculate the phase difference between them in degrees.

Solution : The phase of current i_1 is $\pi/3$ radians i.e. 60° while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 4.26.

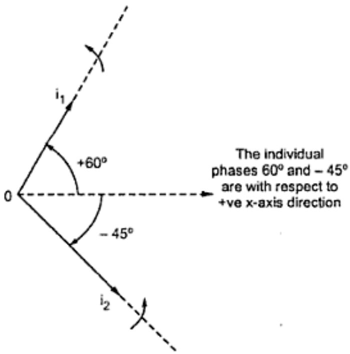


Fig. 4.26

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = 105°$$

And i_2 lags i_1.

For I_{m1}, its X component is I_{m1} and Y component is zero. So, **in rectangular form, it is represented as I_{m1} + j 0.**

For I_{m2}, its X component is $-I_{m2}\cos\left(\frac{\pi}{2}-\theta\right)$ while its Y component is $I_{m2}\sin\left(\frac{\pi}{2}-\theta\right)$

So, in rectangular form, it is represented as,

$$-I_{m2}\cos\left(\frac{\pi}{2}-\theta\right)+j\,I_{m2}\sin\left(\frac{\pi}{2}-\theta\right)$$

Analytically, the resultant of the two can be obtained by adding X components of all the phasors together and Y components of all the phasors.

$$\therefore \quad \boxed{\text{Resultant} = \Sigma\,\text{X components} \pm j\,\Sigma\,\text{Y components}}$$

This is rectangular representation of the resultant.

Following are the **steps for analytical method** :

1) Express all the phasors in rectangular form.
2) Find all X-components i.e. real components and Y-components i.e. imaginary components.
3) Add all X-components and Y-components algebraically to obtain resultant X-component and Y–component.
4) The resultant phasor can be expressed in terms of the resultant X and Y components.

4.14.3 Mathematical Representation of Phasor

Any phasor can be represented mathematically in two ways,

1) Polar co-ordinate system and 2) Rectangular co-ordinate system

Let $i = I_m \sin(\omega t + \phi)$

The phase of current i is ϕ The phase is always with respect the x-axis as shown in the Fig. 4.29.

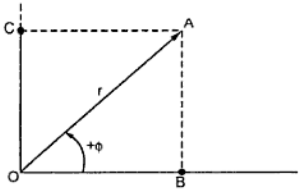


Fig. 4.29

l(OA) = I_m = r and phase = + ϕ

In **polar system,** the phasor is represented as $r \angle \pm \phi$ So current i above is represented as $I_m \angle +\phi$ in polar system.

In **rectangular system,** the phasor is divided into x and y components i.e. real and imaginary components as $x \pm j\,y$. The current i above is represented as $I_m \cos\phi + j\,i_m \sin\phi$ in rectangular system.

Converting it to polar form,

$$I = \mathbf{47.1699\angle 57.99^\circ\,A} = I_{rms} \angle \phi \text{ A}$$

$\therefore$ r.m.s. value of current = **47.1699 A**

Phase = **57.99°**

Key Point: *To obtain phase, express the equation in sine form if given in cosine as,*

If $\qquad e = E_m \cos(\omega t)$

then $\qquad e = E_m \sin(\omega t + 90^\circ)$ as $\sin(90^\circ + \theta) = \cos\theta$

Thus the phase is 90° and not zero.

In general,	$e = E_m \cos(\omega t \pm \phi)$	
then	$e = E_m \sin(\omega t + 90^\circ \pm \phi)$	
$\therefore$	The phase = $90^\circ \pm \phi$	

Example 4.9 : *A voltage is defined as* $- E_m \cos \omega t$. *Express it in polar form.*

Solution : To express a voltage in polar form express it in the form, $e = E_m \sin \omega t$

Now $\qquad e = -E_m \cos \omega t = -E_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad$ as $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$

$$= E_m \sin\left(\omega t + \frac{3\pi}{2}\right) \qquad \text{as } \sin(\pi + \theta) = -\sin\theta$$

Now it can be expressed in polar form as,

$$e = E_m \angle + \frac{3\pi}{2}\text{ rad} = E_m \angle + 270^\circ \text{ V}$$

But + 270° phase is nothing but – 90°

$$\therefore \qquad e = \mathbf{E_m \angle -90^\circ\ V}$$

Example 4.10 : *Find the resultant of the three voltages e_1, e_2 and e_3 where,*

$$e_1 = 20 \sin(\omega t), \quad e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right) \text{ and } e_3 = \cos\left(\omega t + \frac{\pi}{6}\right)$$

Solution : Express all the voltages interms of $\sin(\omega t \pm \phi)$.

$$e_1 = 20 \sin(\omega t + 0^\circ)$$

$$e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right) = 30 \sin(\omega t - 45^\circ)$$

$$e_3 = 40 \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$= 40 \sin\left(\frac{\pi}{2} + \omega t + \frac{\pi}{6}\right) \qquad \text{as } \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$$

$$= 40 \sin\left(\omega t + \frac{2\pi}{3}\right) = 40 \sin(\omega t + 120^\circ)$$

In polar form the three voltages are,

$$e_1 = 20 \angle 0^\circ,\ e_2 = 30 \angle -45^\circ,\quad e_3 = 40 \angle +120^\circ$$

In rectangular form the three voltages are,

$$e_1 = 20 + j\,0,\ e_2 = 21.213 - j\,21.213,\ e_3 = -20 + j\,34.641$$

Hence the resultant is,

$$e_R = \bar{e}_1 + \bar{e}_2 + \bar{e}_3 = 20 + j0 + 21.213 - j\,21.213 - 20 + j\,34.641$$
$$= 21.213 + j\,13.428\ \text{V} = 25.1058 \angle 32.33^\circ\ \text{V}$$

Thus magnitude of the resultant is **25.1058 V** and phase **32.33°**.

And the expression for the instantaneous value of the resultant is,

$$\mathbf{e_R = 25.1058 \sin(\omega t + 32.33^\circ)\ V}$$

Example 4.11 : *Draw a neat sketch, in each case (not to scale), of the waveform and write the equation for instantaneous value, for the following :-*
i) Sinusoidal current of 10 A (r.m.s.), 50 Hz passing through its zero value at $\omega t = \pi/3$ radians and rising positively,
ii) Sinusoidal current of amplitude of 8 A, 50 Hz passing through its zero value at $\omega t = -\pi/6$ and rising positively.

Solution : i) Zero at $\pi / 3$ rad = 60°, $I_m = \sqrt{2} \times 10 = 14.142$ A

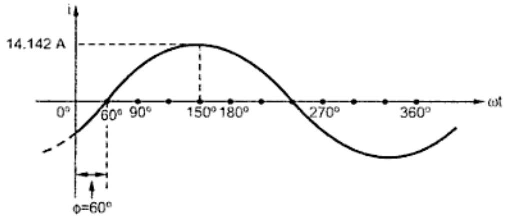


Fig. 4.30 (a)

$\therefore \quad i = I_m \sin(\omega t - \phi) = 14.142 \sin\left(100\pi t - \frac{\pi}{3}\right) A$

ii) $\quad I_m = 8$ A, zero at $\omega t = -\frac{\pi}{6}$ rad $= -30°$

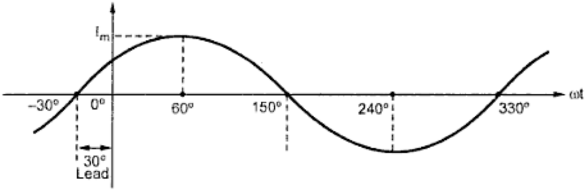


Fig. 4.30 (b)

$$\therefore \quad i = I_m \sin(\omega t + \phi)$$

$$= 8 \sin\left(100\pi t + \frac{\pi}{6}\right) A$$

4.15 Multiplication and Division of Phasors

In the last section, the addition and subtraction of phasors is discussed, which is to be carried out using rectangular form of phasors. But the rectangular form is not suitable to perform multiplication and division of phasors. Hence multiplication and division **must be performed** using **polar form** of the phasors.

Let P and Q be the two phasors such that,

$$P = x_1 + jy_1 \quad \text{and} \quad Q = x_2 + jy_2$$

To obtain the multiplication $P \times Q$ both must be expressed in polar form

$$\therefore \quad P = r_1 \angle \phi_1 \quad \text{and} \quad Q = r_2 \angle \phi_2$$

Then
$$P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2$$

Key Point : *Thus in multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.*

The result then can be expressed back to rectangular form, if required. Now consider the division of the phasors P and Q.

$$\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$$

Key Point: *Thus in division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.*

Note : For converting polar to rectangular form, the students can use the function $P \leftrightarrow R$ on calculators without using basic conversion expressions. Similarly for rectangular to polar conversion, the students can use the function $R \leftrightarrow P$ on calculators without using basic conversion expressions.

4.15.1 Another Way of Complex Number Representation

In complex number analysis, the number may be expressed as $|r|e^{j\phi}$ where ϕ may be in degrees or radians. The mathematically $e^{j\phi} = \cos\phi + j\sin\phi$ while $e^{-j\phi} = \cos\phi - j\sin\phi$. Hence corresponding rectangular form can be obtained.

For example if a particular current is given as $50\ e^{-j30^\circ}$ then,

$$50\ e^{-j30^\circ} = 50\ [\cos 30^\circ - j \sin 30^\circ] = 43.3012 - j\ 25\ A$$

while a particular voltage given by $150\ e^{+j100^\circ}$ is,

$$150\ e^{+j100^\circ} = 150\ [\ \cos 100^\circ + j \sin 100^\circ] = -26.047 + j\ 147.721\ V$$

If ϕ is given in radians, sin and cos must be calculated in radian mode.

Infact $|r|e^{\pm j\phi}$ can be directly expressed in the polar form as $|r|\angle \pm\phi$ where ϕ may be in degrees or radians.

$$\therefore \quad 50\ e^{-j30^\circ} = 50\angle -30^\circ\ A$$

$$\text{while} \quad 150\ e^{+j100^\circ} = 150\angle +100^\circ\ V$$

This can be crosschecked by using rectangular to polar conversion.

Thus $$\boxed{|r|\ e^{\pm j\phi} = |r|\ \angle \pm\phi}$$

Example 4.12 : *Two currents $I_1 = 10\ e^{j50^\circ}$ and $I_2 = 5\ e^{-j100^\circ}$ flow in a 1-ph A.C. circuit. Estimate :-*

i) $I_1 + I_2$ ii) $I_1 - I_2$ and iii) I_1/I_2, in complex form.

Solution : $I_1 = 10\ e^{j\,50^\circ} A$ and $I_2 = 5\ e^{-j\,100^\circ}\ A$

Now

$$I_1 = 10\ [\cos 50^\circ + j \sin 50^\circ]$$

$$= 6.4278 + j\ 7.66\ A = 10\ \angle\ 50^\circ\ A$$

$$I_2 = 5\ [\cos 100^\circ - j \sin 100^\circ]$$

$$= -0.8682 - j\ 4.924\ A = 5\ \angle\ -100^\circ\ A$$

i) $I_1 + I_2 = 5.5596 + j\ 2.736\ A = \mathbf{6.196\ \angle\ 26.2^\circ\ A}$

ii) $I_1 - I_2 = 7.296 + j\ 12.584\ A = \mathbf{14.546\ \angle\ 60^\circ\ A}$

iii) $I_1/I_2 = \dfrac{10\angle 50^\circ}{5\angle -100^\circ} = \mathbf{2\ \angle\ +150^\circ\ A}$

Remember :

While addition and subtraction, use rectangular form.

While multiplication and division, use polar form.

Examples with Solutions

➡ **Example 4.13 :** *The mathematical expression for the instantaneous value of an alternating current is* $i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$ *A. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the waveform from* $t = 0$ *over one complete cycle.*

Solution : The given current is, $i = 7.071 \sin\left(157.08\,t - \frac{\pi}{4}\right)$ ... Amp

Comparing this with, $i = I_m \sin(\omega t - \phi)$ ... Amp

We get, $I_m = 7.071$ A

$$\text{Effective value} = \frac{I_m}{\sqrt{2}} = I_{rms}$$

$$\therefore \quad I_{rms} = \frac{7.071}{\sqrt{2}} = \mathbf{5\ A}$$

$$\omega = 157.08 \quad \text{i.e.} \quad \frac{2\pi}{T} = 157.08$$

$$\therefore \quad T = \frac{2\pi}{157.08} = 0.04 \text{ sec} = \mathbf{40\ msec}$$

Positive maximum is, $i = I_m = +7.071$ A

$$\therefore \quad 7.071 = 7.071 \sin\left(157.08\,t - \frac{\pi}{t}\right)$$

$$\therefore \quad \text{Sin}\left(157.08\,t - \frac{\pi}{t}\right) = 1$$

$$\therefore \quad 157.08\,t - \frac{\pi}{t} = 1.5707$$ Use radian mode

$$\therefore \quad 157.08\,t = 2.3561$$

$$\therefore \quad t = 0.015 \text{ sec} = \mathbf{15\ msec}$$

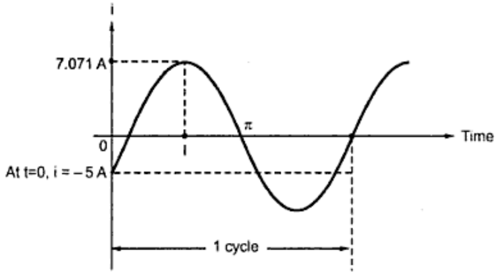


Fig. 4.31

Example 4.14 : *Two sinusoidal sources of e.m.f. have r.m.s. values E_1 and E_2 and a phase difference α. When connected in series, the resultant voltage is 41.1 V.*
When one of the source is reversed, the resultant e.m.f. is 17.52 V. When phase displacement is made zero, the resultant e.m.f. is 42.5 V. Calculate E_1, E_2 and α.

Solution : Let the two e.m.f.s be such that E_2 lags E_1 by α so that there is phase difference of α between them,

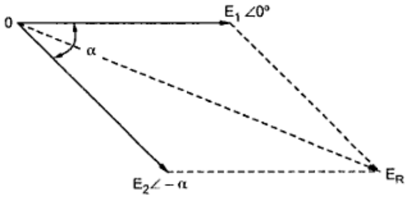


Fig. 4.32

$$e_1 = E_1 \sin \omega t = E_1 \angle 0°$$

and $e_2 = E_2 \sin(\omega t - \alpha) = E_2 \angle -\alpha°$

In the rectangular form the two voltages are,

$$E_1 \angle -0° = E_1 + j0$$

and $E_2 \angle -\alpha = E_2 \cos\alpha - j\, E_2 \sin\alpha$

Remember that
$r \angle -\phi = r\cos\phi - jr \sin\phi$

In series, $E_R = \overline{E}_1 + \overline{E}_2 = (E_1 + E_2\cos\alpha) - j\,(E_2 \sin\alpha)$

Magnitude of $E_R = \sqrt{x^2+y^2} = \sqrt{(E_1+E_2\cos\alpha)^2 + (E_2 \sin\alpha)^2}$

$$\therefore \quad 41.1 = \sqrt{(E_1+E_2\cos\alpha)^2 + (E_2\sin\alpha)^2}$$

$$\therefore \quad (41.1)^2 = E_1^2 + 2E_1E_2\cos\alpha + E_2^2\cos^2\alpha + E_2^2\sin^2\alpha$$

$$\therefore \quad 1689.21 = E_1^2 + 2E_1E_2\cos\alpha + E_2^2 \quad \ldots (1)$$

Example 4.16 : *Calculate the average and effective values of the saw tooth waveform shown in Fig. 4.35.*
The voltage completes the cycle by falling back to zero instantaneously after regular interval of time.

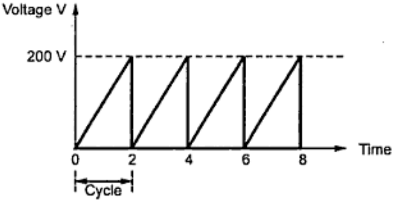


Fig. 4.35

Solution : Let us calculate equation for the instantaneous value of the voltage. The voltage increases linearly from 0 to 200 V in two seconds. So slope between 0 to 2 seconds is,

$$= \frac{200-0}{2} = 100$$

$\therefore$ Equation for the instantaneous value is,

$$V = 100\ t$$

$$\text{The average value} = \frac{\text{Area under curve}}{\text{base}} = \int_0^2 \frac{(100\ t)\ dt}{2}$$

$$= \frac{1}{2}\left[100\frac{t^2}{2}\right]_0^2 = 50 \times 2 = \mathbf{100\ volts.}$$

The r.m.s. value = Root of the mean of square

$$= \sqrt{\frac{\int_0^2 (100\ t)^2\ dt}{2}} = \sqrt{\frac{\frac{1}{2}\times(100)^2\times\left[\frac{t^3}{3}\right]_0^2}{2}}$$

$$= \sqrt{5000\times\frac{8}{3}} = \mathbf{115.47\ volts}$$

Example 4.17 : *Calculate the average value, r.m.s. value and form factor of the output of a half wave rectifier when input to rectifier is purely sinusoidal alternating current.*

(GTU : Dec.-2008)

Solution : Input to rectifier $i = I_m \sin\theta$

Half-wave rectifier is one which rectifies the half cycle of the input applied and its output is as shown in the Fig. 4.36.

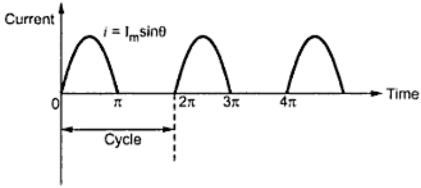


Fig. 4.36

Since the waveform is unsymmetrical, the average and r.m.s. values must be calculated for one complete cycle.

$$\text{The average value} = \frac{\text{Area of curve over a cycle}}{\text{Length of base over a cycle}}$$

$$= \frac{\int_0^{\pi} i\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{\pi} i\,d\theta$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\,d\theta = \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{2\pi}[2] = \frac{I_m}{\pi}$$

$$I_{av} = \mathbf{0.318\ I_m}$$

R.M.S. value = Root of mean of square

$$= \sqrt{\frac{\text{Area of curve over a square wave cycle}}{\text{Length of base over a cycle}}}$$

$$= \sqrt{\frac{\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} (0)^2 d\theta}{2\pi}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta} = \sqrt{\frac{I_m^2}{4\pi}\left[\frac{\theta - \sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}[\pi]} = \sqrt{\frac{I_m^2}{4}} = \frac{I_m}{2}$$

$\therefore \quad I_{r.m.s.} = \mathbf{0.5\ I_m}$

$\therefore \quad \text{Form factor } K_f = \frac{\text{r.m.s.}}{\text{average}} = \frac{0.5\ I_m}{0.318\ I_m} = \mathbf{1.5723}$

Example 4.18 : *At t = 0, the instantaneous value of a 60-Hz sinusoidal current is + 5 ampere and increases in magnitude further. Its r.m.s. value is 10 A.*
i) Write the expression for its instantaneous value.
ii) Find the current at t = 0.01 and t = 0.015 second.
iii) Sketch the waveform indicating these values.

Solution : $t = 0,\ i = 5\ A,\ f = 60\ Hz,\ I_{rms} = 10\ A$

i) $I_m = \sqrt{2}\, I_{rms} = 10\sqrt{2}\ A = 14.1421\ A$

Let the equation for instantaneous value is,

$$i = I_m \sin(2\pi f t + \phi)$$

Now $\quad 5 = 14.1421 \sin(2\pi \times 60 \times 0 + \phi)$

$\therefore \quad \phi = \sin^{-1}\frac{5}{14.1421} = 20.704° = 0.3613\ \text{rad}$

$\therefore \quad i = 14.1421\ \sin(120\pi t + 20.704°)\ A$

i.e. $\quad \mathbf{i = 14.1421\ \sin(120\pi t + 0.3613)\ A}$

ii) To find i, calculate sin in radian mode.

At $t = 0.01$, $\quad i = 14.1421 \sin(120\pi \times 0.01 + 0.3613) = \mathbf{-11.8202\ A}$

At $t = 0.015$, $\quad i = 14.1421 \sin(120\pi \times 0.015 + 0.3613) = \mathbf{-3.7314\ A}$

iii) The waveform is as shown in Fig. 4.37.

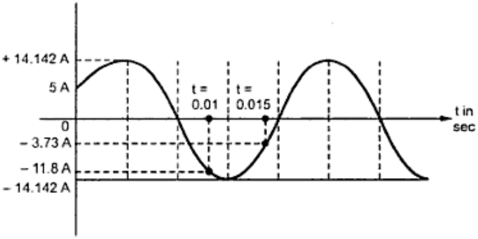


Fig. 4.37

Example 4.19 : *A 50-Hz sinusoidal voltage applied to a single phase circuit has its r.m.s. value of 200 V. Its value at t = 0 is $\left(\sqrt{2}\times 200\right)$ V positive. The current drawn by the circuit is 5 ampere (rms) and lags behind the voltage by one-sixth of a cycle. Write the expressions for the instantaneous values of voltage and current. Sketch their wave-forms, and find their values at t = 0.0125 second.*

Solution : $f = 50$ Hz, $V_{rms} = 200$ V

$$V_m = \sqrt{2}\ V_{rms} = 282.842 \text{ V}$$

The equation for voltage is $V = V_m \sin(2\pi f t + \phi)$

At t = 0, $V = 200\times\sqrt{2}$ V

$\therefore$ $200\times\sqrt{2} = 282.842 \sin(0+\phi)$

$\therefore$ $\phi = \frac{\pi}{2}$ rad = 1.5707 rad = 90°

$\therefore$ $\mathbf{V = 282.842 \sin(100\pi t + 1.5707)\ V}$

Now $I_{rms} = 5$ A hence $I_m = \sqrt{2}\ I_{rms} = 7.071$ A

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

I lags by $\frac{1}{6}^{th.}$ of cycle i.e. by $\frac{T}{6}$ i.e. $\frac{0.02}{6} = 3.333\times 10^{-3}$ sec

$\therefore$ I lags by angle $= \omega t = 2\pi f t = 100\pi\times 3.3333\times 10^{-3} = 1.04709$ rad

$\therefore$ $\theta = 1.04709$ rad. where θ = angle by which I lags V

$\therefore$ $i = I_m \sin(2\pi f t + \phi - \theta)$

$\therefore$ $i = 7.071 \sin(100\pi t + 1.5707 - 1.04709)$

$\therefore$ $\mathbf{i = 7.071 \sin(100\pi t + 0.5236)\ A}$

At t = 0.0125 sec, find v and i. **Use sin in radians.**

$\therefore$ $V = 282.842 \sin(100\pi\times 0.0125 + 1.5707) = \mathbf{-200\ V}$

$i = 7.071 \sin(100\pi\times 0.0125 + 0.5236) = \mathbf{-6.83\ A}$

The waveforms are shown in the Fig. 4.38.

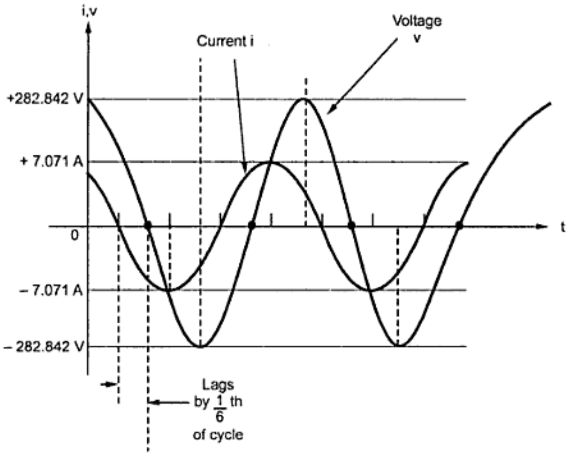


Fig. 4.38

Example 4.20 : *A 50 Hz sinusoidal current has peak factor 1.4 and form factor 1.1. Its average value is 20 Amp. The instantaneous value of current is 15 Amp at t = 0 sec. Write the equation of current and draw its waveform.*

Solution : $f = 50$ Hz, $K_p = 1.4$, $K_f = 1.1$, $I_{av} = 20$ A

$$K_p = \frac{I_m}{I_{rms}} \quad \text{while} \quad K_f = \frac{I_{rms}}{I_{av}}$$

$$\therefore \quad K_f = \frac{(I_m / K_p)}{I_{av}} \qquad \text{...from } K_p$$

$$\therefore \quad 1.1 = \frac{I_m / 1.4}{20}$$

$$\therefore \quad I_m = 20 \times 1.1 \times 1.4 = 30.8 \text{ A}$$

The current has $i = 15$ A at $t = 0$ hence its equation is,

$$i = I_m \sin(\omega t + \phi) = I_m \sin(2\pi f t + \phi)$$

At $t = 0$, $\quad 15 = 30.8 \sin(0 + \phi)$

$\therefore \qquad \phi = \sin^{-1}\left(\frac{15}{30.8}\right) = \mathbf{29.1444°} = \frac{29.1444 \times \pi}{180}$ rad

$= \mathbf{0.50866\ rad.}$

$\therefore \qquad \mathbf{i = 30.8\ sin(100\ \pi\ t + 0.50866)\ A}$ ...Equation

Its waveform is shown in the Fig. 4.39.

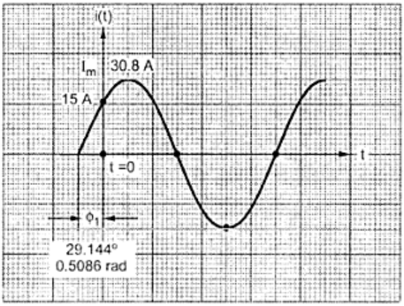


Fig. 4.39

Examples from G.U. and G.T.U. Papers

Example 4.21 : *Three currents i_1, i_2, i_3 are expressed by the expressions 10 sin(314t + 0°), 20 sin(314t + $\pi/12$) and 10 sin(314t + $\pi/2$) respectively are added together. Find : i) Expression of total current ii) R.M.S. value of total current iii) Frequency.*

[GU : June-2000]

Solution : From the given expressions,

$I_{m1} = 10$ A i.e. $I_1 = \frac{I_{m1}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071$ A (RMS)

$I_{m2} = 20$ A i.e. $I_2 = \frac{I_{m2}}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.1421$ A (RMS)

$I_{m3} = 10\text{ A}$ i.e. $I_3 = \frac{I_{m3}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071\text{ A (RMS)}$

$\therefore$ $\bar{I}_1 = 7.071\angle 0°\text{ A} = 7.071 + j\,0\text{ A}$

$\bar{I}_2 = 14.1421\angle\frac{\pi}{12} = 14.1421\angle 15°\text{ A} = 13.6602 + j\,3.6602\text{ A}$

$\bar{I}_3 = 7.071\angle\frac{\pi}{2} = 7.071\angle 90°\text{ A} = 0 + j\,7.071\text{ A}$

i) $\bar{I}_R = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 7.071 + j\,0 + 13.6602 + j\,3.6602 + 0 + j\,7.071$

$= 20.7312 + j\,10.7312\text{ A} = 23.3439\angle 27.3676°\text{ A}$

$\therefore$ $I_{RM} = \text{Maximum value} = 23.3439 \times \sqrt{2} = 33.0132\text{ A}$

$\therefore$ $i_R = I_{RM}\sin(\omega t + \phi) = \mathbf{33.0132\sin(314t + 27.3676°)\text{ A}}$

ii) R.M.S. value of total current = **23.3439 A**

iii) $\omega = 314\text{ rad/sec} = 2\pi f$

$\therefore$ $f = \frac{314}{2\pi} = \mathbf{50\text{ Hz}}$ ... Frequency

Example 4.22 : *A sinusoidal alternating voltage has an r.m.s. value of 200 V and a frequency of 50 Hz. It crosses the zero axis in a positive direction when t = 0. Determine i) the time when voltage first reaches the instantaneous value of 200 V and ii) the time when voltage after passing through its maximum positive value reaches the value of 141.4 V.*

[GU : June-2000]

Solution : V = 200 V (RMS), f = 50 Hz.

$\therefore$ $V_m = \text{Maximum value} = \sqrt{2}\,V = \sqrt{2} \times 200 = 282.8427\text{ V}$

$\omega = 2\pi f = 2\pi \times 50 = 100\pi\text{ rad/sec.}$

$\therefore$ $v = V_m\sin(\omega t) = 282.8427\sin(100\pi t + 0°)\text{ V}$ ... (1)

i) v = 200 V

$\therefore$ $200 = 282.8427\sin(100\pi t)$ i.e. $\sin(100\pi t) = 0.7071$

$\therefore$ $100\pi t = \sin^{-1}(0.7071) = 0.78539$ ... Use **radian mode**

$\therefore$ $t = \mathbf{2.5\text{ ms}}$

Example 4.24 : *Find average value and RMS value of the resultant current in a wire which carries simultaneously a direct current of 10 A and sinusoidal alternating current with a peak of 10 A.* **[GU : June/July-2004]**

Solution : i) The resultant is shown in the Fig. 4.41.

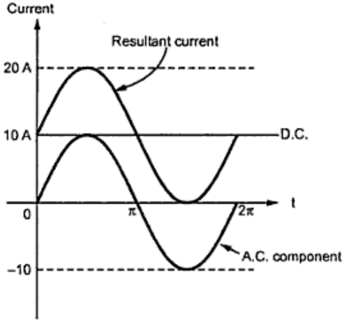


Fig. 4.41

ii) For d.c., $I_{dc} = 10$ A

For a.c. $i = I_m \sin\theta = 10 \sin\theta$

So the resultant is,

$$i_R = I_{dc} + i = \mathbf{10 + 10 \sin\theta}$$

This is the expression for the resultant wave.

iii) Now $i_R = 10 + 10\sin\theta$

The average value can be obtained as,

$$i_R(\text{average}) = \frac{1}{2\pi}\int_0^{2\pi} i_R \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}[10+10\sin\theta]\,d\theta$$

$$= \frac{1}{2\pi}[10\theta - 10\cos\theta]_0^{2\pi}$$

$$= \frac{1}{2\pi}[10(2\pi-0)-10(\cos 2\pi - \cos 0)]$$

$$= \mathbf{10\ A}$$

iv) The r.m.s. value is given by,

$$i_R\ (\text{r.m.s.}) = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_R^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(10+10\sin\theta)^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}[100+200\sin\theta+100\sin^2\theta]\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[100+200\sin\theta+100\left(\frac{1-\cos 2\theta}{2}\right)\right]d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[100\theta-200\cos\theta+100\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)\right]_0^{2\pi}}$$

Example 4.26 : *The following three vectors are given : A = 20 + j20, B = 30 ∠120° and C = 10 + j0. Perform the following indicated operations : i) AB/C ii) BC/A*

[GU : July-2005]

Solution : The three vectors are, A = 20 + j20 = 28.2842 ∠45° ,

B = 30 ∠120° = – 15 + j 25.9807, C = 10 + j 0 = 10 ∠0°

i) $$\frac{AB}{C} = \frac{28.2842\angle 45^\circ \times 30\angle 120^\circ}{10\angle 0^\circ} = \mathbf{84.8526\angle 165^\circ}$$

ii) $$\frac{BC}{A} = \frac{30\angle 120^\circ \times 10\angle 0^\circ}{28.2842\angle 45^\circ} = \mathbf{10.6066\angle 75^\circ}$$

Example 4.27 : *The three vectors are given as : A = 30 – j60, B = 40 + j40, C = 0 + j50. Perform the following operations i) A · C/B ii) BC – 2 A* **[GU : Nov.-2005]**

Solution : The given vectors are, A = 30 – j60 = 67.082 ∠ – 63.435°

B = 40 + j40 = 56.5685 ∠45°, C = 0 + j50 = 50 ∠90°

i) **A · C/B :**

$$\frac{A\cdot C}{B} = \frac{67.082\angle -63.435^\circ \times 50\angle 90^\circ}{56.5685\angle 45^\circ} = \mathbf{59.2927\angle -18.435^\circ}$$

ii) **BC – 2 A :**

$$BC = 56.5685\angle 45^\circ \times 50\angle 90^\circ = 2828.425\angle 135^\circ = -2000 + j2000$$

$$2\,A = 2\times 67.082\angle -63.435^\circ = 134.164\angle -63.435^\circ = 60 - j120$$

$$\therefore \quad BC - 2\,A = -2000 + j2000 - (60 - j120)$$

$$= -2060 + j2120 = \mathbf{2956.01\angle 45.8223^\circ}.$$

Example 4.28 : *An alternating current varying sinusoidally with a frequency of 50 Hz has a r.m.s. value of 21.2132 A. Write down the equation for the instantaneous value and find this value at i) 0.0025 sec. ii) 0.0125 sec. after passing through a positive maximum value. iii) Draw the waveform. iv) At what time measured from a positive maximum value will the instantaneous value of current be 21.2 Amp.*

[GU : Nov.-2002, May-2006]

Solution : $f = 50$ Hz, $I_m = \sqrt{2}\times I_{rms} = \sqrt{2}\times 21.2132 = 30$ A

$$\therefore \quad \omega = 2\pi f = 2\pi\times 50 = 100\,\pi \text{ rad/sec.}$$

Assuming that its phase is zero,

$$i = I_m \sin\omega t = \mathbf{30\sin(100\,\pi t + 0^\circ)\ A} \qquad \text{... Equation}$$

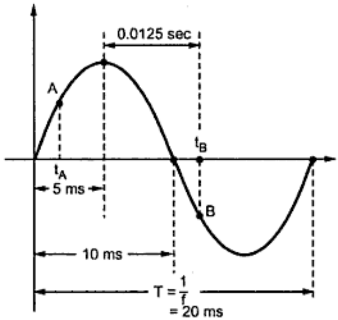


Fig. 4.43

i) t_A = 0.0025 sec

$\therefore$ i = 30 sin (100 π × 0.0025)

... Radian mode

= **21.2132 A**

ii) t_B = 0.0125 sec

after passing through maximum positive

= 5 ms + 0.0125 sec

= 0.0175 sec

$\therefore$ i = 30 sin (100 π × 0.0175)

= **–21.2132 A**

iii) The waveform is shown in the Fig. 4.43 (a).

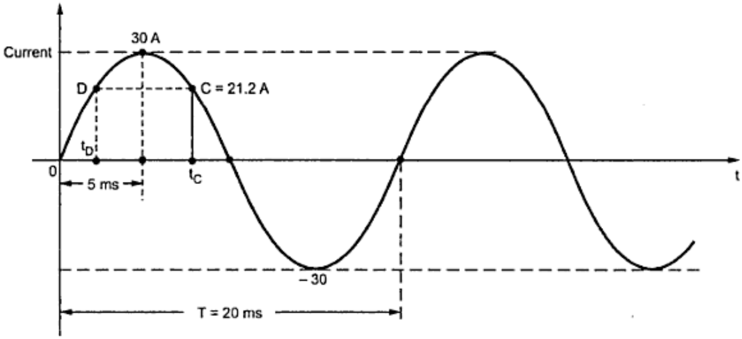


Fig. 4.43 (a)

iv) At the instant t_C and t_D, i = 21.2 A. We want to find t_C . Now i = 21.2 A for the first time at D.

$$\therefore \quad 21.2 = 30 \sin\left(100\pi t_D\right) \quad \text{i.e.} \quad t_D = 2.498 \text{ ms}$$

The time difference between D and time for positive maximum is 5 – 2.498 = 2.502 ms.

Same must be the time difference between time t_C and time for positive maximum.

$\therefore$ Time measured from positive maximum will be **2.502 ms** when instantaneous current will be 21.2 A.

Example 4.29 : *$i_1 = I_{m1} \sin \omega t$ and $i_2 = I_{m2} \sin(\omega t - 45°)$. If $I_{m1} = I_{m2} = 5\sqrt{2}$ then find out, a) $i_1 + i_2$ b) $i_1 - i_2$* **[GU : Nov.-2006]**

Solution :

$$I_{m1} = I_{m2} = 5\sqrt{2} \text{ A}, \quad I_1 = I_2 = \frac{5\sqrt{2}}{\sqrt{2}} = 5 \text{ A} \qquad \text{... (R.M.S.)}$$

$$\therefore \quad \bar{I}_2 = 5\angle -45° \text{ A} = 3.5355 - j3.5355 \text{ A} \qquad \text{... From equation}$$

$$\therefore \quad \bar{I}_1 = 5\angle 0° \text{ A} = 5 + j0 \text{ A} \qquad \text{... From equation}$$

a) $$\bar{I}_1 + \bar{I}_2 = 5 + j0 + 3.5355 - j3.5355 = 8.5355 - j3.5355$$

$$= 9.2385\angle -22.497° \text{ A}$$

$\therefore$ Maximum value of $(i_1 + i_2) = \sqrt{2} \times 9.2305 = 13.0652$ A

$$\therefore \quad i_1 + i_2 = \mathbf{13.0652 \sin(\omega t - 22.497°) \text{ A}}$$

b) $$\bar{I}_1 - \bar{I}_2 = 5 + j0 - (3.5355 - j3.5355) = 1.4645 + j3.5355$$

$$= 3.8268 \angle 67.499° \text{ A}$$

$\therefore$ Maximum value of $(i_1 - i_2) = \sqrt{2} \times 3.8268 = 5.4119$ A

$$\therefore \quad i_1 - i_2 = \mathbf{5.4119 \sin(\omega t + 67.499°) \text{ A.}}$$

Example 4.30 : *$i_1 = 3\sqrt{2} \sin \omega t$, $i_2 = 4\sqrt{2} \sin(\omega t - 90°)$. Find out $(i_1 - i_2)$ and $(i_1 + i_2)$* **[GU : June-2006; July-2007]**

Solution : From the given equations, $I_{1m} = 3\sqrt{2}$ A, $I_{2m} = 4\sqrt{2}$ A

$$\therefore \quad I_1 = \frac{I_{1m}}{\sqrt{2}} = 3 \text{ A}, \quad I_2 = \frac{4\sqrt{2}}{\sqrt{2}} = 4 \text{ A} \qquad \text{... (R.M.S. values)}$$

$$\therefore \quad \bar{I}_1 = 3\angle 0° \text{ A} = 3 + j0 \text{ A}, \quad \bar{I}_2 = 4\angle -90° \text{ A} = 0 - j4 \text{ A}$$

i) $$\bar{I}_1 - \bar{I}_2 = 3 + j0 - (0 - j4) = 3 + j4 \text{ A} = 5\angle 53.13° \text{ A}$$

$\therefore$ Maximum value of $\bar{I}_1 - \bar{I}_2 = 5\sqrt{2}$ A

$$\therefore \quad i_1 - i_2 = \mathbf{5\sqrt{2} \sin(\omega t + 53.13°) \text{ A}}$$

ii) $$\bar{I}_1 + \bar{I}_2 = 3 + j0 + 0 - j4 = 3 - j4 \text{ A} = 5\angle -53.13° \text{ A}$$

$$\therefore \quad i_1 + i_2 = \mathbf{5\sqrt{2} \sin(\omega t - 53.13°) \text{ A.}}$$

Example 4.31 : *An alternating current has maximum value of 100 amp., the frequency being 50 Hz. Calculate the rate of change of current in ampere per second when i) t = 0.00166 sec ii) t = 0.01 sec and iii) t = 0.015 sec.* **[GU : Nov.-2006]**

Solution : $I_m = 100$ A , f = 50 Hz.

$$\therefore \quad \omega = 2\pi f = 100\pi \text{ rad/sec.}$$

$$\therefore \quad i = I_m \sin \omega t = 100 \sin (100\pi t) \text{ A}$$

$$\therefore \quad \frac{di}{dt} = 100\times 100\pi[+\cos(100\pi t)] = +10000\pi\cos(100\pi t) \text{ A / s}$$

i) At t = 0.00166 sec

$$\frac{di}{dt} = 10000\pi\cos(100\pi\times 0.00166) = \mathbf{+\ 27239.829\ A/s}$$

ii) At t = 0.01 sec

$$\frac{di}{dt} = 10000\pi\cos(100\pi\times 0.01) = \mathbf{-\ 31415.926\ A/s}$$

iii) At t = 0.015 sec

$$\frac{di}{dt} = 10000\pi\cos(100\pi\times 0.015) = \mathbf{0\ A/s}$$

Key Point : *Use **radian mode** to calculate cosine values.*

Example 4.32 : *If the waveform of a voltage has a form factor of 1.15 and peak factor of 1.5, and if the maximum value of a voltage is 4500 volt. Calculate the average and r.m.s. values of the voltage.* **[GU : July-2007]**

Solution : The factors are given as,

$$K_f = \frac{\text{R.M.S.}}{\text{Average}} = 1.15 \text{ and } K_p = \frac{\text{Maximum}}{\text{R.M.S.}} = 1.5$$

$$\therefore \quad 1.15 = \frac{V}{V_{av}} \quad \text{and} \quad 1.5 = \frac{V_m}{V} \qquad \text{... } V_m = 4500 \text{ V (given)}$$

$$\therefore \quad 1.15\times 1.5 = \frac{V}{V_{av}}\times\frac{V_m}{V} = \frac{V_m}{V_{av}}$$

$$\therefore \quad V_{av} = \frac{V_m}{1.15\times 1.5} = \frac{4500}{1.15\times 1.5} = \mathbf{2608.6956\ V} \qquad \text{... Average}$$

$$\therefore \quad V = 1.15\times V_{av} = 1.15\times 2608.6956 = \mathbf{3000\ V} \qquad \text{... R.M.S.}$$

Example 4.33 : *A certain waveform has a form factor of 1.2 and a peak factor of 1.5. If the maximum value is 100, find the r.m.s. and average values.* **[GTU : Dec.-2008]**

Solution : The factors given are,

$$K_f = \frac{\text{R.M.S.}}{\text{Average}} = 1.2 \quad \text{and} \quad K_p = \frac{\text{Maximum}}{\text{R.M.S.}} = 1.5$$

$$\therefore \quad 1.2 = \frac{V}{V_{av}} \quad \text{and} \quad 1.5 = \frac{V_m}{V} \quad \ldots V_m = 100$$

$$\therefore \quad 1.2 \times 1.5 = \frac{V}{V_{av}} \times \frac{V_m}{V} \quad \text{i.e.} \quad 1.8 = \frac{100}{V_{av}}$$

$$\therefore \quad V_{av} = \mathbf{55.555\ V} \quad \text{and} \quad V = \mathbf{66.666\ V}$$

Example 4.34 : *Prove that if a d.c. current of I amperes is superimposed in a conductor by an a.c. current of maximum value of I amperes, the r.m.s. value of the resultant is* $\left(\sqrt{3}/\sqrt{2}\right)I$. **[GTU : March-2009]**

Solution : The direct current is I and a.c. current with $I_m = I$.

$$\therefore \quad I_T = I + I_m \sin\theta = I + I\sin\theta$$

The r.m.s. value is given by,

$$I_R = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_T^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}[I + I\sin\theta]^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi} \times I^2 \int_0^{2\pi}[1 + \sin\theta]^2\, d\theta}$$

$$= \frac{I}{\sqrt{2\pi}} \times \sqrt{\int_0^{2\pi}\left(1 + 2\sin\theta + \sin^2\theta\right) d\theta}$$

$$= \frac{I}{\sqrt{2\pi}} \times \sqrt{[\theta - 2\cos\theta]_0^{2\pi} + \int_0^{2\pi}\left(\frac{1 - \cos 2\theta}{2}\right) d\theta}$$

$$= \frac{I}{\sqrt{2\pi}} \times \sqrt{2\pi - 2[\cos 2\pi - \cos 0] + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

University Questions

Q.1 *Define i) Form factor ii) Peak factor. Obtain the r.m.s. value and average value of half wave rectified sinusoidal voltage wave.* **[GTU : Dec.-2008, 8 Marks]**

Q.2 *Prove that if a d.c. current of 'I' amperes is superimposed in a conductor by an a.c. current of maximum value 'I' amperes, the root mean square (r.m.s.) value of the resultant is* $\left(\sqrt{3} / \sqrt{2}\right) I$. **[GTU : March-2009, 7 Marks]**

Q.3 *Define following terms in connection with a.c. waveforms : i) Frequency ii) Phase and phase difference iii) Time period iv) Form factor v) R.M.S. value vi) Average value* **[GTU : June-2009, 7 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

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Premier12

5

Single Phase A.C. Circuits

5.1 Introduction

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyze any electric circuit, it is necessary to understand the following three cases,

1) A.C. through pure resistive circuit.
2) A.C. through pure inductive circuit.
3) A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin(\omega t)$ is applied to the circuit. The equation for the current, power and phase shift are developed in each case. The voltage applied having zero phase angle is assumed reference while plotting the phasor diagram in each case.

Once the behaviour of pure R, L and C is discussed, then the various series and parallel combinations of R, L and C are discussed in this chapter. The concept of impedance, admittance, susceptance, phasor diagrams of series and parallel circuits and resonance in series and parallel circuits are also included in this chapter.

5.2 A.C. through Pure Resistance

Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$. The circuit is shown in the Fig. 5.1.

According to Ohm's law, we can find the equation for the current i as,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \quad \text{i.e.} \quad i = \left(\frac{V_m}{R}\right) \sin(\omega t)$$

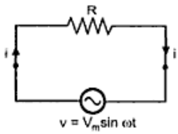


Fig. 5.1 Pure resistive circuit

This is the equation giving instantaneous value of the current.

Comparing this with standard equation,

$$i = I_m \sin(\omega t + \phi)$$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0}$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum (positive and negative) and zero whenever voltage is going to achieve its maximum (positive and negative) and zero values.

> **Key Point :** *In purely resistive circuit, the current and the voltage applied are in phase with each other.*

The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 5.2 (a) and (b).

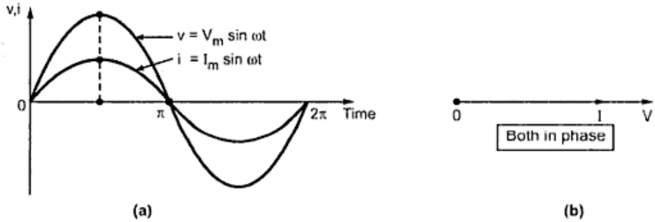


Fig. 5.2 A.C. through purely resistive circuit

In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the **r.m.s. values** of alternating quantities.

5.2.1 Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t = V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t)}$$

From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2}\cos(2\omega t)\right]$ having frequency, double the frequency of the applied voltage.

Now, the average value of the fluctuating cosine component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$.

$$\therefore \qquad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\therefore \qquad \boxed{P_{av} = V_{rms} \times I_{rms} \quad \text{watts}}$$

Generally, r.m.s. values are indicated by capital letters

$$\therefore \qquad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

The Fig. 5.3 shows the waveforms of voltage, current and power.

