Basics of Electrical Engineering

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I.K. International

Published by
I.K. International Publishing House Pvt. Ltd.
S-25, Green Park Extension
Uphaar Cinema Market
New Delhi-110 016 (India)
E-mail: ik_in@vsnl.net

Branch Offices:
A-6, Royal Industrial Estate, Naigaum Cross Road
Wadala, Mumbai-400 031 (India)
E-mail: ik_mumbai@vsnl.net

G-4 "Embassy Centre", 11 Crescent Road
Kumara Park East, Bangalore-560 001 (India)
E-mail: ik_bang@vsnl.net

ISBN 978-81-89866-25-9

Published by Krishan Makhijani for I.K. International Publishing House Pvt. Ltd., S-25, Green Park Extension, Uphaar Cinema Market, New Delhi-110 016. Printed by Rekha Printers Pvt. Ltd., Okhla Industrial Area, Phase II, New Delhi-110 020.

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Alternating Current Fundamentals and Circuit

1.1 ALTERNATING QUANTITY

An alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite intervals of time. When a simple loop revolves in a magnetic field, an alternating emf is induced in the loop. If the loop revolves with an uniform angular velocity the induced alternating emf is sinusoidal in nature. The alternating quantity may have various other wave forms like triangular, semicircular, stepped, distorted, etc. as shown in Fig. 1.1(a), (b), (c) and (d), respectively. The graph repeats after regular intervals. One complete set of positive and negative values of an alternating quantity is called a cycle. The important alternating quantities, $f(t)$ that will be discussed in the chapter are current and voltage.

1.2 ALTERNATING VOLTAGE

Alternating voltage may be generated by

(a) By rotating a coil in a stationary magnetic field.
(b) By rotating a magnetic field within a stationary coil.

The value of the voltage generated in each case depends on:

(i) The number of turns in the coils.
(ii) The strength of the field.
(iii) The speed at which the coil or magnetic field rotates.
   (a) Maximum flux links with the coil when its plane is in vertical position (perpendicular) to the direction of flux between the poles.
   (b) When the plane of a coil is horizontal no flux links with the coil.

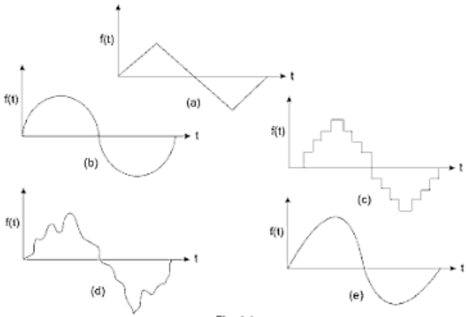


Fig. 1.1

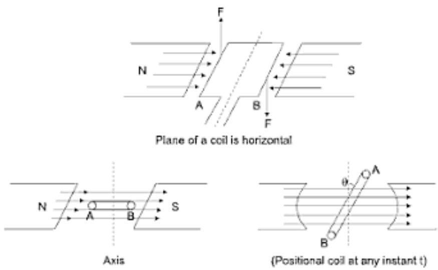


Fig. 1.2

(c) If at any instant the flux linked ϕ is given by

$$\phi = \phi_m \cos \theta$$

where

ϕ_m = Max flux which can link with the coil.

t = Time taken by coil to move through an angle θ from the vertical position and w is the angular velocity, then $\theta = wt$ and $\phi = \phi_m \cos wt$.

$$\text{Instantaneous emf } e = -N\frac{d\phi}{dt}$$

$$e = wN\,\phi_{max}\sin wt$$

$$\boxed{e = E_m \sin wt}$$

- Number of cycles/sec is called the *frequency* of the alternating quantity.
- Time taken by an alternating quantity to complete the cycle is called its time period.

$$T = 1/f\,.$$

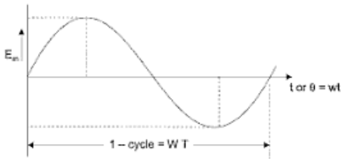


Fig. 1.3

1.3 AC WAVEFORMS

When an alternator produces AC voltage, the voltage switches polarity over time, but does so in a very particular manner. When graphed over time, the "wave" traced by this voltage of alternating polarity from an alternator takes on a distinct shape, known as a *sine wave:*

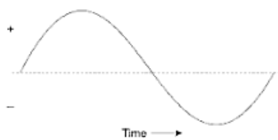


Fig. 1.4 Graph of AC voltage over time (the sine wave)

In the voltage plot from an electromechanical alternator, the change from one polarity to the other is a smooth one, the voltage level changes most rapidly at the zero ("crossover") point and most slowly at its peak. If we were to graph the trigonometric function of "sine" over a horizontal range of 0 to 360 degrees, we would find the exact same pattern:

Angle in degrees		Sine (angle)	
0	...	0.0000	— zero
15	...	0.2588	
30	...	0.5000	
45	...	0.7071	
60	...	0.8660	
75	...	0.9659	
90	...	1.0000	— positive peak
105	...	0.9659	
120	...	0.8660	
135	...	0.7071	
150	...	0.5000	
165	...	0.2588	
180	...	0.0000	— zero
195	...	−0.2588	
210	...	−0.5000	
225	...	−0.7071	
240	...	−0.8660	
255	...	−0.9659	
270	...	−1.0000	
285	...	−0.9659	
300	...	−0.8660	
315	...	−0.7071	
330	...	−0.5000	
345	...	−0.2588	
360	...	0.0000	— zero

The reason why an electromechanical alternator produces output sine-wave AC is due to the physics of its operation. The voltage produced by the stationary coils by the motion of the rotating magnet is proportional to the rate at which the magnetic flux is changing perpendicular to the coils (Faraday's Law of Electromagnetic Induction). That rate is greatest when the magnet poles are closest to the coils, and least when the magnet poles are furthest from the coils. Mathematically, the rate of magnetic flux change due to a rotating magnet follows that of a sine function, so the voltage produced by the coils follows that same function.

If we were to follow the changing voltage produced by a coil in an alternator from any point on the sine wave graph to that point when the wave shape begins to repeat itself, we would have marked exactly one cycle of that wave. This is most easily shown by spanning the distance between identical peaks, but may be measured between any corresponding points on the graph. The degree marks on the horizontal axis of the graph represent the domain of the trigonometric sine function and also the angular position of our simple two-pole alternator shaft as it rotates.

1.4 ADVANTAGES OF SINE WAVE

1. Any periodic non-sinusoidal wave can be expressed as the sum of a number of sine wave of different frequencies.
2. Sine wave can be expressed in a simple mathematical form.
3. The resultant of two or more quantities varying sinusoidally at the same frequency is another sinusoidal quantity of same frequency.
4. Rate of change of any sinusoidal quantity is also