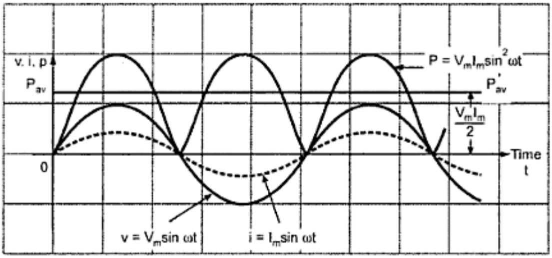


Fig. 5.3 v, i and p for purely resistive circuit

5.3 A.C. through Pure Inductance

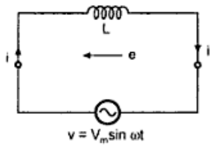


Fig. 5.4 Purely inductive circuit

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 5.4.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

The Fig. 5.5 shows the waveforms and the corresponding phasor diagram.

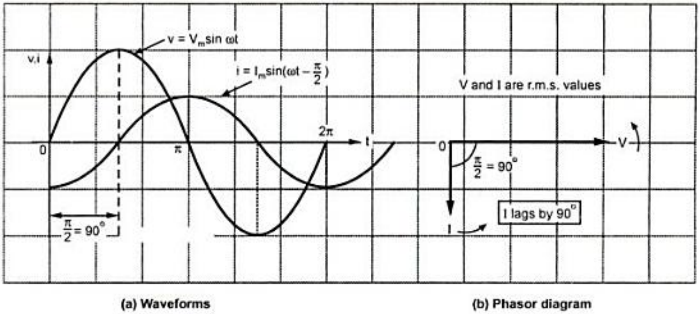


Fig. 5.5 A.C. through purely inductive circuit

Key Point: *In purely inductive circuit, current lags voltage by 90°.*

5.3.1 Concept of Inductive Reactance

We have seen that in purely inductive circuit,

$$I_m = \frac{V_m}{X_L}$$

where

$$\boxed{X_L = \omega L = 2\pi f L\ \Omega}$$

The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**

So, **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

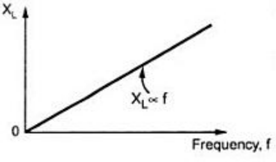


Fig. 5.6 X_L Vs f

It is measured in ohms and it depends on the frequency of the applied voltage.

The inductive reactance is directly proportional to the frequency for constant L.

$X_L \propto f$, for constant L

So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 5.6.

Key Point : *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

5.3.2 Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t) \cos(\omega t) \quad \text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \quad \boxed{P = -\frac{V_m I_m}{2} \sin(2\omega t)} \quad \text{as } 2 \sin \omega t \cos \omega t = \sin 2\omega t$$

Key Point : *This power curve is a sine curve of frequency double than that of applied voltage.*

The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$

The Fig. 5.7 shows voltage, current and power waveforms.

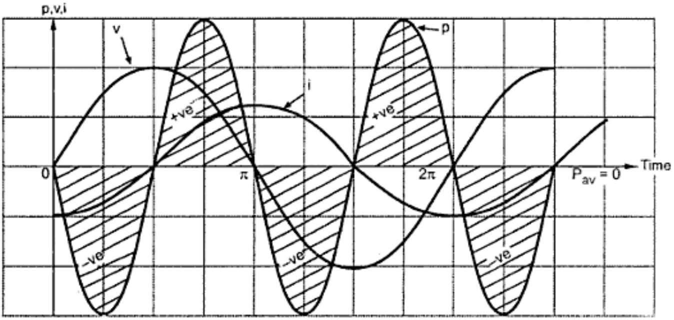


Fig. 5.7 Waveforms of voltage, current and power

It can be observed from it that when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

Key Point : *Pure inductance never consumes power.*

5.4 A.C. through Pure Capacitance

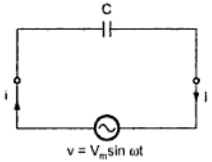


Fig. 5.8 Purely capacitive circuit

Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 5.8.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C\,v$$

$$\therefore \quad q = C\,V_m \sin \omega t$$

Now, current is rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t)$$

$$\therefore \quad i = C\,V_m \frac{d}{dt}(\sin \omega t) = C\,V_m\,\omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $$\boxed{I_m = \frac{V_m}{X_C}}$$

where $$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.

This means **current leads voltage applied by 90°**. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor by 90°.

The Fig. 5.9 shows waveforms of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

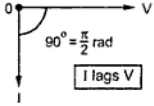


Fig. 5.12 (b)

The phasor diagram is shown in the Fig. 5.12 (b).

Case 3 : $C = 50\ \mu F$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$

$\therefore \quad I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33\ A$

In pure capacitive circuit, current leads voltage by 90°.

$\therefore \quad \phi = \text{Phase Difference} = 90° = \frac{\pi}{2}\ \text{rad}$

$\therefore \quad i = I_m \sin(\omega t + \phi)$

$\therefore \quad i = \mathbf{3.33 \sin\left(100\pi t + \frac{\pi}{2}\right) A}$

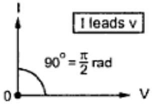


Fig. 5.12 (c)

The phasor diagram is shown in the Fig. 5.12 (c).

All the phasor diagrams represent r.m.s. values of voltage and current.

Example 5.2 : *A voltage $v = 141 \sin\{314t + \pi/3\}$ is applied to*

i) Resistor of 20 ohms ii) Inductance of 0.1 henry iii) Capacitance of 100 μF.

Find in each case rms value of current and power dissipated.

Draw the phasor diagram in each case.

Solution : Comparing given voltage with $v = V_m \sin(\omega t + \theta)$ we get,

$$V_m = 141\ V \text{ and hence } V = V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 99.702\ V$$

$$\omega = 314 \text{ and hence } f = \frac{\omega}{2\pi} = 50\ Hz,\ \theta = \frac{\pi}{3} = 60°$$

Hence the polar form of applied voltage becomes,

$$V = 99.702 \angle 60°\ V$$

Case 1 : $R = 20\ \Omega$

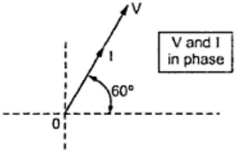


Fig. 5.13 (a)

$$I = \frac{V}{R} = \frac{99.702\angle 60^\circ}{20\angle 0^\circ} = 4.9851\ \angle 60^\circ\ A$$

$\therefore$ $I_{r.m.s.}$ = **4.9851 A**

The phase of both V and I is same for pure resistive circuit. Both are in phase.

P = VI = 99.702×4.9851 = **497.0244 W**

The phasor diagram is shown in the Fig. 5.13(a).

Case 2 : $L = 0.1\ H$

$\therefore$ $X_L = \omega L = 314\times 0.1 = 31.4\ \Omega$

$\therefore$ $I = \frac{|V|}{X_L} = \frac{99.702}{31.4} = \mathbf{3.1752\ A}$

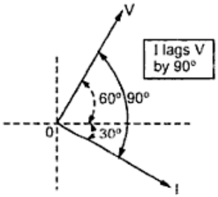


Fig. 5.13 (b)

This is r.m.s. value of current. It has to lag the applied voltage by 90° in case of pure inductor.

Hence phasor diagram is shown in the Fig. 5.13 (b).

The individual phase of I is – 30°.

In polar form I can be represented as $3.1752\angle -30^\circ$ A.

Pure inductor never consumes power so **power dissipated is zero.**

Case 3 : $C = 100\ \mu F$

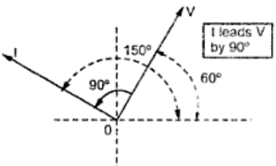


Fig. 5.13 (c)

$$\therefore\ X_C = \frac{1}{\omega C} = \frac{1}{314\times 100\times 10^{-6}} = 31.8471\ \Omega$$

$$\therefore\ I = \frac{|V|}{X_C} = \frac{99.702}{31.8471} = \mathbf{3.1306\ A}$$

This is r.m.s. value of current.

It has to lead the applied voltage by 90° in case of pure capacitor.

Hence phasor diagram is shown in the Fig. 5.13 (c).

The individual phase of I is 150°. In polar form I can be represented as $3.1306\angle +150^\circ$. A Pure capacitor never consumes power and hence **power dissipated is zero.**

5.5 A.C. through Series R-L Circuit

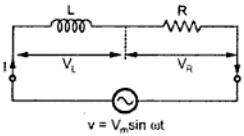


Fig. 5.14 (a) Series R-L circuit

Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L henries as shown in the Fig. 5.14 (a).

The series combination is connected across a.c. supply given by $v = V_m \sin \omega t$.

Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $V_R = I \times R$

b) Drop across pure inductance, $V_L = I \times X_L$ where $X_L = 2 \pi f L$

I = r.m.s. value of current drawn

V_R , V_L = r.m.s. values of the voltage drops.

The Kirchhoff's voltage law can be applied to the a.c circuit but only the point to remember is the addition of voltages should be a phasor (vector) addition and no longer algebraic as in case of d.c.

$$\therefore \quad \bar{V} = \bar{V}_R + \overline{V_L} \quad \text{(Phasor addition)}$$

$$\therefore \quad \bar{V} = \overline{IR} + \overline{I X_L}$$

Let us draw the phasor diagram for the above case.

Key Point : *For series a.c. circuits, generally, current is taken as the reference phasor as it is common to both the elements.*

Following are the **steps to draw the phasor diagram :**

1) Take current as a reference phasor.
2) In case of resistance, voltage and current are in phase, so V_R will be along current phasor.
3) In case of inductance, current lags voltage by 90°. But, as current is reference, V_L must be shown leading with respect to current by 90°.
4) The supply voltage being vector sum of these two vectors V_L and V_R obtained by law of parallelogram.

From the voltage triangle, we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (I \times X_L)^2} = I\sqrt{(R)^2 + (X_L)^2}$$

$$\therefore \quad V = I Z$$

Where

$$Z = \sqrt{(R)^2 + (X_L)^2} \quad \text{... Impedance of the circuit.}$$

The impedance Z is measured in ohms.

$$i = I_m \sin(\omega t - \phi) \text{ as current lags voltage.}$$

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi) = V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$\therefore \quad \boxed{P = V I \cos \phi \text{ watts}}$$ where V and I are r.m.s. values

If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

$$\therefore \quad \overline{VI} = \overline{V \cos \phi I} + \overline{V \sin \phi I}$$

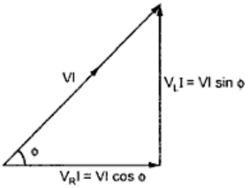


Fig. 5.16 Power triangle

From this equation, power triangle can be obtained as shown in the Fig. 5.16.

So, three sides of this triangle are,

1) VI, 2) VI cos ϕ, 3) VI sin ϕ

These three terms can be defined as below.

5.5.3 Apparent Power (S)

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\therefore \quad \boxed{S = V I \quad \text{VA}}$$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

5.5.4 Real or True Power (P)

It is defined as the product of the applied voltage and the active component of the current.

It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

$$\boxed{P = V I \cos \phi \quad \text{watts}}$$

5.5.5 Reactive Power (Q)

It is defined as product of the applied voltage and the reactive component of the current.

It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$Q = V I \sin \phi \quad \text{VAR}$$

Apparent power, $S = V I$ VA

True power $P = V I \cos \phi$ W (Average power)

Reactive power $Q = V I \sin \phi$ VAR

5.5.6 Power Factor (cos ϕ)

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{V I \cos \phi}{V I} = \cos \phi$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.

It is also defined as the ratio of resistance to the impedance.

$$\cos \phi = \frac{R}{Z}$$

Key Point: *The nature of power factor is always determined by position of current with respect to the voltage.*

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. $\phi = 0$. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\text{Power factor} = \cos \phi$$

ϕ is the angle between supply voltage and current.

Key Point: *Nature of power factor always tells position of current with respect to voltage.*

Example 5.3 : *An alternating current, $i = 414 \sin (2 \pi \times 50 \times t)$ - A, is passed through a series circuit consisting of a resistance of 100-ohm and an inductance of 0.31831 henry. Find the expressions for the instantaneous values of the voltages across (i) The resistance, (ii) The inductance and (iii) The combination.*

Solution : The circuit is shown in the Fig. 5.17.

$i = 1.414 \sin (2 \pi \times 50\ t)$ A

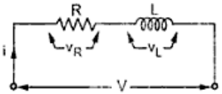


Fig. 5.17

$\therefore \quad \omega = 2 \pi \times 50 = 2 \pi f$

$\therefore \quad f = 50$ Hz, $\quad R = 100\ \Omega, \quad L = 0.31831$ H

$\therefore \quad X_L = 2 \pi f L = 2 \pi \times 50 \times 0.31831 = 100\ \Omega$

i) The voltage across the resistance is,

$$v_R = i R = 1.414 \sin (2 \pi \times 50\ t) \times 100 = \mathbf{141.4 \sin (2 \pi \times 50\ t)\ V}$$

ii) The voltage across L leads current by 90° as current lags by 90° with respect to voltage.

$$\therefore \quad v_L = i X_L \text{ but leading current by } 90^\circ = \mathbf{141.4 \sin (2 \pi \times 50\ t + 90^\circ) V}$$

iii) From the expression of V_R we can write,

$$\text{r.m.s. value of } V_R = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 0^\circ$$

$$\therefore \quad V_R = 100 \angle 0^\circ = 100 + j\,0 \text{ V}$$

$$\text{r.m.s. value of } V_L = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 90^\circ$$

$$\therefore \quad V_L = 100 \angle 90^\circ = 0 + j\,100 \text{ V}$$

$$\therefore \quad V = \bar{V}_R + \bar{V}_L = 100 + j0 + 0 + j100$$

$$= 100 + j\,100 = 141.42 \angle 45^\circ \text{ V}$$

$$\therefore \quad V_m = \sqrt{2} \times 141.42 = 200 \text{ V}$$

Hence expression of instantaneous value of resultant voltage is,

$$v = \mathbf{200 \sin (2 \pi \times 50\ t + 45^\circ)\ V}$$

Where $X_C = \frac{1}{2\pi f\ C}$ and I, V_R, V_C are the r.m.s. values

The Kirchhoff's voltage law can be applied to get,

$$V = \overline{V_R} + \overline{V_C} \quad \text{... (Phasor addition)}$$

$$\therefore \quad \overline{V} = \overline{IR} + \overline{I X_C}$$

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the **steps to draw the phasor diagram :**

1) Take current as reference phasor.
2) In case of resistance, voltage and current are in phase. So, V_R will be along current phasor.
3) In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4) The supply voltage being vector sum of these two voltages V_C and V_R obtained by completing parallelogram.

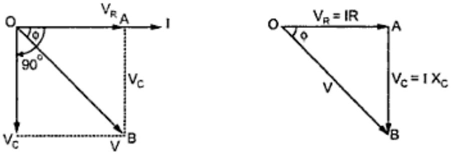


Fig. 5.20 Phasor diagram and voltage triangle

From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (I X_C)^2} = I\sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = I Z$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2}}$ is the impedance of the circuit.

5.6.1 Impedance

Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current. It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_C)^2}$ where $X_C = \frac{1}{2\pi f C}\ \Omega$ called **capacitive reactance.**

In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 5.20.

From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**

Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.

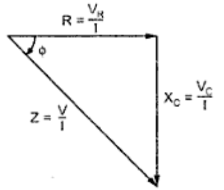


Fig. 5.21 Impedance triangle

The X component of impedance is R and is given by

$$R = Z\cos\phi$$

and Y component of impedance is X_C and is given by

$$X_C = Z\sin\phi$$

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - j X_C \ \Omega}$$

While in polar form, it is denoted as,

$$\boxed{Z = |Z| \angle -\phi \ \Omega}$$

$$Z = R - j X_C = |Z| \angle -\phi$$

where $\quad |Z| = \sqrt{R^2 + X_C^2}, \ \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$

Key Point: *Thus ϕ is negative for capacitive impedance.*

5.6.2 Power and Power Triangle

The current leads voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = V I \cos\phi \text{ watts}} \quad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\therefore \quad \overline{VI} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

Hence, the power triangle can be shown as in the Fig. 5.22.

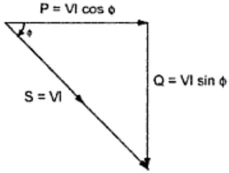


Fig. 5.22

Thus, the various powers are,

Apparent power,	$S = V I$	VA
True or average power,	$P = V I \cos\phi$	W
Reactive power,	$Q = V I \sin\phi$	VAR

Remember that, X_L term appears positive in Z.

$$Z = R + j X_L = |Z| \angle \phi \qquad \phi \text{ is positive for inductive Z}$$

While X_C term appears negative in Z.

$$Z = R - j X_C = |Z| \angle -\phi \qquad \phi \text{ is negative for capacitive Z}$$

For any single phase a.c. circuit, the average power is given by,

$$\mathbf{P = V I \cos\phi \quad watts}$$

Where **V, I are r.m.s. values**

$\cos\phi$ = Power factor of circuit

$\cos\phi$ is **lagging for inductive** circuit and $\cos\phi$ is **leading for capacitive** circuit.

Example 5.5 : *Calculate the resistance and inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60 Hz.*

i) (12 + j 30) ohms ii) – j 60 ohms iii) 20 ∠ 60° ohms.

Comparing with, $i = I_m \sin(\omega t + \phi)$

$$I_m = 10 \quad \text{and} \quad \phi = \frac{\pi}{6} = 30^\circ$$

$$\therefore \quad I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

$$|Z| = \frac{V}{I} = \frac{84.85}{7.07} = 12\ \Omega$$

As current leads voltage by 30°, the circuit is R-C series circuit, capacitive in nature. As impedance is capacitive, ϕ must be negative.

$$\therefore \quad Z = 12\angle -30^\circ\Omega = 10.393 - j\,6\ \Omega \ \ldots \text{ use P} \rightarrow \text{R}$$

Comparing with $Z = R - j\,X_C$

$$R = \mathbf{10.393\ \Omega} \quad \text{and } X_C = 6\ \Omega$$

Now $X_C = \frac{1}{2\pi fC}$

$$\therefore \quad 6 = \frac{1}{2\pi \times 50 \times C}$$

$$\therefore \quad C = \mathbf{530.45\ \mu F}$$

$$P = VI\cos\phi = 84.85 \times 7.07 \times \cos(30^\circ) = \mathbf{519.52\ W}$$

The waveforms are shown in the Fig. 5.23 (a), while phasor diagram is shown in the Fig. 5.23 (b).

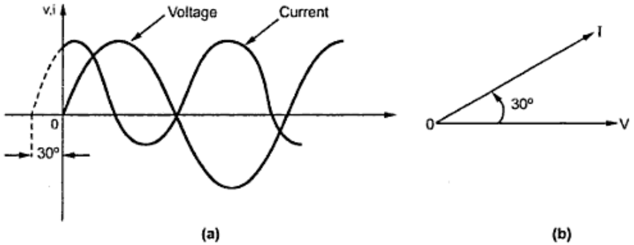


Fig. 5.23

Example 5.7 : *A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a A.C. voltage source. If a current of 0.9 A is flowing in the circuit find out (i) Power factor, (ii) Supply voltage (iii) Voltages across resistance and capacitance (iv) Active power and reactive power.*

$\therefore \quad V = I Z$

Where $\quad Z = R$

5.7.4 Impedance

In general, for RLC series circuit impedance is given by,

$$Z = R + j X$$

Where $X = X_L - X_C$ = total reactance of circuit

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan \phi = \left[\frac{X_L - X_C}{R}\right], \; \cos \phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

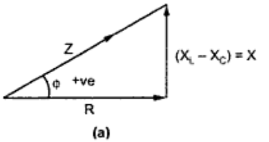
5.7.5 Impedance Triangle

The impedance is expressed as,

$$Z = R + j X \quad \text{where} \quad X = X_L - X_C$$

For $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in the Fig. 5.29 (a).

For $X_L < X_C$, $X_L - X_C$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig. 5.29 (b).



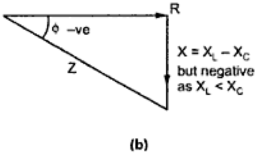


Fig. 5.29 Impedance triangles

In both the cases, $\quad R = Z \cos \phi \quad$ and $\quad X = Z \sin \phi$

5.7.6 Power and Power Triangle

The average power consumed by the circuit is,

P_{av} = Average power consumed by R+ Average power consumed by L + Average power consumed by C

But, pure L and C never consume any power.

$\therefore \quad P_{av}$ = Power taken by R $= I^2 R = I (I R) = I V_R$

But, $\quad V_R = V \cos \phi$ in both the cases

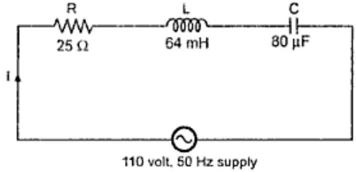


Fig. 5.31

Solution : From Fig. 5.31,

$$R = 25\ \Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 64 \times 10^{-3} = 20.10\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.78\ \Omega$$

$$Z = R + j X_L - j X_C = 25 + j\,20.10 - j\,39.78$$

$$Z = (25 - j19.68)\ \Omega$$

$$I = \frac{V}{Z} = \frac{110\ \angle 0^\circ}{25 - j\,19.68} = \frac{110\ \angle 0^\circ}{31.81\ \angle -38.20^\circ}$$

$$I = \mathbf{3.4580\ \angle 38.20^\circ\ A}$$

$\therefore$
$$|I| = 3.4580\ \text{A}$$

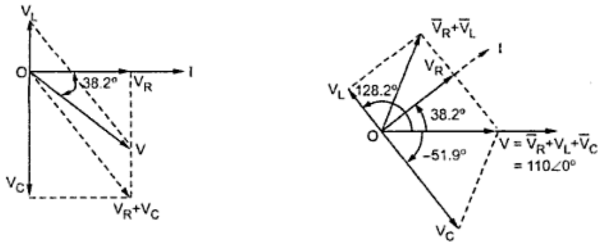
$$V_R = IR = (3.4580\angle 38.20^\circ)(25) = 86.45\angle 38.20^\circ\ \text{volts}$$

$$V_L = I(jX_L) = (3.4580\angle 38.20^\circ)\ (j\,20.10)$$
$$= (3.4580\ \angle 38.20^\circ)\ (20.10\ \angle 90^\circ) = \mathbf{69.50\ \angle 128.2^\circ}\ \text{volts}$$

$$V_C = I\,(-jX_C) = (3.4580\ \angle 38.20^\circ)\ (-j\,39.78)$$
$$= (3.4580\ \angle 38.20^\circ)\ (38.78\ \angle -90^\circ) = \mathbf{134.10\ \angle -51.9^\circ\ volts}$$

$$V = 110\ \angle 0^\circ\ \text{volts}$$

Overall p.f., $\cos\phi = \cos 38.20^\circ = \mathbf{0.7858}$ **leading.**



(a) With I reference (b) With V reference

Fig. 5.32

Example 5.9 : *A series circuit having pure resistance of 40 ohms, pure inductance of 50.07 mH and a capacitor is connected across a 400 V, 50 Hz, A.C. supply. This R, L, C combination draws a current of 10 A. Calculate (i) Power factor of the circuit and (ii) Capacitor value.*

Solution : The arrangement is shown in the Fig. 5.33.

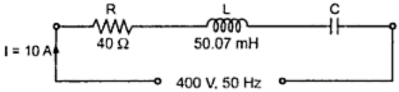


Fig. 5.33

$$X_L = 2\pi f L = 2\pi \times 50 \times 50.07 \times 10^{-3} = 15.73\ \Omega, \quad X_C = \frac{1}{2\pi f C}$$

$$Z = R + j(X_L - X_C)$$

$$\therefore \quad |Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (15.73 - X_C)^2} \quad \text{... (1)}$$

$$|Z| = \frac{|V|}{|I|} = \frac{400}{10} = 40\ \Omega \quad \text{... (2)}$$

$$\therefore \quad 40 = \sqrt{R^2 + (15.73 - X_C)^2} \quad \text{i.e.} \quad 1600 = R^2 + (15.73 - X_C)^2 \quad \text{... (3)}$$

Substitute R = 40 Ω in equation (3), $1600 = 1600 + (15.73 - X_C)^2$

$$\therefore \quad (15.73 - X_C)^2 = 0$$

$$\therefore \quad X_C = 15.73\ \Omega \quad \text{i.e.} \quad 15.73 = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \mathbf{2.023 \times 10^{-4}\ F}$$

$$\therefore \quad Z = 40 + j(15.73 - 15.73) = 40 + j\,0\ \Omega$$

$$= 40 \angle 0^\circ\ \Omega$$

$$\therefore \quad \text{p.f.} = \cos(0^\circ) = \mathbf{1}$$

5.8 Complex Power

As seen earlier in a.c. circuits there are three types of powers exist. These are apparent power (S), active power (P) and reactive power (Q). The P and Q are the components of apparent power (S) such that,

$$\boxed{|S| = \sqrt{P^2 + Q^2}}$$

While $\phi = \tan^{-1}\left[\frac{\sin\phi}{\cos\phi}\right] = \tan^{-1}\left[\frac{VI\sin\phi}{VI\cos\phi}\right] = \tan^{-1}\left[\frac{Q}{P}\right]$

Where $P = VI\cos\phi$ and $Q = VI\sin\phi$

Thus the apparent power can be expressed in the rectangular form as,

$$\boxed{S = P \pm jQ}$$

This is called **complex power** where,

Real part = Active, true or real power in watts (W)

Imaginary part = Reactive power in reactive volt-amp (VAR)

Key Point : *The reactive power Q may be positive or negative, depending upon nature of the circuit.*

The postive sign indicates lagging nature of reactive power while negative sign indicates leading nature of reactive power.

The complex power is generally indicated in a **power triangle** as shown in the Fig. 5.34.

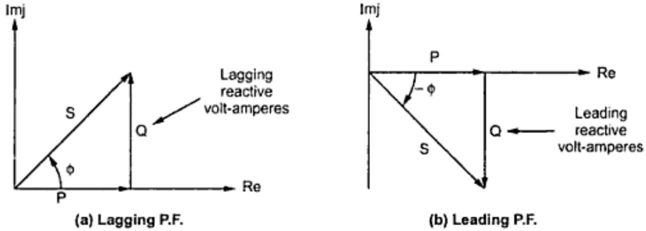


Fig. 5.34 Power triangle

The angle ϕ is p.f. angle i.e. angle between V and I.

In general if	$V = V_1 \angle \theta_1$ and $I = I_1 \angle \theta_2$	
Then	$\phi = \theta_1 - \theta_2$	
And	$P = VI \cos(\theta_1 - \theta_2)$ W	... Active power
	$Q = VI \sin(\theta_1 - \theta_2)$ VAR	... Reactive power
	$S = P + jQ$ VA	... Complex power

If $\theta_1 - \theta_2 > 0$, Q is positive indicating lagging p.f. while

If $\theta_1 - \theta_2 < 0$, Q is negative indicating leading p.f.

Sign of reactive power	Nature of power factor	Nature of load
Q is positive	Lagging	Inductive
Q is negative	Leading	Capacitive

Table 5.2

Physical significance of reactive power :

The reative power is that component of power which is supplied to the reactive components of the load from the source during positive half cycle while it is returned back to supply from the components to the source during negative half cycle. It is rate of change of energy with time which keeps on flowing from the source to reative components and back from the components to the source, alternately. The reactive power charges and discharges the reactive components alternately.

> **Key Point** : *Thus reactive power never gets consumed by the circuit but flows alternately back and forth from the source to the reactive components and vice-versa.*

Example 5.10 : *Find the complex power delivered by the source.*

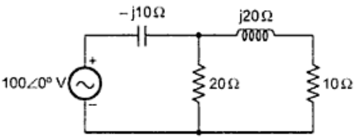


Fig. 5.35

Solution : The circuit can be analysed as,

Applying KVL to the two loops,

$$-(I)(-j10) - 20I_1 + 100\angle 0^\circ = 0$$

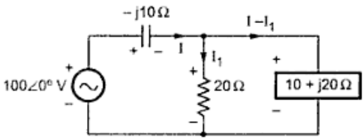


Fig. 5.35 (a)

i.e. $(-j10)\,I + 20\,I_1 = 100\angle 0^\circ$ ... (1)

$$-(10 + j20)(I - I_1) + 20\,I_1 = 0$$

$\therefore (10 + j20)\,I + (-30 - j20)\,I_1 = 0$ ... (2)

$$D = \begin{vmatrix} -j10 & 20 \\ 10+j20 & -30-j20 \end{vmatrix} = +j300 - 200 - 200 - j400 = -400 - j100$$

$$= 412.3105 \angle -165.96^\circ$$

$$D_1 = \begin{vmatrix} 100\angle 0^\circ & 20 \\ 0 & -30-j20 \end{vmatrix} = (100 \angle 0^\circ)(-30-j20)$$

$$= (100 \angle 0^\circ)(36.055 \angle -146.3^\circ) = 3605.55 \angle -146.3^\circ$$

$$\therefore \quad I = \frac{D_1}{D} = \frac{3605.55\angle -146.3^\circ}{412.3105\angle -165.96^\circ} = 8.7447 \angle 19.66^\circ \text{ A}$$

$$\therefore \quad S = VI = \text{Complex power} = 100 \angle 0^\circ \times 8.7447 \angle 19.66^\circ \text{ VA}$$

$$= 874.47 \angle 19.66^\circ \text{ VA} = \mathbf{(823.4933 + j294.204)\ VA}$$

5.9 Resonance in Series R-L-C Circuit

We know that both X_L and X_C are the functions of frequency f. When f is varied both X_L and X_C also get varied. At a certain frequency, X_L becomes equal to X_C. Such a condition when $X_L = X_C$ for a certain frequency is called **series resonance**. At resonance the reactive part in the impedance of RLC series circuit is zero. The frequency at which the resonance occurs is called **resonant frequency** denoted as ω_r rad/sec or f_r Hz.

5.9.1 Characteristics of Series Resonance

In a series resonance, the voltage applied is constant and frequency is variable. Hence following parameters of series RLC circuit get affected due to change in frequency :

1) X_L 2) X_C 3) Total reactance X 4) Impedance Z 5) I 6) cos ϕ

As $X_L = 2\pi fL$. As frequency is changed from 0 to ∞, X_L increases linearily and graph of X_L against f is straight line passing through origin.

As $X_C = \frac{1}{2\pi f C}$, as frequency is changed from 0 to ∞, X_C reduces and the graph of X_C against f is rectangular hyperbola. Mathematically sign of X_C is opposite to X_L hence graph of X_L Vs f is shown in the first quadrant while X_C Vs f is shown in the third quadrant.

At $f = f_r$, the value of $X_L = X_C$ at this frequency.

As $X = X_L - X_C$, the graph of X against f is shown in the Fig. 5.36.

For **$f < f_r$**, the $X_C > X_L$ and net reactance X is **capacitive** while for $f > f_r$, the $X_L > X_C$ and net reactance X is **inductive**.

Now $Z = R + jX = R + j(X_L - X_C)$ but at $f = f_r$, $X_L = X_C$ and $X = 0$ hence the net impedance Z = R which is purely resistive. So **impedance** is **minimum** and **purely resistive** at series resonance. The graph of Z against f is also shown in the Fig. 5.36

Key Point: *As impedance is minimum, the current I = V/Z is maximum at series resonance.*

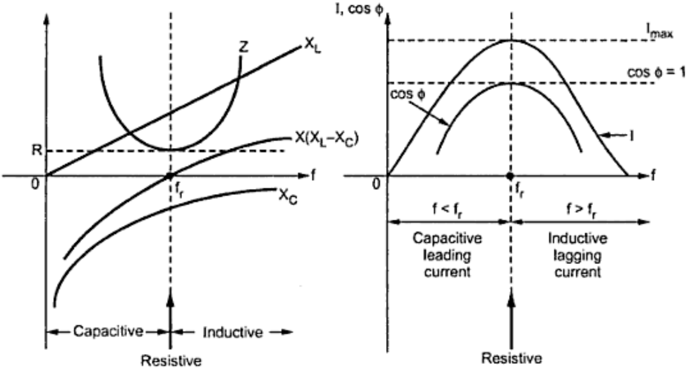


Fig. 5.36 Characteristics of series resonance

Now power factor $\cos \phi = R/Z$ and at $f = f_r$ as $Z = R$, the **power factor** is **unity** and at its **maximum** at series resonance. For $f < f_r$ it is **leading** in nature while for $f > f_r$ it is **lagging** in nature.

5.9.2 Expression for Resonant Frequency

Let f_r be the resonant frequency in Hz at which,

$$X_L = X_C \quad \text{... Series resonance}$$

$$\therefore \quad 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore \quad (f_r)^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

i.e.

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$$

5.9.3 Bandwidth of Series R-L-C Circuit

At series resonance, current is maximum and impedance Z is minimum. Now power consumed in a circuit is proportional to square of the current as $P = I^2R$. So at series resonance as current is maximum, power is also at its maximum i.e. P_m. The Fig. 5.37 shows the graph of current and power against frequency.

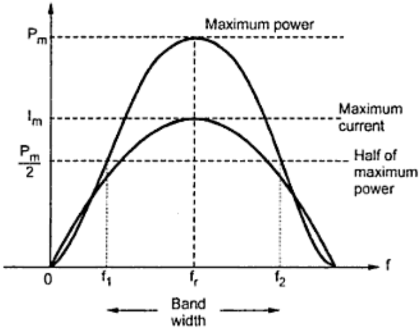


Fig. 5.37 Bandwidth

It can be observed that at two frequencies f_1 and f_2 the power is half of its maximum value. These frequencies are called **half power frequencies**.

Definition of Bandwidth :

The difference between the half power frequencies f_1 and f_2 at which power is half of its maximum is called bandwidth of the series R-L-C circuit.

$$\therefore \quad \boxed{B.W. = f_2 - f_1}$$

In the bandwidth, the power is more than half the maximum value. The bandwidth decides the selectivity. The **selectivity** is defined as the ratio of the resonant frequency to the bandwidth.

$$\boxed{\text{Selectivity} = \frac{f_r}{B.W.} = \frac{f_2 - f_1}{B.W.}}$$

Key Point: *Thus if the bandwidth is more, the selectivity of the circuit is less.*

Out of the two half power frequencies, the frequency f_2 is called **upper cut-off frequency** while the frequency f_1 is called **lower cut-off frequency**.

5.9.4 Expressions for Lower and Upper Cut-off Frequencies

The current in a series RLC circuit is given by the equation,

$$I = \frac{V}{Z} \quad \text{but } Z = R + j\,(X_L - X_C)$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \dots (1)$$

At resonance,

$$I_m = \frac{V}{R} \quad \text{(maximum value)} \quad \dots (2)$$

And

$$P_m = I_m^2 R$$

At half power point,

$$P = \frac{P_m}{2} = \frac{I_m^2}{2} R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$\therefore \quad I = \frac{I_m}{\sqrt{2}}$ at half power frequency

Equating equations (1) and (2),

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2} \cdot R}$$

$$\therefore \quad \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\, R$$

$$\therefore \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2\, R^2$$

$$\therefore \quad \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\therefore \quad \omega L - \frac{1}{\omega C} = \pm R \qquad \ldots(3)$$

From the equation (3) we can find two values of half power frequencies which are ω_1 and ω_2 corresponding to f_1 and f_2.

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = +R \qquad \ldots(4)$$

And $$\omega_1 L - \frac{1}{\omega_1 C} = -R \qquad \ldots(5)$$

$$\therefore \quad (\omega_1 + \omega_2)\, L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0 \qquad \ldots \text{adding equations (4) and (5)}$$

$$\therefore \quad (\omega_1 + \omega_2)\, L = \frac{(\omega_1 + \omega_2)}{\omega_1\, \omega_2} \cdot \frac{1}{C}$$

$$\therefore \quad \omega_1\, \omega_2 = \frac{1}{LC} \qquad \ldots(6)$$

But $$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{\begin{aligned} \omega_1\, \omega_2 &= (\omega_r)^2 \\ f_1 f_2 &= (f_r)^2 \end{aligned}} \qquad \ldots(7)$$

The equation (7) shows that **the resonant frequency is the geometric mean of the two half power frequencies.**

$$\therefore \quad \boxed{f_r = \sqrt{f_1\, f_2}} \qquad \ldots(8)$$

5.9.5 Quality Factor

The quality factor of R-L-C series circuit is the voltage magnification in the circuit at resonance.

$$\text{Voltage magnification} = \frac{\text{Voltage across L or C}}{\text{Supply voltage}}$$

Now V_L = Voltage across L = $I_m X_L = I_m \omega_r L$ at resonance

And at resonance, $I_m = \frac{V}{R}$ and $V_L = \frac{V\omega_r L}{R}$

$$\therefore \quad \text{Voltage magnification} = \frac{\frac{V\omega_r L}{R}}{V} = \frac{\omega_r L}{R}$$

This is nothing but quality factor Q.

$$\therefore \quad \boxed{Q = \frac{\omega_r L}{R}} \quad \text{but} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{while} \quad \text{B.W.} = \frac{R}{2\pi L}$$

$$\therefore \quad Q \times \text{B.W.} = \frac{1}{R}\sqrt{\frac{L}{C}} \times \frac{R}{2\pi L} = \frac{1}{2\pi\sqrt{LC}} = f_r$$

$$\therefore \quad \boxed{Q = \frac{f_r}{\text{B.W.}}}$$

The **significance of quality factor** can be stated as,

1. It indicates the selectivity or sharpness of the tuning of a series circuit.

2. It gives the correct indication of the selectivity of such series R-L-C circuit which are used in many radio circuits.

> **Key Point :** *At the resonant frequency, the impedance is minimum and hence the circuit is known as acceptor circuit at resonance.*

➡ **Example 5.11 :** *A RLC series circuit with a resistance of 10 Ω, impedance of 0.2 H and a capacitance of 40μF is supplied with a 100 V supply at variable frequency. Find the following w.r.t the series resonant circuit :*

i) The frequency at resonance ii) The current iii) Power iv) Power factor v) Voltage across R, L, C at that time vi) Quality factor of the circuit vii) Half power points viii) Phasor diagram.

Solution : The given values are, R = 10 Ω, L = 0.2 H, C = 40 μF and V = 100 V

i) $$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = \mathbf{56.2697\ Hz}$$

ii) $I_m = \frac{V}{R} = \frac{100}{10} = \mathbf{10\ A}$ ... Current is maximum at resonance

iii) $P_m = I_m^2\ R = (10)^2 \times 10 = \mathbf{1000\ W}$

iv) Power factor is **unity**, as impedance is **purely resistive** at resonance

v) $V_R = I_m R = 10 \times 10 = \mathbf{100\ V}$

$X_L = 2\pi f_r L = 2\pi \times 56.2697 \times 0.2 = 70.7105\ \Omega$

$\therefore$ $V_L = I_m X_L = 10 \times 70.7105 = \mathbf{707.105\ V}$

And $X_C = \frac{1}{2\pi f_r C} = \frac{1}{2\pi \times 56.2697 \times 40 \times 10^{-6}} = 70.7105\ \Omega$

$\therefore$ $\boxed{V_C = I_m X_C = \mathbf{707.105\ V}}$

Thus $V_L = V_C$ at resonance

vi) $Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \mathbf{7.071}$

vii) $\Delta f = \frac{R}{4\pi L} = \frac{10}{4\pi \times 0.2} = 3.9788$

$\therefore$ $f_1 = f_r - \Delta f = 56.2697 - 3.9788 = \mathbf{52.2909\ Hz}$

and $f_2 = f_r + \Delta f = 56.2697 + 3.9788 = \mathbf{60.2485\ Hz}$

viii) B.W. $= f_2 - f_1 = 60.2485 - 52.2909 = \mathbf{7.9576\ Hz}$

The phasor diagram is shown in the Fig. 5.39.

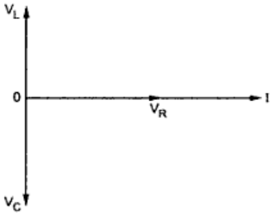


Fig. 5.39

Example 5.12 : *A series R-L-C circuit is connected to 230 V a.c. supply. The current drawn the circuit at the resonance is 25 A. The voltage drop across the capacitor is 4000 V, at the series resonance.*

Calculate the resistance, inductance if capacitance is 5 μF, also calculate the resonant frequency.

Solution : At resonance,

$R = \frac{V}{I} = \frac{230}{25} = \mathbf{9.2\ \Omega}$

Voltage drop across capacitor

$V_C = I\ X_C$ i.e. $4000 = 25 \times X_C$

$X_C = \mathbf{160\ \Omega}$

At resonant frequency, $f = f_r$ And $X_C = \frac{1}{2\pi f_r C}$

$$\therefore \quad 160 = \frac{1}{2\pi f_r \times 10^{-6}}$$

$$\therefore \quad f_r = \mathbf{198.943\ Hz}$$

At resonance $\quad X_L = X_C = 160\ \Omega$

$$2\pi f_r L = 160$$

$$\therefore \quad L = \frac{160}{2\pi f_r} = \frac{160}{2\pi \times 198.943}$$

$$\therefore \quad L = \mathbf{0.128}\ \text{H}$$

5.10 A.C. Parallel Circuit

A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. 5.40 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

> **Key Point** : *The voltage across all the impedances is same as supply voltage of V volts.*

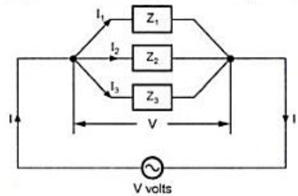


Fig. 5.40 A.C. parallel circuit

The current taken by each impedance is different.

Applying Kirchhoff's law, $\quad \bar{I} = \overline{I_1} + \overline{I_2} + \overline{I_3}$ ... (Phasor addition)

$$\therefore \quad \frac{\overline{V}}{\overline{Z}} = \frac{\overline{V}}{\overline{Z_1}} + \frac{\overline{V}}{\overline{Z_2}} + \frac{\overline{V}}{\overline{Z_3}}$$

$$\therefore \quad \frac{1}{\overline{Z}} = \frac{1}{\overline{Z_1}} + \frac{1}{\overline{Z_2}} + \frac{1}{\overline{Z_3}}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

5.10.1 Two Impedances in Parallel

If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\bar{I}_1 = \bar{I}_T \times \frac{\overline{Z_2}}{\overline{Z_1} + \overline{Z_2}}$$

$$\bar{I}_2 = \bar{I}_T \times \frac{\overline{Z_1}}{\overline{Z_1} + \overline{Z_2}}$$

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1},\quad \bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2},\ldots\ldots,\ \bar{I}_n = \frac{\bar{V}}{\bar{Z}_n}$$

While the individual phase angles can be calculated by the relation,

$$\tan\phi_1 = \frac{X_1}{R_1},\ \tan\phi_2 = \frac{X_2}{R_2},\ \ldots,\ \tan\phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in **rectangular form**. This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit..

5.10.2 Concept of Admittance

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit **siemens or mho.**

Now, current equation for the circuit shown in the Fig. 5.41 is,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$\bar{I} = \bar{V}\times\left(\frac{1}{Z_1}\right) + \bar{V}\times\left(\frac{1}{Z_2}\right) + \bar{V}\times\left(\frac{1}{Z_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where **Y** is the **admittance** of the total circuit. **The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 5.41.

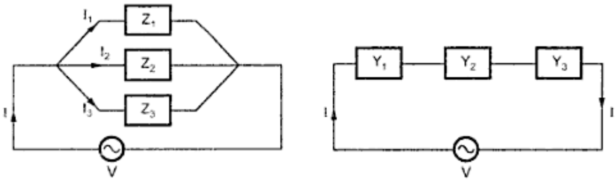


Fig. 5.41 Equivalent parallel circuit using admittances

Solution : The circuit is shown in the Fig. 5.43.

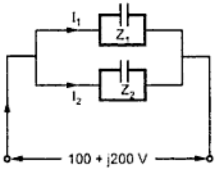


Fig. 5.43

$$V = 100 + j\,200 = 223.607 \angle 63.43° \text{ V}$$

$$Z_1 = 5 - j\,13.1 = 14.021 \angle -69.109° \ \Omega$$

$$Z_2 = 8.57 + j\,6.42 = 10.71 \angle +36.83° \ \Omega$$

i) $$I_1 = \frac{V}{Z_1} = \frac{223.607 \angle 63.43°}{14.021 \angle 69.109°}$$

$$= 15.948 \angle 132.539° \text{ A}$$

$$= \mathbf{-10.782 + j\,11.75 \text{ A}}$$

$$I_2 = \frac{V}{Z_2} = \frac{223.607 \angle 63.43°}{10.71 \angle +36.83°} = 20.878 \angle 26.6° \text{ A} = \mathbf{18.668 + j\,9.3483 \text{ A}}$$

$$\therefore \quad I_T = \bar{I}_1 + \bar{I}_2 = -10.782 + j\,11.75 + 18.668 + j\,9.3483$$

$$= 7.886 + j\,21.0983 \text{ A}$$

$$= \mathbf{22.5239 \angle 69.5° \text{ A}}$$

$\therefore \quad \phi_T$ = Angle between V and I_T

$$= 69.5 - 63.43 = \mathbf{6.075°} \text{ leading}$$

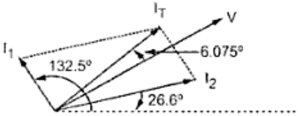


Fig. 5.44

$\therefore \quad P_T = VI_T \cos\phi_T = 223.607 \times 22.5239 \times \cos(6.075) = \mathbf{5008.212 \text{ W}}$

The phasor diagram is shown in the Fig. 5.44.

Example 5.14 : *Two admittances, $Y_1 = (0.167 - j0.167)$ siemen, and $Y_2 = (0.1 + j\,0.05)$ siemen are connected in parallel across a 100 V, 50Hz single-phase supply. Find the current in each branch and the total current. Also find the power-factor of the combination. Sketch a neat phasor diagram.*

Solution : The circuit is shown in the Fig. 5.45.

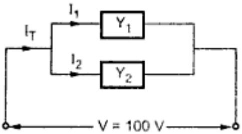


Fig. 5.45

Let $V = 100 \angle 0°$ V

$$Y_1 = 0.167 - j\,0.167 = 0.23617 \angle -45°$$

$$Y_2 = 0.1 + j\,0.05 = 0.1118 \angle 26.56°$$

$$I_1 = VY_1$$

$$= 100 \angle 0° \times 0.23617 \angle -45°$$

$$= 23.617 \angle -45° \text{ A}$$

$$= \mathbf{16.699 - j\,16.699 \text{ A}}$$

$$I_2 = VY_2 = 100 \angle 0° \times 0.1118 \angle +26.56°$$

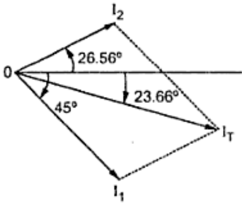


Fig. 5.46

$= 11.18 \angle +26.56° \quad A = \mathbf{10 + j\,5\ A}$

$\therefore \quad I_T = \bar{I}_1 + \bar{I}_2 = 16.699 - j\,16.699 + 10 + j\,5$

$= 26.699 - j\,11.699\ A$

$= \mathbf{29.15 \angle -23.66°\ A}$

p.f. = cos (– 23.66°) = **0.9159 lagging**

The phasor diagram is shown in the Fig. 5.46.

➡ **Example 5.15 :** *Two impedances Z_1 and Z_2 are connected in parallel across applied voltage of $(100+j200)$ volts. The total power supplied to the circuit is 5 kW. The first branch takes a leading current of 16 A and has a resistance of 5 ohms while the second branch takes a lagging current at 0.8 power factor. Calculate*
i) Current in second branch ii) Total current iii) Circuit constants.

Solution : The circuit is shown in the Fig. 5.47.

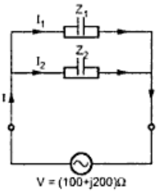


Fig. 5.47

$V = 100 + j\,200 = 223.60 \angle 63.43°$ volts

$I_1 = 16$ A , $R_1 = 5\ \Omega$, $\cos\phi_2 = 0.8$

Now, $|Z_1| = \frac{V}{I_1} = \frac{223.60}{16} = 13.975\ \Omega$

$R_1 = Z_1 \cos\phi_1$

$\therefore \cos\phi_1 = \frac{R_1}{Z_1} = \frac{5}{13.975} = 0.3577$

$\therefore \quad \phi_1 = -69.03°$ (... negative, as leading in nature)

$\therefore \quad \sin\phi_1 = 0.9338$

$X_1 = Z_1 \sin\phi_1 = (13.975)(0.9338) = 13.04\ \Omega$

Power consumed in Z_1, $P_1 = I_1^2\ R_1 = (16)^2 (5) = 1280$ watt

Power consumed in Z_2 = Total power supplied – Power consumed in Z_1

$\therefore \quad P_2 = 5000 - 1280 = 3720$ watt

Power consumed in Z_2, $P_2 = V\ I_2\ \cos\phi_2$

$$\therefore \quad I_2 = \frac{P_2}{V\cos\phi_2}$$

$$\therefore \quad I_2 = \frac{3720}{223.60 \times 0.8}$$

$\therefore \quad I_2 = \mathbf{20.79\ A}$

Now we have, $P_2 = I_2^2 \; R_2$

$\therefore \quad R_2 = \frac{P_2}{I_2^2} = \frac{3720}{(20.79)^2} = 8.60 \; \Omega$

Now, $\cos \phi_2 = 0.8$

$\therefore \quad \sin \phi_2 = 0.6$

$\therefore \quad X_2 = |Z_2| \sin \phi_2 = \frac{V}{I_2} \sin \phi_2 = \frac{223.60}{20.79} \times 0.6$

$X_2 = 6.4531 \; \Omega$

$\therefore \quad Z_2 = R_2 + j X_2 = 8.60 + j 6.4531 = 10.75 \angle 36.88° \; \Omega$

$\therefore \quad \dot{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{223.60 \angle 63.43°}{10.75 \angle 36.88°} = 20.79 \angle 26.55° \; A$

Similarly, $Z_1 = R_1 - j X_1 = (5 - j\, 13.04) = 13.965 \angle -69.02° \; \Omega$

$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{223.60 \angle 63.43°}{13.965 \angle -69.02°} = 16 \angle 132.45° \; A$

$\therefore$ Total current, $\bar{I} = \bar{I}_1 + \bar{I}_2 = (16 \angle 132.45°) + (20.79 \angle 26.55°)$

$= (-10.799 + j\, 11.80) + (18.59 + j\, 9.29) = 7.8 + j\, 21.09$

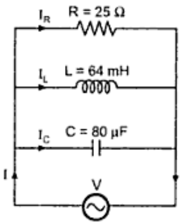
$\therefore \quad \bar{I} = 22.48 \angle 69.70° \; A$

$\therefore$ Total current = **22.48 A**

Current in branch 2 = **20.79 A**

Circuit constants are $R_1 = 5 \; \Omega$, $R_2 = 8.60 \; \Omega$, $X_1 = 13.04 \; \Omega$, $X_2 = 6.4531 \; \Omega$

Example 5.16 : *A parallel circuit of 25 Ω resistor, 64 mH inductor and 80 μF capacitor connected across a 110 V, 50 Hz, single phase supply, is shown in Fig. 5.46. Calculate the current in individual element, the total current drawn from the supply and the overall p.f. of the circuit. Draw a neat phasor diagram showing* $\bar{V}$, $\bar{I}_R$, $\bar{I}_L$, $\bar{I}_C$ *and* $\bar{I}$.



110 volt, 50 Hz supply

Fig. 5.48

Solution : From Fig. 5.48,

$R = 25 \; \Omega, \quad X_L = 2\pi f L = 2\pi \times 50 \times 64 \times 10^{-3}$

$\therefore \quad X_L = 20.10 \; \Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.78 \; \Omega$

Let $V = 110 \angle 0° \;$ volts

$$I_R = \frac{V}{R} = \frac{110\angle 0^\circ}{25} = 4.4\angle 0^\circ \text{ A}$$

$$I_L = \frac{V}{jX_L} = \frac{110\angle 0^\circ}{20.10\angle 90^\circ} = 5.47\angle -90^\circ \text{ A}$$

$$I_C = \frac{V}{-jX_C} = \frac{110\angle 0^\circ}{39.78\angle -90^\circ} = 2.76\angle 90^\circ \text{ A}$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$= (4.4\angle 0^\circ) + (5.47\angle -90^\circ) + (2.76\angle 90^\circ)$$

$$= (4.4 + j0) + (0 - j5.47) + (0 + 2.76)$$

$$= (4.4 - j2.71) \text{ A}$$

$$\therefore \quad \bar{I} = 5.1676\angle -31.62^\circ \text{ A}$$

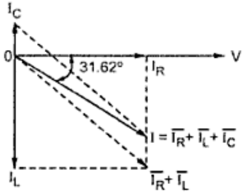


Fig. 5.49

Overall p.f. = cos ϕ = cos (31.62) = **0.851 lagging.**

Phasor diagram : The phasor diagram is shown in the Fig. 5.49.

5.12 Resonance in Parallel Circuit

Similar to a series a.c. circuit, there can be a resonance in parallel a.c. circuit. When the power factor of a parallel a.c. circuit is unity i.e. the voltage and total current are in phase at a particular frequency then the parallel circuit is said to be at **resonance**. The frequency at which the parallel resonance occurs is called **resonant frequency** denoted as f_r Hz.

5.12.1 Characteristics of Parallel Resonance

Consider a practical parallel circuit used for the parallel resonance as shown in the Fig. 5.50.

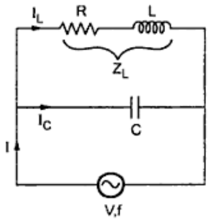


Fig. 5.50 Practical parallel circuit

The one branch consists of resistance R in series with inductor L. So it is series R-L circuit with impedance Z_L. The other branch is pure capacitive with a capacitor C. Both the branches are connected in parallel across a variable frequency constant voltage source.

The current drawn by inductive branch is I_L while drawn by capacitive branch is I_C.

$$I_L = \frac{V}{Z_L} \quad \text{where } Z_L = R + jX_L$$

And

$$I_C = \frac{V}{X_C} \quad \text{where } X_C = \frac{1}{2\pi f C}$$

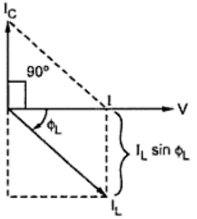


Fig. 5.51

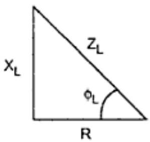


Fig. 5.52 Impedance triangle

The current I_L lags voltage V by angle ϕ_L which is decided by R and X_L while the current I_C leads voltage V by 90°. The total current I is phasor addition of I_L and I_C. The phasor diagram is shown in the Fig. 5.51.

For the parallel resonance V and I must be in phase. To achieve this unity p.f. condition,

$$I = I_L \cos \phi_L$$

and

$$I_C = I_L \sin \phi_L$$

From the impedance triangle of R-L series circuit we can write,

$$\tan \phi_L = \frac{X_L}{R}, \cos\phi_L = \frac{R}{Z_L}, \sin \phi_L = \frac{X_L}{Z_L}$$

As frequency is increased, $X_L = 2\pi f L$ increases due to which $Z_L = \sqrt{R^2 + X_L^2}$ also increases. Hence cos ϕ_L decreases and sin ϕ_L increases. As Z_L increases, the current I_L also decreases.

At resonance $f = f_r$ and $I_L \cos \phi_L$ is at its minimum. Thus at resonance **current is minimum** while the total **impedance** of the circuit is **maximum.** As admittance is reciprocal of impedance, as frequency is changed, **admittance** decreases and is **minimum** at resonance. The three curves are shown in the Fig. 5.53 (a), (b) and (c).

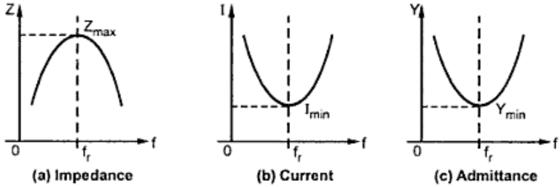


Fig. 5.53 Characteristics of parallel resonance

5.12.2 Expression for Resonant Frequency

At resonance $I_C = I_L \sin \phi_L$

$$\therefore \quad \frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L} = \frac{V X_L}{Z_L^2}$$

$$\therefore \quad Z_L^2 = X_L X_C$$

$$\therefore \quad R^2 + (2\pi f_r L)^2 = (2\pi f_r L) \times \frac{1}{2\pi f_r C} \quad \text{as } f = f_r$$

$$\therefore \quad R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$\therefore \quad (2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$\therefore \quad (2\pi f_r)^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Thus if R is very small compared to L and C, $\frac{R^2}{L^2} << \frac{1}{LC}$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

This is same as that for series resonance.

Key Point : *The net susceptance of the whole circuit is zero at resonance.*

5.12.3 Dynamic Impedance at Resonance

The impedance offered by the parallel circuit at resonance is called **dynamic impedance** denoted as Z_D. This is maximum at resonance. As current drawn at resonance is minimum, the parallel circuit at resonance is called **rejector circuit**. This indicates that it rejects the unwanted frequencies and hence it is used as filter in radio receiver.

From $I_C = I_L \sin \phi_L$ we have seen that,

$$Z_L^2 = \frac{L}{C}$$

while

$$I = I_L \cos \phi_L = \frac{V}{Z_L} \cdot \frac{R}{Z_L} = \frac{VR}{Z_L^2}$$

$$\therefore \quad I = \frac{VR}{\frac{L}{C}} = \frac{V}{(L/RC)}$$

$$\therefore \quad I = \frac{V}{Z_D}$$

Where

$$\boxed{Z_D = \frac{L}{RC} = \text{Dynamic impedance}}$$

5.12.4 Quality Factor of Parallel Circuit

The parallel circuit is used to magnify the current and hence known as current resonance circuit.

The quality factor of the parallel circuit is defined as the current magnification in the circuit at resonance.

The current magnification is defined as,

$$\text{Current magnification} = \frac{\text{Current in the inductive branch}}{\text{Current in supply at resonance}} = \frac{I_L}{I}$$

$$= \frac{\frac{V}{Z_L}}{\frac{V}{Z_D}} = \frac{Z_D}{Z_L} = \frac{\frac{L}{RC}}{\sqrt{\frac{L}{C}}} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{as} \quad Z_L = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}}$$

This is nothing but the quality factor at resonance.

$$\therefore \qquad \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Example 5.17 : *An inductive coil of resistance 10 Ω and inductance 0.1 henries is connected in parallel with a 150 μF capacitor to a variable frequency, 200 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with the supply voltage. Also find the value of this current. Draw the phasor diagram.*

Solution : The circuit is shown in the Fig. 5.54.

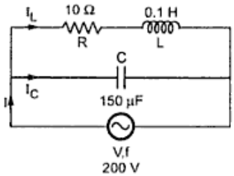


Fig. 5.54

The resonant frequency is,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 150 \times 10^{-6}} - \frac{(10)^2}{(0.1)^2}}$$

$$= \mathbf{37.8865 \quad Hz}$$

Now $Z_L = R + j\,X_L = 10 + j\ (2\pi f_r\, L)$

$$= 10 + j\,23.805 = 25.82 \angle 67.21^\circ\ \Omega$$

$$\therefore \qquad \bar{I}_L = \frac{V}{Z_L} = \frac{200\angle 0^\circ}{25.82\angle 67.21^\circ} = \mathbf{7.7459 \angle -67.21^\circ\ A}$$

and $$I_C = \frac{V}{X_C} = \frac{200\angle 0^\circ}{\frac{1}{2\pi f_r\, C}\angle -90^\circ} = \frac{200\angle 0^\circ}{28\angle -90^\circ} = \mathbf{7.143 \angle +90^\circ\ A}$$

Examples with Solutions

➡ **Example 5.18 :** *Two impedances Z_1 and Z_2 having same numerical value are connected in series. If Z_1 is having p.f. of 0.866 lagging and Z_2 is having p.f. of 0.6 leading. Calculate the p.f. of the series combination.*

Solution : Z_1 has p.f. $\cos \phi_1 = 0.866$ lagging

$$Z_2 \text{ has p.f. } \cos \phi_2 = 0.6 \text{ leading}$$

$$|Z_1| = |Z_2| = Z$$

$$\cos \phi_1 = 0.866 \text{ and } \sin \phi_1 = 0.5$$

$$\cos \phi_2 = 0.6 \text{ and } \sin \phi_2 = 0.8$$

$$Z_1 = [Z \cos \phi_1 + j \sin \phi_1] = Z\,[0.866 + j\,0.5]$$

$$Z_2 = Z \cos \phi_2 + j\, Z \sin \phi_2 = Z\,[0.6 + j\,0.8]$$

$$\therefore \quad Z_T = Z\,[1.466 - j\,0.3] = Z\,1.4963 \angle -11.565°$$

∴ Power factor of the combination is cos (– 11.565) i.e.

$$\cos \phi_T = \mathbf{0.9796\ leading.}$$

➡ **Example 5.19 :** *A heater operates at 100 V, 50 Hz and takes current of 8 A and consumes 1200 W power. A choke coil is having ratio of reactance to resistance as 10, is connected in series with the heater. The series combination is connected across 230 V, 50 Hz a.c. supply. Calculate the*

i) Resistance of choke coil *ii) Reactance of choke coil*

iii) Power consumed by choke coil *iv) Total power consumed.*

Solution : For heater $V_{rated} = 100$ V and $I = 8$ A

Heater is purely resistive load.

$$\therefore \quad R_{heater} = \frac{V}{I} = \frac{100}{8} = 12.5\ \Omega$$

$$\text{For choke coil } \frac{X_L}{r} = 10$$

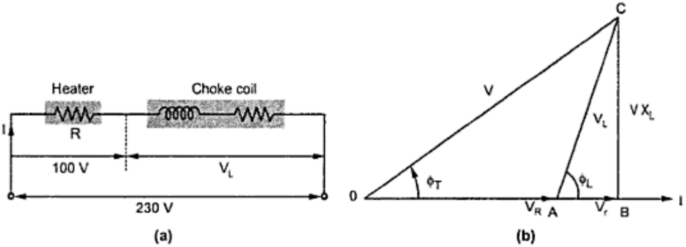


Fig. 5.56

$$Z_{coil} = r + j X_L = (r) + j\ (10r)$$

$$|Z| = \sqrt{(r)^2 + (10r)^2} = (10.049\ r) \angle 84.289°$$

And $\qquad \phi_L = \tan^{-1}\left(\frac{10\,r}{r}\right)$

$\therefore \qquad \phi_L = 84.289°$

This is shown in the Fig. 5.56 (b)

Resolving V_L into its two components,

i) $\qquad V_r$ = Drop across resistance of coil = $V_L \cos \phi_L$

ii) $\qquad V_{X_L}$ = Drop across reactance of coil = $V_L \sin \phi_L$

Consider triangle OBC as shown in Fig. 5.56 (b)

$\therefore \qquad (OC)^2 = (OB)^2 + (BC)^2$

$\therefore \qquad (OC)^2 = (OA + AB)^2 + (BC)^2$

$\therefore \qquad V^2 = (V_R + V_r)^2 + (V \times L)^2$

$\therefore \qquad (230)^2 = (100)^2 + 2 \times 100 \times V_r + (V_r)^2 + (V \times L)^2$

$\therefore \qquad (230)^2 = (100)^2 + 200 \times (V_L \cos \phi_L) + (V_L \cos \phi_L)^2 + (V_L \sin \phi_L)^2$

$\therefore \quad (230)^2 = (100)^2 + 200 \times (V_L)(\cos 84.28) + (V_L)^2 (\cos \phi_L^2 + \sin \phi_L^2)$

$\therefore \qquad (230)^2 = (100)^2 + 19.93\ V_L + (V_L)^2$

$$(V_L)^2 + 10.93\ V_L - 42900 = 0$$

Solving for V_L we get, $\qquad V_L = 197.397$ V

$$V_r = V_L \cos \phi_L = 19.643 \text{ V}$$

$$V \times L = V_L \sin \phi_L = 196.417 \text{ V}$$

i) $V_r = I \times r$

$\therefore \quad r = \frac{V_r}{I} = \frac{196.417}{8} = \mathbf{2.4552\ \Omega}$

ii) $V_{X_L} = I \times X_L$

$\therefore \quad X_L = \frac{V \times L}{I} = \frac{196.417}{8} = \mathbf{24.552\ \Omega}$

iii) Power consumed by coil is $= I^2 \times r = (8)^2 \times 2.4532 = \mathbf{157.13\ W}$

Or $\quad P_{coil} = V_L \times I \times \cos \phi_L = 197.387 \times 8 \times \cos (84.28) = \mathbf{157.13\ W}$

iv) Total power consumed $= I^2 \times R_{heater} + I^2\, r = I^2\,(R_{heater} + r)$

$= 64 \times 14.9552 = \mathbf{957.1328\ W.}$

Example 5.20 : *A coil has a resistance of 4 Ω and inductance of 0.05 H, forms one branch of parallel circuit. The other branch has a similar coil but in series with it is R-C combination. If current in the two branches are equal in magnitude but have a phase difference of* $\frac{1}{4}^{th}$ *time period of 50 Hz voltage applied. Calculate values of R and C in second branch. Also find total current and total power factor if V = 200 V a.c.*

Solution :

$R = 4\ \Omega$ and $X_L = 2\pi \times 50 \times 0.05 = 15.707\ \Omega$

$Z_1 = 4 + j\,15.707\ \Omega = 16.21 \angle -75.72°\ \Omega$

$Z_1 = 16.21\ \Omega,\ \phi_1 = +75.72°$

$\frac{V}{Z_1} = I_1 = \left(\frac{V}{16.21}\right) \angle -75.72°$

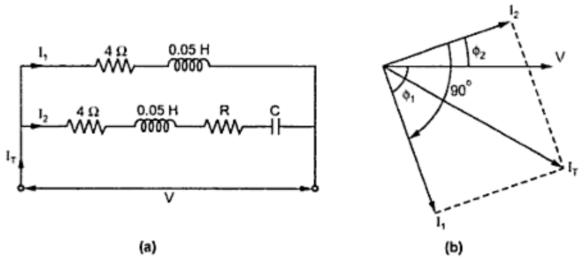


Fig. 5.57

$\therefore$ I_2 must be leading V by angle ϕ_2 in such a way that phase difference between I_1, and I_2 is $\frac{1}{4}^{th}$ of time period of 50 Hz, voltage.

$$\therefore \quad \overline{I_T} = \overline{I_1} + \overline{I_2} = 3.0433 - j\,11.956 + 11.956 + j\,3.0433$$

$$= 15 - j\,8.9127\text{ A} = \mathbf{17.45 \angle -30.71^\circ\text{ A}}$$

The resultant current of 17.45 A, lags voltage by 30.71°.

$\therefore$ Total power factor = cos (30.71) = **0.8596 lagging**.

Example 5.21 : *Two impedances Z_1 and Z_2 when connected separately across 230 volts, 50 Hz, ac supply consumed 100 watts and 60 watts at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply find.*

i) Total power absorbed and overall power factor.

ii) Value of the impedance to be added in series to raise the overall power factor to unit.

Solution : When Z_1 alone is connected across the supply.

$$\cos\phi = 0.5 \text{ lagging}$$

$$P = 100\text{ W}$$

$$P = VI\cos\phi$$

$$\therefore \quad 100 = 230 \times I \times 0.5$$

$$\therefore \quad I = 0.8695\text{ A}$$

$$\therefore \quad |Z_1| = \frac{V}{I} = \frac{230}{0.8695} = 264.5198\ \Omega$$

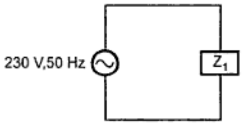


Fig. 5.58

$$\phi = \cos^{-1} 0.5 = 60^\circ$$

$$Z_1 = |Z_1| \angle +\phi = 264.5198 \angle +60^\circ\Omega = 132.29 + j\,229.1328\ \Omega$$

Similarly when Z_2 alone is connected.

$$\cos\phi = 0.6 \text{ leading}$$

$$P = 60\text{ W}$$

$$P = VI\cos\phi$$

$$\therefore \quad 60 = 230 \times I \times 0.6$$

$$\therefore \quad I = 0.4347\text{ A}$$

$$|Z_2| = \frac{V}{I} = \frac{230}{0.4347} = 529$$

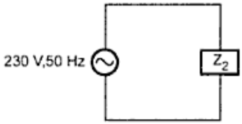


Fig. 5.59

$$\phi = -53.13^\circ \qquad \text{negative as leading}$$

$\therefore \quad I_T = 12.755 \angle -11.47° A$

Now $\quad P_A = V I_A \cos\phi_A$

$\therefore \quad 1600 = 200 \times 10 \times \cos\phi_A$

$\therefore \quad \cos\phi_A = 0.8$ lagging $\qquad$ hence $\phi = -\cos^{-1} 0.8 = -36.86°$

$\therefore \quad I_A = 10 \angle -36.86°$ A

And $\quad |Z_A| = \dfrac{|V|}{|I_A|} = \dfrac{200}{10} = 20$ A

$\therefore \quad X_A = |Z_A| \sin\phi = 20 \sin(-36.86°) = 12\ \Omega$

$\therefore \quad Z_A = 16 + j\,12\ \Omega = 20 \angle 36.86°\ \Omega$

i) $\therefore \quad Y_A = \text{Admittance} = \dfrac{1}{Z_A} = \dfrac{1}{20\angle 36.86°} = \mathbf{0.05 \angle -36.86°}$ **mho**

ii) $\quad P_B = \text{Power of branch B} = P_T - P_A = 2500 - 1600 =$ **900 W**

$\overline{I_B} = \overline{I_T} - \overline{I_A} = [12.5 - j\,2.5364] - [8 - j6]$

$= 4.5 + j\,3.4636$ A$= 5.6786 \angle 37.585°$ A

$\therefore \quad \cos\phi_B = \cos(37.858) = 0.7924$ leading

$\therefore \quad \sin\phi_B = 0.61$

$\therefore \quad Q_B = \text{Reactive volt-ampere of branch B} = V I_B \sin\phi_B$

$= 200 \times 5.6786 \times 0.61 =$ **692.7892 VAR leading**

Example 5.23 : *A series circuit consisting of a 12 Ω resistance, 0.3 henry inductance and a variable capacitor is connected across 100 V, 50 Hz a.c. supply. The capacitance value is adjusted to obtain maximum current. Find this capacitance value and the power drawn by the circuit under this condition.*

Now, the supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of capacitor C_1 to be connected across the above series circuit, so that current drawn from supply is the minimum.

Solution : **Case 1 :** The circuit diagram is shown in the Fig. 5.60.

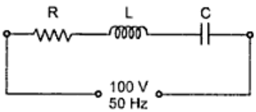


Fig. 5.60

For I_{max}, there must be resonance.

$$\therefore \quad X_L = X_C$$

Now $$X_L = 2\pi f L = 2\pi \times 50 \times 0.3 = 94.2477\ \Omega = X_C$$

$$\therefore \quad \frac{1}{2\pi \times 50 \times C} = 94.2477$$

$$\therefore \quad C = \mathbf{33.7737\ \mu F}$$

And $$P = VI = \frac{V^2}{R} \quad \text{... as } I_{max} = \frac{V}{R} \text{ under resonance}$$

$$= \frac{(100)^2}{12} = \mathbf{833.333\ W}$$

Case 2 : f = 60 Hz, V = 100 V

The circuit is shown in the Fig. 5.61.

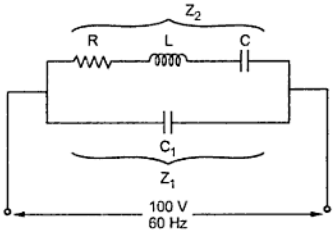


Fig. 5.61

At 60 Hz,

$$X_L = 2\pi f L = 113.0973\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = 78.5398\ \Omega$$

$$\therefore \quad Z_2 = R - j X_C + j X_L$$

$$= 12 + j\,34.5575\ \Omega$$

$$\doteq 36.5817 \angle 70.8505°\ \Omega$$

$$\therefore \quad Y_2 = \frac{1}{Z_2} = \frac{1}{36.5817 \angle 70.8505°} = 0.02733 \angle -70.8505°$$

$$= 8.9671 \times 10^{-3} - j\,0.02582 \text{ mho}$$

and $Z_1 = -j X_{C1} = X_{C1} \angle -90°$

$$\therefore \quad Y_1 = \frac{1}{X_{C1} \angle -90°} = \frac{1}{X_{C1}} \angle +90° = +j\left[\frac{1}{X_{C1}}\right]$$

$$Y_T = Y_1 + Y_2 = 8.9671 \times 10^{-3} + j\left(\frac{1}{X_{C1}} - 0.02582\right)$$

For current minimum, there must be parallel resonance and circuit must be having unity p.f. so **imaginary part of Y_T must be zero.**

$$\therefore \quad \left(\frac{1}{X_{C1}}\right) - 0.02582 = 0$$

$$\therefore \quad \left(\frac{1}{X_{C1}}\right) = 0.02582\ \Omega$$

$$\therefore \quad X_{C1} = 38.7296 = \frac{1}{2\pi f C_1}$$

$$\therefore \quad C_1 = \mathbf{68.4896\ \mu F}$$

Examples from G.U. and G.T.U. Papers

Example 5.24 : *A resistance R, an inductance L = 0.5 H and a capacitance C are connected in series. When a voltage v = 350 cos (3000 t – 20°) volts is applied to this series combination, the current flowing is 15 cos (3000 t – 60°) ampere. Find the values of R and C.*

[GU : 1998]

Solution : $v = 350 \cos(3000\,t - 20°) = 350 \sin(3000\,t - 20° + 90°)$

$$v = 350 \sin(3000\,t + 70°)\ V$$

$$i = 15 \cos(3000\,t - 60°) = 15 \sin(3000\,t - 60° + 90°)$$

$$\therefore \quad i = 15 \sin(3000\,t + 30°)\ A$$

$$\therefore \quad V = \frac{V_m}{\sqrt{2}} \angle \phi_1 = \frac{350}{\sqrt{2}} \angle 70° = 247.4873 \angle 70°\ V$$

$$\therefore \quad I = \frac{I_m}{\sqrt{2}} \angle \phi_2 = \frac{15}{\sqrt{2}} \angle 30° = 10.6066 \angle 30°\ A$$

$$\therefore \quad Z = \frac{V}{I} = \frac{247.4873 \angle 70°}{10.6066 \angle 30°} = 23.3333 \angle +40°\ \Omega$$

$$= 17.8743 + j\,15\ \Omega = R + j\,X$$

$$\therefore \quad R = \mathbf{17.8743\ \Omega},\ X = X_L - X_C = 15$$

$$\omega = 3000 \text{ rad/sec} \quad \text{and hence} \quad X_L = \omega L = 3000 \times 0.5 = 1500\,\Omega$$

$$\therefore \quad 15 = 1500 - X_C \quad \text{i.e.} \quad X_C = 1500 - 15 = 1485\,\Omega = \frac{1}{\omega C}$$

$$\therefore \quad 1485 = \frac{1}{\omega C} \quad \text{i.e.} \quad C = \frac{1}{3000 \times 1485} = \mathbf{0.2244\,\mu F}$$

Example 5.25 : *A series circuit consists of a non-inductive resistance of 10 ohm, an inductance of 0.159 H and capacitance of 106 μF. The circuit is supplied by 230 V, 50 Hz mains. Calculate (i) Current (ii) Power factor (iii) Voltage across each element.*
[GU : Dec.-1998, 2000]

Solution : $R = 10\,\Omega$, $L = 0.159$ H, $C = 106\,\mu F$, $V = 230$ V, $f = 50$ Hz.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.159 = 49.9513\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 106 \times 10^{-6}} = 30.0292\,\Omega$$

$$\therefore \quad Z = R + j(X_L - X_C) = 10 + j19.9221\,\Omega = 22.291\angle 63.345^\circ\,\Omega$$

i) $$I = \frac{V}{Z} = \frac{230\angle 0^\circ}{22.291\angle 63.345^\circ} = \mathbf{10.318\angle -63.345^\circ\,A}$$

ii) $$\cos\phi = \cos(-63.345^\circ) = \mathbf{0.4486 \text{ lagging.}}$$

iii) $$|V_L| = |I| \times |X_L| = 10.318 \times 49.9513 = \mathbf{515.3975\ V.}$$

$$|V_C| = |I| \times |X_C| = 10.318 \times 30.0292 = \mathbf{309.8412\ V.}$$

Example 5.26 : *A series circuit has resistance of 10 ohms, inductance 200/π mH and capacitance 1000/π micro-farad. Calculate :*
i) The current, flowing in the circuit of supply voltage 250 V, 50 Hz
ii) Power factor for the circuit.
iii) Power drawn from the supply.
iv) Also draw the phasor diagram.
[GU : June - 2000]

Solution : $R = 10\,\Omega$, $L = \frac{200}{\pi}$ mH, $C = \frac{1000}{\pi}\,\mu F$, $V = 250$ V, $f = 50$ Hz

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{200}{\pi} \times 10^{-3} = 20\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times \frac{1000}{\pi} \times 10^{-6}} = 10\,\Omega$$

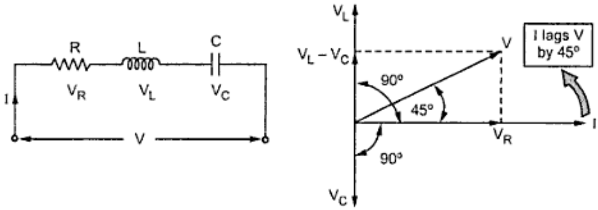
$$\therefore \quad Z = R + j(X_L - X_C) = 10 + j(20-10) = 10 + j10\,\Omega = 14.1421\angle 45^\circ\,\Omega$$

i) $I = \frac{V}{Z} = \frac{250\angle 0°}{14.1421\angle 45°} = \mathbf{17.6777\angle -45° A}$

ii) $\cos\phi = \cos(-45°) =$ **0.7071 lagging.**

iii) $P = V I \cos\phi = 250 \times 17.6777 \times 0.7071 =$ **3124.975 W.**

iv) The phasor diagram is shown in the Fig. 5.62.



(a) Circuit (b) Phasor diagram

Fig. 5.62

Example 5.27 : *A series R-L circuit takes a powers of 384 watts at a power factor 0.8 from a 120 V, 60 Hz supply. Calculate the values of R and L.* **[GU : June -2000]**

Solution : P = 384 W, cos ϕ = 0.8 lagging, V = 120 V, f = 60 Hz

$$P = V I \cos\phi \quad \text{i.e.} \quad 384 = 120 \times I \times 0.08$$

$$I = \frac{384}{120 \times 0.8} = 4 \text{ A} \quad \text{and} \quad \phi = \cos^{-1} 0.8 = -36.869°$$

Key Point : *ϕ is negative as current is lagging.*

$\therefore \quad I = 4\angle -36.869° \text{ A} \quad \text{and} \quad V = 120\angle 0° \text{ V}$

$\therefore \quad Z = \frac{V}{I} = \frac{120\angle 0°}{4\angle -36.869°} = 30\angle +36.869° \Omega = 24 + j18 \; \Omega$

But $Z = R + j X_L$ hence $\mathbf{R = 24\ \Omega}$ and $\mathbf{X_L = 18\ \Omega}$

$\therefore \quad X_L = 2\pi f L$ hence $L = \frac{18}{\pi \times 60} =$ **47.746 mH**

Now a capacitor is connected in parallel with the series circuit so circuit becomes as shown in the Fig. 5.64 (a)

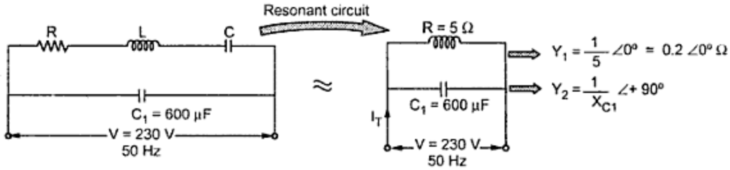


Fig. 5.64 (a)

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 50 \times 600 \times 10^{-6}} = 5.3051\ \Omega$$

$$\therefore \quad Y_2 = \frac{1}{5.3051} \angle +90° = 0.1884 \angle +90° = 0 + j0.1884 \text{ mho}$$

$$Y_T = Y_1 + Y_2 = (0.2 + j\,0) + (0 + j\,0.1884) = 0.2 + j\,0.1884 \text{ mho}$$

$$= 0.2747 \angle 43.2893° \text{ mho}$$

$$\therefore \quad I_T = VY_T = 230 \angle 0° \times 0.2747 \angle 43.2893°$$

$$= \mathbf{63.181 \angle 43.2893° A} \quad \text{... New current}$$

$$\therefore \quad \cos\phi_T = \cos(43.2893°) = \mathbf{0.7279 \text{ leading}}$$

Example 5.31 : *A capacitor connected to a 230 V, 50 Hz supply draws 10 amp. What current it will draw for a frequency of 20 Hz ? Assume voltage remains unchanged.*

[GU : Dec. - 2001]

Solution : The capacitor is shown in the Fig. 5.65.

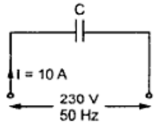


Fig. 5.65

$$|I| = \frac{|V|}{|X_C|} \quad \text{i.e.} \quad 10 = \frac{230}{|X_C|}$$

$$\therefore \quad |X_C| = 23\ \Omega = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 23} = \mathbf{0.1384 \text{ mF}}$$

At frequency f' = 20 Hz, $|X'_C| = \frac{1}{2\pi \times 20 \times 0.1384 \times 10^{-6}} = 57.5\ \Omega$

$$\therefore \quad |I'| = \frac{|V|}{|X'_C|} = \frac{230}{57.5} = \mathbf{4 A} \quad \text{... New current}$$

Example 5.32 : *The two impedances (14 + j 5) Ω and (18 + j 10) Ω are connected in parallel across a 200 V, 50 Hz supply. Determine*

i) The admittance of each branch and of the entire circuit.

ii) Total current, power and power factor.

iii) Capacitance which when connected in parallel with the original circuit will make the resultant power factor unity. **[GU : May/June - 2001]**

Solution : The circuit is shown in the Fig. 5.66.

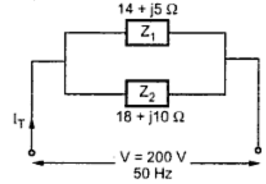


Fig. 5.66

$$Z_1 = 14 + j\,5\ \Omega = 14.866\ \angle\ 19.653°\ \Omega$$

$$Z_2 = 18 + j\,10\ \Omega = 20.591\ \angle\ 29.054°\ \Omega$$

i) $$Y_1 = \frac{1}{Z_1} = \frac{1}{14.866\angle 19.653°}$$

$$= \mathbf{0.0672\ \angle -19.653°\ \mho}$$

$$= \mathbf{0.06328 - j0.0226\ \mho}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{20.591\angle 29.054°}$$

$$= \mathbf{0.04856\ \angle -29.054°\ \mho = 0.0424 - j0.0235\ \mho}$$

$$Y_T = Y_1 + Y_2 = 0.10568 - j\,0.0461\ \mho = \mathbf{0.1153\ \angle -23.56°\ \mho}$$

ii) $$I_T = VY_T = 200\ \angle 0° \times 0.1153\ \angle -23.56°$$

$$= \mathbf{23.06\ \angle -23.56°\ A} \quad \text{... Total current}$$

$$\cos\phi_T = \cos(-23.56°) = \mathbf{0.92\ lagging}$$

$$P = V\,I_T\cos\phi_T = 200 \times 23.06 \times 0.92 = \mathbf{4243.04\ W.}$$

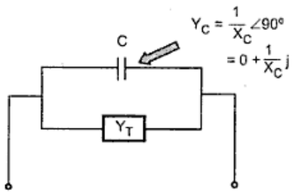


Fig. 5.67

iii) Now a capacitance is connected in parallel with original circuit. The original circuit is represented by its equivalent admittance Y_T as shown in the Fig. 5.67.

$$\therefore \quad Y_{eq} = Y_T + Y_C$$

$$= 0.10568 - j0.0461 + j\frac{1}{X_C}$$

$$= 0.10568 + j\left[\frac{1}{X_C} - 0.0461\right]\mho$$

For the power factor to be unity, the imaginary part of Y_{eq} must be zero.

$$\therefore \quad \frac{1}{X_C} - 0.0461 = 0 \qquad \text{i.e. } X_C = \frac{1}{0.0461} = 21.692\ \Omega$$

$$\therefore \quad X_C = \frac{1}{2\pi fC} \quad \text{i.e.} \quad 21.692 = \frac{1}{2\pi\times 50\times C}$$

$$\therefore \quad C = \mathbf{146.741\ \mu F.}$$

Example 5.33 : *A choke coil having a resistance of 10 Ω and a inductance of 63.7 mH is connected in series with a resistance of 5 ohms. The circuit is connected across 230 V, 50 Hz supply. Calculate (i) Current (ii) Voltage across the coil (iii) Power factor (iv) Voltage across 5 ohm resistor (v) Power. Draw complete phasor diagram.* **[GU : Dec.-2001]**

Solution : The circuit is shown in the Fig. 5.68. Assume voltage as reference, $V = 230 \angle 0^\circ$ V.

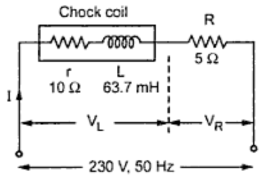


Fig. 5.68

$$X_L = 2\pi f L = 2\pi\times 50\times 63.7\times 10^{-3}$$

$$= 20.01\ \Omega$$

$$\therefore \quad Z_T = (R+r) + j\,X_L = 15 + j\,20.01\ \Omega$$

$$= 25 \angle 53.13^\circ\ \Omega$$

$$\text{i)} \quad I = \frac{V}{Z_T} = \frac{230\angle 0^\circ}{25\angle 53.13^\circ}$$

$$\therefore \quad I = \mathbf{9.2 \angle -53.13^\circ\ A.}$$

$$\text{ii)} \quad Z_L = r + j\,X_L = 10 + j\,20.01\ \Omega = 22.37 \angle 63.44^\circ\ \Omega$$

$$\therefore \quad V_L = I\times Z_L = 9.2 \angle{-53.13^\circ} \times 22.37 \angle 63.44^\circ = \mathbf{205.804 \angle 10.3^\circ V}$$

$$\text{iii)} \quad \cos\phi_T = \cos(53.13^\circ) = \mathbf{0.6\ lagging}$$

$$\text{iv)} \quad V_R = I\times R = 9.2 \angle{-53.13^\circ} \times 5 \angle 0^\circ = \mathbf{46 \angle -53.13^\circ\ V}$$

$$\text{v)} \quad P = V\,I\cos\phi_T = 230\times 9.2\times 0.6 = \mathbf{1269.6\ W}$$

The phasor diagram is shown in the Fig. 5.69. which is drawn by taking current as a reference, though in calculations voltage is assumed to be reference.

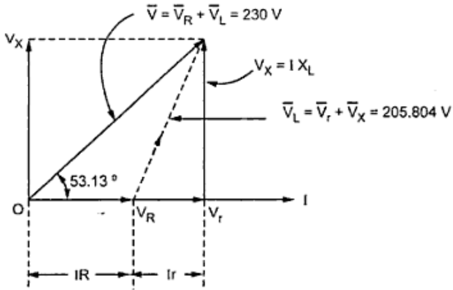


Fig. 5.69

Example 5.35 : *A capacitor of 25 μF is connected in series with a variable resistor. It is connected across 230 V, 50 Hz mains. Find the value of the resistor for a condition when the voltage across capacitor is half the supply voltage, also calculate, (i) Current drawn (ii) Power factor and (iii) Power of the said conditions.* **[GU : June-2001]**

Solution : The circuit is shown in the Fig. 5.72

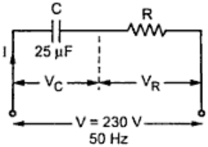


Fig. 5.72

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times 50\times 25\times 10^{-6}}$$

$$= 127.3239\ \Omega$$

$$|V_C| = |I|\times|X_C| = \left|\frac{V}{2}\right| \qquad \text{... Magnitude}$$

$$\therefore\quad |I| \times 127.3239 = \frac{230}{2}$$

$$\therefore\quad |I| = \frac{115}{127.3239} = 0.9032\ \text{A} \qquad \text{... Current}$$

Now $$|I| = \frac{|V|}{|Z|} \quad \text{where} \quad Z = R - j\,X_C,\ |Z| = \sqrt{R^2 + X_C^2}$$

$$\therefore\quad 0.9032 = \frac{230}{\sqrt{R^2 + (127.3239)^2}} \quad \text{i.e. } R^2 + (127.3239)^2 = 254^2$$

$$R = \mathbf{220.5314}\ \text{V and } |Z| = 254.6477\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{220.5314}{254.6477} = \mathbf{0.866\ leading}$$

$$P = VI\cos\phi = 230 \times 0.9032 \times 0.866 = \mathbf{179.9046\ W}$$

Example 5.36 : *The parallel circuit comprises respectively (i) A coil of resistance 20 ohm and inductance 0.07 H and (ii) A condenser of capacitance 60 microfarad in series with a resistance of 50 ohm. Calculate the current drawn from mains and power factor of the arrangement when connected across a 230 volts, 50 Hz supply. Draw the phasor diagram showing branch current and total current.* **[GU: Nov.-2001, July-2002]**

Solution : The circuit is shown in the Fig. 5.73.

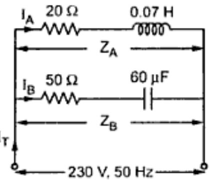


Fig. 5.73

$$X_{LA} = 2\pi f L_A = 2\pi \times 50 \times 0.07 = 21.9911\ \Omega$$

$$\therefore\quad Z_A = R_A + j X_{LA} = 20 + j21.9911\ \Omega$$

$$= 29.7255 \angle 47.7147°\ \Omega$$

$$X_{CB} = \frac{1}{2\pi fC_B} = \frac{1}{2\pi\times 50\times 60\times 10^{-6}} = 53.051\ \Omega$$

Example 5.40 : *A series circuit having a resistance of 10 Ω, and inductance of (1/2π) H and a variable capacitor is connected to 100 V, 50 Hz supply. Calculate the value of capacitor to form series resonance. Calculate resonant current, power and power factor.*
[GU : June/July-2003]

Solution : The circuit is shown in the Fig. 5.79.

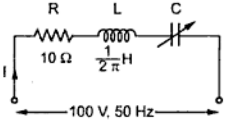


Fig. 5.79

For a series resonance, $X_L = X_C$

$$X_L = 2\pi fL = 2\pi \times 50 \times \frac{1}{2\pi} = 50\ \Omega$$

$$\therefore \quad X_C = 50\ \Omega = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi \times 50 \times 50} = \mathbf{63.6619\ \mu F}$$

$$I = \frac{V}{R} = \frac{100}{10} = \mathbf{10\ A}$$

$$P = I^2R = 10^2 \times 10 = \mathbf{1000\ W}$$

$\cos\phi = \mathbf{1}$ ... Circuit is purely resistive

Example 5.41 : *A choke coil of negligible resistance is connected across 230 V, 50 Hz, supply. It draws 5 amp from mains. Calculate the inductance of a coil. If the frequency of supply is reduced to 40 Hz, what will be the current drawn from mains?*
[GU : Dec.-2001, June-2004]

Solution : The coil is shown in the Fig. 5.80.

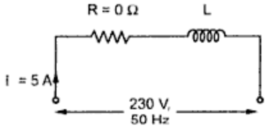


Fig. 5.80

$$|I| = \frac{|V|}{|X_L|} \quad \text{... } R = 0\ \Omega$$

$$\therefore \quad 5 = \frac{230}{|X_L|}$$

$$|X_L| = \frac{230}{5} = 46\ \Omega$$

But $|X_L| = 2\pi f L$ i.e. $L = \frac{46}{2\pi \times 50} = \mathbf{0.14642\ H.}$

Now new frequency is $f' = 40$ Hz

$$\therefore \quad |X'_L| = 2\pi f' L \times 40 \times 0.14642 = 36.8\ \Omega$$

$$\therefore \quad |I| = \frac{|V|}{|X'_L|} = \frac{230}{36.8} = \mathbf{6.25\ A}$$ ... New current

➠ **Example 5.42** : *A resistor and a capacitor are connected in series across 230 V, ac supply. The current taken by the circuit is 6 A for 50 Hz frequency. The current is reduced to 5 A, when the frequency of supply is decreased to 40 Hz. Determine the value of resistor.*

[GU : Dec.-2001, July-2004]

Solution : **Case 1 :** $f_1 = 50$ Hz, $|I_1| = 6$ A

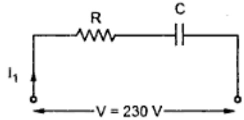


Fig. 5.81

$$Z_1 = R - j X_{C1}$$

$$\therefore \quad |I_1| = \frac{|V|}{|Z_1|}$$

$$\therefore \quad 6 = \frac{230}{\sqrt{R^2 + X_{C1}^2}}$$

$$\therefore \quad R^2 + X_{C1}^2 = 1469.4444 \qquad \text{... (1)}$$

Case 2 : $f_2 = 40$ Hz, $|I_2| = 5$ A, $Z_2 = R - j X_{C2}$

$$\therefore \quad |I_2| = \frac{V}{|Z_2|} \quad \text{i.e} \quad 5 = \frac{230}{\sqrt{R^2 + X_{C2}^2}}$$

$$\therefore \quad R^2 + X_{C2}^2 = 2116 \qquad \text{...(2)}$$

Subtracting equation (1) from (2) equation,

$$X_{C2}^2 - X_{C1}^2 = 2116 - 1469.4444 = 646.5556$$

$$\therefore \quad \left[\frac{1}{2\pi f_2 C}\right]^2 - \left[\frac{1}{2\pi f_1 C}\right]^2 = 646.5556$$

$$\therefore \quad \left[\frac{1}{2\pi \times 40C}\right]^2 - \left[\frac{1}{2\pi \times 50 \times C}\right]^2 = 646.5556$$

$$\therefore \quad \frac{1}{C^2} = [1.583143 \times 10^{-5} - 1.0131 \times 10^{-5}] = 646.5556$$

$$\therefore \quad C^2 = 8.8181 \times 10^{-9} \quad \text{i.e} \quad C = \mathbf{93.905\ \mu F}$$

Using equation (1), $R^2 = 1469.4444 - \left[\frac{1}{2\pi \times 50 \times 93.905 \times 10^{-6}}\right]^2 = 320.437$

$$\therefore \quad R = \mathbf{17.9\ \Omega}$$

Example 5.43 : *A coil of power factor 0.8 is in series with a 100 μF capacitor. The p.d. across the coil is equal to the p.d. across the capacitor when the combination is connected across a 50 Hz supply. Determine the resistance and inductance of the coil.*

[GU : June/July-2004]

Solution: The circuit is shown is the Fig. 5.82.

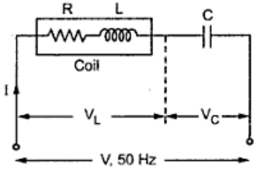


Fig. 5.82

$$\cos\phi_L = 0.8,\ C = 100\ \mu F$$

$$|V_L| = |V_C| \quad \text{i.e.} \quad |I|\,|Z_L| = |I|\,|X_C|$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 31.8309\ \Omega$$

$$\therefore\ |Z_L| = |X_C| = 31.8309\ \Omega$$

But $Z_L = R + j\,X_L$ hence $|Z_L| = \sqrt{R^2 + X_L^2}$

$$\therefore \quad \sqrt{R^2+X_L^2} = 31.8309 \quad \text{i.e.} \quad R^2 + X_L^2 = 1013.2062 \qquad ...(1)$$

$$\therefore \quad \cos\phi_L = \frac{R}{|Z_L|} = \frac{R}{\sqrt{R^2+X_L^2}} \text{ and } \cos\phi_L = 0.8 \text{ (given)}$$

$$\therefore \quad 0.8 = \frac{R}{\sqrt{R^2+X_L^2}} \quad \text{i.e.} \quad R^2 + X_L^2 = 1.5625\ R^2 \qquad ...(2)$$

Equating equations (1) and (2), $1.5625\ R^2 = 1013.2062$

$$\therefore \quad R^2 = 648.4519 \quad \text{i.e.} \quad R = \mathbf{25.4647\ \Omega}$$

Using equation (1), $X_L^2 = 1013.2062 - 648.4519 = 364.7543$

$$\therefore \quad X_L = 19.0985\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{19.0985}{2\pi \times 50} = \mathbf{0.0608\ H}$$

Example 5.44 : *Two impedances Z_1 and Z_2 are connected in parallel with Z_3 in series across 230 V, 1-phase a.c. supply. Calculate the equivalent impedance and current drawn from the supply . What is this power factor?* **[GU : June-1999, July-2004]**

Given : $Z_1 = 2 + j\,3.5\ \Omega$, $Z_2 = 1 + j\,3\ \Omega$, $Z_3 = 3 - j\,2\ \Omega$,

Solution : The circuit is shown in the Fig. 5.83.

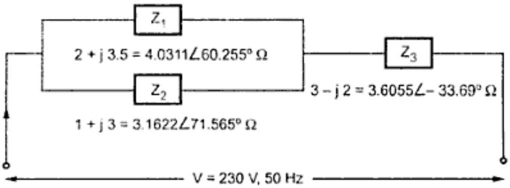


Fig. 5.83

$$Z_{eq} = (Z_1 || Z_2) + Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3$$

$$= \frac{4.0311\angle 60.255° \times 3.1622\angle 71.565°}{2 + j3.5 + 1 + j3} + 3 - j2 = \frac{12.747\angle 131.82°}{(3 + j6.5)} + 3 - j2$$

$$= \frac{12.747\angle 131.82°}{7.1589\angle 65.225°} + 3 - j2 = [1.7805\angle 66.595°] + 3 - j2$$

$$= 0.707 + j\,1.634 + 3 - j2 = \mathbf{3.707 - j\,0.366\ \Omega = 3.725\angle -5.638°\ \Omega}$$

$$I_T = \frac{V}{Z_{eq}} = \frac{230\angle 0°}{3.725\angle -5.638°} = \mathbf{61.75\angle + 5.638°\ A.}$$

$\therefore \quad \cos\phi_T = \cos(5.638°) =$ **0.9951 leading** ...Power factor

Example 5.45 : *A capacitor of 50 μF is connected in parallel with a coil that has a resistance of 20 Ω and inductance of 0.05 H. If this parallel combination is connected across 200 V, 50 Hz supply, calculate : (i) The line current (ii) Power factor and (iii) Power consumed.* **[GU : June-2000, Nov.-2004]**

Solution : The circuit is shown in the Fig. 5.84.

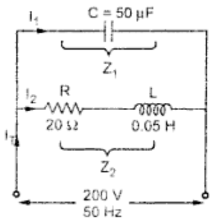


Fig. 5.84

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.6619\ \Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.05$$

$$= 15.708\ \Omega$$

$$\therefore \quad Z_1 = 0 - j X_C = 0 - j\,63.6619\Omega$$

$$= 63.6619\angle -90°\ \Omega$$

$$Z_2 = 20 + j\,15.708\ \Omega = 25.4311\angle 38.146°\ \Omega$$

Let supply voltage be reference,

$$V = 200\angle 0^\circ \text{ V}$$

$$\bar{I}_1 = \frac{V}{Z_1} = \frac{200\angle 0^\circ}{63.6619\angle -90^\circ} = 3.1415\angle +90^\circ \text{ A} = 0 + j\,3.1415 \text{ A}$$

$$\bar{I}_2 = \frac{V}{Z_2} = \frac{200\angle 0^\circ}{25.4311\angle 38.146^\circ} = 7.8643\angle -38.146^\circ\text{A} = 6.1847 - j4.857 \text{ A}$$

i) $\bar{I}_T = \bar{I}_1 + \bar{I}_2 = 0 + j\,3.1415 + 6.1847 - j4.857$

$= 6.1847 - j\,1.7155 \text{ A} = \mathbf{6.4182\angle -15.502^\circ\ A}$ ... Line current

ii) $\cos\phi_T = \cos(-15.502^\circ) = \mathbf{0.9636}$ **lagging** ...Power factor

iii) $P = V\,I_T\cos\phi_T = 200 \times 6.4182 \times 0.9636 = \mathbf{1236.94\ W}$

Example 5.46 : *A coil is connected across a variable frequency a.c. supply of 110 V. An ammeter in the circuit showed 15.6 A at 80 Hz and 19.7 A at 40 Hz. Find the value of parameters of the coil.* **[GU : May/June-2005]**

Solution : The coil is shown in the Fig. 5.85.

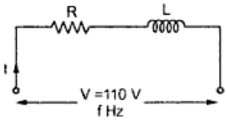


Fig.5.85

Case 1 : $f_1 = 80$ Hz, $|I_1| = 15.6$ A

$$Z_1 = R + j\,X_{L1} = R + j\,(2\pi f_1 L)$$

$$\therefore \quad |Z_1| = \sqrt{R^2 + (2\pi f_1 L)^2}$$

$$\therefore \quad |I_1| = \frac{|V|}{|Z_1|} \quad \text{i.e. } 15.6 = \frac{110}{\sqrt{R^2 + (2\pi \times 80 \times L)^2}}$$

$$\therefore \quad R^2 + (502.654L)^2 = 49.7205 \quad \text{... (1)}$$

Case 2 : $f_2 = 40$ Hz, $|I_2| = 19.7$ A

$$Z_2 = R + j\,X_{L2} = R + j\,(2\pi f_2 L)$$

$$|Z_2| = \sqrt{R^2 + (2\pi \times 40\,L)^2}$$

$$\therefore \quad |I_2| = \frac{|V|}{|Z_2|} \quad \text{i.e } 19.7 = \frac{110}{\sqrt{R^2 + (251.3274\,L)^2}}$$

$$\therefore \quad R^2 + (251.3274\,L)^2 = 31.1783 \quad \text{... (2)}$$

Solution : The circuit is shown in the Fig. 5.87.

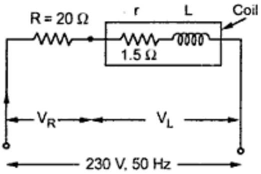


Fig. 5.87

$$Z_T = (R+r) + j\,X_L$$
$$= (20+1.5) + j\,X_L$$
$$= 21.5 + j\,X_L\ \Omega$$
$$|I| = \frac{|V|}{|Z_T|} \quad \text{and} \quad I = 8\ A$$

$$\therefore \quad 8 = \frac{230}{\sqrt{21.5^2 + X_L^2}} \quad \text{i.e.} \quad 21.5^2 + X_L^2 = \left(\frac{230}{8}\right)^2$$

$$\therefore \quad X_L^2 = 364.3125 \quad \text{i.e.} \quad X_L = 19.0869\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{19.0869}{2\pi \times 50} = 0.0607\ \mathbf{H}$$

$$\therefore \quad |V_L| = |I| \times |Z_L \quad \text{where} \quad |Z_L| = \sqrt{r^2 + X_L^2}$$

$$\therefore \quad |V_L| = 8 \times \sqrt{(1.5)^2 + (19.0869)^2} = \mathbf{153.166\ V} \quad \text{... Voltage across coil}$$

Example 5.49 : *Two capacitors of 80 μF and 50 μF are connected in series. Calculate the current when a 220 V, 50 Hz supply is applied across the circuit.* **[GU : July-2005]**

Solution: The circuit is shown in the Fig. 5.88.

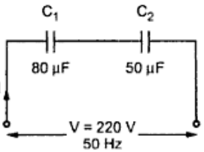


Fig. 5.88

$$|X_{C1}| = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.7887\ \Omega$$

$$|X_{C2}| = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.6619\ \Omega$$

In polar form, the reactances are,

$$X_{C1} = 0 - j\,39.7887\ \Omega, \quad X_{C2} = 0 - j\,63.6619\ \Omega$$

$$\therefore \quad X_C = X_{C1} + X_{C2} = 0 - j\,103.4506 = 103.4506 \angle -90^\circ\ \Omega$$

Rationalize,

$$Z_{eq} = \frac{(R_1^2 + j R_1(X_L - X_C) + X_L X_C)[2R_1 - j(X_L - X_C)]}{[2R_1 + j(X_L - X_C)][2R_1 - j(X_L - X_C)]}$$

$$= \frac{2R_1^3 + j2R_1^2(X_L - X_C) + 2R_1 X_L X_C - jR_1^2(X_L - X_C) + R_1(X_L - X_C)^2 - jX_L X_C(X_L - X_C)}{(2R_1)^2 + (X_L - X_C)^2}$$

$$= \frac{2R_1^3 + 2R_1 X_L X_C + R_1(X_L - X_C)^2}{(2R_1)^2 + (X_L - X_C)^2} + \frac{j(X_L - X_C)[2R_1^2 - R_1 - X_L X_C]}{(2R_1)^2 + (X_L - X_C)^2})$$

At resonance, the circuit must be purely resistive hence imaginary part must be zero.

$$\therefore \quad \frac{(X_L - X_C)(2R_1^2 - R_1 - X_L X_C)}{(2R_1)^2 + (X_L - X_C)^2} = 0 \qquad \text{i.e.} \qquad X_L - X_C = 0$$

$$\therefore \quad 2\pi f L = \frac{1}{2\pi f C} \quad \text{i.e.} \quad f^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore \quad \boxed{f = \frac{1}{2\pi\sqrt{LC}}} \qquad \text{... Resonance frequency.}$$

Example 5.53 : *A coil having resistance of 5 Ω and inductance of 0.0250 H is connected in series with a condenser of such a value that combination resonates at frequency of 50 Hz. Determine the value of condenser. If this combination is connected across 230 V, 25 Hz supply, calculate the current drawn, power and nature of power factor.*

[GU : May/June-2006]

Solution : The circuit is shown in the Fig. 5.92.

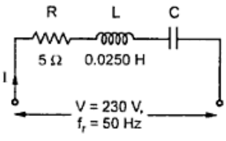


Fig. 5.92

$$f_r = 50 \text{ Hz} = \frac{1}{2\pi\sqrt{LC}}$$

$$50 = \frac{1}{2\pi\sqrt{0.0250 \times C}}$$

$$C = \mathbf{405.284\ \mu F.}$$

Now the combination is connected across **230 V, 25 Hz.**

$$\therefore \quad X_L = 2\pi f' L = 2\pi \times 25 \times 0.025 = 3.9269\ \Omega$$

$$\therefore \quad X_C = \frac{1}{2\pi f' C} = \frac{1}{2\pi \times 25 \times 405.284 \times 10^{-6}} = 15.708\ \Omega$$

$$Z = R + j(X_L - X_C) = 5 - j\ 11.781\ \Omega = 12.798 \angle -67^\circ\ \Omega$$

$$I = \frac{V}{Z} = \frac{230\angle 0^\circ}{12.798\angle -67^\circ} = \mathbf{17.9715 \angle +67^\circ\ A}$$

Solution : The branches are shown in the Fig. 5.95.

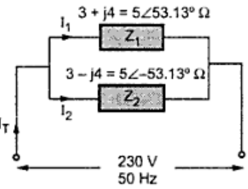


Fig. 5.95

i) As both are in parallel, the voltage across them is equal. Let supply voltage be the reference.

$\therefore \quad V = 230 \angle 0^\circ \text{ V}$

$\therefore \quad \bar{I}_1 = \dfrac{V}{Z_1} = \dfrac{230\angle 0^\circ}{5\angle 53.13^\circ}$

$= 46 \angle -53.13^\circ \text{ A} = 27.6 - j\,36.8 \text{ A}$

$\therefore \quad \bar{I}_2 = \dfrac{V}{Z_2} = \dfrac{230\angle 0^\circ}{5\angle -53.13^\circ} = 46\angle +53.13^\circ\text{A} = 27.6 + j36.8 \text{ A}$

$\therefore \quad \bar{I}_T = \bar{I}_1 + \bar{I}_2 = 27.6 - j\,36.8 + 27.6 + j\,36.8 = \mathbf{55.2 + j\,0\ A}$

$\therefore \quad \bar{I}_T = \mathbf{55.2 \angle 0^\circ\ A}$ ... Total current

ii) $\cos\phi_T = \cos 0^\circ = \mathbf{1}$ ...Power factor I_T

iii) The phasor diagram is shown in the Fig. 5.96.

Note that $\bar{I}_T = \bar{I}_1 + \bar{I}_2$

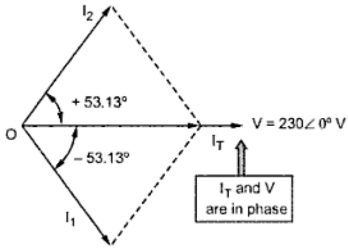


Fig. 5.96

➠ **Example 5.58 :** *The circuit having two impedances in parallel of $Z_1 = 8 + j\,15\ \Omega$ and $Z_2 = 6 - j8\ \Omega$, connected to a single phase a.c. supply and the current drawn is 10 A. Find each branch current, both in magnitude and phase and also the supply voltage*

[GTU : June-2009]

Solution : The circuit is shown in the Fig. 5.97.

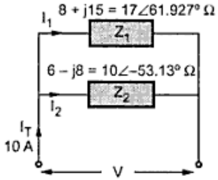


Fig. 5.97

Assume total current is reference.

$$\therefore \quad I_T = 10 \angle 0^\circ \text{ A}$$

Using current distribution rule,

$$\bar{I}_1 = I_T \times \frac{Z_2}{Z_1 + Z_2}$$

$$= 10\angle 0^\circ \times \frac{10\angle -53.13^\circ}{8 + j15 + 6 - j8}$$

$$= \frac{100\angle -53.13^\circ}{(14 + j7)} = \frac{100\angle -53.13^\circ}{15.6524\angle 26.565^\circ} = \mathbf{6.3887 \angle -79.695^\circ \text{ A}}$$

$$\bar{I}_2 = I_T \times \frac{Z_1}{Z_1 + Z_2} = 10\angle 0^\circ \times \frac{17\angle 61.927^\circ}{(14 + j7)}$$

$$= \frac{10\angle 0^\circ \times 17\angle 61.927^\circ}{15.6524\angle 26.565^\circ} = \mathbf{10.861 \angle 35.362^\circ \text{ A}}$$

Key Point: *Check that* $\bar{I}_T = \bar{I}_1 + \bar{I}_2$

$$\therefore \quad V = \overline{I_1 Z_1} = \overline{I_2 Z_2} = 6.3887\angle -79.695^\circ \times 17\angle 61.927^\circ$$

$$= \mathbf{108.6079 \angle -17.768^\circ \text{ V}}$$ ...Supply voltage

Example 5.59 : *A series R-L-C circuit having a resistance of 8 Ω, inductance of 80 mH and capacitance of 100 μF is connected across 150 V, 50 Hz supply. Calculate, a) Current b) Power factor and c) Voltage drops in the coil and capacitance.*

[GTU : June-2009]

Solution : The circuit is shown in the Fig. 5.98.

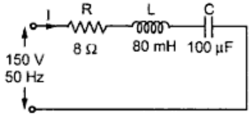


Fig. 5.98

$$V = 150 \text{ V}, \quad f = 50 \text{ Hz}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 80 \times 10^{-3}$$

$$= 25.1327 \ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 31.8309 \ \Omega$$

$$\therefore \quad Z = R + jX_L - jX_C = 8 + j\,25.1327 - j\,31.8309$$

$$= 8 - j\,6.6982 \ \Omega = 10.4338 \angle -39.9385^\circ \ \Omega$$

a) $$I = \frac{V}{Z} = \frac{150\angle 0^\circ}{10.4338\angle -39.9385^\circ} = \mathbf{14.3769 \angle 39.9385^\circ \text{ A}}$$

b) $\cos\phi = \cos(39.9385°) =$ **0.7667 leading** ... $X_C > X_L$

c) $|V_L| = I \times X_L = 14.3769 \times 25.1327 =$ **361.3303 V** ... **Only magnitude**

$|V_C| = I \times X_C = 14.3769 \times 31.8309 =$ **457.6296 V** ... **Only magnitude**

Review Questions

1. *Show that current through purely resistive circuit is in phase with the applied voltage.*
2. *Show that current through pure inductance lags applied voltage by 90°.*
3. *Show that current through pure capacitor leads applied voltage by 90°.*
4. *Obtain an expression for the average power consumed by an a.c. circuit in terms of r.m.s. values of voltage, current and power factor.*
5. *What is power factor ? Explain its significance.*
6. *Show that average power consumed by pure inductor and pure capacitor is zero.*
7. *Show that*
   i) Current lags voltage in R-L series circuit
   ii) Current leads voltage in R-C series circuit.
8. *Draw the phasor diagram for a series R-L-C circuit energized by a sinusoidal voltage showing the relative positions of the current, component voltage and the applied voltage for the following cases :-*
   (a) When $X_L > X_C$; (b) When $X_L < X_C$; and (c) When $X_L = X_C$.
9. *Explain the concept of admittance.*
10. *Define the following terms :*
    i) Admittance ii) Conductance iii) Susceptance.
11. *Derive the expressions to calculate conductance and susceptance.*
12. *What is resonance ? State the characteristics of series resonant circuit.*
13. *What is Q factor ? How it is related to bandwidth and selectivity ?*
14. *Derive the expression for the resonating frequency of series resonant circuit.*
15. *Derive the expression for the resonating frequency of parallel resonant circuit.*
16. *Derive the expressions for upper and lower cut-off frequencies for series circuit.*
17. *State the characteristics of parallel resonating circuit.*
18. *Compare series and parallel resonating circuits.*
19. *Two voltage sources have equal emf's and a phase difference of α. When they are connected in series, the voltage is 200 V, when one source is reversed the voltage is 15 V. Find their emf's and phase angle α.* **(Ans. : 100.26 V, 8.58°)**
20. *The current in series circuit R = 5 Ω and L = 30 mH lags the applied voltage by 80°. Determine the source frequency and the impedance Z.* **(Ans. : 150.47 Hz, 28.8 ∠ 80° Ω)**

26. *In parallel RC circuit shown in the Fig. 5.99,*
i_R = 15 cos (5000 t − 30°) *amperes. Obtain the current in capacitance.*

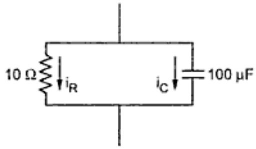


Fig. 5.99

(Ans. : 75 cos (5000 t + 60°) A)

27. *A choking coil and a pure resistor are connected in series across a supply of 230 V, 50 Hz. The voltage drop across the resistor is 100 V, and that across the choking coil is 150 V. Find graphically the voltage drop across the inductance and resistance of the choking coil. Hence find their values if the current is 1 A.* **(Ans. : 92.5 Ω, 0.366 H)**

28. *In a particular circuit, a voltage of 10 V at 25 Hz produces a current of 100 mA, while the same voltage at 75 Hz, produces 60 mA. Draw the circuit diagram and insert the values of the components.* **(Ans. : 88.277 Ω, 0.3 H)**

29. *In the network shown in the Fig. 5.100 , source frequency is 500 rad/sec and current I_2 is 1.25 ∠ 60° A.*

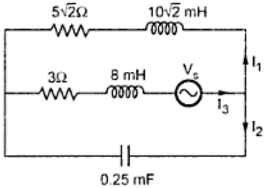


Fig. 5.100

Find,

a) Currents I_1 and I_3

b) Source voltage V_s

c) Source power factor

d) Draw phasor diagram

Find magnitudes and positions of all the quantities.

(Ans. : 1 ∠ −75° A, 0.8914 ∠ 7.51° A, 10.9 ∠ −5.86° V, 0.97 lead)

30. *In the network shown in the Fig. 5.101,*

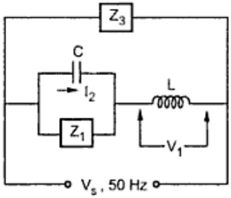


Fig. 5.101

$L = 0.153$ H, $C = 0.3183$ mF, $I_2 = 5\angle 60°$ A

and $V_1 = 250 \angle +90°$ V

when Z_3 *is not connected, find :*

1. Z_1 *and its components*
2. V_s *in the form* $V_m \sin(\omega t + \theta)$
3. *Power loss in the circuit*

Now calculate Z_3 *with its components so that overall p.f. of the circuit is unity without adding to the circuit power loss.*

Draw the phasor diagram.

(Ans. : 10 ∠ 30° Ω, 324.03 sin (100 (100πt +79.1°) V, 216.51 W, 68.2 μF)

University Questions

Q.1 *Discuss resonance in R-L-C series circuit. Explain how p.f.,* X_L *and R vary with frequency.* **[GTU : Dec.-2008, 10 Marks]**

Q.2 *Define power factor. What is the power factor of a pure inductor? Give the difference between active and reactive power.* **[GTU : Dec.-2008, 4 Marks]**

Q.3 *Give the comparison of series resonance and parallel resonance.* **[GTU : Mar.-2009, 5 Marks]**

Q.4 *Define the term 1) Reactance, 2) Inductive reactance and 3) Capacitive reactance and explain how it depends on frequency in an A.C. circuit.* **[GTU : June-2009, 7 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

6

Three Phase Circuits

6.1 Introduction

We have seen that a single phase a.c. voltage can be generated by rotating a turn made up of two conductors, in a magnetic field. Such an a.c. producing machine is called single turn alternator. But voltage produced by such a single turn is very less and not enough to supply practical loads. Hence number of turns are connected in series to form one winding in a practical alternator. Such a winding is called armature winding. The sum of the voltages induced in all the turns is now available as a single phase a.c. voltage, which is sufficient to drive the practical loads.

But in practice there are certain loads which require polyphase supply. **Phase** means **branch, circuit** or **winding** while **poly** means **many**. So such applications need a supply having many a.c. voltages present in it simultaneously. Such a system is called **polyphase system**.

To develop polyphase system, the armature winding in an alternator is divided into number of phases required. In each section, a separate a.c. voltage gets induced. So there are many independent a.c. voltages present equal to number of phases of armature winding. The various phases of armature winding are arranged in such a manner that the magnitudes and frequencies of all these voltages is same but they have definite phase difference with respect to each other. The phase difference depends on number of phases in which armature is divided. For example, if armature is divided into three coils then three separate a.c. voltages will be available having same magnitude and frequency but they will have a phase difference of $360°/3 = 120°$ with respect to each other. All three voltages with a phase difference of 120° are available to supply a three phase load. Such a supply system is called **three phase system**. Similarly by dividing armature into various number of phases, a 2 phase, 6 phase supply system also can be obtained. A phase difference between such voltages is $360°/n$ where n is number of phases.

Key Point: *In practice a three phase system is found to be more economical and it has certain advantages over other polyphase systems. Hence three phase system is very popularly used everywhere in practice.*

This chapter explains the various three phase circuits, the analysis of star and delta circuits and three phase active, reactive and apparent power. Let us see the advantages of three phase system first.

6.2 Advantages of Three Phase System

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 120° between each other. Such a three phase system has following advantages over single phase system.

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.
4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5) Three phase systems give steady output.
6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.
7) Power factor of single phase motors is poor than three phase motors of same rating.
8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard system throughout the world.

6.3 Generation of Three Phase Voltage System

It is already discussed that alternator consisting of one group of coils on armature produces one alternating voltage. But if armature coils are divided into three groups such that they are **displaced by the angle 120°** from each other, three separate alternating voltages get developed.

Consider armature of alternator divided into three groups as shown in the Fig. 6.1. The coils are named as R_1- R_2 , Y_1 -Y_2 and B_1- B_2 and mounted on same shaft. The ends of each coil are brought out through the slipring and brush arrangement to collect the induced e.m.f.

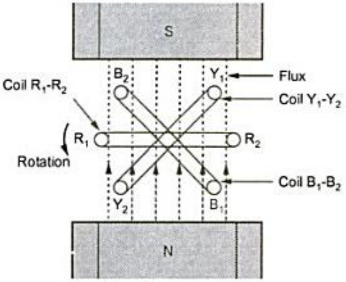


Fig. 6.1 Generation of 3 phase

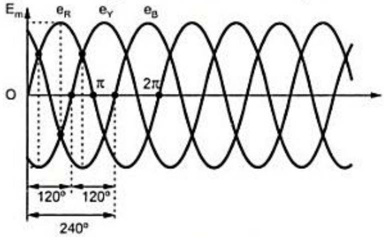


Fig. 6.2 Waveforms of 3 phase voltages

Let e_R, e_Y and e_B be the three independent voltages induced in coils R_1-R_2, Y_1-Y_2 and B_1-B_2 respectively. All are alternating voltages having same magnitude and frequency as they are rotated at uniform speed. All of them will be displaced from one other by 120°.

Suppose e_R is assumed to be the reference and is zero for the instant shown in the Fig. 6.2. At the same instant e_Y will be displaced by 120° from e_R and will follow e_R while e_B is ahead of e_R by angle 120°. i.e. if e_R is reference then e_Y will attain its maximum and minimum position 120° later than e_R and e_B will attain its maximum and minimum position 120° later than e_Y i.e. 120°+120° = 240° later with respect to e_R. All coils together represent three phase supply system.

The equations for the induced voltages are :

$$e_R = E_m \sin(\omega t)$$
$$e_Y = E_m \sin(\omega t - 120°)$$
$$e_B = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

The waveforms are shown in the Fig. 6.2.

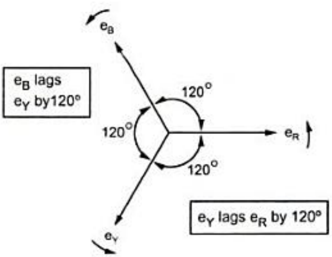


Fig. 6.3 Phase sequence

The phasor diagram of these voltages can be shown as in the Fig. 6.3. As phasors rotate in anticlockwise direction, we can say that e_Y lags e_R by 120° and e_B lags e_Y by 120°.

If we add three voltages vectorially, it can be observed that the sum of the three voltages at any instant is zero.

Mathematically this can be shown as :

$$e_R + e_Y + e_B$$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120°) + E_m \sin(\omega t + 120°)$$

$$= E_m [\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°]$$

$$= E_m [\sin \omega t + 2 \sin \omega t \cos 120°] = E_m \left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$$

$$\therefore \quad \boxed{\bar{e}_R + \bar{e}_Y + \bar{e}_B = 0}$$

Key Point: *The phasor addition of all the phase voltages at any instant in three phase system is always zero.*

6.4 Important Definitions Related to Three Phase System

Some terms are commonly used while analysing three phase system which are defined below

1) Symmetrical system : It is possible in polyphase system that magnitudes of different alternating voltages are different. But a three phase system in which the three voltages are of same magnitude and frequency and displaced from each other by 120° phase angle is defined as **symmetrical system**.

2) Phase sequence : The sequence in which the voltages in three phases reach their maximum positive values is called **phase-sequence**. Generally the phase sequence is R-Y-B.

Key Point: *The phase sequence is important in determining direction of rotation of a.c. motors, parallel operation of alternators etc.*

6.5 Three Phase Supply Connections

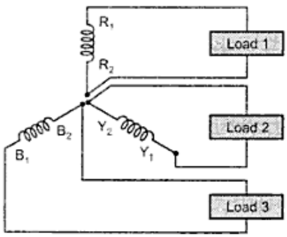


Fig. 6.4 Three phase connections

In single phase system, two wires are sufficient for transmitting voltage to the load i.e. phase and neutral. But in case of three phase system, two ends of each phase i.e. R_1-R_2, Y_1-Y_2 and B_1-B_2 are available to supply voltage to the load. If all six terminals are used independently to supply voltage to load as shown in the Fig. 6.4, then total six wires will be required and it will be very much costly.

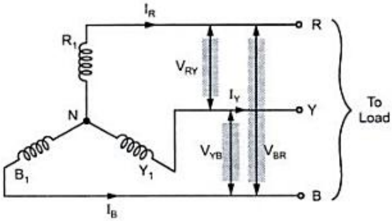


Fig. 6.7

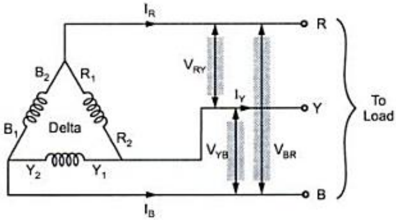


Fig. 6.8

Line voltages are denoted by V_L. These are V_{RY}, V_{YB} and V_{BR}. Line currents are denoted by I_L. These are I_R, I_Y and I_B.

Similarly for delta connected system we can show the line voltages and line currents as in the Fig. 6.8.

Line voltages V_L are V_{RY}, V_{BR}, V_{YB}.

While Line currents I_L are I_R, I_Y and I_B.

6.7 Concept of Phase Voltages and Phase Currents

Now to define the phase voltages and phase currents let us see the connections of the three phase load to the supply lines. Generally Red, Yellow and Blue coloured wires are used to differentiate three phases and hence the names given to three phases are R, Y and B.

The load can be connected in two ways, i) Star connection, ii) Delta connection

The **three phase load** is nothing but three different impedances connected together in star or delta fashion

i) Star connected load : There are three different impedances and are connected such that one end of each is connected together and other three are connected to supply terminalis R-Y-B. This is shown in the Fig. 6.9.

Key Point : *The voltage across any branch of the three phase load i.e. across Z_{ph1}, Z_{ph2} or Z_{ph3} is called **phase voltage** and current passing through any branch of the three phase load is called **phase current**.*

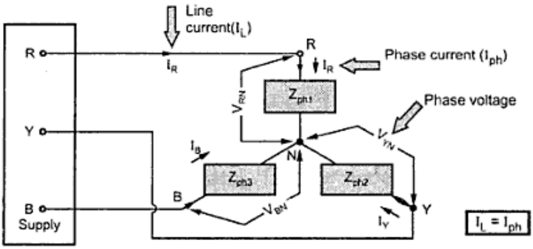


Fig. 6.9 Star connected load

In the diagram shown in the Fig. 6.9 V_{RN}, V_{YN} and V_{BN} are phase voltages while I_R, I_Y and I_B as shown in the Fig. 6.9 are phase currents. The phase voltages are denoted as V_{ph} while the phase currents are denoted as I_{ph}. Generally suffix N is not indicated for phase voltages in star connected load. So $V_{ph} = V_R = V_Y = V_B$

It can be seen from the Fig. 6.9 that,

$$I_{ph} = I_R = I_Y = I_B$$

But same are the currents flowing through the three lines also and hence defined as line currents. Thus we can conclude that for star connection $I_L = I_{ph}$.

$$\boxed{I_L = I_{ph} \quad \text{for star connection}}$$

ii) Delta connected load : If the three impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are connected such that starting end of one is connected to terminating end of other, to form a closed loop it is called delta connection of load. The junction points are connected to supply terminals R-Y-B. This is shown in the Fig. 6.10.

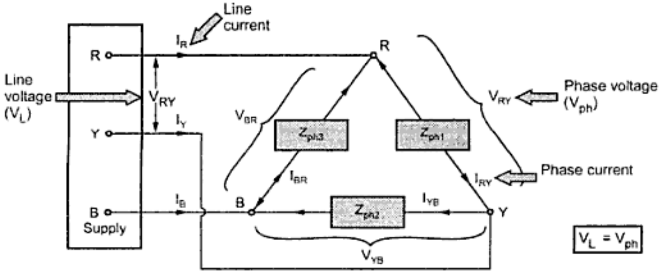


Fig. 6.10 Delta connected load

Key Point : *The line values do not decide the impedance angle or power factor angle.*

$$\phi = V_{ph} \wedge I_{ph} \neq V_L \wedge I_L$$

The complete phasor diagram for lagging power factor load is shown in the Fig. 6.13.

$$Z_{ph} = R_{ph} + j\, X_{Lph} = |Z_{ph}| \angle \phi \ \Omega$$

Each I_{ph} lags corresponding V_{ph} by angle ϕ

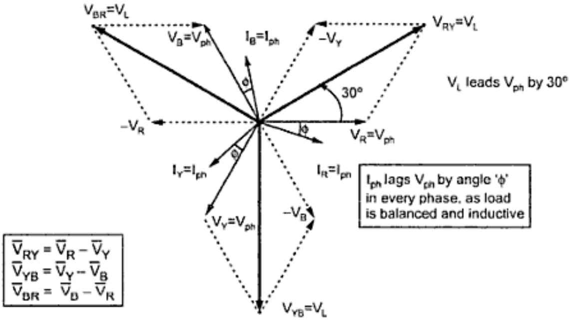


Fig. 6.13 Star and lagging p.f. load

All line voltages are also displaced by 120° from each other.

Key Point: *Every line voltage leads the respective phase voltage by 30°.*

Power : The power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph}\, I_{ph} \cos\phi$$

For balanced load, all phase powers are equal. Hence total three phase power consumed is,

$$P = 3P_{ph} = 3\, V_{ph}\, I_{ph} \cos\phi = 3\, \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$\therefore \quad P = \sqrt{3}\; V_L I_L \cos\phi$$

For star connection, to draw phasor diagram, use

$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y, \quad \bar{V}_{YB} = \bar{V}_Y - \bar{V}_B \quad \text{and} \quad \bar{V}_{BR} = \bar{V}_B - \bar{V}_R$$

Example 6.1 : *Three inductive coils each having resistance of 16 ohm and reactance of 12 ohm are connected in star across a 400 V, three-phase 50 Hz supply. Calculate :*
i) Line voltage, ii) Phase voltage, iii) Line current,
iv) Phase current, v) Power factor, vi) Power absorbed.
Draw phasor diagram

Solution : $R_{ph} = 16\ \Omega$, $X_L = 12\ \Omega$ per ph, Star connection $V_L = 400$ V

$$\therefore \quad Z_{ph} = R_{ph} + j X_L = 16 + j\,12\ \Omega = 20 \angle +36.86°\ \Omega$$

Using rectangular to polar conversion on calculator.

i) Line voltage $V_L = \mathbf{400\ V}$

ii) Phase voltage $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = \mathbf{230.94\ V}$ ...Star $V_L = \sqrt{3}\, V_{ph}$

iii) $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230.94}{20} = \mathbf{11.547\ A}$

For star connection, $I_L = I_{ph}$ i.e. Line current = **11.547 A**

iv) Phase current = **11.547 A**

v) Power factor $\cos\phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{16}{20} = \mathbf{0.8\ lagging}$

...Inductive hence lagging

vi) Power absorbed $P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 11.547 \times 0.8 = \mathbf{6400\ W}$

The phasor diagram can be shown as in the Fig. 6.14.

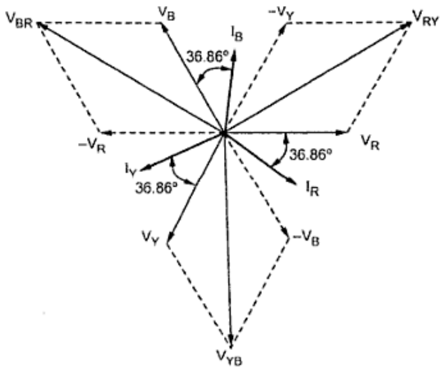


Fig. 6.14

6.9 Relations for Delta Connected Load

Consider the balanced delta connected load as shown in the Fig. 6.15.

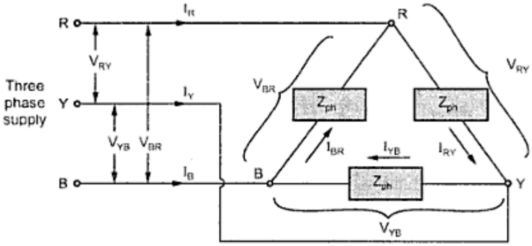


Fig. 6.15 Delta connected load

Line voltages $V_L = V_{RY} = V_{YB} = V_{BR}$

Line currents $I_L = I_R = I_Y = I_B$

Phase voltages $V_{ph} = V_{RY} = V_{YB} = V_{BR}$

Phase currents $I_{ph} = I_{RY} = I_{YB} = I_{BR}$

As seen earlier, $V_{ph} = V_L$ for delta connected load. To derive the relation between I_L and I_{ph}, apply the KCL at the node R of the load shown in the Fig. 6.15.

$$\sum I_{entering} = \sum I_{leaving} \text{ at node R}$$

$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\therefore \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \quad ...(1)$$

Applying KCL at node Y and B, we can write equations for line currents I_Y and I_B as,

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \quad ...(2)$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} \quad ...(3)$$

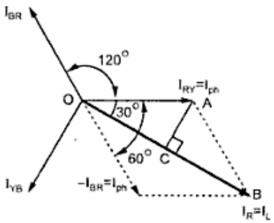


Fig. 6.16

The phasor diagram to obtain line current I_R by carrying out vector subtraction of phase currents I_{RY} and I_{YB} is shown in the Fig. 6.16.

The three phase currents are displaced from each other by 120°.

I_{BR} is reversed to get $-I_{BR}$ and then added to I_{RY} to get I_R.

Each I_{ph} lags respective V_{ph} by angle ϕ

Key Point: *Every line current lags the respective phase current by 30°.*

Power : Power consumed in each phase is single phase power given by,

$$\boxed{P_{ph} = V_{ph} I_{ph} \cos\phi}$$

Total power $P = 3P_{ph} = 3V_{ph}I_{ph}\cos\phi = 3V_L \frac{I_L}{\sqrt{3}}\cos\phi$

$\therefore$ $P = \sqrt{3}\, V_L I_L \cos\phi$

Key Point: *The expression for power is same but values of line currents are different in star and delta connected load which must be correctly determined to obtain power.*

6.10 Power Triangle for Three Phase Load

Total apparent power $S = 3 \times \text{Apparent power per phase}$

$\therefore$ $S = 3\, V_{ph} I_{ph} = 3\,\frac{V_L}{\sqrt{3}} I_L = 3\, V_L \frac{I_L}{\sqrt{3}}$

$\therefore$ $S = \sqrt{3}\, V_L I_L$ volt-amperes (VA) or kVA

Total active power $P = \sqrt{3}\, V_L I_L \cos\phi$ watts (W) or kW

Total reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi$ reactive volt amperes (VAR) or kVAR

Hence power triangle is as shown in the Fig. 6.18.

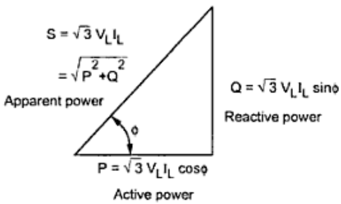


Fig. 6.18 Power triangle

6.11 Steps to Solve Problems on Three Phase Systems

While solving three phase problems :

1) Given supply voltages are always line voltages.
2) Determine phase voltage depending on whether load is star or delta connected.
3) Then determine phase current,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

4) Determine line current depending on whether load is star or delta connected.
5) ϕ is angle between V_{ph} and I_{ph}. Value can be obtained from given Z_{ph}.
6) The total power consumed is $\sqrt{3}\ V_L I_L \cos\phi$

Example 6.2 : *Three identical coils, each having resistance of 10 Ω and inductance of 0.03 H are connected in delta across a three-phase, 400 V, 50 Hz supply. Calculate :*

i) The phase current, ii) The line current, iii) The total power consumed, iv) p.f. and p.f. angle. Draw a neat phasor diagram.

Solution : $R_{ph} = 10\ \Omega$, $L_{ph} = 0.03$ H,

$$\therefore \quad X_{Lph} = \pi f L = 2\pi \times 50 \times 0.03 = 9.425\ \Omega$$

$$\therefore \quad Z_{ph} = R_{ph} + j X_{Lph} = 13.7415 \angle 43.304^\circ\ \Omega = 10 + j\ 9.425\ \Omega$$

As connection is delta, $V_L = V_{ph}$ and $I_L = \sqrt{3}\ I_{ph}$

Now $V_L = 400$ V

i) Phase current : $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{13.7415} = \mathbf{29.1089\ A}$

ii) Line current : $I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 29.1089 = \mathbf{50.4181\ A}$

ii) Total power consumed : $P = \sqrt{3}\ V_L\ I_L\ \cos\phi = \sqrt{3} \times 400 \times 50.4181 \times \cos(43.304^\circ)$

$= \mathbf{25.42\ kW}$

iv) p.f. and p.f. angle : p.f. angle = 43.304°

and p.f. = cos (43.304°) = **0.7277 lagging**

6.13 About Wattmeter

It is a device which gives power reading, when connected in single phase or three phase system, directly in watts.

It consists of two coils.

i) Current Coil : This senses the current and always to be connected in series with the load. Similar to ammeter, the resistance of this coil is as small as possible and hence its cross-sectional area is large and it has less number of turns.

ii) Voltage Coil : This is also called **pressure coil**. This senses the voltage and always to be connected across the supply terminals. Similar to voltmeter, the resistance of this coil is very large and hence its cross-sectional area is small and it has large number of turns.

> **Key Point:** *It is important to note that wattmeter senses the angle between current phasor which is sensed by its current coil and voltage phasor which is sensed by its voltage coil.*

It will not read phase angle 'ϕ' all the time. It depends on how we connect its current and voltage coils in the system. As 'ϕ' is the angle between V_{ph} and I_{ph}, if wattmeter has to sense this, its current coil must carry phase current I_{ph} and its voltage coil must sense phase voltage V_{ph}. In general if I_c is the current through its current coil (may be phase or line depends on its connection) and V_{pc} is voltage across its pressure coil (may be phase or line depends on its connection) then wattmeter reading is,

$$W = V_{pc} \times I_c \times \cos\left(I_c \wedge V_{pc}\right) \text{ watts}$$

Angle between V_{pc} and I_c is to be decided from the phasor diagram.

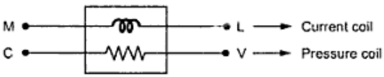


Fig. 6.20

If $I_c = I_{ph}$ and $V_{pc} = V_{ph}$ then $I_c \wedge V_{pc} = I_{ph} \wedge V_{ph} = \phi$ and then only wattmeter reads per phase power which is $V_{ph} I_{ph} \cos\phi$

A wattmeter can be represented symbolically as shown in Fig. 6.20.

The terminologies used to denote current and pressure coil are,

M = From mains, L = To load = For current coil

C = Common, V = Voltage = For voltage coil

6.14 Examples of Wattmeter Connections and Corresponding Readings

Case i) Consider delta connected inductive load and wattmeter connected as shown to measure power.

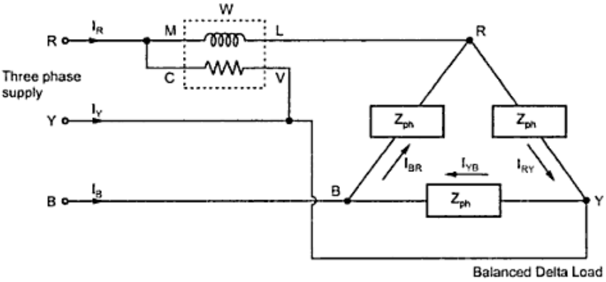


Fig. 6.21

Find the reading on it. Will it measure phase power ? Let us find it out.

For wattmeter : $I_c = I_R = I_L$ = Line current

while $V_{pc} = V_{RY} = V_L = V_{ph}$ as delta load

$$\therefore \quad W = V_{pc} I_c \cos(V_{pc} \hat{} I_c)$$

$$= V_{RY} \cdot I_R \cdot \cos(V_{RY} \hat{} I_R)$$

Now $I_R = I_L$ and $V_{RY} = V_L = V_{ph}$

Hence angle between V_{RY} and I_R is not 'ϕ'.

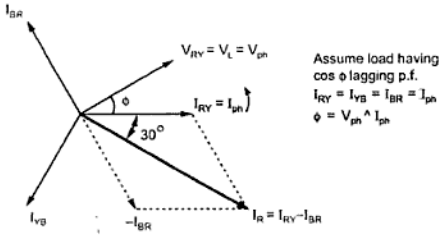


Fig. 6.22

To find $V_{RY} \hat{} I_R$ let us draw phasor diagram as shown in the Fig. 6.22.

For delta connected load, $\overline{I_R} = \overline{I_{RY}} - \overline{I_{BR}}$

Phase current I_{RY} lags phase voltage V_{RY} assuming that load p.f. is cos ϕ lagging.

From phasor diagram it is clear that,

$$V_{RY} \wedge I_R = (30+\phi)^\circ$$

$$\therefore \quad W = V_{RY} \times I_R \times \cos(30+\phi)$$

$$\therefore \quad \boxed{W = V_L I_L \cos(30+\phi)\ W}$$

This is not a phase power reading.

Case ii) Now let us shift the same wattmeter in such a way that it has to read phase power $V_{ph}\ I_{ph} \cos\phi$. For this $I_c = I_{ph} = I_{RY}$ or I_{YB} or I_{BR} and $V_{pc} = V_{ph} = V_L = V_{RY}$ or V_{YB} or V_{BR}. Accordingly wattmeter coils must be connected such that $I_c = I_{ph}$ and $V_{pc} = V_{ph}$, as $I_{ph} \wedge V_{ph} = \phi$ and then it will read $V_{ph}\ I_{ph} \cos\phi$ which is phase power. The connections can be shown as in the Fig. 6.23.

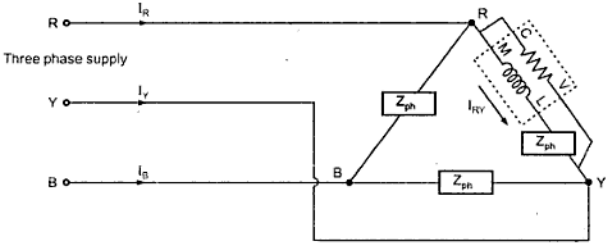


Fig. 6.23

For wattmeter : $I_c = I_{ph} = I_{RY}$ and $V_{pc} = V_{RY} = V_{ph}$

$\therefore$ Wattmeter will read, $W = I_c V_{pc} \cos(I_c \wedge V_{pc}) = I_{RY}\ V_{RY} \cos(I_{RY} \wedge V_{RY})$

$$\therefore \quad \boxed{W = I_{ph}\ V_{ph} \cos\phi = \textbf{phase power}}$$

Key Point: *Wattmeter will not always measure phase power. Its reading depends on the angle between current sensed by its current coil and voltage sensed by its pressure coil. This depends on its connections in the circuit.*

Three wattmeters measuring power in three phases may be connected to measure power. But connecting wattmeter to measure phase power is not always possible if neutral point of star connected load is not available out side. Similarly in delta connected load to measure phase current it is necessary to open delta load to insert current coil of the wattmeter is discussed above, which is not practicable. The best method of measuring power whether load is star or delta connected, balanced or unbalanced, neutral is available or not is, using only two wattmeters. The method is called Two Wattmeter Method.

6.15 Two Wattmeter Method

Method of Connection :

The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeter is connected between its own current coil terminal and the line without a current coil. For example, the current coils are inserted in the lines R and Y then the pressure coils are connected between R – B for one wattmeter and Y– B for other wattmeter, as shown in the Fig. 6.23.

The connections are same for star or delta connected load. It can be shown that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power dissipated in the three phase circuit.

If W_1 and W_2 are the two wattmeter readings then total power

$$W = W_1 + W_2 = \text{Three phase power}$$

6.15.1 Proof of Two Wattmeter Method for Star Connected Load

Consider star connected load and two wattmeters connected as shown in the Fig. 6.24.

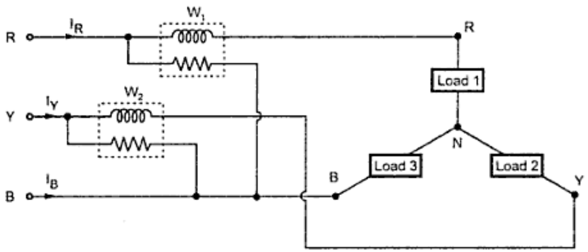


Fig. 6.24

Let us consider the r.m.s. values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.

$$W_1 = I_R \times V_{RB} \times \cos\left(I_R \,^\wedge V_{RB}\right)$$

$$W_2 = I_Y \times V_{YB} \times \cos\left(I_Y \,^\wedge V_{YB}\right)$$

To find angle between (I_R and V_{RB}) and (I_Y and V_{YB}) let us draw phasor diagram. (Assuming load p.f. be cos ϕ lagging)

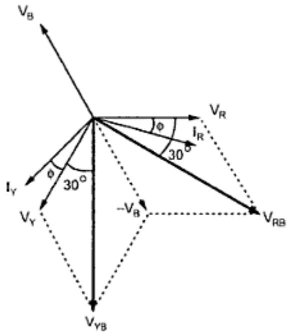


Fig. 6.25

$$\bar{V}_{RB} = \bar{V}_R - \bar{V}_B$$

and $$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$$

$$V_R \wedge I_R = \phi \text{ and } V_Y \wedge I_Y = \phi$$

$$V_R = V_Y = V_B = V_{ph}$$

and $$V_{RB} = V_{YB} = V_L$$

$$I_R = I_Y = I_L = I_{ph}$$

From Fig. 6.25, $I_R \wedge V_{RB} = 30 - \phi$

and $$I_Y \wedge V_{YB} = 30 + \phi$$

∴ $$W_1 = I_R V_{RB} \cos(30-\phi)$$

∴ $$\boxed{W_1 = V_L I_L \cos(30-\phi)}$$

and $$W_2 = I_Y V_{YB} \cos(30+\phi)$$

$$\boxed{W_2 = V_L I_L \cos(30+\phi)}$$

∴ $$W_1 + W_2 = V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$\therefore \quad 0.9 = \frac{15 \times 735.5}{P_{in}}$ as 1 H.P. = 735.5 watts

$\therefore \quad P_{in} = 12258.33 \text{ W}$

But $\quad P_{in} = \sqrt{3} V_L I_L \cos\phi$

$\therefore \quad 12258.33 = \sqrt{3} V_L I_L \times 0.8$

$\therefore \quad V_L I_L = 8846.9^{\cdot}$ and $\phi = \cos^{-1} 0.8 = 36.86°$

$\therefore \quad W_1 = V_L I_L \cos(30-\phi) = 8846.69 \cos(30-36.86)$

$= \mathbf{8783.1737 \text{ watts}}$

and $\quad W_2 = V_L I_L \cos(30+\phi) = 8846.69 \cos(30+36.86)$

$= \mathbf{3476.569 \text{ watts}}$

6.16 Power Factor Calculation by Two Wattmeter Method

In case of balanced load, the p.f. can be calculated from W_1 and W_2 readings.

For balanced, lagging p.f. load, $W_1 = V_L I_L \cos(30-\phi)$

$$W_2 = V_L I_L \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad \ldots(1)$$

$$W_1 - W_2 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi]$$

$$= V_L I_L [2 \sin 30 \sin\phi] = V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right]$$

$$\therefore \quad W_1 - W_2 = V_L I_L \sin(\phi) \quad \ldots(2)$$

Taking ratio of (1) and (2),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\therefore \quad \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\therefore \quad \phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\therefore \quad \boxed{\text{p.f. } \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}}$$

For **leading p.f.** we get **tan ϕ negative**. But cosine of negative angle is positive.

Key Point: *So cos ϕ is always positive but its nature must be determined by observing sign of tan ϕ.*

6.17 Effect of P.F. on Wattmeter Readings

For a lagging p.f.
$$W_1 = V_L I_L \cos(30-\phi)$$
$$W_2 = V_L I_L \cos(30+\phi)$$

Consider different cases,

Case i) $\cos\phi = 0 \quad \phi = 90°$

$$\therefore \quad W_1 = V_L I_L \cos(30-90) = +\frac{1}{2} V_L I_L$$
$$W_2 = V_L I_L \cos(30+90) = -\frac{1}{2} V_L I_L$$

i.e. $$W_1 + W_2 = 0$$
$$|W_1| = |W_2| \qquad \text{but} \quad W_2 = -W_1$$

Note : Wattmeter can not show negative reading as it has only positive scale. Indication of negative reading is that pointer tries to deflect in negative direction i.e. to the left of zero. In such case, reading can be converted to positive by interchanging either pressure coil connections i.e. $(C \leftrightarrow V)$ or by interchanging current coil connections $(M \leftrightarrow L)$. Remember that interchanging connections of both the coils will have no effect on the wattmeter reading.

Key Point: *Such a reading obtained by interchanging connections of either of the two coils will be positive on wattmeter but must be taken as negative for calculations.*

In the case discussed above W_1 will show positive reading with normal connections while W_2 will try to deflect in negative direction and hence W_2 reading must be obtained by reversing connections of either of the two coils and must be taken as negative.

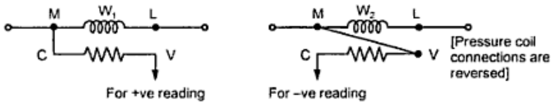


Fig. 6.29 Negative reading on wattmeter

So on wattmeter $W_1 = W_2$ but W_2 must be taken as negative as this reading will be obtained by reversing connections of any one coil.

Case ii) $\cos\phi = 0.5, \ \phi = 60°$

$$\therefore \quad W_1 = V_L I_L \cos(30-60) = V_L I_L \cos 30$$

$$= \text{positive}$$

$$W_2 = V_L I_L \cos(30+60) = 0$$

$$\therefore \quad W_1 + W_2 = W_1 = \text{total power.}$$

One wattmeter shows zero reading for $\cos\phi = 0.5$. For all power factors between 0 to 0.5 W_2 shows negative and W_1 shows positive, **for lagging p.f.**

Case iii) $\cos\phi = 1, \ \phi = 0°$

$$W_1 = V_L I_L \cos(30+0) = V_L I_L \cos 30 = +\text{ve}$$

$$W_2 = V_L I_L \cos(30-0) = V_L I_L \cos 30 = +\text{ve}$$

∴ Both W_1 and W_2 are equal and positive. For all power factors between 0.5 to 1 both wattmeter gives +ve reading.

In short, the result can be summarised as,

Range of p.f.	Range of ' ϕ '	W_1 sign	W_2 sign	Remark
$\cos\phi = 0$	$\phi = 90°$	positive	negative	$\lvert W_1 \rvert = \lvert W_2 \rvert$
$0 \angle \cos\phi \angle 0.5$	$90° \angle \phi \angle 60°$	positive	negative	
$\cos\phi = 0.5$	$\phi = 60°$	positive	0	
$0.5 \angle \cos\phi \angle 1$	$60° \angle \phi \angle 0°$	positive	positive	
$\cos\phi = 1$	$\phi = 0°$	positive	positive	$W_1 = W_2$

Table 6.1

Key Point: *The Table 6.1 is applicable for lagging power factors but same table is applicable for leading power factors by interchanging columns of W_1 and W_2.*

The curve of p.f. against K is shown in the Fig. 6.30 where,

$$K = \frac{\text{smaller reading}}{\text{larger reading}}$$

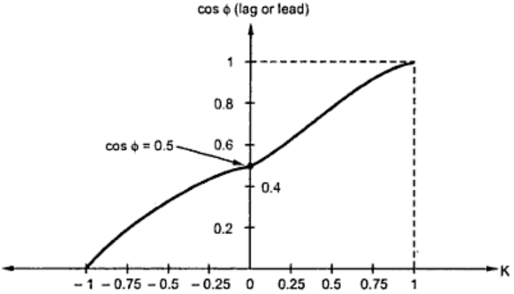


Fig. 6.30 Effect of p.f. on wattmeter readings

6.18 Reactive Volt-Amperes by Two Wattmeter Method

We have seen that,

$$W_1 - W_2 = V_L I_L \sin\phi$$

The total reactive volt-amperes for a 3 phase circuit is given by,

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\,(W_1 - W_2) \quad \text{VAR}$$

Thus reactive volt-amperes of a 3 phase circuit can be obtained by multiplying the difference of two wattmeter readings by $\sqrt{3}$.

So, S = apparent power $\sqrt{3}\, V_L I_L$ VA or kVA

P = active power = $\sqrt{3} V_L I_L \cos\phi = W_1 + W_2$ W or kW

$$\boxed{Q = \text{reactive power} = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\,(W_1 - W_2)} \quad \text{VAR or kVAR}$$

6.19 Advantages of Two Wattmeter Method

The various advantages of two wattmeter method are,

1. The method is applicable for balanced as well as unbalanced loads.
2. Neutral point for star connected load is not necessary to connect the wattmeters.
3. The delta connected load, need not be opened for connecting the wattmeters.
4. Only two wattmeters are sufficient to measure total 3 phase power.
5. If the load is balanced not only the power but power factor also can be determined.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 50\times 198\times 10^{-6}}$$

$$= 16.076\ \Omega$$

$\therefore\quad Z_{ph} = 6 - j\ 16.076\ \Omega = 17.159\ \angle -69.53^\circ\ \Omega$

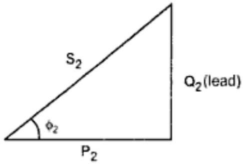


Fig. 6.31 (c)

$$V_{ph} = V_L = 415\ V$$

... delta connection

$\therefore\quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{17.159} = 24.1855\ A$

$\therefore\quad I_L = \sqrt{3}\ I_{ph} = \sqrt{3}\times 24.1855 = 41.89\ A$

$\therefore\quad P_2 = \sqrt{3}\ V_L\ I_L \cos\phi = \sqrt{3}\times 415\times 41.89\times\cos(-69.53^\circ)$

$= 10515.405\ W$

and $\quad Q_2 = \sqrt{3}\ V_L\ I_L \sin\phi = \sqrt{3}\times 415\times 41.89\times\sin(-69.53^\circ)$

$= -28209.257\ VAR$ ... – ve as leading

$\therefore\quad P_T = P_1 + P_2 = 16265.813\ W$

$Q_T = Q_1 + Q_2$

$= 4312.8065 - 28209.257$

$= -23896.4505\ VAR$

... Leading

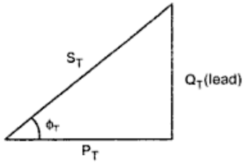


Fig. 6.31 (d)

$$\tan\phi_T = \frac{Q_T}{P_T} = \frac{23896.4505}{16265.813} = 1.4699$$

$\therefore\quad \phi_T = 55.77^\circ$ i.e. $\cos\phi_T = 0.5624$ leading

i) Total power consumed = P_T = **16265.813 W**

ii) $P_T = \sqrt{3}\ V_L\ I_L \cos\phi_T$

$\therefore\quad 16265.813 = \sqrt{3}\times 415\times I_L\times 0.5624$

$\therefore\quad I_L =$ **40.23 A** ...Total line current

iii) $Z_{ph1} = \frac{V_{ph}}{I_{ph}} = \frac{239.6}{10} = 23.96$ A and $\phi_1 = \cos^{-1} 0.8 = 36.86^\circ$

$$Z_{ph1} = \mathbf{23.96 \angle +36.86^\circ \Omega}$$

iv) p.f. of load B = cos (– 69.53) = **0.3497 leading**

Example 6.9 : *A balanced star connected load of (8+j 6) Ω/phase is connected to a 3 phase, 440 V supply. The line voltages are*

$V_{RY} = 440 \angle 0^\circ$ V, $V_{YB} = 440 \angle -120^\circ$ V, $V_{BR} = 440 \angle +120^\circ$ V

Find the phasor expressions for the line currents I_R, I_Y and I_B. Draw the phasor diagram.

Solution : Given : Star connection, $Z_{ph} = 8 + j\,6\ \Omega$, $V_L = 440$ V

To find : Phasor expressions for I_R, I_Y and I_B.

$$Z_{ph} = 8 + j\,6 = 10 \angle 36.8698^\circ\ \Omega$$

For star connection $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.034\ \text{V}$$

$$\therefore \quad \bar{I}_{ph} = \frac{\overline{V}_{ph}}{\overline{Z}_{ph}}$$ But in this V_{ph} is not reference.

This indicates that angle of V_{ph} is not zero.

This is clear from the fact that line voltages are given by,

$V_{RY} = 400 \angle 0^\circ$ V, $V_{YB} = 440 \angle -120^\circ$ V, $V_{BR} = 440 \angle +120^\circ$ V

Angle of V_{ph} can be calculated from phasor diagram as in the Fig. 6.32.

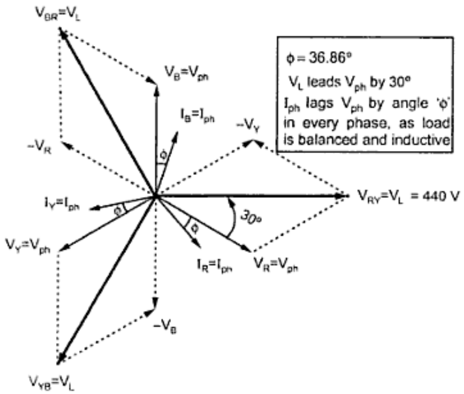


Fig. 6.32

$$I_{ph1} = \frac{\overline{V}_{ph1}}{\overline{Z}_{ph}} = \frac{254.034\angle-30^\circ}{10\angle 36.8698^\circ} = 25.4034 \angle\ 66.8698^\circ \text{ A}$$

$$I_{ph2} = \frac{\overline{V}_{ph2}}{\overline{Z}_{ph}} = \frac{25.034\angle-150^\circ}{10\angle 36.8698^\circ} = 25.4034 \angle\ -186.8698^\circ \text{ A}$$

$$I_{ph3} = \frac{\overline{V}_{ph3}}{\overline{Z}_{ph}} = \frac{254.034\angle+90^\circ}{10\angle 36.8698^\circ} = 25.4034 \angle\ 53.1302^\circ \text{ A}$$

Now for star, $I_{ph} = I_L$

$\therefore \quad I_R = I_{ph1} = I_L$

$\therefore \quad I_Y = I_{ph2} = I_L$

$\therefore \quad I_B = I_{ph3} = I_L$

Phasor expressions for I_R, I_Y, I_B

$$I_R = 25.4034 \angle -66.8698^\circ \text{ A}$$
$$I_Y = 25.4034 \angle 186.8698^\circ \text{ A}$$
$$I_B = 25.4034 \angle +53.1302^\circ \text{ A}$$

Angle between V_L and V_{ph} is always 30°, V_{ph} lags V_L by 30° for star connection.

Key Point: *In this problem V_L is taken as reference, generally V_{ph} is taken as reference.*

Example 6.10 : *In a 3 phase, 4 wire system, the line voltage is 400 V and purely resistive loads of 5 kW, 11 kW are connected between lines and neutral. Draw the circuit diagram, calculate current in each line, current in the neutral.*

Solution : $\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$ (vector addition)

cos ϕ for all loads is unity as all are resistive in nature.

$$V_{RN} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V} = V_{ph}$$

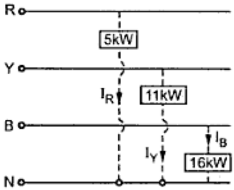


Fig. 6.33 Circuit diagram

$\therefore \quad P \quad Vph\ I_{ph} \cos\phi$

$\therefore \quad 5 \times 10^3 = 230.94 \times I_{ph} \times 1$

$\therefore \quad I_{ph} = I_R = \mathbf{21.65\ A}$

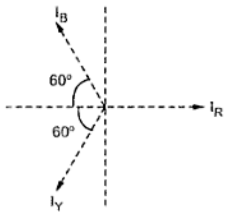


Fig. 6.34

Similarly $I_Y = \dfrac{11 \times 10^3}{230.94 \times 1}$

$= \mathbf{47.63\ A}$

and $I_B = \dfrac{16 \times 10^3}{230.94 \times 1}$

$= \mathbf{69.282\ A}$

Now these currents can be represented as in the Fig. 6.34.

To find $\bar{I}_R + \bar{I}_Y + \bar{I}_B$ let us find horizontal and vertical components of each current.

	Vertical	Horizontal
I_R	0	+ 21.65
I_Y	− 47.63 sin 60 = − 41.24	− 47.63 × cos 60 = − 23.815
I_B	+ 69.28 sin 60 = + 59.99	− 69.28 × cos 60 = − 34.64

$\therefore$ Total Horizontal Component $= 21.65 - 23.815 - 34.64 = -36.805$

Total Vertical Component $= 0 - 41.24 + 59.99$

$= +18.75$

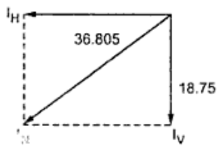


Fig. 6.35

$\therefore I_{N\ magnitude} = \sqrt{(I_H)^2 + (I_V)^2}$

$= \sqrt{(36.805)^2 + (18.75)^2}$

$= \mathbf{41.3058\ A}$

Example 6.11 : *A balanced three-phase star connected load of 100 kW takes a leading current of 80 Amp, when connected across 3−φ, 1100 volt, 50 Hz supply. Find the value of resistance / phase and capacitance / phase of load and p.f. of load. If the same load is connected in delta, calculate power consumed.*

Solution : $P_T = 100$ kW, $I_L = 80$ A, $V_L = 1100$ V, f = 50 Hz, star

$\therefore \quad I_{ph} = \dfrac{V_{ph}}{R_{ph}} = \dfrac{230.9401}{100} = 2.3094 \text{ A}$

$I_L = I_{ph} = 2.3094 \text{ A}$

$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 2.3094 \times 1 =$ **1600 W**

b) Delta : $V_{ph} = V_L = 400 \text{ V}$

$\therefore \quad I_{ph} = \dfrac{V_{ph}}{R_{ph}} = \dfrac{400}{100} = 4 \text{ A}$

$\therefore \quad I_L = \sqrt{3} I_{ph} = 4\sqrt{3} \text{ A}$

$\therefore \quad P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 4\sqrt{3} \times 1 =$ **4800 W**

If now one of the three resistances is open circuited,

a) Star :

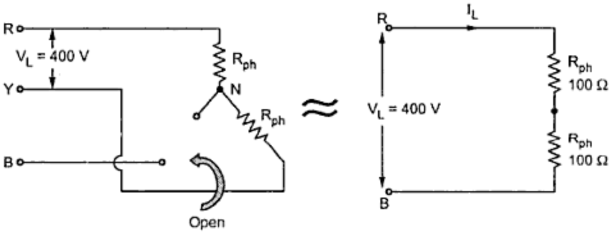


Fig. 6.36 (a)

$\therefore \quad I_L = \dfrac{V_L}{2R_{ph}} = \dfrac{400}{2 \times 100} =$ **2 A**

$\therefore \quad P = I_L^2 R_{ph} + I_L^2 R_{ph} = 4 \times 100 + 4 \times 100 =$ **800 W**

b) Data :

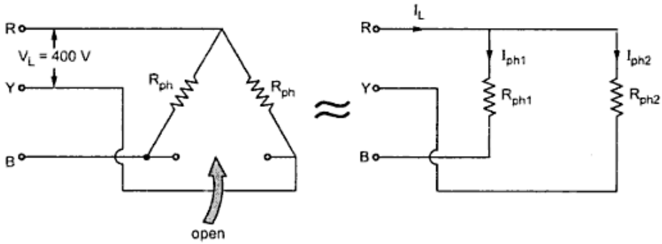


Fig. 6.36 (b)

$$I_{ph1} = \frac{V_{RB}}{R_{ph}} = \frac{400}{100} = 4 \text{ A}$$

$$I_{ph2} = \frac{V_{RY}}{R_{ph}} = \frac{400}{100} = 4 \text{ A}$$

$$\therefore \quad \text{Total } I_L = I_{ph1} + I_{ph2} = \mathbf{8\ A}$$

$$\therefore \quad P = I_{ph1}^2 \times R_{ph} + I_{ph2}^2 R_{ph}$$

$$= 16 \times 100 + 16 \times 100 = \mathbf{3200\ W}$$

Examples from G.U. and G.T.U. Papers

Example 6.13 : *Two wattmeters connected to measure 3-phase power for star connected read 5.185 kW and 10.37 kW. The line current 10 A. Calculate i) Line and phase voltages ii) Resistance and reactance per phase.* **[GU : 1999]**

Solution : $W_1 = 10.37$ kW, $W_2 = 5.185$ kW, $I_L = 10$ A , star

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(10.37 - 5.185)}{(5.185 + 10.37)}\right]\right\} = \cos\left\{\tan^{-1}[+0.5773]\right\}$$

$$= \cos(+30^\circ) = \mathbf{0.866\ lagging.}$$

$$P_T = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

$$\therefore \quad (10.37 + 5.185) \times 10^3 = \sqrt{3} \times V_L \times 10 \times 0.866$$

$$\therefore \quad V_L = \mathbf{1037.03\ V} \qquad \text{... Line voltage}$$

$$\therefore \quad V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1037.03}{\sqrt{3}} = \mathbf{598.7297\ V} \qquad \text{... Phase voltage}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{598.7297}{10} = 59.872\ \Omega \qquad \text{... } I_L = I_{ph} = 10 \text{ A}$$

$$\therefore \quad R_{ph} = Z_{ph}\cos\phi = 59.872 \times 0.866 = \mathbf{51.849\ \Omega}$$

$$X_{ph} = Z_{ph}\sin\phi = 59.872 \times \sin 30 = \mathbf{29.936\ \Omega}$$

Example 6.14 : *Three similar coils each of resistance 28 Ω and inductance of 0.7 H are connected in i) star and ii) delta. If the supply voltage is 230 V, 50 Hz, calculate the line current and the total power consumed.* **[GTU : Dec.-2008]**

Solution : $V_L = 230$ V, f = 50 Hz, L = 0.7 H, R = 28 Ω, $X_L = 2\pi fL = 219.911\ \Omega$

$$\therefore \quad Z_{ph} = R + j X_L = 28 + j\,219.911\ \Omega = 221.686 \angle 82.74°\ \Omega$$

i) Star connection hence $V_{ph} = \frac{V_L}{\sqrt{3}} = 132.7905$ V

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.7905 \angle 0°}{221.686 \angle 82.74°} = 0.6 \angle -82.74°\ A$$

$$\therefore \quad I_L = I_{ph} = \mathbf{0.6\ A}$$

$$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 230 \times 0.6 \times \cos(82.74°) = \mathbf{30.2058\ W}$$

ii) Delta connection hence $V_{ph} = V_L = 230$ V

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230 \angle 0°}{221.686 \angle 82.74°} = 1.0375 \angle -82.74°\ A$$

$$\therefore \quad I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 1.0375 = \mathbf{1.797\ A}$$

$$\therefore \quad P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 1.797 \times 230 \times \cos(82.74°) = \mathbf{90.466\ W}$$

Example 6.15 : *The power taken by a 3-phase, 415 V, delta connected motor is measured by two wattmeter method and readings of two wattmeters are 3 kW and 1 kW respectively. Estimate : i) power factor and ii) line current of motor.*

[GU : June-1998; Aug.-2001]

Solution : Delta, $V_L = 415$ V, $W_1 = 3$ kW, $W_2 = 1$ kW

i)
$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(3-1)}{(3+1)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 2}{4}\right]\right\} = \cos(40.8933°) = \mathbf{0.7559\ lagging.}$$

ii)
$$P_T = W_1 + W_2 = 3 + 1 = 4\text{ kW} \quad \text{but } P_T = \sqrt{3} V_L I_L \cos\phi$$

$$\therefore \quad 4 \times 10^3 = \sqrt{3} \times 415 \times I_L \times 0.7559$$

$$\therefore \quad I_L = \mathbf{7.3615\ A}$$

Example 6.16 : *A balanced inductive load connected in star across 415 V, 50 Hz, 3-phase supply takes line current of 25 A. The phase sequence is RYB. A single phase watt meter has its current coil connected in R line and its voltage coil across the line YB. With*

this connection, the reading is 8 kW. Draw the phasor diagram and determine.
I) the kW ii) the kVAR iii) the kVA iv) the p.f. of load. **[GU : May/June-2001]**

Solution : V_L = 415 V, I_L = 25 A, star

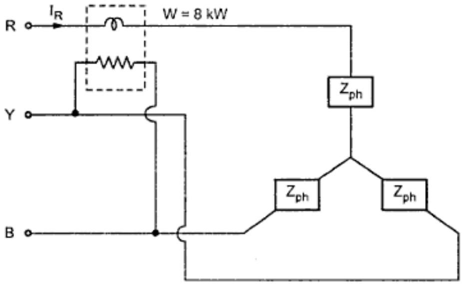


Fig. 6.37 (a)

For W,

$$I_{cc} = I_R, \quad V_{pc} = V_{YB}$$

$$\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$$

In each phase, I_{ph} lags V_{ph} by angle ϕ.

The phasor diagram is shown in the Fig. 6.37 (b).

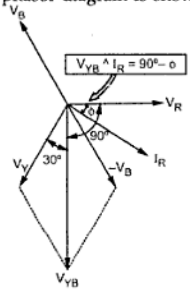


Fig. 6.37 (b)

$$\therefore \quad W = I_R V_{YB} \cos(I_R \wedge V_{YB})$$

$$8 \times 10^3 = I_L V_L \cos(90° - \phi)$$

$$\therefore \quad 8 \times 10^3 = 25 \times 415 \times \sin\phi$$

$$\therefore \quad \sin\phi = 0.77108$$

$$\therefore \quad \cos\phi = 0.6367$$

$$\phi = 50.4538°$$

i) $P_T = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 415 \times 25 \times 0.6367$ = **11.4415 kW.**

ii) $Q_T = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 415 \times 25 \times 0.77108$ = **13.8563 kVAR.**

iii) $Q = \sqrt{P_T^2 + Q_T^2} = \sqrt{(11.4415)^2 + (13.8563)^2}$ = **17.9695 kVA.**

iv) $\cos\phi$ = **0.6367 lagging**

Example 6.17 : *A 3-phase, 400 V, induction motor has a power factor of 0.5 lagging. Two wattmeters are connected to measure the input power. Find the readings of each wattmeter if the input power is 25 kW.* **[GU : May/June-2001]**

Solution : $V_L = 400$ V, $\cos\phi = 0.5$ lag, $P_T = 25$ kW

Now, $$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$\therefore \quad 0.5 = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$\therefore \quad 60° = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right] \qquad \text{but } P_T = W_1 + W_2 = 25 \text{ kW.}$$

$$\therefore \quad \tan 60° = \frac{\sqrt{3}(W_1 - W_2)}{25\times 103} \qquad \text{i.e. } W_1 - W_2 = 25000 \qquad ...(1)$$

While $$W_1 + W_2 = 25000 \qquad ...(2)$$

$$\therefore \quad 2W_1 = 50000 \qquad \text{i.e. } W_1 = \mathbf{25\ kW},\ W_2 = \mathbf{0\ kW.}$$

Example 6.18 : *A balanced star connected load is supplied from a symmetrical 3 phase, 440 volts, 50 Hz supply. The current in each phase is 20 Amps and lags behind its phase voltage by an angle of 30°.*
Calculate : i) Phase voltage ii) Resistance iii) Impedance iv) Reactance v) Power in 3 phase circuit vi) Power factor vii) Reading of two wattmeters.
[GU : June-1998, July-2002, Dec.-2002]

Solution : $V_L = 440$ V, $I_{ph} = 20$ A, lagging V_{ph} by 30°, star

i) $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = \mathbf{254.034\ V} \qquad \text{... Star}$$

Let V_{ph} be the reference i.e. $V_{ph} = 254.034 \angle 0°$ V

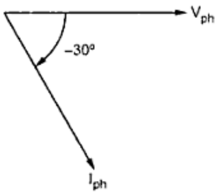


Fig. 6.38

$$I_{ph} = 20 \angle -30° \text{ A} \qquad \text{... Given}$$

$$\therefore \quad Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254.034\angle 0°}{20\angle -30°}$$

$$= \mathbf{12.7017 \angle +30°\ \Omega}$$

$$= \mathbf{11 + j\ 6.3508\ \Omega}$$

But $$Z_{ph} = R_{ph} + jX_{Lph}$$

$\therefore \quad P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3}\times 415\times 47.92\times 0.6 = \mathbf{20.666\ kW.}$

Example 6.23 : *Three coils each of inductance and resistance of 1 H and 100 ohms respectively are connected in delta and joined to a 3 phase, 440 V, 50 Hz a.c. supply. Calculate : i) Power factor of the circuit ii) Total power iii) Impedance per phase iv) Line current v) Phase current.* **[GU : June-2000, 2004; Dec.-2001, 2004]**

Solution : $L_{ph} = 1$ H, $R_{ph} = 100\ \Omega$, Delta, $V_L = 440$ V, f = 50 Hz

$$X_{Lph} = 2\pi f L_{ph} = 2\pi\times 50\times 1 = 314.1592\ \Omega$$

$$\therefore \quad Z_{ph} = R_{ph} + jX_{Lph} = 100 + j\,314.1592\ \Omega = 329.6907\angle 72.343^\circ\ \Omega$$

$$\therefore \quad \phi = 72.343^\circ$$

$$\therefore \quad \cos\phi = \cos(72.343^\circ) = \mathbf{0.3033\ lagging} \qquad \text{... Power factor}$$

$$V_{ph} = V_L = 440\ V \qquad \text{... Delta connection}$$

$$\therefore \quad |I_{ph}| = \frac{|V_{ph}|}{|Z_{ph}|} = \frac{440}{329.6907} = 1.3345\ A \qquad \text{... phase current}$$

$$\therefore \quad I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\times 1.3345 = 2.3115\ A \qquad \text{... Line current}$$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3}\times 440\times 2.3115\times 0.3033 = \mathbf{534.293\ W.}$$

Example 6.24 : *A balanced star-connected load each phase having a resistance of 10 Ω and inductive reactance of 30 Ω is connected to 400 V, 50 Hz supply. The phase sequence is RYB. Wattmeters connected to read total power have their current coil in the red phase and blue phase respectively. Calculate the reading on each wattmeter and draw a phasor diagram.* **[GU : July-2005]**

Solution : The circuit diagram is shown in the Fig. 6.39 (a).

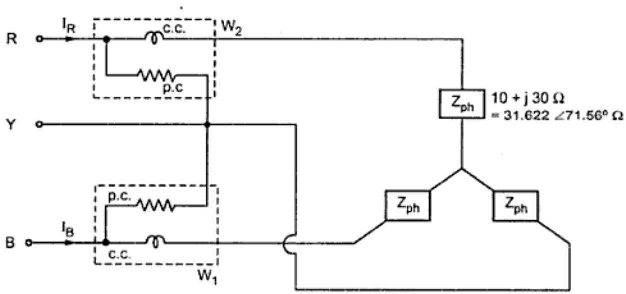


Fig. 6.39 (a)

For W_1, $V_{pc} = V_{BY} = \bar{V}_B - \bar{V}_Y, \; I_{cc} = I_B$

For W_2, $V_{pc} = V_{RY} = \bar{V}_R - \bar{V}_Y, \; I_{cc} = I_R$

In each phase, I_{ph} lags V_{ph} by angle 71.56°. The phasor diagram is shown in the Fig. 6.39(b).

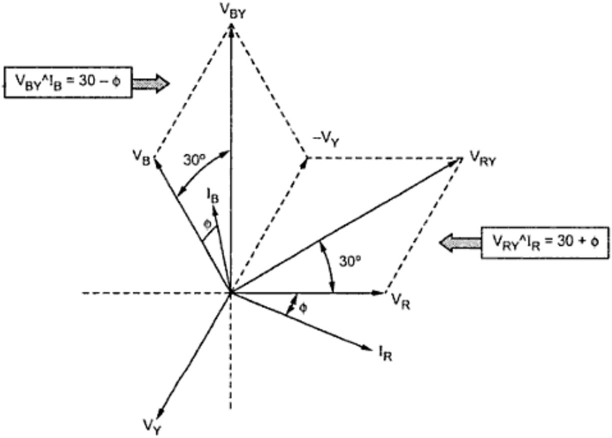


Fig. 6.39 (b)

$$W_1 = V_{BY} I_B \cos(V_{BY} \hat{} I_B) = V_L I_L \cos(30 - \phi)$$

$$W_2 = V_{RY} I_R \cos(V_{RY} \hat{} I_R) = V_L I_L \cos(30 + \phi)$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9401 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.9401}{31.622} = 7.3031 \text{ A} = I_L$$

$$\therefore \quad W_1 = 400 \times 7.3031 \times \cos(30° - 71.56°) = \mathbf{2185.8511 \; W}$$

$$\therefore \quad W_2 = 400 \times 7.3031 \times \cos(30° + 71.56°) = \mathbf{-585.398 \; W}$$

Key Point: *Cross check that* $P_T = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$.

Example 6.25 : *A star connected three phase alternator supplies a delta connected load. The impedance of each load branch is 8+j6 ohm. The line voltage is 230 V. Determine : i) Current in the branch. ii) Power consumed by the load iii) Power factor of the load iv) Reactive power of the load.* **[GU : Nov.-2005]**

Solution : Delta connected load, $V_L = 230$ V

$$Z_{ph} = 8 + j6\ \Omega = 10 \angle 36.869^\circ\ \Omega,\quad \phi = 36.869^\circ$$

$$V_{ph} = V_L = 230 \angle 0^\circ \text{ V} \qquad \text{... Delta}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230 \angle 0^\circ}{10 \angle 36.869^\circ} = \mathbf{23 \angle -36.869^\circ\ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 23 = 39.8371 \text{ A}$$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 230 \times 39.8371 \times \cos(36.869^\circ)$$

$$= \mathbf{12.696\ kW} \qquad \text{... Active power}$$

$$Q_T = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3} \times 230 \times 39.8371 \times \sin(36.869^\circ)$$

$$= \mathbf{9.521\ kVAR} \qquad \text{... Reactive power}$$

$$\cos\phi = \cos(36.869^\circ) = \mathbf{0.8\ lagging} \qquad \text{... Power factor.}$$

Example 6.26 : *The input power to a three phase load is measured by two wattmeter method. The ratio of the readings of the two wattmeters connected for 3 phase balanced load is 4 : 1. The load is inductive. Find the load power factor.* **[GTU : March-2009]**

Solution : $W_1 : W_2$ is 4 : 1 hence $W_1 = 4\,W_2$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(4\,W_2 - W_2)}{(4\,W_2 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3} \times 3}{5}\right]\right\} = \cos\left\{\tan^{-1}[1.0392]\right\}$$

$$= \cos(46.1021^\circ) = \mathbf{0.6933\ lagging} \qquad \text{... Power factor}$$

Review Questions

1. *What are the advantages of a three phase system ?*
2. *Explain in short the three phase generation.*
3. *Define : Symmetrical system and phase sequence.*
4. *Explain the concept of balanced load.*

5. *Explain the difference between a line voltage and phase voltage, similarly line current and phase current.*
6. *Derive the relationship between a line current and a phase current and a line voltage and a phase voltage related to a star connected and delta connected load.*
7. *State the power relations for a three phase system and explain a power triangle.*
8. *A balanced 3 phase load connected in delta, draws a power of 10.44 kW at 200 V at a p.f. of 0.5 lead, find the values of the circuit elements and the reactive volt amperes drawn.*

   (Ans. : 2.8736 Ω, 639.17 μF, 18.0825 kVAR)
9. *A balanced 3 phase star connected load of 100 kW takes a leading current of 80 A, when connected across a 3 phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.*

   (Ans. : 5.208 Ω, 530.516 μF)
10. *Three similar choke coils are connected in star to a three phase supply. If the line current is 15 A, the total power consumed is 11 kW and the volt ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.*

    If these coils are now connected in delta calculate phase and line currents, active and reactive power. **(Ans. : 10.197 kVAR, 30.592 kVAR)**
11. *Three pure elements connected in star, draw -x kVAR. What will be the value of elements that will draw the same kVAR, when connected in delta across the same supply ?*

    (Ans. : Three times than original)
12. *A 3 ϕ, star connected source feeds 1500 kW at 0.85 p.f. lag to a balanced mesh connected load. Calculate the current, it's active and reactive components in each phase of the source and the load. The line voltage is 2.2 kV.*

    (Ans. : 463.112 A, 227.27 A, 140.91 A, 303.64 A, 244.06 A)
13. *For a balanced three phase wye connected load the phase voltage V_R is $100 \angle -45°$ and it draws a line current I_Y of $5 \angle +180°$. i) Find the complex impedance per phase ii) draw a power triangle and identify its all sides with magnitudes and appropriate units. Assume phase sequence R-Y-B.*

    (Ans. : $20 \angle 15° \Omega$)
14. *Each leg of a balanced delta connected load consists 7 ohms resistance in series with 4 ohms inductive reactance. Line-to-line voltages are : $E_{ab} = 2360 \angle 0° V$, $E_{bc} = 2360 \angle -120° V$, $E_{ca} = 2360 \angle +120° V$*

    Determine : a) Phase current I_{ab}, I_{bc} and I_{ca} (both magnitudes and phase)

    b) Each line current and its associated phase angle.

    c) Load power factor.

    d) Draw with the instruments, phasor diagram based on circuit diagram and clearly indicate on both circuit diagram and phasor diagram :

i) Line currents. *ii) Phase currents* *iii) Line voltages.*

iv) Phase voltages *v) Load phase angle*

e) Find the impedance per phase that draws the same power at the same power factor.

(Ans. : 292.724 $292.724\angle-29.74°A, 292.724\angle-149.74°A, 292.724\angle+90.26°A$, $507.012A, 0.8682 lag, 8.0622\angle-29.24°\Omega$)

15. *With the help of connection diagram and phasor diagram, show that two wattmeters are sufficient to measure active power in a three phase three wire system with,*

    i) balanced star connected load

    ii) balanced delta connected load.

16. *Explain how the power factor can be calculated by two wattmeter method.*

17. *Explain the effect of power factor on two wattmeter readings in a two wattmeter method.*

18. *Two wattmeter method is used to measure power in a 3-phase balanced load. Find the power factor if-*

    i) the two readings are equal and have the same sign,

    ii) the two readings are equal and have opposite sign.

    iii) the readings of one wattmeter is zero.

    iv) the reading of one wattmeter is half of the other wattmeter.

    (Ans. : (i) =1, (ii) = 0, (iii) = 0.5, (iv) = 0.8660)

19. *Find the power and power factor of the balanced circuit in which the wattmeter readings are 5 kW and 0.5 kW, the latter being obtain after the reversal of the current coil terminal of the wattmeter.*

    (Ans. : Power = 4.5 kW, Power factor = 0.4271 lagging)

20. *A 3-phase, 220 V , 50 Hz, 11.2 kW induction motor has a full load efficiency of 88 percent and draws a line current of 38 Amps under full load, when connected to 3-phase 220 V supply. Find the reading on two wattmeters connected in the circuit to measure the input to the motor. Determine also the power factor at which the motor is operating.*

    (Ans. : W_1 = 8357.0699 watts, W_2= 4370.2028 watts, 0.8789 lagging)

21. *A 10 HP, 3 phase Induction Motor having an efficiency of 90% and power factor of 0.87 is connected to a 400 V supply. When running on full load, determine the reading on each of the wattmeters connected to measure the input.* **(Ans. : 5423.026 W, 2749.219 W)**

22. *Two wattmeters are connected to measure the input power in a three phase, balanced star connected circuit. The two wattmeters W_1 and W_2 read + 2 kW and − 2 kW respectively. If the circuit is fed by three phase, 50 Hz, 400 volt balanced supply. Find (i) the total power input. (ii) power factor of the load (iii) line current (iv) phase current. (v) per phase power. (iv) if $W_1 = W_2 = +2$ kW, find the p.f. of load.* **(Ans. : 0 W, 0, 10 A, 5.773 A, 0 W, 1)**

23. *Two wattmeters are connected to measure total power in a balanced delta connected load Impedance of each branch of load is (6 + j8) ohms. If the load is connected to a 400 volt, 50 Hz, 3 phase supply, calculate : (i) Reading of the 2 wattmeters. (ii) Line current. (iii) Phase current (iv) Power factor. (v) Total reactive and active power.*

(Ans. : 25485.13 W, 3314.923 W, 69.282 A, 40 A, 0.6 lag, 38.46 kVAR)

University Questions

Q.1 *Prove that power in a 3-phase balanced circuit can be deduced from the readings of two wattmeters. Draw the relevant connection and phasor diagrams. Discuss the nature of power factor i) when two readings are equal and positive ii) when two readings are equal but opposite in sign iii) when one wattmeter reads zero.* **[GTU : Dec.-2008, 10 Marks]**

Q.2 *Write down the line value and phase value relationship of voltages and currents in 3 phase star and delta connected systems.* **[GTU : March-2009, 7 Marks]**

Q.3 *Derive an expression for the total power for a balanced 3 phase star OR delta connected load in terms of line voltage line current and power factor.* **[GTU : June-2009, 6 Marks]**

Note : The examples asked in the previous university papers are solved and included in the chapter.

❑❑❑

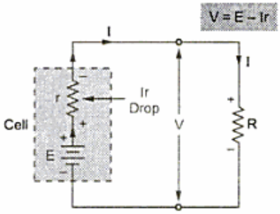


Fig. 7.3 Terminal voltage

3. Terminal voltage : When an external resistance is connected across the terminals of the cell, the current I flows through the circuit. There is voltage drop 'Ir' across the internal resistance of the cell. The cell e.m.f. E has to supply this drop. Hence practically the voltage available at the terminals of the cell is less than E by the amount equal to 'Ir'. This voltage is called the **terminal voltage** V. This is shown in the Fig. 7.3.

Mathematically, the terminal voltage is given by,

$$V = E - Ir$$

From external resistance side we can write,

$$V = IR$$

Key Point: *Practically internal resistance of the cell must be as small as possible.*

It can also be observed that on no load i.e. external resistance not connected, the open circuit terminal voltage is same as e.m.f. of the cell, as current I = 0.

$$\therefore \quad V = E \quad \text{... on no load i.e. open circuit}$$

7.4 Primary Cells

It is seen that the primary cell is that which is required to replace by new one when it is run down.

The oldest types of primary cell are simple voltaic cell, Daniell cell, Leclanche cell etc. Some commonly used primary cells are,

1. Dry cell [Zinc-Carbon]
2. Mercury cell
3. Zinc-Chloride cell
4. Lithium cell
5. Alkaline Zinc-mercury oxide cell

Let us discuss primary cell in detail.

7.4.1 Dry Zinc-Carbon Cell

This is most common type of dry cell. It is the type of Leclanche cell.

Negative electrode → Zinc cup lined with paper
Positive electrode → Centrally located carbon rod

The space between the paper and carbon rod is filled with a paste of sal ammoniac, zinc chloride, manganese dioxide and carbon dust.

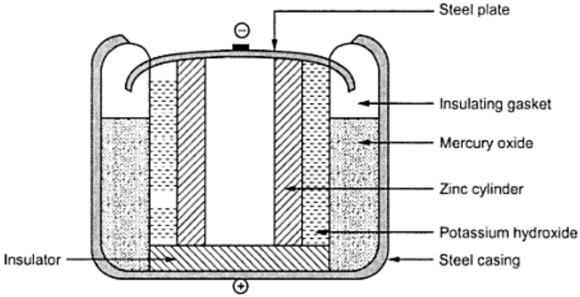


Fig. 7.5 Mercury Cell

The zinc electrode acting as negative terminal is made in the form of a hollow cylinder. The steel casing acts as a positive terminal. The layer of mercury oxide covers the electrolyte which is solution of KOH and ZnO. The cell is sealed with the help of insulating gasket.

7.4.2.1 Chemical Reaction

The net chemical reaction involved in the cell is,

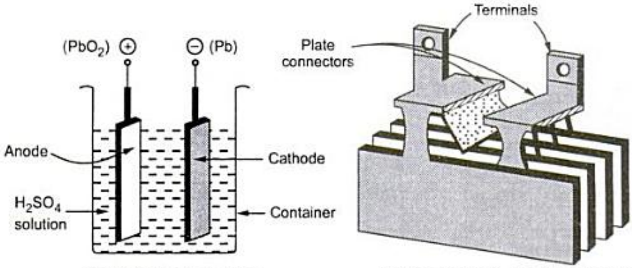
$$Zn + HgO \longrightarrow ZnO + Hg$$

7.4.2.2 Features of the Cell

1. The chemical reaction does not evolve any gas hence no polarisation.
2. The cell maintains its e.m.f. for longer time in working condition.
3. The terminal voltage is about 1.2 to 1.3 V.
4. It has long life.
5. It has high ratio of output energy to weight of about 90 – 100 Wh/kg.
6. Costlier than dry cell.
7. It has high energy to volume ratio of about 500 – 600 Wh/L.
8. It has high efficiency.
9. Good resistance to shocks and vibrations.
10. Disposal is difficulty due to presence of poisonous materials inside.

8. **Vent-plug :** These are made up of rubber and screwed to the cover of the cell. Its function is to allow the escape of gases and prevent escape of electroyte.

The Fig. 7.6 shows the construction of lead acid battery.



(a) Battery construction

(b) Assembly of – ve and + ve plates

Fig. 7.6 Construction of lead acid battery

The various plates are welded to the plate connectors. The plates are immersed in H_2SO_4 solution. Each plate is a grid or frame work. Except some special assemblies, wide space between the plates is provided. In an alternate assembly of plates, the **negative plate is one more in number than positive**. So all the positive plates can work on both the sides.

7.6.1 Functions of Separators

The **separators** used have the following functions in the construction of lead acid battery :

1. Acting as mechanical spacer preventing the plates to come in contact with each other.
2. Prevent the growth of lead trees which may be formed on the negative plates and due to heavy accumulation may reach to positive plate to short circuit the cell.
3. Help in preventing the plates from shedding of the active material. The separators must be mechanically strong and must be porous to allow diffusion of the electrolyte through them.

7.6.2 Chemical Action in Lead Acid Battery

The chemical action in the lead acid battery can be divided into three processes :

1. First charging 2. Discharging 3. Recharging

Let us discuss these processes in detail.

1. First charging : When the current is passed for the first time through electrolyte, the H_2O in the electrolyte is electrolysed as,

$$H_2O \longrightarrow 2\,H^+ + O^-$$

The hydrogen ions as positively charged get attracted towards one of the electrodes which acts as cathode (negative). The hydrogen does not combine with lead and hence cathode retains its original state and colour.

The oxygen ion as negatively charged gets attracted towards the other lead plate which acts as anode (positive). But this oxygen chemically combines with the lead (Pb) to form lead peroxide (PbO_2). Due to the formation of lead peroxide the anode becomes dark brown in colour.

Thus anode is dark brown due to the layer of lead peroxide deposited on it while the cathode is spongy lead electrode.

So there exists a potential difference between the positive anode and the negative cathode which can be used to drive the external circuit. The electrical energy obtained from chemical reaction is drawn out of the battery to the external circuit, which is called discharging.

2. Discharging : When the external supply is disconnected and a resistance is connected across the anode and cathode then current flows through the resistance, drawing an electrical energy from the battery. This is **discharging**. The direction of current is opposite to the direction of current at the time of first charging. The discharging is shown in the Fig. 7.7.

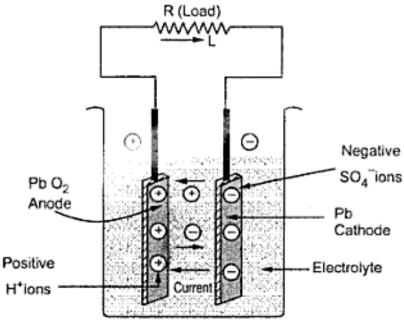


Fig. 7.7 Discharging

During the discharging, the directions of the ions are reversed. The H^+ ions now move towards anode and the SO_4^{--} ions move towards cathode.

This is because H_2SO_4 decomposes as,

$$H_2SO_4 \rightarrow 2H^+ + SO_4^{--}$$

At the anode, the hydrogen ions become free atoms and react with lead peroxide alongwith the H_2SO_4 and ultimately lead sulphate $PbSO_4$ results as,

$$PbO_2 + H_2SO_4 + 2H \rightarrow PbSO_4 + 2H_2O \qquad \text{... At Anode}$$

At the cathode, each SO_4^{--} ion become free SO_4 which reacts with the metallic lead to get lead sulphate.

$$Pb + SO_4 \rightarrow PbSO_4 \qquad \text{... At Cathode}$$

Thus discharging results into formation of whitish lead sulphate on both the electrodes.

3. Recharging : The cell provides the discharge current for limited time and it is necessary to recharge it after regular time interval. Again an e.m.f. is injected through the cell terminals with the help of an external supply.

The charging is shown in the Fig. 7.8

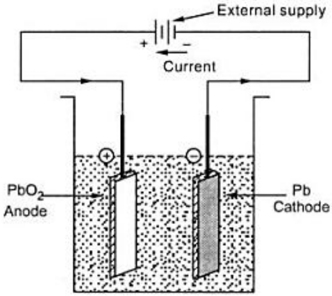


Fig. 7.8 Recharging the lead acid battery

Due to this recharging current flows and following reactions take place,

At the cathode, $PbSO_4 + 2H \rightarrow Pb + H_2SO_4$

At the anode, $PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4$

Thus the PbO_2 gets formed at anode while lead sulphate layer on the cathode is reduced and gets converted to grey metallic lead. So the strength of the cell is regained. It can be seen from the reaction that water is used and H_2SC_4 is created. Hence the specific gravity of H_2SO_4 which is the charging indicator of battery, increases.

Key Point: *More the specific gravity of H_2SO_4, better is the charging.*

The specific gravity is 1.25 to 1.28 for fully charged battery while it is about 1.17 to 1.15 for fully discharged battery. The voltage also can be used as a charging indicator. For fully charged battery it is 2.2 to 2.5 volts.

The chemical reaction during charging and discharging can be represented using single equation as,

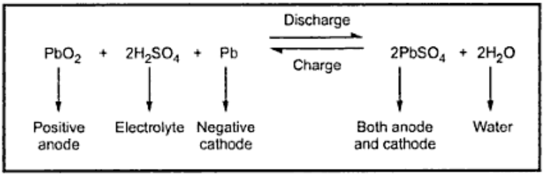


Fig. 7.9

7.6.3 Features of Lead Acid Battery

The various features of lead acid battery are,

1. The capacity is about 100 to 300 ampere-hours.
2. The voltage is 2.2 V for fully charged condition.
3. The cost is low.
4. The internal resistance is very low.
5. The current ratings are high.
6. The ampere-hour efficiency is about 90 to 95% with 10 hour rate.

7.6.4 Conditions of a Fully Charged Battery

For identifying whether the battery is fully charged or not, following conditions must be observed,

1. The specific gravity of H_2SO_4 must be 1.25 to 1.28.
2. The voltage stops to rise and its value is about 2.2 to 2.5 V.
3. Violent gasing starts as battery is fully charged.
4. The colour of positive plate becomes dark brown while the colour of negative plate becomes slate grey.

7.6.5 Maintenances and Precautions to be taken for Lead Acid Battery

The following steps must be taken in the maintenance of the lead acid battery,

1. The battery must be recharged immediately when it discharges.
2. The level of the electrolyte must be kept above the top of plates so the plates remain completely immersed.
3. The rate of charge and discharge shold not be exceeded as specified by the manufacturers.
4. Maintain the specific gravity of the electrolyte between 1.28 to 1.18.
5. The loss of water due to evaporation and gasing must be made up using only distilled water.
6. The connecting plugs should be kept clean and properly tightened.
7. It should not be discharged till its voltages falls below 1.8 V.
8. When not in use, it should be fully charged and stored in a cool and dry place.
9. It should not be kept long in discharged condition. Otherwise $PbSO_4$ gets converted to hard substance which is difficult to remove by charging. This is called **sulphating.** Thus sulphating should be avoided.
10. The battery must be given periodic overcharge at half the normal rate to remove white sulphate.
11. The temperature of the battery should not exceed 45 °C otherwise plates deteriorate rapidly.
12. The battery terminals should not be shorted to check whether battery is charged or not.
13. Always keep the surface of the container dry.
14. No sulphuric acid should be added till it is sure that low specific gravity is due to under charge and not due to white sulphate formed on plates.
15. The acid used must be pure without any impurity and colourless.
16. The sparks and flames must be kept away from the battery.

7.6.6 Testing Procedure for Lead Acid Battery

1. Using hydrometer : The testing basically involves the checking of specific gravity of the sulphuric acid. It can be checked by the use of **hydrometer**. The hydrometer consists of a glass float with a calibrated stem placed in a syringe. The readings on hydrometer are shown in the Fig. 7.10

7.9 Charge and Discharge Curves

The behaviour of battery voltage with respect to the time in hours of charging or discharging at normal rate is indicated by the curves called charge and discharge curves.

During discharge of the lead acid cell, the voltage decreases from about 2.1 V to 1.8 V, when cell is said to be completely discharged. The discharge rate is always specified as 8 hours, 10 hours etc.

During charging of the lead acid cell, the voltage increases from 1.8 V to about 2.5 V to 2.7 V, when cell is said to be completely charged. If the discharge rate is high, the curve is more drooping as voltage decreases faster. Such typical charge and discharge curves for lead-acid cell are shown in the Fig. 7.13. While discharging the voltage decreases to 2 V very fast, then remains constant for long period and at the end of discharge period falls to 1.8 V. While charging, initially it rises quickly to 2.1 to 2.2 V and then remains constant for long time. At the end of charging period it increases to 2.5 to 2.7 V.

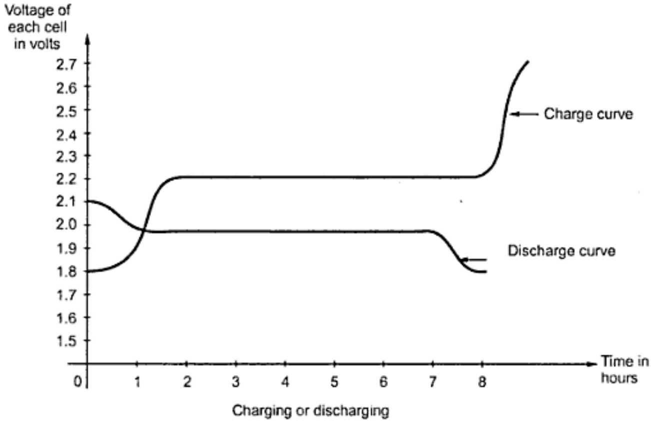


Fig. 7.13

From the given charging and discharging curves, the time of discharge for a specified voltage level can be obtained. This is nothing but T_D. Hence for a specific discharge current, battery life can be estimated.

7.10 Battery Charging

During charging, the chemical action takes place which is exactly opposite to that of discharging. Thus current in opposite direction to that at the time of discharge, is passed through the battery. For this the voltage applied is in excess of the voltage of the battery or cell. The battery voltage acts in opposite direction to that of the applied voltage and hence called back e.m.f. The charging current can be obtained as,

$$\text{Charging current} = \frac{E_a - E_b}{R+r}$$

Where E_a = Applied voltage

E_b = Back e.m.f. i.e. battery voltage

R = External resistance in the circuit

r = Internal resistance of the battery

Simple battery charging circuit used to charge the battery from d.c. supply is shown in the Fig. 7.14.

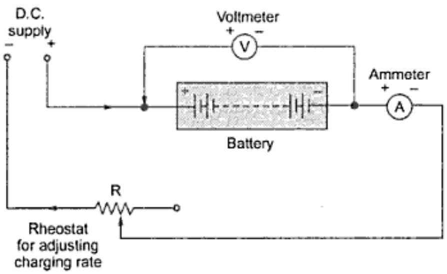


Fig. 7.14 Battery charging

The ammeter measures the charging current which is called **charging rate**, which can be adjusted using the external resistance R. The voltmeter measures the battery voltage. It is necessary that the positive terminal of the battery must be connected to the positive of the D.C. supply.

The charging current must be adjusted such that the temperature of the electrolyte will not increase beyond 100° to 110° F.

7.10.1 Indications of Fully Charged Battery

The various indications of the fully charged cells are,

1. **Specific gravity** : The specific gravity of the fully charged cell increases upto 1.28 from about 1.18.
2. **Gassing** : When the cell is fully charged, it starts liberating the gas freely. In lead acid battery the hydrogen is liberated at cathode while oxygen at the anode. Gassing is a good indication of fully charged battery. Some acid particles may go out with the gases hence the charging room must be kept well ventilated.
3. **Voltage** : The voltage of the fully charged cell is about 2.7 V.
4. **Colour** : The colour of the plates changes for fully charged cell. Colour of the positive plate changes to dark chocolate brown while that of negative plate changes to grey colour. But as plates are immersed in the electrolyte, this indication is not clearly visible.

7.11 Charging Methods

The main methods of battery charging are,

1. Constant current method
2. Constant voltage method
3. Rectifier method

7.11.1 Constant Current Method

When the supply is high voltage but battery to be charged is of low voltage, then this method is used. The number of batteries which can be charged are connected in series across the available d.c. voltage. The constant current is maintained through the batteries with the help of variable resistor connected in series. The circuit is shown in the Fig. 7.15.

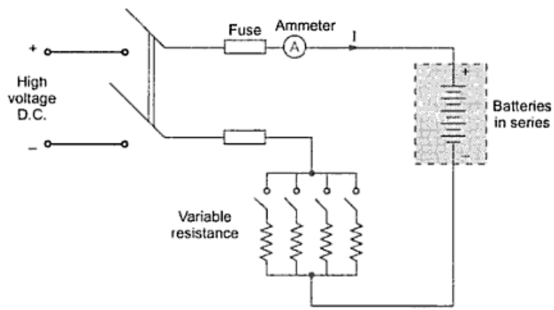


Fig. 7.15 Constant current method

The charging time required in this method is comparatively large. Hence in modern charger the number of charging circuits are used to give a variation of charging rates. Initially higher charging rate is used and later on lower charging rate is preferred.

7.11.2 Constant Voltage Method

In this method, the constant voltage is applied across the cells, connecting them in parallel. The charging current varies according to the state of the charge of each battery. The batteries to be charged are connected in 6 or 12 volt units across the positive and negative busbars i.e. mains supply. When the battery is first connected, a high charging current flows but as the terminal voltage of the battery increases, the charging current reduces automatically. At the end of the full charge, the voltage of the battery is equal to the voltage of the busbars and no current flows. The charging time required is much less in this method.

Another practically used method is called **trickle charge**. In this method, the charging current is maintained slightly more than the load current, through the battery. The load is constantly connected to the battery. So battery remains always in fully charged condition.

The Fig. 7.16 shows the cascade resistances used for the charging of batteries on d.c. mains supply.

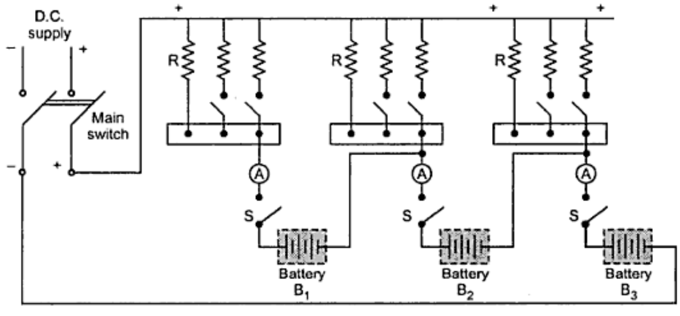


Fig. 7.16

The Fig. 7.17 shows the parallel charging circuit in which 2 separate groups each of 4 cells in series are connected in parallel across the mains.

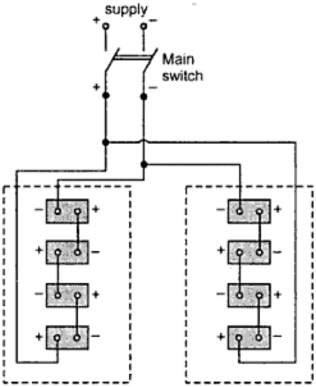


Fig. 7.17 Parallel charging circuit

7.11.3 Rectifier Method

When battery is required to be charged from a.c. supply, the rectifier method is used. The rectifier converts a.c. supply to d.c. Generally bridge rectifier is used for this purpose. The Fig. 7.18 shows the circuit used for rectifier method.

The step down transformer lowers the a.c. supply voltage as per the requirement. The bridge rectifier converts this low a.c. voltage to d.c. this is used to charge the battery.

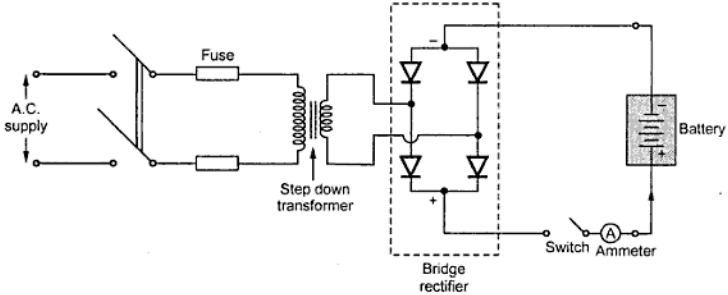


Fig. 7.18 Rectifier method

7.12 Grouping of Cells

The single cell is not sufficient to provide necessary voltage in many cases. Practically number of cells are grouped to obtain the **battery** which provides necessary voltage or current. The cells are grouped in three ways,

1. Series grouping
2. Parallel grouping
3. Series-parallel grouping

7.12.1 Series Grouping

The Fig. 7.19 shows the series grouping of cells so as to obtain the battery. There are n cells connected in series.

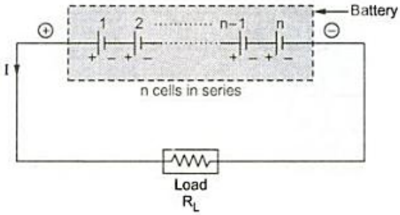


Fig. 7.19 Series grouping of cells

Let E = E.M.F. of each cell

r = Internal resistance of each cell

∴ V = Total voltage available = n × E volts

R_T = Total circuit resistance = load + cells

$= R_L + n \times r$

∴ $$I = \frac{\text{Total voltage}}{\text{Total resistance}} = \frac{V}{R_T} = \frac{nE}{R_L + nr} \text{ A}$$

Key Point : *In series circuit, current remains same. So this method does not improve current capacity. The current capacity is same as that of each cell connected in series. But voltage can be increased by increasing number of cells n.*

7.12.2 Parallel Grouping

In this method, positive terminals of cells are connected together and negative terminals are connected together as shown in the Fig. 7.20.

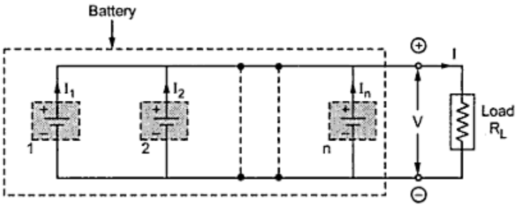


Fig. 7.20 Parallel grouping of cells

The terminal e.m.f. of each battery must be same as E.

$\therefore$ V = Battery voltage = E = e.m.f. of each cell

r = Internal resistance of each cell

I_n = Current through nth branch

$\therefore$ I = Total current

$$\therefore \quad \boxed{I = I_1 + I_2 + I_3 + \ldots + I_n}$$

Key Point: *It can be seen that in parallel grouping, the voltage remains same but by increasing number of cells the current capacity can be increased.*

In series grouping current rating of each cell must be same while in parallel grouping voltage rating of each cell must be same.

7.12.3 Series-Parallel Grouping

Practically the various groups can be connected in parallel where each group is a series combination of cells as shown in the Fig. 7.21.

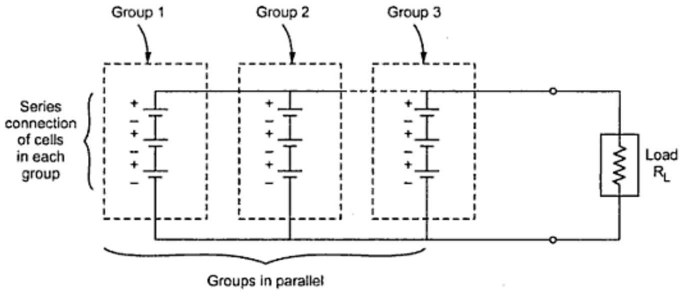


Fig. 7.21 Series-parallel grouping

This is used to satisfy both voltage and current requirement of the load.

7.13 Alkaline Cells

The secondary cells can be alkaline cells. These are of two types.

1. Nickel - iron cell or Edison cell
2. Nickel - cadmium or Nife Cell or Junger cell

7.14 Nickel - Iron Cell

In this cell,

Positive Plate → Nickel hydroxide [($Ni(OH)_3$]
Negative Plate → Spongy iron (Fe)

The electrolyte is an alkali of 21 % solution of **potassium hydroxide solution** (KOH).

The insulated rods are used to seperate the positive and negative plates.

The Fig. 7.22 shows the construction of Nickel-iron cell.

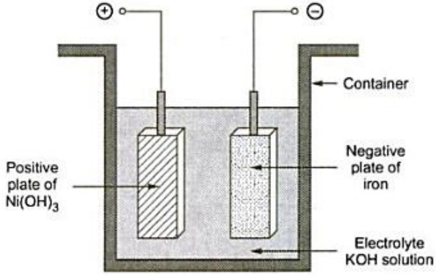


Fig. 7.22 Construction of Nickel-iron cell

7.14.1 Chemical Reaction

In a charged condition, the material of positive plate is $Ni(OH)_3$ and that of negative plate is iron. When it is connected to load and starts discharging, the nickel hydroxide gets converted to lower nickel hydroxide as $Ni(OH)_2$ while the iron on negative plate gets converted to ferrous hydroxide $Fe(OH)_2$. When charged again, reversible reaction takes place, regaining the material on each plate.

7.14.3 Capacity

It is mentioned that electrolyte does not undergo any chemical change for this cell. Thus specific gravity of the electrolyte remains constant for long periods. Hence rate of discharge does not affect ampere-hour capacity of this cell significantly. **Thus Ah capacity of Nickel-iron cell remains almost constant.** But it does get affected by the temperature. The Fig. 7.25 shows the Ah capacity against discharging time curve for Nickel-iron cell.

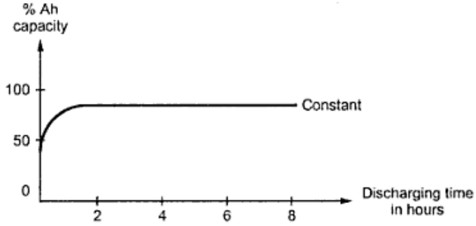


Fig. 7.25 Ah capacity against discharge time curve

7.14.4 Efficiency

The internal resistance of Nickel-iron cell is higher than lead acid cell hence both the efficiencies ampere-hour as well as watt-hour are less than that of lead acid cell. The ampere-hour efficiency is about 80 % while the watt-hour efficiency is about 60 %.

7.14.5 Advantages

The various advantages of Nickel-iron cell are,

1. Light in weight compared to lead acid cell.
2. Compact construction.
3. Mechanically strong and can sustain considerable vibrations.
4. Free from sulphatation and corrosion.
5. Less maintenance is required
6. Do not evolve dangerous attacking fumes.
7. Gives longer service life.

7.14.6 Disadvantages

The various disadvantages of Nickel-iron cell are,

1. High initial cost.
2. Low voltage per cell of about 1.2 V.

3. High internal resistance.
4. Lower operating efficiency.

7.14.7 Applications

The Nickel-iron batteries are used in,

1. Mine locomotives and mine safety lamps
2. Space ship
3. Repeater wireless station
4. To supply power to tractors, submarines, aeroplanes etc.
5. In the railways for lighting and airconditioning purposes.

7.15 Nickel - Cadmium Cell

The construction of this cell is similar to the nickel-iron cell except the active material used for the negative plate.

Positive plate → Nickel hydroxide [$Ni(OH)_3$]
Negative plate → Cadmium (Cd)

The electrolyte used is again 21 % solution of potassium hydroxide (KOH) in distilled water. The specific gravity of the electrolyte is about 1.2.

Little iron is added to cadmium to get negative plate. The iron prevents the caking of active material and losing its porosity.

The Fig. 7.26 shows the construction of Nickel-cadmium cell.

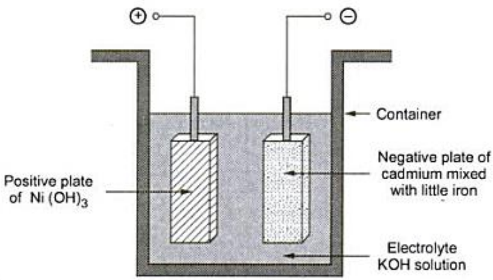


Fig. 7.26 Construction of Nickel-Cadmium cell

7.15.1 Chemical Reaction

In this cell also, in working condition $Ni(OH)_3$ gets converted to lower nickel hydroxide as $Ni(OH)_2$ while cadimum hydroxide $Cd(OH)_2$ gets formed at the negative plate. During charging reverse reaction takes place. The electrolyte does not undergo any chemical change.

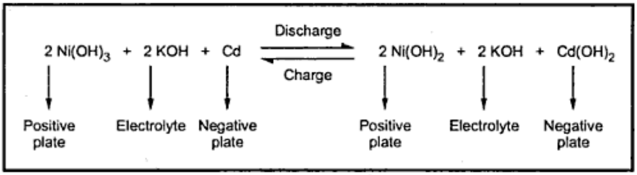


Fig. 7.27 Total reaction

7.15.2 Features

1. The electrical characeristics are similar to the Nickel-iron cell.
2. Due to use of cadmium, internal resistance is low.
3. The efficiencies are little bit higher than Nickel-iron cell.
4. Advantages and disadvantages are same as that of Nickel-iron cell.
5. The various charging methods such as constant current, constant voltage, trickel charging can be used.

7.15.3 Applications

The various applications of Nickel-cadmium battery are,

1. In railways for lighting and air conditioning systems.
2. In millitary aeroplanes, helicopters and commerical airlines for starting engines and provide emergency power supply.
3. In photographic equipments such as movie cameras and photoflash.
4. In electric shavers.
5. Due to small size in variety of cordless electronic devices.

7.16 Comparison of Various Batteries

Sr. No.	Particular	Lead acid cell	Nickel-Iron cell	Nickel-Cadmium cell
1.	Positive plate	Lead peroxide (PbO_2)	Nickel hydroxide $Ni(OH)_3$	Nickel hydroxide $Ni\ (OH)_3$
2.	Negative plate	Lead (Pb)	Iron (Fe)	Cadmium (Cd)
3.	Electrolyte	Sulphuric acid H_2SO_4	Potassium hydroxide KOH	Potassium hydroxide KOH
4.	Average e.m.f.	2.0 V/cell	1.2 V/cell	1.2 V/cell
5.	Internal resistance	Low	High	Low
6.	Ah efficiency	90 to 95 %	70-80 %	70-80 %
7.	Wh efficiency	72 to 80%	55-60%	55-60%
8.	Ah capacity	Depends on discharge rate and temperature	Depends only on temperature	Depends only on temperature
9.	Cost	Less expensive	Almost twice the lead acid cell	Almost twice the lead acid cell
10.	Life	1250 charges and discharges	About 8 to 10 years	Very long life
11.	Weight	Moderate	Light	More heavy
12.	Mechanical strength	Poor	Good	Good

7.17 Comparison of Primary and Secondary Cells

Sr. No.	Primary cells	Secondary cells
1.	Electrical energy is directly obtained from chemical energy.	Electrical energy is present in the cell in the form of chemical energy and then converted to electrical energy.
2.	The chemical actions are irreversible.	The chemical actions are reversible.
3.	Cell is completely replaced when it goes down.	The cell is recharged back when it goes down.

supply system, in addition to 3 main conductors, a neutral is also included in the cable. But due to possibility of unbalanced heavy neutral currents, the cross-section of neutral is same as that of main conductors hence neutral is treated to be full core. Such a cable is called **4 core cable.**

7.20 Types of Cables

The type of a cable is basically decided based on the voltage level for which it is manufactured and the material used for the insulation such as paper, cotton, rubber etc. The classification of cables according to the voltage levels is,

1. Low tension cables (L.T. cables) : These are used for the voltage levels upto 6.6 kV. The electrostatic stresses in L.T. cables are not severe hence no special construction is used for L.T. cables. The paper is used as an insulation in these cables. Sometimes resin is also used which increases the viscosity and helps to prevent drainage.

The Fig. 7.29 shows the cross-section of a single core L.T. cable. It consists of a circular core of stranded copper or aluminium. The conductor is insulated by impregnated paper. Over the paper insulation, the lead sheath is provided. Then a layer of compounded fibrous material is provided. Then armouring is provided and finally covered again with a layer of fibrous compounded material.

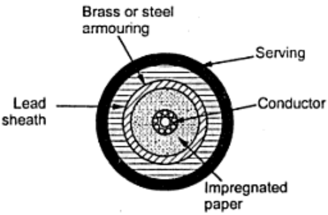


Fig. 7.29 Single core L.T. cable

Many a times, L.T. cables are not provided with armouring, to avoid excessive sheath losses. The simple construction and the availability of more copper section are the advantages of L.T. single core cable.

2. Medium and high tension cables (H.T. cables) : The three phase medium and H.T. cables are three core cables. For voltages upto 66 kV, the three core cables i.e. multicore cables are used. These cables are classified as,

a. H.T. cables upto 11 kV level which are **belted type.**

b. Super tension (S.T.) cables for 22 kV and 33 kV levels which are **screened cables.**

c. Extra high tension (E.H.T.) cables for voltage levels from 33 kV to 66 kV which are **pressure cables.**

Let us see the constructional features of these types of three core cables.

7.20.1 Belted Cables

As mentioned earlier, these are used for the voltage levels upto 11 kV. The construction of belted cable is shown in the Fig. 7.30.

The cores are not circular in shape. The cores are insulated from each other by use of impregnated paper. The three cores are grouped together and belted with the help of a paper belt. The gaps are filled with fibrous material like jute. This gives circular cross-sectional shape to the cable. The belt is covered with lead sheath which protects cable from moisture and also gives mechanical strength. The lead sheath is finally covered by jute like fibrous compounded material.

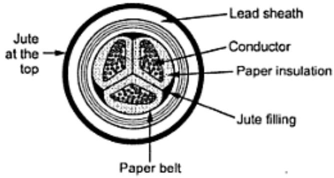


Fig. 7.30 Belted 3 core cable

The electric field in single core cable is radial while it is tangential in case of three core cables. Hence the insulation is subjected to tangential electrical stresses rather than radial one. The paper has good radial strength but not tangential strength. Similarly paper resistance along the radius is much larger than resistance along tangential path. The same is true for dielectric strength also. The fibrous material is also subjected to the tangential electrical stresses, for which, the material is weak. Hence under high voltage cases, the cumulative effect of tangential electrical stresses is to form spaces inside the cable due to leakage currents. Such air spaces formed inside the insulation is called **void formation.** This void formation is dangerous because under high voltage, spaces are ionized which deteriorates the insulation which may lead to the breakdown of the insulation. Hence the belted cables are not used for the high voltage levels. Another disadvantage of the belted cable is large diameter of paper belt. Due to this, wrinkles are formed and gaps may be developed if the cable is bended. To overcome all these difficulties, the screened type cables are used.

7.20.2 Screened Type Cables

These cables are used for the voltage levels of 22 kV and 33 kV. The two types of screened cables are (1). H type cables and (2). S.L. type cables.

7.20.2.1 H-Type Cables

The cable is designed by M. Hochstadter and hence the name given to it is H-type cable. There is no paper belt in this type of cable. Each conductor in this cable is insulated with a paper, covered with a metallic screen which is generally an aluminium foil. The construction is shown in the Fig. 7.31. The metallic screen touches each other. Instead of paper belt, the three cores are wrapped with a conducting belt which is

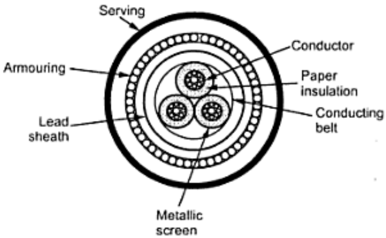


Fig. 7.31 H-type cable

usually copper woven fabric tape. Then there is lead sheath. The conducting belt is in electrical contact with the metallic screen and lead sheath. After lead sheath there are layers of bedding, armouring and serving. The metallic screen helps to completely impregnate the cable which avoids the possibility of formation of voids and spaces. The conducting belt, the three metallic screens and lead sheath are at earth potential, due to which electrical stresses are radial in nature. This keeps the dielectric losses to minimum. Another advantage of metallic screens is increase in the heat dissipation which reduces the sheath losses. Due to these advantages, current carrying capacity of these cables increases. In special cases, the use of these cables can be extended upto the 66 kV level.

7.20.2.2 S.L. Cables

The name S.L. stands for separate lead screened cables. In this cable, each core is insulated with an impregnated paper and each one is then covered by separate lead sheath. Then there is a cotton tape covering the three cores together using a proper filler material. Then there are the layers of armouring and serving. The difference between H-type and S.L. type cable is that in S.L. type a common lead sheath covering all the three cores is absent while each core is provided with separate lead sheath. This allows bending of the cables as per the requirement. The construction of S.L. type cable is shown in the Fig. 7.32.

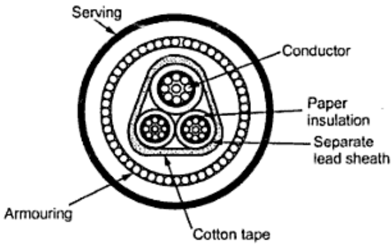


Fig. 7.32 S.L. type cable

The three cores in this type of cable are as good as three separate single core cables.

The various advantages of S.L. type cable are,

1. Due to individual lead sheath, core to core fault possibility gets minimised.
2. The electrical stresses are radial in nature.
3. Due to absence of overall lead sheath, bending of cable is easy.
4. The dielectric which gets subjected to electric stresses is paper which is homogeneous hence there is no possibility of formation of voids.
5. Metal sheath increases the heat dissipation which increases the current carrying capacity.

A combination of H-type and S.L. type cable called H.S.L. cable also can be used.

The limitations of **screened cables** which are also called **solid type cables** are,

1. It uses solid insulation only like paper. When the conductor temperature increases, the paper gets expanded. This eventually stretches the lead sheath.

2. When the load on the cable decreases, it cools down and there is contraction of lead sheath. Due to this air may be drawn into the cable forming voids. This deteriorates the cable insulation.
3. Moisture may be drawn in alongwith the air which deteriorates the dielectric strength of dielectric.
4. Mechanical shock can cause voids. The breakdown strength of voids is much less than insulation. Hence voids can cause permanent damage to the cables.

7.20.3 Super Tension (S.T.) Cables

In solid type cables separate arrangement for avoiding void formation and increasing dielectric strength is not provided. Hence those cables are used maximum upto 66 kV level. The S.T. cables are intended for 132 kV to 275 kV voltage levels.

In such cables, the following methods are specially used to eliminate the possibility of void formation :

1. Instead of solid type insulation, low viscosity oils under pressure is used for impregnation. The channels are used for oil circulation and oil is always kept under pressure. The pressure eliminates completely, the formation of voids.

2. Using inert gas at high pressure in between the lead sheath and dielectric.

Such cables using oil or gas under pressure are called **pressure cables** and are of two types,

a. Oil filled cables b. Gas pressure cables.

7.20.4 Oil Filled Cables

In case of oil filled cables, the channels or ducts are provided within or adjacent to the cores, through which oil under pressure is circulated.

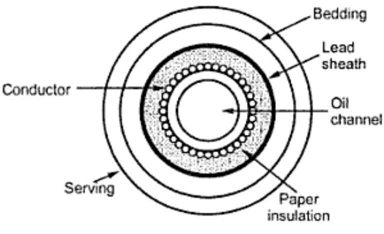


Fig. 7.33 Conductor channel single core oil filled cable

The Fig. 7.33 shows the construction of single core oil filled cable. It consists of concentric stranded conductor but built around a hollow cylindrical steel spiral core. This hollow core acts as a channel for the oil. The oil channel is filled in a factory and the pressure is maintained in the oil by connecting the oil channel to the tanks which are placed at the suitable distances along the path of the cable.

The oil pressure compresses the paper insulation, eliminating the

7.20.4.2 Disadvantages

The disadvantages of oil filled cables are,

1. The initial cost is very high.
2. The long lengths are not possible.
3. The oil leakage is serious problem hence automatic signalling equipment is necessary.
4. The laying of cable is difficult and must be done very carefully.
5. Maintenance of the cables is also complicated.

7.20.5 Gas Pressure Cables

In case of gas pressure cables, an inert gas like nitrogen at high pressure is introduced lead sheath and dielectric. The pressure is about 12 to 15 atmospheres. Due to such a high pressure there is a radial compression due to which the ionization is totally eliminated. The working power factors of such cables is also high.

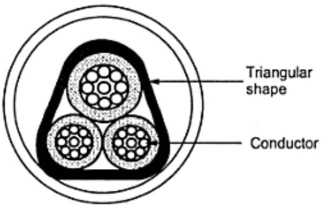


Fig. 7.36 Gas pressure cable

The Fig. 7.36 shows the section of a gas pressure cable. The cable is triangular in shape and installed in the steel pipe. The pipe is filled with the nitrogen at 12 to 15 atmospheric pressure. The remaining construction is similar to that of solid type cable but the thickness of lead sheath is 75% of that of solid type cable. There is no bedding and serving. The pressure cable was firstly designed by Hochstadter, Vogel and Bowden.

The triangular shape lead sheath acts as a pressure membrane. The shape reduces the weight and provides the low thermal resistance. The high pressure creates the radial compression to close any voids. The steel pipe is coated with a paint to avoid corrosion.

During heating, the cable compound expands and a sheath which acts as a membrane becomes circular in such a case. When cable cools down the gas pressure acting via sheath forces compound to come back to the noncircular normal shape. Due to good thermal characteristics, fire quenching property and high dielectric strength, the gas SF_6 is also used in such cables.

7.20.5.1 Advantages

The various advantages of gas pressure cables are,

1. Gas pressure cables can carry 1.5 times the normal load current and can withstand double the voltage. Hence such cables can be used for ultra high voltage (UHV) levels.

2. Maintenance cost is small.

3. The nitrogen in the steel tube, helps in quenching any fire or flame.
4. No reservoirs or tanks required.
5. The power factor is improved.
6. The steel tubes used make the cable laying easy.
7. The ionization and possibility of voids is completely eliminated.

The only disadvantage of this type of cables is very high initial cost.

7.21 Insulating Materials for Cables

Number of layers of the various materials is used around the actual conductor in a cable. To isolate the conductor from the surroundings, the conductor is provided with an insulation around it. The materials like paper, vulcanized rubber, PVC etc. are used for providing such an insulation.

The material to be used as an insulation must have the following properties,

1. To prevent leakage current, its insulation resistance must be very high.
2. To avoid electrical breakdown, its dielectric strength must be very high.
3. To withstand the mechanical injuries, it must be mechanically very strong.
4. It should be flexible.
5. It should be non-hygroscopic so that it will not absorb the moisture from the surroundings.
6. It should be non-inflammable.
7. It should be unaffected by acids and alkalies.
8. It should be capable of withstanding high breakdown voltages.
9. It should have high temperature withstanding capability.

Practically it is not possible to have all these properties in a single material. Hence insulation material is selected depending upon the use of the cable and the quality of insulation required for it. Some changes are done at the time of design depending upon the nature of material selected. For example if the material is hygroscopic then a layer of waterproof covering is provided around it so that moisture can not reach to the insulation. The main insulating materials which are in use are,

1. Poly Vinyl Chloride (PVC)
2. Paper
3. Cross Linked Polythene
4. Vulcanized India Rubber (VIR)

7.21.1 Poly Vinyl Chloride (PVC)

It is thermo plastic synthetic compound. It is available in the powder form and is obtained from polymerisation of acetylene. This powder is chemically inert, non-inflammable, odourless, tasteless and insoluble. It is combined with plastic compound and a gel is used over the conductor to obtain the insulation.

It has following characteristics,

1. Good dielectric strength of 17 kV/mm
2. Chemically inert.
3. Non-hygroscopic.
4. Resistant to corrosion.
5. Maximum continuous temperature rating of 75°C.
6. High electrical resistivity.

The mechanical properties like elasticity of PVC are not as good as rubber so PVC cables are used for low and medium voltage domestic, industrial lights and power installations.

7.21.2 Paper

The paper is very cheap insulating material. Its dielectric strength is also high but it is hygroscopic in nature. When it is dry its insulation resistance is very high but a small amount of moisture reduces its insulation resistance to a very low value. Thus it is impregnated in an insulating oil. After impregnating also it has a tendency to absorb the moisture. Hence paper cables are never left unsealed and provided with the protective covering. When not in use, paper cable ends are temporarily covered with wax or tar.

The paper has following characteristics,

1. High dielectric strength of 20 kV/mm.
2. Higher thermal conductivity.
3. Low capacitance
4. High durability
5. Low cost
6. Maximum continuous temperature rating of 80°C
7. High insulation resistance when dry.

It is used in high voltage power cable manufacturing. The paper cables are preferred when the cable route has very few joints and hence generally used for low voltage distribution in thickly populated areas.

7.21.3 Cross Linked Polythelene

The cables using cross linked polythelene as the insulating material are called XLPE cables.

The low density polythelene is treated specially due to which there occurs cross linking of carbon atoms in it. This results into a new material which has following properties,

1. High dielectric strength of 20 to 40 kV/mm
2. Non-inflammable : If at all the continuous flame is applied its burning stops after very few centimetres away from the flame.
3. Extremely high melting point.
4. Light in weight and flexible.
5. Mechanically strong.
6. High temperature withstanding capability.
7. Low moisture absorption.
8. Maximum continuous temperature rating of 90°C.

XLPE cables are directly laid on soil bed and are used for the voltages upto and including 33 kV.

7.21.4 Vulcanized India Rubber (VIR)

This is the most olden insulating material developed during 1880-1930. The pure rubber is very soft and it can not withstand high temperatures hence it is 20 to 40% of India rubber mixed with mineral matter such as zinc oxide, red lead etc. with a little bit of sulphur in it.

It has following characteristics,

1. Good dielectric strength of 15 kV/mm.
2. Good mechanical strength.
3. Durable and wear resistant.
4. Good insulation resistance.
5. Remain more elastic than pure rubber.

But it has number of drawbacks such as,

1. It absorbs moisture, slightly.
2. It has low melting point.
3. The sulphur content attack the copper conductor and changes the VIR insulation colour. Hence copper conductors to be used with VIR insulation must be tinned.
4. Short span of life.

The use of VIR is very limited nowadays and is used for low and moderate voltage cables i.e. distribution systems only.

Review Questions

1. *What is cell and battery ? State and explain the various types of cells.*
2. *Explain the construction of lead acid battery.*
3. *Explain first charging, discharging and recharging in case of lead acid battery.*
4. *State the features and application areas of lead acid battery.*
5. *State the maintenance procedure for lead acid batteries.*
6. *How battery capacity is defined ? On which factors it depends ?*
7. *What is battery efficiency? In how many ways battery efficiency is expressed ?*
8. *State and explain what is ampere-hour efficiency and watt-hour efficiency.*
9. *Write a note on charge and discharge curves for lead acid battery.*
10. *With a basic charging circuit, explain the battery charging.*
11. *Explain the two methods of battery charging.*
12. *State the indications for fully charged battery.*
13. *State the requirements of an underground cable.*
14. *Draw and explain the general construction of an underground cable.*
15. *Explain the various types of underground cables.*
16. *Write short notes on :*
    a) Oil filled cables *b) Gas pressure cables*
17. *Which are the important insulating materials used for the cables ?*

University Questions

Q.1 *Explain the following methods of charging a battery i) Constant current method ii) Constant voltage method.* **[GTU. : Dec.-2008, 6 Marks]**

Q.2 *Explain the construction and working of any type of battery you know. What is its voltage when it is fully charged?* **[GTU. : March-2009, 7 Marks]**

Q.3 *How do you estimate the life of a battery when charging and discharging characteristics are available?* **[GTU. : June-2009, 4 Marks]**

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8.3 Specification of Wires

Similar to the cables the various types of wires also use more than one conductors in it. The wires using more than one strand of conductors in it are called **multistranded wires.** The number of strands commonly used in various wires are 3, 7, 19, 37, 61, 91, 127 and 169. This number of strands ensures that the cross-section of the conductor or wire remains circular in shape.

The Fig. 8.4 shows the cross-section of **single strand, three stranded, seven stranded** and **nineteen stranded** wires.

Fig 8.4 Multistranded wires

The multistranded construction increases the current carrying capacity of the wires. As current through conductor increases, heat produced is more. In case of single solid conductor, majority of current flows near the surface and hence surface becomes very hot and insulation is under high temperature stress. Due to multistrands, the current gets divided into number of paths and larger surface area is available for heat dissipation than the single solid conductor. Hence for the same temperature limit, multistranded wire can carry more current than single solid conductor. The multistranded wire is more flexible as cross section of each strand is much less. It makes it easy for the wiring. If at all there is open circuit in one of the strands, other strands can carry current. The heat produced gets dissipated earlier in multistranded wires. These are the few advantages of multistranded wires.

Thus the one way of specifying wires is the **number of strands of conductors** used in it.

Secondly, various insulations can withstand different temperatures and depending upon the **type of insulation,** wires are specified. As mentioned earlier, various insulations are vulcanized india rubber, cab tyre, tough rubber, poly vinyl chloride etc. So wires are specified as V.I.R., T.R.S, C.T.S., P.V.C. wires.

Key Point: *Now a days P.V.C. insulated wires are very commonly used.*

The size of the strand of the conductor used also is important from the specification point of view. The size determines the current carrying capacity of the conductor. To specify the size of conductor various methods, are used.

The Table 8.1 gives the specifications of P.V.C. insulated aluminium conductor wires.

Cross-sectional area in mm^2	Number of strands Diameter in mm	Current carrying capacity in Amp	
		Single phase	Three phase
1.5	1/1.4	12	11
2.5	1/1.8	17	15
4.0	1/2.24	23	19
6	1/2.8	29	24
10	1/3.55	40	33
16	7/1.7	54	46
25	7/2.24	69	58
35	7/2.5	83	71
50	7/3.0	105	89

Table 8.1

8.4 Factors Affecting Wiring System

The method of wiring to be adopted depends on various factors such as:

a) Durability : The wiring scheme selected must be sufficiently durable. It should fulfill the requirements of the consumer. It should be according to proper specifications and should not give any problems frequently.

b) Safety : This is most important factor. As there is a danger of loosing life in case of electric shocks, the safety must be observed strictly while choosing wiring scheme. The wiring should be fully shock proof and leakage proof.

c) Appearance : Due to wiring scheme the beauty of the house and premises should not get spoiled. Hence it is necessary to select proper wiring scheme such that it enhances the beauty of house, where it is to be carried out.

d) Cost : The funds made available by the consumer should be considered before recommending particular type of wiring scheme. There should be proper balance between convenience and look of the wiring, from the installation cost point of view .

e) Maintenance : The maintenance of the wiring should be as minimum as possible. There should be scope for further extension of the wiring. Renewal of the wiring should

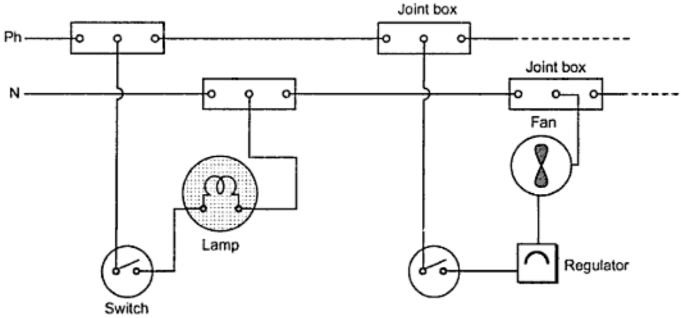


Fig. 8.10

The length of wire required in this system is less but additional cost is involved in joint boxes. To locate the fault, all the joint boxes are required to be examined. Thus finding fault location is difficult in this system. The task of fixing joint boxes requires a skilled labour and its appearance is not good.

This method of wiring was in use earlier for residential wiring but now used only in temporary installations.

8.6.2 Looping in System

This system is commonly used for residential installations. Here the phase wire is directly connected to terminal of switch and looped in to the next switch terminal. The neutral wire is looped in from one point to another within the same subcircuit.

The main advantage of this method is no junction boxes are required. Also the connections are accessible for inspection. Thus the fault can be easily located. But the drawback of this method is it requires more length of cable which further increases voltage drop and copper losses in it. The looping through ceiling rose and looping in both phase and neutral from switch board is shown in the Fig. 8.11 (a) and Fig. 8.11 (b).

8.7 Wiring Schemes

In the domestic wiring, the various appliances and lamps are connected in parallel but for the various advantages the various combinations of switches and lamps are also used. Such combinations are called **wiring systems or schemes.** Let us see the wiring diagrams of various wiring schemes.

8.7.1 Control of One Lamp from One Switch

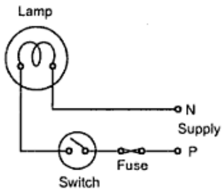


Fig. 8.12

For a lamp, one live i.e. phase and one neutral is necessary. To control the supply to the lamp, switch is introduced in the live wire and neutral is directly connected to the lamp.When switch is ON, a full voltage gets applied to the lamp and it glows.

When the switch is turned off, the circuit gets opened and lamp gets switched off. This is controlling of one lamp by one switch. The circuit is shown in the Fig 8.12. The switch is always connected in phase wire so as to ensure the safety of the personnel doing the maintenance at the lamp holder.

8.7.2 Two Way Control of Lamps or Staircase Wiring

This is also called as **staircase wiring** as it is commonly used for stair cases and corridor lighting. It consist of two way switches. A two way switch operates always in one of the two possible positions. The circuit is shown in the Fig. 8.13. A lamp in this case is controlled independently from two switches located at different positions.

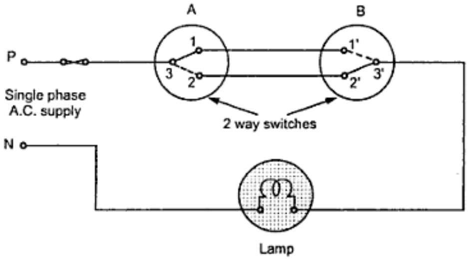


Fig. 8.13 Control of one from two points

The phase wire is connected to terminal 3 of the first two way switch A. The movement of switch makes connection from terminal 3 to either of the terminals 1 or 2. The similar two way switch is fixed at another position. With the positions of switches shown in the Fig. 8.13 , the lamp remains OFF.

Following table shows the positions of switches and condition of lamp.

Switch A	Switch B	Lamp
1	1'	ON
1	2'	OFF
2	2'	ON
2	1'	OFF

Assume that lamp is on first floor. Switch A is on first floor and B is on second floor. In the position shown, the lamp is OFF.

When person changes position of switch A from (1) to (2) then lamp gets phase through switches A and B and it gets switched ON as shown in the Fig 8.14.

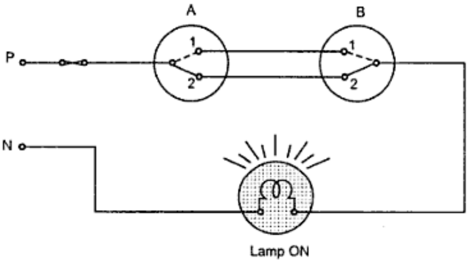


Fig. 8.14 ' ON ' position of lamp

When person reaches on second floor, the lamp is required to be switched OFF. So person will change switch B from (2) to (1), due to which, phase connection reaching to the lamp gets opened and lamp will be switched OFF as shown in the Fig 8.15.

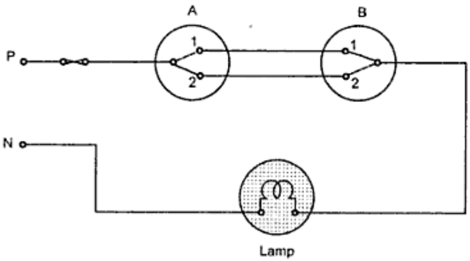


Fig. 8.15 ' OFF ' position of lamp

8.7.3 Control of Two Lamps by Individual Switches

The two lamps can be controlled independently with the help of two switches. The circuit is as shown in the Fig. 8.16.

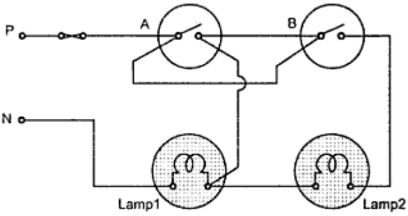


Fig. 8.16

8.7.4 Three Way Control of Lamps

This is also a type of staircase wiring. It consist of two way switches A and B and one intermediate switch C. The circuit used to have three way control of lamps is shown in the Fig. 8.17.

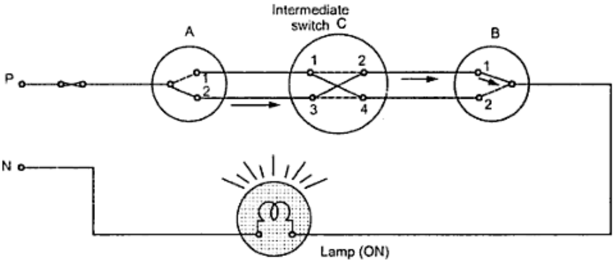


Fig. 8.17

The **intermediate switch** can have positions to connect points 1-4, 3-2 as shown or 1-2 and 3-4 shown dotted. The switch A is on first floor and switch B is on third floor say.

In the position shown in the Fig. 8.17, the lamp is ON.

When person from floor 2 changes switch C position to have connections 1-2, and 3-4 then it can be seen from the Fig 8.18 that there is open circuit in the connection. The position 1 of first two way switch becomes open circuit and similarly the position 2 also becomes open circuit. Thus the lamp gets switched OFF.

Fuses cannot be used for large currents as they bear low breaking capacity. The other limitation of this type of fuse is possibility of rewiring of fuse with wrong size or material.

The kit-kat type is of semienclosed type of fuse or also called as **rewirable fuse**. The main advantage of this type is if it is blown off, top can be taken out and fuse wire can be changed. The material used for fuse element is copper, aluminium, tin-lead alloy or silver. Tin-lead alloy fuse wires are used upto 30 A.

8.8.2.1 H.R.C. Fuse

Another type of fuse is used to break the circuit where fault current level is very high. In such case fuse has to withstand heavy stresses hence the construction is of totally closed type. The fuse is called as high rupturing capacity commonly called as H.R.C. fuse. The fuse wire is enclosed in a ceramic cartridge. The ends of fuse wire are connected to metal caps. The body of cartridge is filled with powdered quartz. When fuse melts, it reacts with quartz powder forming a substance having high resistance like insulator. This also restricts the arc formation. This type of fuse can not be rewired once operated.

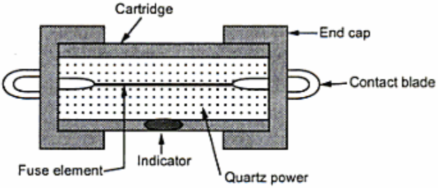


Fig. 8.24 H.R.C. Fuse

The construction of H.R.C. fuse is shown in the Fig. 8.24.

Advantages :

i) The operation is highly reliable.

ii) The speed of fusing the element is high.

iii) The operation is without arc, noise, flame or smoke.

iv) The high levels of fault current can be cleared.

v) The fuse element is totally enclosed, hence it is totally protected from atmospheric conditions.

Disadvantages :

i) It is very costly.

ii) It is not rewirable. The whole cartridge is to be replaced.

Applications :

Very commonly used in chemical industries, low voltage distribution systems, petrochemical industries and majority of industrial installations. It is must for the industries related to chemical gases, petroleum gases etc. This is because fire hazards can

3 phase, 4 wire system. A typical incoming connection of electric supply is as shown in the Fig. 8.29.

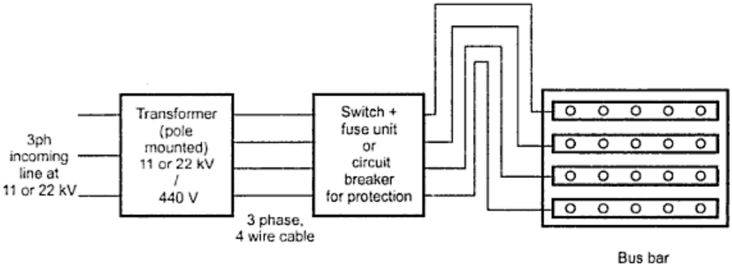


Fig. 8.29

The 3 phase supply available from the transformer with 3 phase, 4 wire system is brought through underground cable is brought to the main distribution board from where the electricity is further distributed to individual consumers. The switch + fuse unit or circuit breaker is used for switching and protection purpose. Through the bus bar arrangement, single phase supply is given to different consumers with protective and measuring instruments.

The Fig. 8.30 shows a schematic diagram explaining the consumer panel or domestic wiring installation.

The consumer's premises is connected to the distributor by a two conductor overhead or underground service cable. The service cable, sealing end box, bus bar arrangement , service fuse, neutral link, energy meters and other switching and protective elements is the property of supply company and is normally sealed.

The installation and maintenance of meter board where meter and neutral link are present is the sole responsibility of electricity board or supply company. The load side connections from the energy meter are then taken to main switch board on which main switch along with consumer's fuse is installed. The energy meter consists of two electromagnets with current and pressure coils. It records the energy consumption in units and supply company accordingly charges the bills to the consumers, based on the tariff.

The service board on which energy meter, neutral link etc. are installed must be located at covered place which is easily accessible for inspection and meter reading and at a sufficient height. The main switch is located after the energy meter at a sufficient height so as to facilitate its easy operation. The main switch is used for isolating the supply from remaining circuit. After the main switch, consumer's fuse and distribution board comes in sequence.

From the main distribution board supply is given to number of different subcircuits with fuse in its phase wire. Under the event of fault, fuse will be blown off and isolating the supply from faulty part without affecting the remaining subcircuits.

The tube lights, fans and other electrical appliances are connected in parallel across the subcircuit. Each of this appliance can be controlled with a single way switch for its ON and OFF operation. Now instead of fuses, miniature circuit breakers are used. Only 3 pin, 5 A socket outlets are to be used in all light and fan subcircuits while 3 pin, 15 A socket outlets are used for all power circuits. All socket outlets are to be controlled by individual switches placed adjacent to it.

The connected load on any subcircuit consisting of tubes, fans etc. should not exceed 800 W with a maximum of 10 points. For a power subcircuit the connected load should not exceed 2000 W subject to maximum of 2 points. The lights, fans and 5 A plug points can be connected on a common subcircuit but separate subcircuits are to be provided for power points. Minimum one spare circuit must be provided on each distribution board. Thus the domestic wiring installation consists of distribution of electrical energy from energy meter board to the various subcircuits connected to electrical appliances through protective devices and network of wires.

8.10 Industrial Electrification

The electrification of industry is different from that of domestic wiring as the total load of industry, current levels of protective devices, etc. is far more than in case of residential load. In industry apart from lighting load consisting of tube lights and fans, there are various power equipments which include a.c. or d.c. motors, rotary converters, rotary condensers, rectifiers, transformers, frequency changers etc. According to Indian Electricity Rules (Nos. 50 and 51), the wiring of electrical motors in industries is to be carried out apart from meeting local requirements on various conditions.

There are certain provisions which must be made while supplying electrical energy at medium voltage. The supply to each motor or other apparatus should be controlled by a suitable linked switch or a circuit breaker placed near to the motor or that apparatus and can be operated easily by the operating personnel. All the conductors are to be completely enclosed in mechanically strong metal casing. Each main switch or switch board should have a clear space of atleast 1 m in width in front. The rating of the feeders which provides power to electrical motors must have minimum current carrying capacity 200 % of the full load current rating of all the motors in that group in addition to single phase or three phase lighting load requirements and future expansion considerations.

The power distribution system in small industries or workshops starts from three phase energy meter to main switch and main switch to power distribution board.

The coils are connected to the circuit through flexible leads called ligaments which do not produce a restoring torque on the moving element, consequently the moving element takes up any position over the scale when the generator handle is stationary.

The current coil is connected in series with resistance R_1 between one generator terminal and the test terminal T_2. The series resistance R_1 protects the current coil in the event of the test terminals getting short circuited and also controls the range of the instrument. The pressure coil, in series with a compensating coil and protection resistance R_2 is connected across the generator terminals. The compensating coil is included in the circuit to ensure better scale proportions. The scale is calibrated reversely means the normal position of pointer indicates infinity while full scale deflection indicates zero resistance.

8.11.3 Working

When the current flows from the generator, through the pressure coil, the coil tends to set itself at right angles to the field of the permanent magnet.

When the test terminals are open, corresponding to infinite resistance, no current flows through deflection coil. Thus the pressure coil governs the motion of the moving element making it move to its extreme anticlockwise position. The pointer comes to rest at the infinity end of the scale.

When the test terminals are short circuited i.e. corresponding to zero resistance, the current from the generator flowing through the current coil is large enough to produce sufficient torque to overcome the counter-clockwise torque of the pressure coil. Due to this, pointer moves over a scale showing zero resistance.

When the high resistance to be tested is connected between terminals T_1 and T_2 the opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale. The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.

The guard ring is provided to eliminate the error due to leakage current. The supply to the meter is usually given by a hand-driven permanent magnet d.c. generator sometimes motor-driven generator may also be used.

8.11.4 Applications

The megger can be used to determine whether there is sufficiently high resistance between the conducting part of a circuit and the ground. This resistance is called insulation resistance.

The megger can also be used to test continuity between any two points. When connected to the two points, if pointer shows full deflection then there is an electrical continuity between them.

8.12 Testing of Wiring Installation

It is necessary from the safety point of view, to carry out the following tests on a wiring installation before satart of its use.

The various tests are :

i) Installation test with respect to earth

ii) Installation test between two conductors

iii) Continuity test

iv) Test for earth resistance

v) Polarity test for single pole switches.

Most of these tests are conducted by an instrument called as **"Megger"**.

8.12.1 Insulation Test with Respect to Earth

The wires used in the system must be sound enough to avoid current leakage which can be checked by this test. Megger terminals are connected to a terminal of wire or main leads and the earth point. Normally 500 V Megger is used for the test.

In this test all lamps, and fuses are in ON position, except main switch which should be OFF. The handle is rotated, the position of pointer in steady state gives the value of the insulation resistance. For the entire installation the minimum value of insulation resistance required is one megaohm but not more than 1 MΩ.

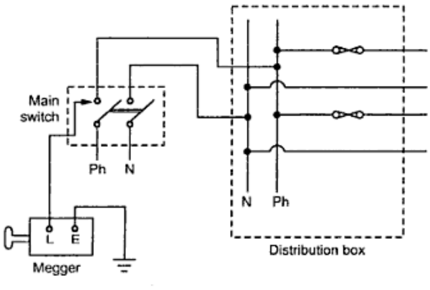


Fig. 8.34 Installation test with respect to earth

The measured resistance for satisfactory wiring must not be less than $\frac{50\ M\Omega}{\text{Number or oultets (points + switches)}}$. For PVC insulated cabels, instead of 50 MΩ a value of 12.5 can be used.

8.12.2 Insulation Test between Two Conductors

In this test, megger terminals are connected to two ends of different conductors which are insulated and placed side by side in the installation. The lamps and fuses must be removed while conducting this test. When megger handle is rotated, it gives the direct reading of an insulation resistance between the two conductors. Generally it should not be more than one megaohms. Such two conductors may be phase and neutral and there must exist a proper insulation between the two. The insulation resistance between two conductors should not be less than $\frac{50\ \text{M}\Omega}{\text{Number of oultets}}$.

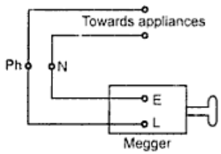


Fig. 8.35 Insulation test between two conductors

8.12.3 Continuity Test

In this test, megger terminals are connected to the ends of a single wire. When the handle is rotated, the pointer should indicate full deflection i.e. zero resistance. This indicates proper continuity between the two ends. If wire is open, megger indicates high resistance in megaohms. Due to this, faulty and borken wires can be detected and leakage current is avoided in metallic conduits which prevents the person from getting shock and ensures safety.

8.12.4 Test for Earth Resistance

In this test, the earth resistance of the user's earthing is measured with the help of a special device called as **megget earth tester**. Such earth resistance should be less than 5 Ω as per I.S.S.

8.12.5 Polarity Test for Single Pole Switches

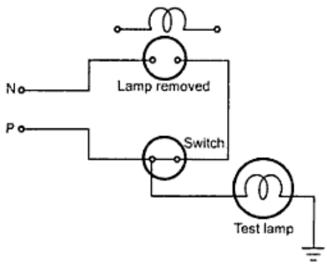


Fig. 8.36 Polarity test

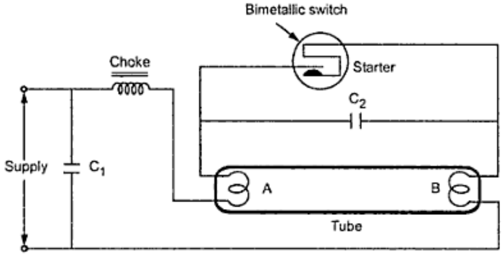


Fig. 9.1 Fluorescent tube

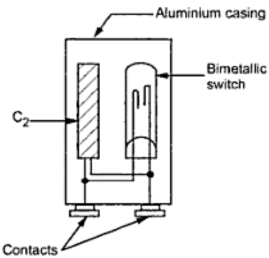


Fig. 9.2 Glow type starter

There are two electrodes A and B made up of coiled tungsten filament coated with an electron emitting material. The control circuit of the tube contains glow type starter, choke L and two capacitors C_1 and C_2.

Fig 9.2 shows a cut section of a glow type starter.

There are two electrodes of which one is fixed while other is U shaped bimetallic strip made of two different metals. These electrodes are sealed in a glass bulb which is filled with a mixture of helium and hydrogen. The contacts are normally open.

9.3.1 Working

When the supply is switched ON, as electric arc is established between the electrodes of the starter due to flow of current through small air gap between the electrodes. Due to this arc, heat is produced which is sufficient to bend the bimetallic strip which makes contact with fixed electrode. This closes the circuit and therefore choke carries large current. Once the electrodes close, arc vanishes and bimetallic strips cools down again.

Now the electrodes A and B become hot and due to cooling the choke circuit opens. The current through the choke coil is suddenly reduced to a small value. This change in current induces an e.m.f. which is very high of the order of 1000 V, in the choke coil.This e.m.f. induced is sufficient for ionizing the gas molecules between electrodes A and B which establishes the discharge between the electrodes A and B through the gas.

The potential difference across the tube falls to about 100-110 V which is sufficient to maintain the discharge but not sufficient to restart the glow in the circuit.

So even if starter is removed from the circuit, discharge continues as the current flows from electrode A and B due to ionization of gas. If the supply voltage is low, there is difficulty in starting the tube as the low voltage is insufficient to establish a glow in the starter.

As choke lowers the power factor, the capacitor C_1 used in the circuit improves the power factor of the circuit.

The capacitor C_2 suppresses the radio interference developed due to arcing. The function of the inductive choke is to supply a large voltage surge for establishing the discharge between the electrodes A and B.

9.3.2 Advantages

1) The light available is much more than the normal incandescent lamp. Fluorescent lamp gives 2200 to 2400 lumens while normal lamp gives 600 lumens.
2) The life of the fluorescent tube is much more than the incandescent lamp.
3) The fluorescent tube gives effect of day light while incandescent lamp gives yellowish light.
4) Low power consumption.
5) High efficiency.
6) Instantaneous switching without any warming period.
7) Using different fluorescent materials various coloured lights can be obtained.

9.3.3 Disadvantages

1. Very high initial cost.
2. Produces radio interference.

9.4 Sodium Vapour Lamp

The Fig. 9.3 shows sodium vapour lamp construction.

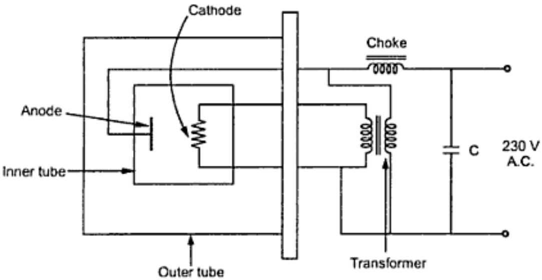


Fig. 9.3 Sodium vapour lamp

9.4.1 Construction

It consist of two glass tube, outer glass tube and inner glass tube. The inner glass tube contains two electrodes. Sodium along with the small quantity of neon or argon gas is filled in inner tube to make discharge self starting. Sodium vapour is chemically very active. The glass of the tube is made up of suitable material to resist this action.

To maintain the correct temperature in the discharge, it is placed in an evacuated outer tube. The outer tube reduces the heat loss. The transformer included in the circuit heats the cathode while the choke stabilises the discharge.

9.4.2 Working

When the lamp is switched on, the discharge is first established through the neon or argon gas, This gives out reddish colour. After some time heat is developed due to this discharge which vaporizes sodium vapour. In this way, lamp starts its normal operation giving yellow colour. Capacitor C is connected to have a better power factor. The operating temperature of this lamp is about 300°C. These lamps are commonly used for illumination of roads, good yards, airports etc.

9.4.3 Advantages

1. Its efficiency is higher than that of the filament lamps.
2. It has a long life.

9.4.4 Disadvantages

1. The bright yellow colour obtained is not suitable for indoor lighting. So it is not useful in houses.
2. For the necessary output, long tubes are required.
3. For giving full output, some time (about 10 minutes) is required.

9.5 Mercury Vapour Lamp

Fig. 9.4 shows mercury vapour lamp.

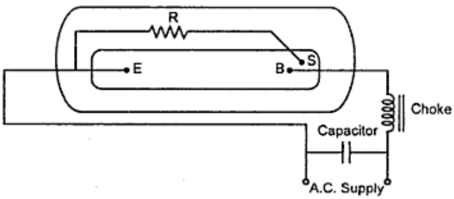


Fig. 9.4 Mercury vapour lamp

Construction : The lamp consists of two bulbs, inner bulb and outer bulb. The electric discharge takes place in the inner bulb. The outer bulb protects the inner bulb and reduces loss of heat. The inner bulb consists of a small amount of mercury and argon gas. The two electrodes E and B are made up of electron emitting material. Three electrodes B, E and S are provided in the inner bulb. The electrode 'E' is connected to electrode 'S' through a high resistance.

Choke (L) and capacitor (C) forms the control circuit of the lamp.

Working of mercury vapour lamp :

When the supply is switched ON, the initial discharge is established between electrodes B and S through the argon gas and then between electrodes B and E. The heat produced due to this discharge is sufficient to vapourise mercury and discharge through the mercury vapour takes place. In this normal operation of the lamp, it emits or radiates its characteristics light.

The electrode 'S' is called as starting electrode or auxilliary electrode. The choke serves to limit the current drawn by the electrodes to a safe limit. The capacitor C improves the power factor of the lamp.

These lamps are widely used for outdoor street lighting where a high illumination is necessary, where the colour of the light is not important.

Advantages :

1) It is high efficiency and gives more output.
2) It has long life.

Disadvantage :

1) The initial time required for warming up is more about 5 minutes.
2) If lamp goes out while in service, cooling is required for restarting. This cooling reduces the vapour pressure.

9.6 Important Definitions Related to Illumination

Before discussing about the various lighting schemes, it is necessary to be familiar with various terms used in illumination engineering.

Light is in the form of radiant energy. Similar to the sound waves, light waves are produced. Natural source of light is sun which emits both light waves and heat waves. In case of artificial light source, when temperature of incandescent material is increased it starts emitting light. It is convenient to measure the wavelength of light in much smaller unit than centimeter or meter. Such unit is called angstrom unit denoted as AU.

$$\boxed{1 \text{ AU} = 10^{-10} \text{ m}}$$

Key Point: *A visible light has wavelength of 4000 AU upto 7000 AU.*

9.6.1 Light

It is defined as that part of energy which is radiated from a body in the form of waves and sensed by human eyes.

9.6.2 Radiant Efficiency

A body radiates energy when its temperature is increased. The entire energy is not in the form of light but some may be in the form of other type of energy such as heat. So radiant efficiency is defined as the ratio of energy radiated as light by a body when its temperature is increased to the total energy radiated by the body.

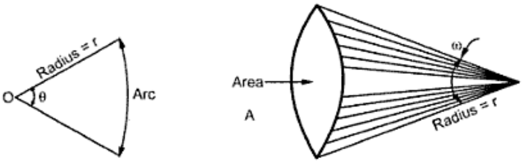
$$\text{Radiant efficiency} = \frac{\text{Energy radiated as a light energy}}{\text{Total energy radiated by the body}}$$

9.6.3 Plane Angle

The angle made by two straight lines meeting at a point lying in the same plane is called plane angle. It is measured in radians.

$$\text{Plane angle } \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{S}{r} \text{ radians}$$

This is shown in the Fig. 9.5 (a).



(a) Plane angle (b) Solid angle

Fig. 9.5

9.6.4 Solid Angle

The angle subtended at a point in space by an area is called solid angle ω.

It is the angle generated by the lines passing through the point in space and periphery of the area. It is measured in **steradians.**

$$\text{Solid angle } \omega = \frac{\text{Area}}{(\text{radius})^2} = \frac{A}{r^2} \text{ steradians}$$

This is shown in the Fig. 9.5 (b).

9.6.5 Luminous Flux

It is the energy in the form of light waves radiated per second from a luminous body (such as lamp). It is denoted by ϕ and measured in lumens.

The whole of the electrical energy input is not converted into luminous flux. Some energy gets lost in conduction, convention and radiation. Out of the remaining radiant flux, only a fraction of flux is available in the form of luminous flux. The value of luminous flux specifies the output and efficiency of a given light source.

9.6.6 Luminous Intensity

It is a measure of the brightness for the source of light in comparison with the standard lamp.

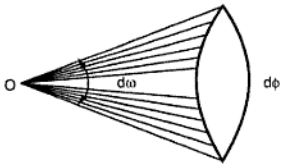


Fig. 9.6 Luminous intensity

The luminous intensity of a point source is defined as the luminous flux radiated out per unit solid angle, in that direction.

Consider a point source 'O' as shown in the Fig. 9.6.

Let $d\phi$ = the luminous flux passing through the solid angle $d\omega$ then the luminous intensity denoted as I is defined as

$$I = \frac{d\phi}{d\omega}$$

It is measured in **candela.**

Lumens emitted by one candela source of light is one lumen per steradian.

9.6.7 Illuminance or Illumination

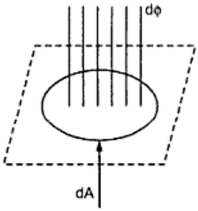


Fig. 9.7 Illumination

When a light or luminous flux falls on a surface gets illuminated. The illuminance is defined as the total luminous flux received by the surface received per unit area.

Consider the surface of area dA and receiving the total luminous flux of $d\phi$ lumens, as shown in the Fig. 9.7. Then the illumination denoted as E is defined as,

$$E = \frac{d\phi}{dA} \text{ lumens/m}^2$$

The unit lumens per square meter is also called **lux.**

The visibility becomes zero by rod cells at about 6000 AU, this effect is known as **purking effect**.

9.6.12 Mean Spherical Candle Power (M.S.C.P.)

It is the mean of candle powers in all the directions in all places.

$$\text{M.S.C.P} = \frac{\text{Total flux in all directions in all planes in lumen}}{4\pi}$$

9.6.13 Mean Half Spherical Candle Power (M.H.S.C.P.)

It is the mean of the candle powers in all the directions below a horizontal plane passing through the light source.

$$\text{M.H.S.C.P.} = \frac{\text{Total flux emitted in the hemispjhere}}{2\pi}$$

9.6.14 Different Units

i) Candela (cd) : It is seen that luminous intensity is measured in candela. One candel gives out luminous flux of 4π lumens in space. So one lumen per steradian is called light source of one candela.

It is also defined as the luminous intensity, in the perpendicular direction, of a surface of 1/600000 square meters of a black body at the temperature of freezing platinum under standard atmospheric pressure.

ii) Lumen (lm) : It is the unit of luminous flux. It is defined as the flux contained per unit solid angle of a source of one candela.

Hence for a uniform point source having a luminous intensity of I candelas, the total flux radiated is solid angle ω is,

$$\phi = I \times \omega \text{ lumens}$$

$$1 \text{ lumen} = 0.0016 \text{ watts}$$

iii) Lux : It is the unit of illumination. It is defined as lumens per square metres, (lm/m^2)

9.7 Laws of Illumination

9.7.1 Inverse Square Law of Illumination

The law states that the illumination at a point on any surface is inversely proportional to the square of the distance between the surface and the light source provided that the distance is sufficiently large so that source can be considered as a point source.

2. Using more number of lamps and providing indirect lighting.
3. Employing wide surface sources of light.

Complete absence of shadows is again not recommendable as soft shadows do help us in recognizing the shapes of three dimensional objects.

9.8.4 Colour Rendering

Light given out by incandescent lamp contains all the wave lengths in visible spectrum. Due to such mixture of the radiations, it causes colour distortion. This type of colour distortion by artificial light makes it sometimes difficult to identify true colour of the object. Lighting arrangements required for buying of food stuffs, preparation of paints, colour matching of fabrics in textile industries must be so as to avoid the colour distortion. Sodium and mercury vapour lamps cause the colour distortion and hence are in used for street lighting where such type of colour distortion would not matter much.

9.8.5 Lamp Fittings

Lamp fittings serve the following functions in good lighting schems :

1. To diffuse the light
2. To cut off the light at certain angles to avoid the glare.
3. To give mechanical protection to the light source against mechanical damage.
4. To increase the aesthetical requirements of the premises.
5. To control the colour of the lights.

Hence lamp fitting should be selected as per the requirements of the application which will serve all or few of the above mentioned functions.

9.8.6 Maintenance

Regular cleaning of lights and light fittings is necessary to maintain their efficiency. The maintenance is necessary against dust, water leakage, dangerous gases which may cause the corrosion of the fittings etc. Hence lamp fittings should therefore be simple and easy from maintenance point of view.

9.9 Factors Affecting Design Procedure of Good Lighting Scheme

The design procedure of a good lighting scheme involves the calculation of the light requirements. For doing such calculations it is important to know the following factors :

9.9.1 Space to Height Ratio

It is defined as the ratio of horizontal distance between lamps and the mounting height of the lamps.

$$\text{Space to height ratio} = \frac{\text{The horizontal distance between lamps}}{\text{Mounting height of lamps}}$$

This ratio depends upon the polar curves of the source of light. This ratio is generally choosen for reflectors as 1 to 2.

9.9.2 Coefficient of Utilization i.e. Utilization Factor

All the light emitted by the source cannot reach on the working plane.

Utilization factor is defined as the ratio of the light actually received on the working place to the total light emitted by the light source.

$$\text{U.F.} = \frac{\text{Total lumens utilised on working planes}}{\text{Total lumens radiated by lamp}}$$

Its value depends on the following factors :

1. The height at which the lamps are fitted.
2. The type of the lighting scheme employed.
3. The shape and size of the room.
4. The colour of the walls.
5. The area of the illuminated.

Its value may vary between 0.1 to 0.9. For direct lighting schemes it lies between 0.4 to 0.8 while for indirect lighting schemes it is between 0.1 to 0.35.

9.9.3 Depreciation Factor

This factor takes into account the effect of ageing of the lamps, accumulation of dirt, dust, smoke on the surface of the lamps etc. Due to these factors the effectiveness of the lamps reduces.

The factor is defined as the ratio of the illumination under normal working conditions to the illumination under the conditions when the lamps are new and everything is perfectly clean.

$$\text{D.F.} = \frac{\text{Illumination under normal working conditions}}{\text{Illumination under new and clean lamp conditions}}$$

The value of the factor is generally between 0.7 to 0.85.

9.9.4 Waste Light Factor

When more lamps are provided than the actual requirements so that the light from various lamps get overlapped. So there is wastage of light. Hence while calculating total lumens required it is necessary to multiply the lumens by this factor called as waste light factor. Value of this is 1.2 for rectangular areas and 1.5 for irregular surfaces.

9.10.1 Direct Lighting

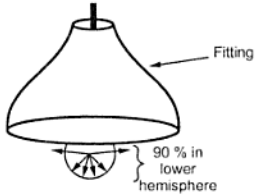


Fig. 9.11 Direct lighting

In this type of lighting, the most of the light from the lamps is directed towards the object using various light fitting. The fitting used in such scheme are as shown in the Fig. 9.11.

Here 90 % to 100 % light of lamp is in lower hemisphere as shown. These fittings are efficient, cheap and give hard light. These fittings create tunneling effect i.e. ceiling remains dark. Commonly used in industries, residential and commercial buildings.

9.10.2 Semi-Direct Lighting

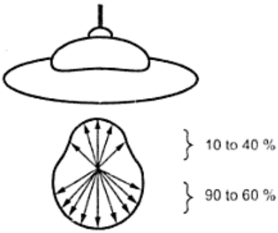


Fig. 9.12 Semi-direct lighting

In such scheme entire light is not directed towards the object or working plane. By using translucent reflectors as shown in the Fig. 9.12, 60 to 90 % of light is directed towards the object. So 60 to 90 % light is in lower hemisphere while 10 to 40 % light is in upper hemisphere as shown in the Fig. 9.12.

These fittings reduce the tunneling effect. This provides more uniform distribution of light. These are less efficient than direct and are more suitable for commercial use.

9.10.3 Indirect Lighting

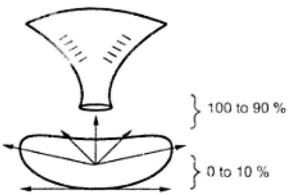


Fig. 9.13 Indirect lighting

In this method, the light does not reach the working plane directly from the lamps. It is all reflected on the walls and ceiling from where it is indirectly reached to the working plane. This is called as diffused reflection.

In this 0 to 10 % of light is in the lower hemisphere and 90 to 100 % is in upper hemisphere as shown in the Fig. 9.13. The advantage of this method is it gives no shadows and no glare. It is used in shops, hotels, drawing offices and workshops.

9.10.4 Semi-Indirect Lighting

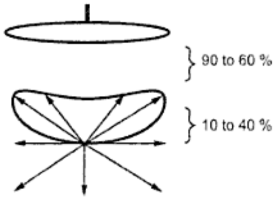


Fig. 9.14 Semi-indirect lighting

In this method, 10 to 40% light is directed towards the working plane and 60 to 90% light is reflected towards the walls and ceiling for diffused reflection.

The inverted type of reflectors are used for this purpose as shown in the Fig. 9.14. This method gives soft shadows and is more efficient than indirect lighting.

9.10.5 General Lighting

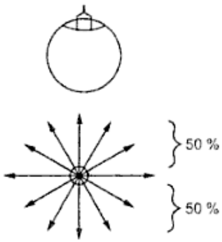
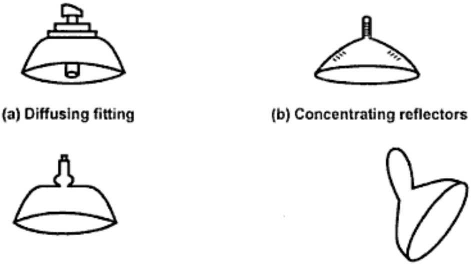


Fig. 9.15 General lighting

This method gives equal distribution of light in all the directions, upward and downwards. The reflectors used are in such a way that 50% of light is in either of hemisphere. This is shown in the Fig. 9.15. This helps in reducing the brightness contrast. This method gives soft light with little shadows. With this method room decoration should be in light colours. Also mounting height of the lamps should be so as to avoid glare. These are used in offices, schools and commercial buildings.

9.11 Factory Lighting

Sufficient lighting of factories is of very much important. Not only it provides improved amenities for the employee but it also provides pleasent working conditions for the workers. The factor lighting should be uniform and glare free. The sufficient lighting increases the production, improves the quality of the work and also reduces the accidents. The proper space to height ratio must be considered which ensures the uniform distribution of the light over the working area. Suitable colour rendering property must be considered i.e. the walls and ceilings must be light coloured preferably white to have maximum reflection of light. If factory has a crane, lamps over the crane may not provide sufficient light. In such cases hanging lamps and side lamps must be provided. Depending upon the nature of factory, the factory environment is likely to be dusty, smoky hence regular maintenance including cleaning of light fittings and lamp surfaces is must. For



(a) Diffusing fitting

(b) Concentrating reflectors

(c) Dispersive reflector

(d) Angle reflectors

Fig. 9.16 Industrial light fittings

9.12.2 Concentrating Reflectors

These have a shape of deep parabola. The space to height ratio required is one to get uniform distribution of light. Waste light factor is more in such cases. It is suitable for high mounting height as required in workshops and industries having electric overhead cranes. Fig. 9.16 (b).

9.12.3 Dispersive Reflectors

This is most commonly used in industries. The space to height ratio is one and half, which gives uniform illumination. These are preferred for moderate ceiling height. The lamps rated 40 to 1500 W are fixed inside the reflectors. Fig. 9.16 (c)

9.12.4 Angle Reflectors

The vertical surfaces can not be illuminated by normal overhead lamps, in such cases angle reflectors are used. These are available in various shapes like parabolic, elliptical etc. The choice depends on the requirement of illumination. Fig. 9.16 (d).

9.13 Flood Lighting

Flood lighting means flooding of large surfaces with light from powerful projections. It is employed to serve one or more of the following purposes.

i) Aesthetic flood lighting : It is used for enhancing beauty of buildings at night, ancient buildings and monuments, churches, gardens etc.

ii) Industrial and commercial flood lighting : It is used for illuminating railway yards, art stadiums, car parking areas etc.

iii) Advertising : It is used for illuminating advertisements and show cases at night

The main element of the flood lighting is the projector.

9.13.1 The Projector

The projector called flood light projector concentrates the light form the lamp into a relatively narrow beam. It is usually installed out of doors and in inaccessible positions.

The most important part of the projector is its reflecting surface. This may be of silvered glass, stainless steel or chromium plates. The polished metal is more robust and preferred for reflecting surfaces.

Beam spread : Beam spread indicates amount of divergence of the beam and is defined as the angle within which the minimum illumination on a surface normal to the axis of the beam is $1/10^{th}$ of the maximum.

The casing and mounting must be arranged so that the illumination of the beam can be varied in both horizontal and vertical directions. For permanent installations cast metal cases are used. For temporary installations sheet metal casings are used. Large wattage lamps produce tremendous heat hence for the projectors using lamps of 100 to 1000 W must be provided the proper ventilating accessories.

9.13.2 Types of Projectors

Projectors are classified according to the beam spread as follows :

i) Narrow beam projectors : These are having beam spread of 12° to 25° and are used for distance above 70 m.

ii) Medium angle projectors : These are having beam spread of 25° to 40°. They are used for distances between 30 to 70 m.

iii) Wide angle projectors : These are having beam spread of 40° to 90° and are used for distances between 3 to 30 m.

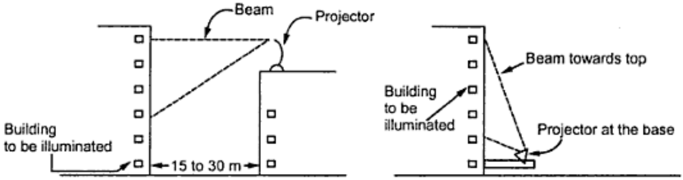
For economic reasons use of wide angle projectors with high wattage lamp is encouraged over narrow beam projectors. High wattage lamp is more efficient than low wattage lamp, used is narrow beam projectors.

9.13.3 Location and Mounting of Projectors

Success of flood lighting depends on choice of suitable sites for the projectors. The source of light must be invisible to the observer and projector must be invisible during the day time. Care must be taken about the shadow cast by projections on the illuminated building.

In long range installation, the projectors are mounted on the adjacent building or at ground level about 15 to 30 m away from the building to be illuminated. The parallel beam is generally used in such situations. This is shown in the Fig. 9.17 (a) . The parabolic reflector is used.

In short range installation, the projector has to be located near the base of the building to be illuminated. An asymmetric reflector is used, which directs more intense light towards the top of the building. This is shown in the Fig. 9.17 (b).



(a) Long range installation (b) Short range installation

Fig. 9.17

9.13.4 Flood Lighting Calculations

i) Illumination level required : The illumination level in lux (lm/m^2) required depends on the type of building, the purpose of flood lighting, the amount of reflecting light in the vicinity etc.

ii) Type of projector : Beam spread, long range or short range installation and light output necessary decides the type of projector to be used.

iii) Number of projectors : For any desired intensity over a definite surface area, the number of projectors can be obtained by using following relation,

$$N = \frac{A \times E \times \text{Depreciation factor} \times \text{Waste light factor}}{\text{Beam factor} \times \text{Utilization factor} \times \text{Wattage of lamp} \times \text{Luminous efficiency of lamps}}$$

where N = Number of projectors

A = Area of surface to be illuminated in m^2

E = Illumination level required in lm/m^2

Depreciation factor must be used in denominator if its value is less than one or in numerator if greater than one.

9.14 Street Lighting

The street lighting is basically essential for the pedestrians and the vehicle drivers to see the obstructions clearly. The features of the street lighting are, Colour and decoration of walls and ceiling is deciding factor for the portion of light to be reflected. Mounting height of lamp also plays an important role. To avoid glare and long shadows, the lamp should be mounted 8 feet above floor. The cleaning and relamping becomes more dificult as ordinary step ladder can not be used above 12 feet height. Hence mounting height of lamps should be between 8 to 12 feet to avoid special arrangements for its access. In certain cases, the positions of beams and trusses is fixed which decides to some extent the spacing of lamps so that mounting height lamps can be decided.

4) Colour of light

Appearance of body colour depends on incident light composition which should be such that colour appears natural. For certain applications colour of light is not important. But if components are to be distinguished from each other by their colours then colour of light is important consideration. In such cases highly efficient discharge lamps may be used.

5) Financial aspects

Economy plays an important role while designing indoor lighting scheme. Installation cost depends on type and fitting used for lamp. Discharge lamps are more efficient but more costly than filament lamps. For lower rate of electricity and for few working hours in a year, better way is to use filament lamp than discharge lamps which may be used for higher rate and more working hours. The cleaning and maintenance cost along with reclamping cost must be taken into account. For more mounting height this cost may be more. Overall cost of lighting scheme should be economical with high energy efficiency, good life span and having good appearance.

By considering all the above factors final choice about the installation and design of lighting scheme must be made after careful study of each installation independently and taking into consideration the previous experience.

Examples with Solutions

Example 9.1 : *An illumination on the working plane of 70 lux is required in a room 72 m × 15 m in size. The lamps are required to hung 4 m above working plane. Assuming suitable space to height ratio, utilisation factor of 0.5, efficiency of lamp as 14 lumens per watt and depreciation of 20 %, estimate the number, rating and position of lamps.*

Solution : Area of room = 72 × 15 = 1080 m^2

Illumination required, E = 70 lux

$$\text{Lumens required} = A \times E = 1080 \times 70$$

$$\text{Utilisation factor} = 0.5$$

$$\text{Depreciation factor} = 1 - \text{depreciation} = 1 - 0.2 = 0.8$$

$$\text{Gross lumens} = \frac{A \times E}{\text{Utilisation factor} \times \text{Depreciation factor}}$$

$$= \frac{1080 \times 70}{0.5 \times 0.8}$$

$$= 189000$$

$$\text{Total wattage required} = \frac{189000}{14} = 13{,}500 \text{ watts}$$

Assuming space to height ratio as 1. As mounting height is 4 m spacing should be also 4 m to get ratio 1. Let us assume number of lamps to be 72 with spacing in between the lamps as 4 m. The spacing is shown in the Fig. 9.19.

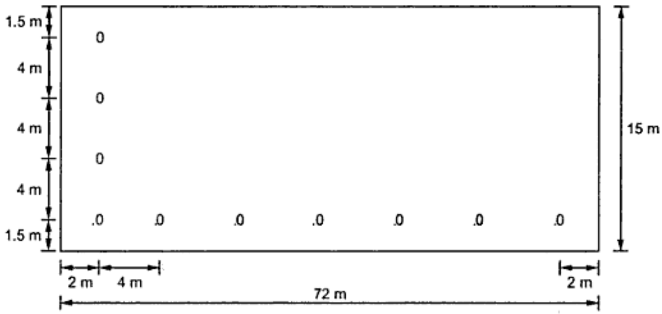


Fig. 9.19

There are 4 rows of lamps. In each row there are 18 lamps situated at a distance of 4 m from each other.

$$\text{Rating of lamp} = \frac{\text{Total wattage required}}{\text{Total no. of lamps}}$$

$$= \frac{13{,}500}{72}$$

$$= 187.5 \text{ watt}$$

Hence the lamps with rating of **200 W** should be selected.

Example 9.2 *Estimate the number and wattage of lamps which would be required to illuminate a work shop 60 × 15 m by means of lamps mounted 5 m above the working plane. The average illumination required is 100 lux with utilisation factor of 0.4. Luminous efficiency of lamp is 16 lumens/watt. Assume a space-height ratio of unity and depreciation factor of 1.2. Show the arrangement of lamps with their spacing.*

Solution : Area of workshop = 60 × 15 = 900 m^2

Illumination = 100 lux

Utilisation factor = 0.4, Depreciation factor = 1.2

Total lumens required = 100 × 900 = 90, 000

$$\text{Gross lumens required} = \frac{\text{Total lumens required} \times \text{Deprecitation factor}}{\text{Utilisation factor}}$$

Sometimes the depreciation factor is defined as,

$$\text{Depreciation factor} = \frac{\text{Illumination under conditions when everything is clean}}{\text{Illumination under normal working conditions}}$$

Key Point: *In this case its value is greater than 1 and it should be considered in the numerator for the formula of gross lumens required.*

$$\therefore \quad \text{Gross lumens required} = \frac{90000 \times 1.2}{0.4} = 270000$$

$$\text{Total wattage required} = \frac{270000}{16} = 16875 \text{ W}$$

As the mounting height is given as 5 m above ground the spacing between lamps should be 5 m to get space to height ratio as 1. One side of workshop is 15 m in which 3 lamps can be fitted while along the length the number of lamps that can be fitted will be say 12. The arrangement is shown in the Fig. 9.20.

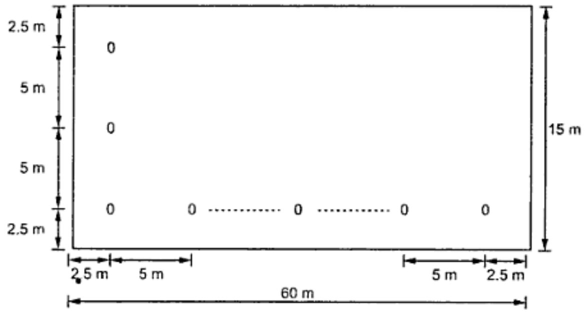


Fig. 9.20

$$\text{Wattage of each lamp} = \frac{16875}{36}$$

$$= 468.75$$

We may select **500 W.**

Key Point: *As it is a design problem everybody will not get the same arrangement. But the factors given in the problem must be satisfied. If some factors are not given then they should be suitably assumed and the problem should be solved. As per number of lamps the position and wattage of lamp will change.*

Example 9.3 : *It is required to provide an illumination of 100 lux in a factory hall of 40 m × 10 m. Assume depreciation factor of 0.8 and coefficient of utilisation of 0.4. The efficiency of lamp is 14 lumens/W. If 300 W lamps are to be used, calculate the number of lamps and their positions.*

Solution : Area of factory = 40 × 10 = 400 m^2

Illumination, E = 100 lux

Total lumens = A × E = 400 × 100 = 40000

Depreciation factor = 0.8, Utilisation factor = 0.4

$$\text{Gross lumens required} = \frac{\text{Total lumens}}{\begin{bmatrix}\text{Depreciation} \\ \text{factor}\end{bmatrix} \times \begin{bmatrix}\text{Utilisation} \\ \text{factor}\end{bmatrix}} = \frac{40000}{0.8 \times 0.4} = 1{,}25{,}000$$

$$\text{Total wattage required} = \frac{\text{Gross lumens required}}{\text{Efficiency of lamp}} = \frac{1{,}25{,}000}{14} = 8928.57 \text{ W}$$

$$\text{Number of lamps required} = \frac{\text{Total wattage required}}{\text{Rating of lamp}} = \frac{8928.57}{300} = 29.76 \approx 30$$

Assuming the space to height ratio to be 1 and let us asume that the mounting height of lamp to be 4 m so spacing between lamps is also 4 m so let's have 3 rows of lamps with 10 lamps in each row. The arrangement is shown in the Fig. 9.21

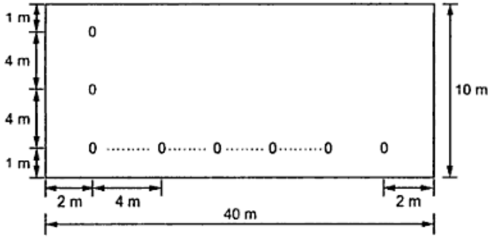


Fig. 9.21

Example 9.5 : *A hall 30 × 12 m is to be illuminated with illumination of 50 lux. Five types of lamps with their lumen outputs as given below are available.*

Watts	100	200	300	500	1000
Lumens	1620	3660	4700	9900	21000

The depreciation factor is 1.3 and utilisation factor is 0.5. Calculate the number of lamps in each case. Select most suitable type and design a suitable scheme. Assume suitable mounting height and calculate space to height ratio.

Solution : Area of hall = $30 \times 12 = 360\ m^2$

$$E = 50 \text{ lux}$$

Depreciation factor = 1.3, Utilisation factor = 0.5

Total lumens required = $A \times E = 360 \times 50 = 18000$

$$\text{Gross lumens required} = \frac{\text{Total lumens} \times \text{Depreciation factor}}{\text{Utilisation factor}} = \frac{18000 \times 1.3}{0.5}$$

$$= 46{,}800 \qquad \text{... Depreciation factor} > 1$$

For 100 W lamps, No. of lamps = $\frac{46800}{1620} = 28.88 \approx 29$

For 200 W lamps, No. of lamps = $\frac{46800}{3660} = 12.78 \approx 13$

For 300 W lamps, No. of lamps = $\frac{46800}{4700} = 9.95 \approx 10$

For 500 W lamps, No. of lamps = $\frac{46800}{9900} = 4.72 \approx 5$

For 1000 W lamps, No. of lamps = $\frac{46800}{21000} = 2.22 \approx 2$

Let the mounting height of lamps be 5 m. With 300 W lamps let us select 2 rows with 5 lamps in each row as shown in the Fig. 9.23.

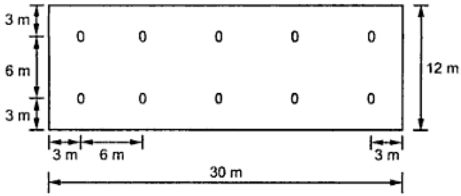


Fig. 9.23

Review Questions

1. *Along with the necessary diagrams, explain the working of fluorescent lamp.*
2. *Along with the necessary diagrams, explain the working of sodium vapour lamp.*
3. *Along with the necessary diagrams, explain the working of mercury vapour lamp.*
4. *Define the terms : luminous flux, luminous intensity, illumination, radiant efficiency, plain angle, solid angle, luminance, luminous efficiency.*
5. *What is purking effect ? Explain.*
6. *State and explain laws of illumination.*
7. *Discuss the general requirements of a good lighting scheme.*
8. *Discuss the following factors :*

   i) Space to height ratio ii) Depreciation factor iii) Waste light factor iv) Utilization factor v) Beam factor.
9. *Write a short note on various lighting schemes.*
10. *What is factory lighting ? Explain the essential requirements of a factory lighting.*
11. *Write a note on industrial fittings.*
12. *What is flood lighting ? Where is it used ? Discuss the different types of projectors used in flood lighting.*
13. *Write a note on street lighting clearly explaining the principles of diffusion principle and specular reflection principle.*
14. *Explain the factors affecting design of indoor lighting scheme.*
15. *A hall of 30 × 15 m with a ceiling height of 5 m is to be provided with a general illumination of 120 lumens/m^2. Taking utilization factor 0.5 and depreciation factor 1.4, determine the number of fluorescent tubes required and their spacing. Take luminous efficiency of tube as 40 lumens/W for 80 W tube.* **(Ans. 48 tubes)**

University Questions

Q.1 *Explain the wiring diagram of a tubelight with choke and glow starter.*
[GTU : Dec.-2008, 7 Marks]

Q.2 *List lumens requirements for various categories of illumination.*
[GTU : June-2009, 4 Marks]

❑❑❑

10 Protective Devices and Safety Precautions

10.1 Introduction

The importance of electric supply is well known. It is very much necessary to protect the power systems, equipments, motors, generators etc. from the dangerous fault conditions in an electric supply. Hence it is necessary to have the arrangements with which all these equipments can be switched on or off under no load or load conditions or even under fault conditions. The collection of various equipments used for the switching and protecting purposes in a power system is called **switchgear**. The various components of a switchgear are switches, fuses, relays, circuit breakers etc. The switchgear protects the system from fault and abnormal conditions and assures continuity of an electric supply.

10.2 Introduction to Relay

An important element of any protective relaying scheme is a relay. It is a device which detects the fault and is responsible to energize the trip circuit of a circuit breaker. This isolates the faulty part from rest of the system.

10.2.1 Basic Trip Circuit Operation

Consider a simplified circuit of a typical relay as shown in the Fig. 10.1. Usually the relay circuit is a three phase circuit and the contact circuit of relays is very much complicated. The Fig. 10.1 shows a single phase simplified circuit to explain the basic action of a relay. Let part A is the circuit to be protected. The current transformer C.T. is connected with its primary in series with the line to be protected. The secondary of C.T. is connected in series with the relay coil. The relay contacts are the part of a trip circuit of a circuit breaker. The trip circuit consists of a trip coil and a battery, in addition to relay contacts. The trip circuit can operate on a.c. or d.c.

If the fault occurs as shown in the Fig. 10.1, then current through the line connected to A increases to a very high value. The current transformer senses this current. Accordingly its secondary current increases which is nothing but the current through a relay coil. Thus the relay contacts get closed mechanically under the influence of such a high fault current. Thus the trip circuit of a circuit breaker gets closed and current starts flowing from battery, through trip coil, in a trip circuit. Thus the trip coil of a circuit breaker gets energised. This activates the circuit breaker opening mechanism, making the circuit breaker open. This isolates the faulty part from rest of the healthy system.

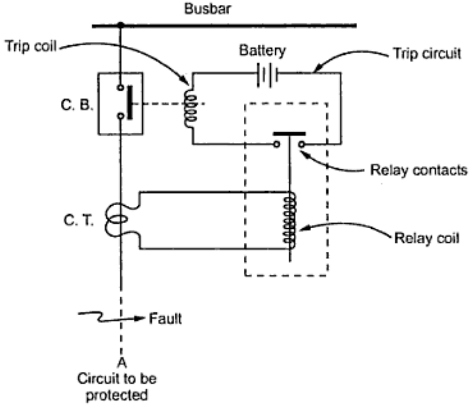


Fig. 10.1 Typical relay circuit

10.2.2 Auxiliary Switch

Another important device in the trip circuit is an auxiliary switch. It is a multipoint switch generally 4 point, 6 point, 12 point or 24 point. This switch is mechanically coupled with operating mechanism of the circuit breaker. Thus when circuit breaker opens, the switch also gets opened. The switch is in the trip circuit and hence when it opens, it breaks the current through the trip circuit. Once the current in the trip circuit is interrupted the relay contacts come to normal position. The advantage of an auxiliary switch is that the breaking of trip circuit takes place only across the switch and hence possible arcing due to current interruption across the relay contacts gets avoided. Such arcing is harmful for relay contacts as relay contacts are delicate and light. To interrupt a current through the inductive circuit like trip circuit a robust mechanical switch is necessary. This purpose is served by an auxiliary switch, protecting delicate relay contacts. In addition to this, indication circuits showing whether circuit breaker is open or close and some other control circuits also get connected or disconnected by an auxiliary switch.

The auxiliary switch is generally placed in the control cabinet of the circuit breaker.

10.3 Tripping Schemes

Two schemes are very popularly used for tripping in circuit breakers which are,

1. Relay with make type contact
2. Relay with break type contact

The relay with make type contact requires auxiliary d.c. supply for its operation while the relay with break type contact uses the energy from the main supply source for its operation. Let us see the details of these two types of schemes.

10.3.1 Relays with Make Type Contact

The schematic diagram representing the arrangement of various elements in a relay with make type contact is shown in the Fig. 10.2.

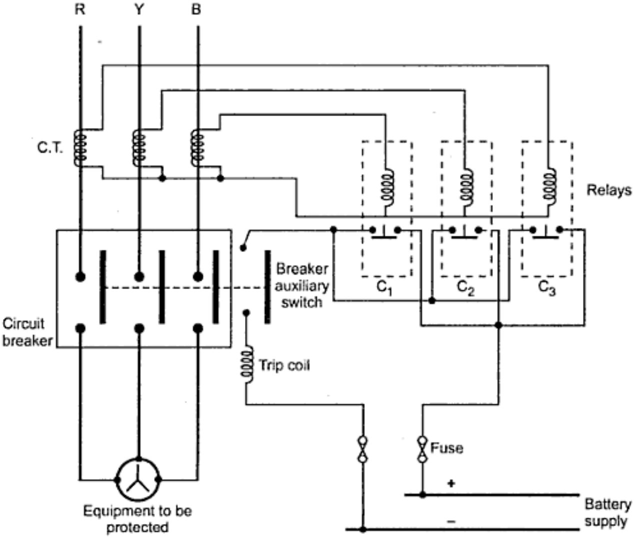


Fig. 10.2 Relay with make type contact

As mentioned earlier, a separate supply is necessary for the relay operation. The relays are connected in star while the relay contacts are connected in parallel. The entire relay contact unit is connected in series with the auxiliary switch, trip coil and the battery. Relay contacts are open in normal position.

Operation : When the fault occurs, the current through relay coils increases to a very high value. Due to this, the normally open relay contacts C_1, C_2 and C_3 get closed. This activates the trip coil of a circuit breaker. The auxiliary switch is initially closed along with the circuit breaker. So when contacts C_1, C_2 and C_3 are closed, the current flows through trip coil of circuit breaker. This activates the trip coil which opens the circuit breaker. As auxiliary switch is mechanically coupled with the circuit breaker, it also gets opened. This interrupts the current through trip coil. Thus supply to faulty part gets interrupted and trip coil also gets de-energized. This brings the relay contacts back to normal position. Due to auxiliary switch, arcing across relay contacts gets avoided. As relay contacts are normally open and they 'make' the circuit to open the circuit breaker hence called make type contact relay.

10.3.2 Relay with Break Type Contact

The schematic arrangement of various elements in a relay with break type contact is shown in the Fig. 10.3.

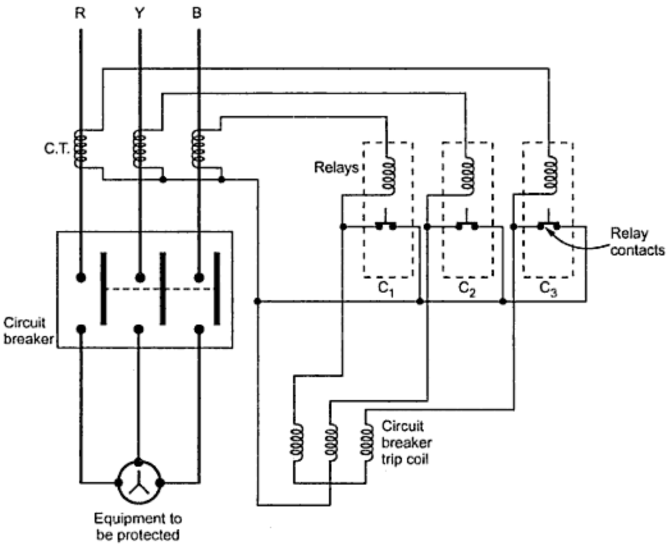


Fig. 10.3 Relay with break type contact (using C.T.s)

This type of relay does not require external battery supply for the tripping. The current transformers (C.T.s) or potential transformers (P.T.s) are used to derive the energy required for the relay from the main supply source. The relay using C.T.s to derive operating energy from the supply is shown in the Fig. 10.3.

In this scheme, the relay coil and trip coil of each phase are connected in series. The three phases are then connected in star. Under normal working, the relay contacts C_1, C_2 and C_3 are closed. The energy for relay coils is derived from supply using C.T.s. The trip coils of circuit breakers are de-energized under normal condition. When the fault occurs, heavy current flows through relay coils due to which relay contacts C_1, C_2 and C_3 break. Thus current flows through trip coils of circuit breaker due to which circuit breaker gets open.

The Fig. 10.4 shows the break type contact relay using P.T. to derive energy to keep relay coils energized.

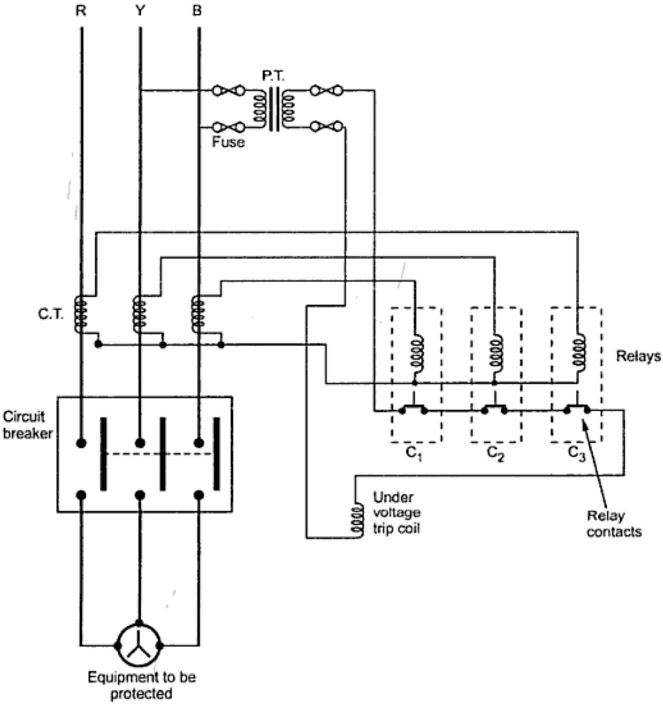


Fig. 10.4 Relay with break type contact (Using P.T.)

In this type, in addition to normal trip coils of circuit breaker, an additional undervoltage trip coil is used. All the relay contacts are in series with the undervoltage trip coil. Through potential transformer, for normal voltage, the undervoltage trip coil is kept energized. When the voltage becomes less than the normal value, the magnetic effect produced by undervoltage trip coil reduces which is responsible for the opening of the circuit breaker. When fault occurs, the normal trip coils of circuit breaker come into the picture and are responsible for the opening of the circuit breaker.

In both the types, relay contacts 'break' to cause the circuit breaker operation hence the relay is called break type contact relay.

10.4 Electromagnetic Attraction Relays

In these relays, there is a coil which energizes an electromagnet. When the operating current becomes large, the magnetic field produced by an electromagnet is so high that it attracts the armature or plunger, making contact with the trip circuit contacts. These are simplest type of relays. The various types of electromagnetic attraction type relays are,

1. Attracted armature relay
2. Solenoid and plunger type relay

10.4.1 Attracted Armature Type Relay

There are two types of structures available for attracted armature type relay which are,

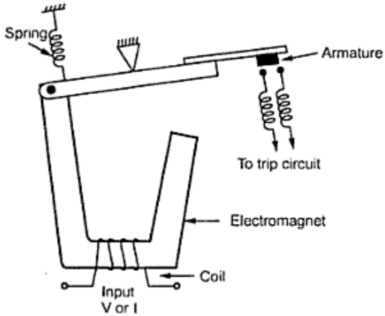
i) Hinged armature type

ii) Polarised moving iron type

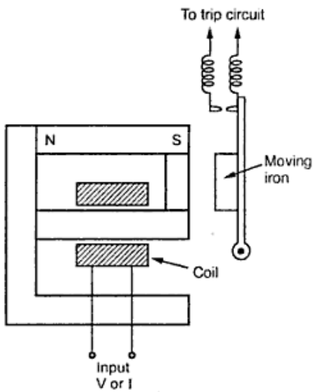
The two types of attracted armature type relays are shown in the Fig. 10.5(a) and (b).

In attracted armature type, there exists a laminated electromagnet which carries a coil. The coil is energized by the operating quantity which is proportional to the circuit voltage or current. The armature or a moving iron is subjected to the magnetic force produced by the operating quantity. The force produced is proportional to the square of current hence these relays can be used for a.c. as well as d.c. The spring is used to produce restraining force. When the current through coil increases beyond the limit under fault conditions, armature gets attracted. Due to this it makes contact with contacts of a trip circuit, which results in an opening of a circuit breaker.

The minimum current at which the armature gets attracted to close the trip circuit is called **pickup current**.



(a) Hinged armature type relay



(b) Polarised moving iron type relay

Fig. 10.5

Generally the number of tappings are provided on the relay coil with which its turns can be selected as per the requirement. This is used to adjust the set value of an operating quantity at which relay should operate.

An important advantage of such relays is their high operating speed. In modern relays an operating time as small as 0.5 msec is possible. The current-time characteristics of such relays is hyperbolic, as shown in the Fig. 10.6.

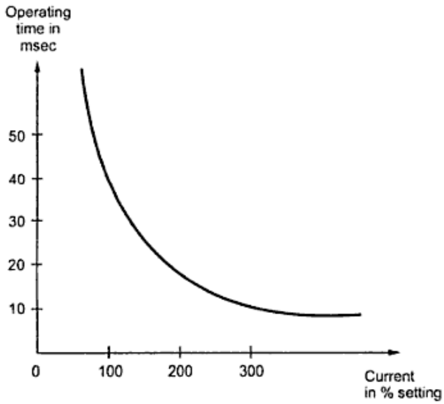


Fig. 10.6 Current-time characteristics

10.4.2 Solenoid and Plunger Type Relay

The Fig. 10.7 shows the schematic arrangement of solenoid and plunger type relay which works on the principle of electromagnetic attraction.

It consists of a solenoid which is nothing but an electromagnet. It also consists a movable iron plunger. Under normal working conditions, the spring holds the plunger in the position such that it cannot make contact with trip circuit contacts.

Under fault conditions when current through relay coil increases, the solenoid draws the plunger upwards. Due to this, it makes contact with the trip circuit contacts, which results in an opening of a circuit breaker.

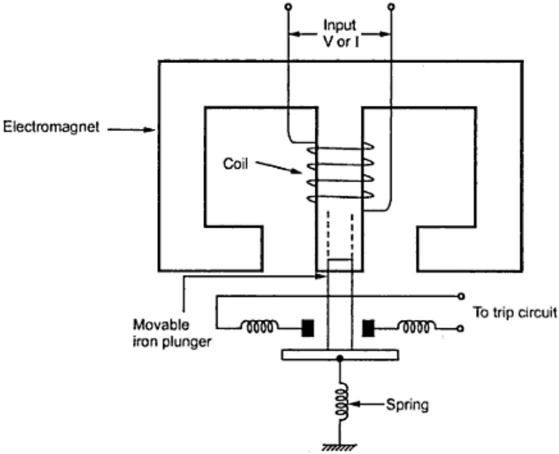


Fig. 10.7 Solenoid and plunger type relay

10.5 Induction Type Relays

The induction type relays are also called magnitude relays. These relays work on the principle of the induction motor or an energy meter. In these relays a metallic disc is allowed to rotate between the two electromagnets. The coils of the electromagnets are energized with the help of alternating currents. The torque is produced in these relays due to the interaction of one alternating flux with eddy currents induced in the rotor by another alternating flux. The two fluxes have same frequency but are displaced in time and space. As the interaction of alternating fluxes is the base of operation of these relays, these are not used for the d.c. quantities. These are widely used for protective relaying involving only a.c. quantities.

Based on the construction, the various types of the induction type relays are,

1. Shaded pole type
2. Watthour meter type
3. Induction cup type

10.6 Thermal Relays

Thermal relays work on the principle of heating effect of an electric current in the relay coil. Instead of the measurement of temperature, these relays sense the temperature rise produced by the current.

In a simplest thermal relay, a bimetallic strip is used. The strip is mounted above a resistance coil carrying current to produce necessary heating effect. The spring is used to make the connection between contacts and the strip. The insulated lever arm is used to carry the contact which is pivoted. To have variable settings, the tension in the spring can be adjusted. The Fig. 10.8 shows the schematic diagram of thermal relay.

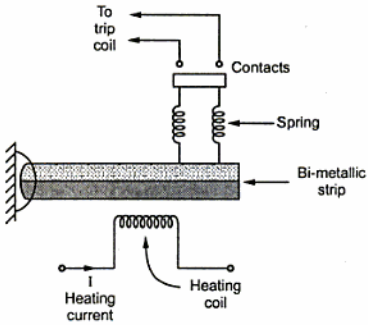


Fig. 10.8

Under normal conditions, the heating due to current I is not enough to heat the strips and contacts remain closed as strip remains straight.

When there is overloading, then the current I increases beyond safe value producing very high I^2R losses and corresponding large heat. Thus the strip gets heated up and bends. Due to the bending of the strip, the spring opens the contacts and current is interrupted.

In some cases, the bimetallic strips themselves carry the current without using a heater coil. These relays are commonly used in protection of low voltage a.c. and d.c. motors. In case of large motors, the bimetallic strip is connected through current transformer.

...ature Circuit Breaker (MCB)

A miniature circuit breaker is an electromechanical device which makes and breaks the circuit in normal operation and disconnects the circuit under the abnormal condition when current reaches a preset limit. It can be used in place of fuse in distribution board. As it is accurate and efficient under the overload and short circuit conditions, it can replace conventional rewirable fuse.

Normally MCB operates at 1.25 times its rated current. If a MCB is rated for 16 A then it will operate at 20 A. With the latest technology, it is tried that MCB will operate at 1.05 times its rated value. MCBs can be reset quickly by hand after its operation without any cost which is not the case with a rewirable fuse and they cannot be reclosed if fault still exists. Manually MCB's can be reclosed after correcting the fault. The MCB which is tripped due to overload or short circuit can be easily identified as its operating knob moves to OFF position.

A typical cross sectional view of MCB is shown in the Fig. 10.9.

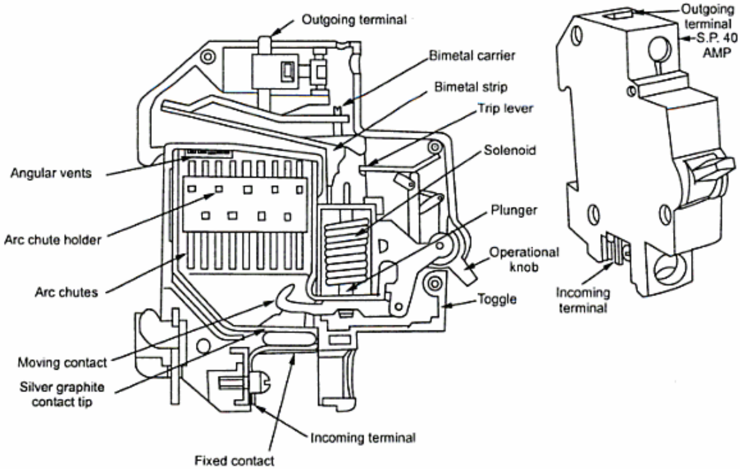


Fig. 10.9

A MCB is a high fault capacity current limiting, trip free automatic switching device with thermal and magnetic operation for protection against overload and short circuit. The thermal operation is possible with the use of bimetallic strip. A bimetallic strip deflects or bends when current exceeds a preset limit due to heating. Due to its deflection, a latch mechanism is released which opens the contacts. For providing protection against short circuit, inverse time current characteristics is used which gives lesser time for larger fault currents. The opening time of MCB is less than one msec for larger currents and possibility of welding of contacts due to heating is eliminated due to its quick operation.

When short circuit occurs in the circuit, the current energizes the solenoid which attracts the plunger which strikes the trip lever which immediately releases the latch mechanism to open the contacts. The instantaneous opening of contacts is achieved due to rapid operation of solenoid.

The arc chute is used to increase the length of arc when it passes through it. Due to lengthening, arc quickly extinguishes. This arc is produced due to separation of fixed and moving contacts during the operation of MCB. Under the influence of magnetic field, the arc is rapidly moved into arc chute where it quickly cools and gets extinguished which ensures longer life of MCB. For ensuring longer life of contacts, eliminating possibility of arc erosion, a non melting material silver graphite is used for contacts. The contact surface must be cleaned after every operation of MCB for ensuring its effective and efficient operation.

The material used for body of MCB must be having high melting point, minimum water absorption and flame resistant. It should also have high dielectric strength of low coefficient of linear thermal expansion.

All moving parts within MCB are placed in a sheet metal casing instead of plastic casing which changes its shape under thermal stress and continuous load. In case of a multipole MCB, all phases should be tripped simultaneous in the event of short circuit or overload.

Generally the MCBs are rated for ac voltage of 240 V for single phase, 415 V for three phase or 220 V dc. The current rating available is from 0.5 A to 63 Amp. It is available as 1 pole, 2 pole, 3 pole or 4 pole with short circuit breaking capacity from 1 kA to 10 kA with rated frequency of 50 or 60 Hz. The mechanical life of MCB is upto or more than 1 lakh operating cycles.

MCBs are classified into two series. L-series MCBs are suitable for resistive loads with low and steady currents. It includes heaters, ovens, geysers, electric irons, G.L.S.lamps etc., The L-series MCBs are also used for protection of distribution equipments such as wires and cables. G-series MCBs are suitable with equipments having high inrush current and requiring overload protection. Such equipments are primarily inductive loads such as motors, air conditioners, transformers, halogen lamps etc.

MCBs are widely used in distribution boards, various panels for protection of individual circuit and complete wiring system. MCBs are costlier than fuse but no cost is involved in their resetting after operation. Also the resetting is quick and simple. MCBs are extensively used and are replacing rewirable fuses.

provided, the child can get a servere shock which can be avoided by using ELCB which typically trip in around 25 msec if the leakage current exceeds its preset value.

The schematic of ELCB is as shown in the Fig. 10.10

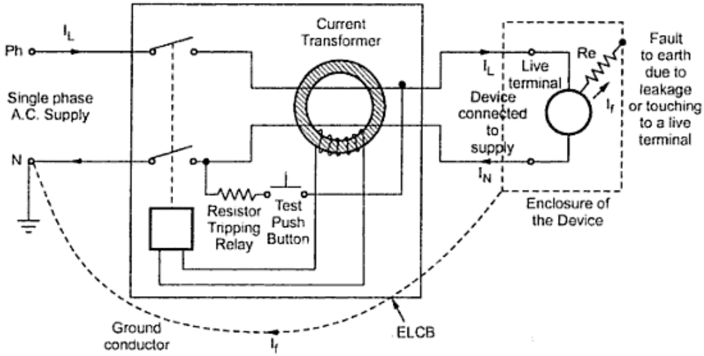


Fig. 10.10

As shown in the Fig. 10.10 ELCB consists of a small current transformer surrounding live and neutral wire. The secondary winding of current transformer is connected to relay circuit which can trip the circuit breaker which is connected in the circuit.

Under normal conditions, the current in line and neutral conductor is same so the net current $(I_L - I_N)$ flowing through the core is zero. Eventually there will not be any production of flux in the core and no induced e.m.f. So the breaker does not trip.

If there is a fault due to leakage from live wire to earth or a person by mistake touching to the live terminal then the net current through the core will no longer remain as zero but equal to $I_L - I_N$ or I_f which will set up flux and emf in C.T. As per the preset value, the unbalance in current is detected by C.T. and relay coil is energized which will give tripping singal for the circuit breaker. As C.T. operates with low value of current, the core must be very permeable at low flux densities.

Thus ELCB provides protection against electric shock when a person comes in contact with live parts. resulting in flow of current from body to earth. A properly connected ELCB detects such small currents in milliamperes flowing to earth through human body or earth wire and breaks the circuit to reduce the risk of electrocution to humans.

In the absence of ELCB and with a continuous flow of small fault or leakage current to earth undetected for a considerable period may create hot spots which may result into fire.

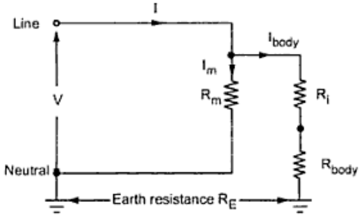


Fig. 10.13 Equivalent circuit

The equivalent electric circuit of the above condition can be shown as in the Fig. 10.13.

In the equivalent circuit we have,

I_m = Machine Current

I_{body} = Current passing through the body of the person.

When a person, standing on the earth touches the machine, current *I* gets an alternative path through the body of the person to earth from the insulation resistance, finally to the neutral of the supply. From the equivalent circuit we can write,

$$I_{body} = \frac{V}{R_i + R_{body} + R_E} \quad ...(1)$$

When the insulation of the machine is perfect, the insulation resistance is of the order of few mega ohms and practically can be considered as infinity.

So $\quad R_i = \infty \quad$ ... .Insulation perfect

$$\therefore \quad I_{body} = \frac{V}{R_E + \infty + R_{body}} = 0 \quad ...(2)$$

So in normal operating conditions, there is no current passing through the body of the person and hence there is no danger of the shock.

But when the insulation becomes weak or defective or if one of the windings is touching to the frame directly due to some fault then R_i i..e. insulation resistance becomes almost zero. Now resistance of body and earth are not very high and hence I_{body} increases to such a high value that the person receives a fatal shock. Such a current is called a leakage current. Hence when the machine is not earthed, there is always a danger of the shock, under certain fault conditions.

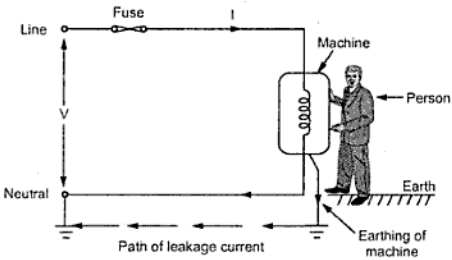


Fig. 10.14 Machine is earthed

Let us see now, what happens due to earthing. In case of earthing, the frame of the machine is earthed as shown in the Fig. 10.14.

The resistance of the path from frame to earth is very

very low. When the person touches to the frame, and if there is a leakage due to fault condition, due to earthing a leakage current takes a low resistance path i.e. path from frame to earth, bypassing the person. So body of the person carries very low current which is not sufficient to cause any shock.

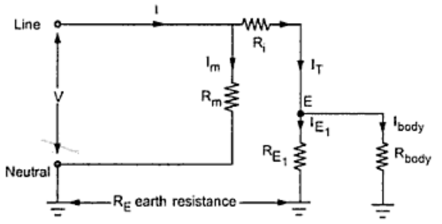


Fig.10.15 Equivalent circuit when machine is earthed

The equivalent circuit of the earthed condition is shown in the Fig. 10.15.

When there is a leakage current due to deterioration of an insulation R_i approaches to zero. So current is sufficiently high to cause a fatal shock. But at point E shown in the Fig.10.15 the current I_T has two paths :

i) One flowing through R_{body} through the person.

ii) Other through new earthing connection having resistance R_{E1}.

The current through the body of the person can be obtained by using the results of current division in a parallel combination.

$$I_{body} = I_T \times \frac{R_{E1}}{R_{body} + R_{E1}} \qquad \text{...(3)}$$

Now R_{E1} is very very small about 5 while R_{body} under worst condition is 1000 Ω but generally higher than 1000 Ω. Hence current I_{body} is negligibly small compared to current I_{E1}. So entire leakage current I_T passes through the earthing contact bypassing the body of the person. The value of I_{body} is not sufficient to cause any shock to the person.

Not only this but the current I_T, is high due to which fuse blows off and thus it helps to isolate the machine from the electric supply.

10.10.1 Uses of Earthing

Apart from basic use of earthing discussed above, the other uses can be stated as

1) To maintain the line voltage constant.

2) To protect tall buildings and structures from atmospheric lightening strikes.

3) To protect all the machines, fed from overhead lines, from atmospheric lightening.

4) To serve as the return conductor for telephone and traction work. In such case, all the complications in laying a separate wire and the actual cost of the wire, is thus saved.

5) To protect the human being from disability or death from shock in case the human body comes into the contact with the frame of any electrical machinery, appliance or component, which is electrically charged due to leakage current or fault.

10.11 Methods of Earthing

Earthing is achieved by connecting the electrical appliances or components to earth by employing a good conductor called **'Earth Electrode'**. This ensures very low resistance path from appliance to the earth. The various methods of earthing are

i) Plate earthing

ii) Pipe earthing

iii) Earthing through water main

iv) Horizontal strip earthing

v) Rod earthing

Let us discuss in detail, the two methods of earthing which are commonly used in practice.

10.11.1 Plate Earthing

The earth connection is provided with the help of copper plate or galvanized iron (G.I.) plate. The copper plate size is 60 cm × 60 cm × 3.18 mm while G.I. plate size is not less than 60 cm × 60 cm × 6.3 mm. The G.I. plates are commonly used now-a-days. The plate is embedded 3 meters (10 feet) into the ground. The plate is kept with its face vertical.

The plate is surrounded by the alternate layer of coke and salt for minimum thickness of about 15 cm. The earth wire is drawn through G.I. pipe and is perfectly bolted to the earth plate. The nuts and bolts must be of copper plate and must be of galvanized iron for G.I. plate.

The earth lead used must be G.I. wire or G.I. strip of sufficient cross-sectional area to carry the fault current safely. The earth wire is drawn through G.I. pipe of 19 mm diameter, at about 60 cm below the ground.

The G.I. pipe is fitted with a funnel on the top. In order to have an effective earthing, salt water is poured periodically through the funnel.

The earthing efficiency, increases with the increases of the plate area and depth of embedding. If the resistivity of the soil is high, then it is necessary to embed the plate vertically at a greater depth into the ground.

The only disadvantage of this method is that the discontinuity of the earth wire from the earthing plate below the earth can not be observed physically. This may cause misleading and may result into heavy losses under fault conditions.

The schematic arrangement of plate earthing is shown in the Fig. 10.16.

The earth wires are connected to the G.I. pipe above the ground level and can be physically inspected from time to time. These connections can be checked for performing continuity tests. This is the important advantage of pipe earthing over the plate earthing. The earth lead used must be G.I. wire of sufficient cross-sectional area to carry fault current safely. It should not be less than electrical equivalent of copper conductor of 12.97 mm^2 cross-sectional area.

The only disadvantage of pipe earthing is that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of high order. This increases the excavation work and hence increased cost. In ordinary soil condition the range of the earth resistance should be 2 to 5 ohms.

In the places where rocky soil earth bed exists, horizontal strip earthing is used. This is suitable as soil excavation required for plate or pipe earthing is difficult in such places. For such soils earth resistance is between 5 to 8 ohms.

10.12 Multimeter

A multimeter is most commonly used laboratory instrument which is used for measurement of current, voltage, resistance. It consists of a balanced bridge d.c. amplifier and PMMC indicating instrument. To get different ranges of voltages and currents, range switch is provided. A function switch is also provided to measure various measurement quantities like voltage, current or resistance. An internal battery is supplied for measurement or resistance. A rectifier used in the circuit converts a.c. to a proportionate d.c. value.

Multimeters are available in two types namely analog multimeter and digital multimeter. Now a days digital multimeters are commonly used. The PMMC indicating instrument mentioned above is the feature of the analog multimeter while LCD (Liquid Crystal Display) of digital type is the feature of digital multimeters.

A multimeter measures a.c. and d.c. voltage, a.c. and d.c. currents and resistance.

The Fig. 10.18 shows the modern laboratory type multimeter.

Fig. 10.18 Laboratory type multimeter

10.12.1 Use of Multimeter for D.C. Voltage Measurement

The Fig. 10.19 shows the arrangement used in multimeter to measure the d.c. voltages.

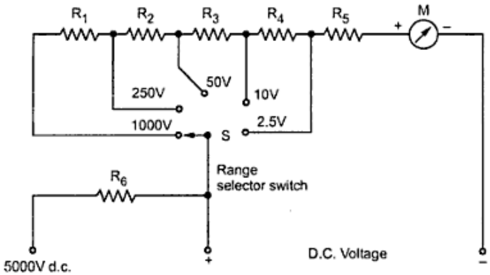


Fig. 10.19

For getting different ranges of voltages, different series resistances are connected in series which can be put in the circuit with the range selector switch. We can get different ranges to measure the d.c. voltages by selecting the proper resistance in series with the basic meter..

10.12.2 Use of Multimeter as an Ammeter

To get different current ranges, different shunts are connected across the meter with the help of range selector switch. The working is same as that of PMMC ammeter.

The Fig. 10.20 shows the arrangement used in the multimeter to use is as an ammeter.

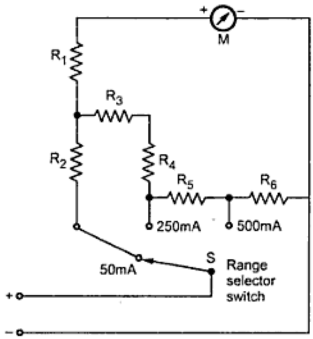


Fig. 10.20

10.13 Safety Precautions

It is necessary to observe some safety precautions while using the electric supply to avoid the serious problems like shocks and fire hazards.

Some of the safety precautions are listed below :

1) Insulation of the conductors used must be proper and in good condition. If it is not so the current carried by the conductors may leak out. The person coming in contact with such faulty insulated conductors may receive a shock.
2) Megger tests should be conducted and insulation must be checked. With the help of megger all the tests discussed above must be performed, on the new wiring before starting use of it.
3) Earth connection should be always maintained in proper condition.
4) Make the mains supply switch off and remove the fuses before starting work with any installation.
5) Fuses must have correct ratings.
6) Use rubber soled shoes while working. Use some wooden supper under the feet. this removes the contact with the earth.
7) Use rubber gloves while touching any terminals or removing insulation layer from a conductor.
8) Use a line tester to check whether a 'live' terminal carries any current still better method is to use a test lamp.
9) Always use insulated screw drivers, pilers, line testers etc.
10) Never touch two different terminals at the same time.
11) Never remove the plug by pulling the wires connected to it.
12) The sockets should be fixed at a height beyond the reach of the children.

10.14 Electric Shock

A sudden agitation of the nervous system of a body, due to the passage of an electric current is called an **electric shock**.

The factors affecting the severity of the shock are,

1. Magnitude of current passed through the body.
2. Path of the current passed through the body.
3. Time for which the current is passed through the body.
4. Frequency of the current.
5. Physical and psychological condition of the affected person.

2.	Dry	100000	0.001	No burns and very light shock.	0.005	Light shock with no burns.	0.1	Death sure but slight burns.
3.	Neither Dry nor Wet	5000	0.02	Painful shock but no injury or burns.	0.1	Death certain with slight burns.	2	Severe burns but may survive.

10.15 Safety Rules

Following are few of the safety rules must be observed while dealing with electricity.

1) All the electrical supply lines shall be sufficient in power and size and of sufficient mechanical strength for the work.
2) All electric supply lines, wires, fittings and apparatus at a consumer's premises should be in a safe condition and in all respects fit for supplying energy.
3) The underground cable must be properly insulated and protected under all the ordinary operating conditions.
4) A suitable earthed terminal should be provided by supplier on the consumer's premises.
5) The bare conductors, if any are ensured that they are inaccessible.
6) The conductor or apparatus, before handled by any person proper precaution is taken by earthing or suitable means to discharge electrically.
7) No person shall work on any live electric supply line or apparatus and no person shall assist such person.
8) Flexible cables shall not be used for portable or transportable motors, generators, transformers, rectifiers, electric drills, welding sets etc. unless they are heavily insulated and adequately protected from mechanical injury.
9) When a.c. and d.c. circuits are installed on the same supports they shall be so arranged and protected that they shall not come into contact with each other when live.
10) First aid boxes must be provided and maintained at generating stations and substations.
11) Fire buckets filled with clean dry sand and ready for immediate use for extinguishing fires.
12) Instructions in English, Hindi and any local languages for the restoration of person suffering from electric shock must be affixed in generating station and substation at a suitable place.
13) Each installation is periodically inspected and tested.

Contents

- D.C. Circuits :- Effect of temperature upon resistance, Solutions of series, parallel in brief, Star-delta combination of resistances, KVL and KCL.
- Electrostatics and Capacitance :- Definitions of electrostatic, Types of capacitors, Series, Parallel combinations and related circuit calculations in brief charging and discharging of capacitor. Energy stored in capacitor.
- Electromagnetics :- Magnetic circuit, Comparison between electric and magnetic circuits, Series/Parallel magnetic circuit calculations, Magnetic hysteresis, Hysteresis and eddy current loss, Magnetic materials, Electromagnetic induction, Statically and dynamically induced e.m.f.s in brief, Fleming's right hand rule - left hand rule, Coefficients of self and mutual inductances, Coefficient of coupling, Series/Parallel combinations of inductances, Rise and decay of current in inductive circuits, Force experienced by current carrying conductor placed in magnetic field.
- Single Phase A.C. Circuits :- Generation of alternating voltages and currents, Their equations, Definitions, R.M.S. and average values, Vector representation of alternating quantities, Addition and subtraction of vectors, Complex algebra, Phasor relations between voltage and current in each of resistance, inductance and capacitance, A.C. series and parallel circuits, Power and power factor, Methods of circuit solution (analytically and vectorially), Resonance in series and parallel circuits.
- Polyphase Circuits :- Generation of polyphase voltages, 3 phase system, Phase sequence, Inter connection of 3 phases, Voltage, Current and power relationships in balanced three phase circuits, Power measurement in single phase and 3 phase circuits.
- Batteries, Cables :- Battery, Life of batteries, Charging and discharging of battery. Cables, 2, 21/2, 3 and 4 core armored and unarmored cables.
- Electrical Wiring :- Connectors and switches, System of wiring, Domestic wiring installation, Sub circuits in domestic wiring, Simple control circuit in domestic installation, Industrial electrification.
- Illumination :- Types of lamps, Fixtures and reflectors, Illumination schemes for domestic, Industrial and commercial premises, Lumen requirements for different categories.
- Safety and Protection :- Safety, Electric shock, First aid for electric shock other hazards of electrical laboratories and safety rules, Use of multimeters, Grounding, Importance of grounding, Equipment of grounding for safety. Circuit protection devices, Fuses, MCB, ELCB and relays.

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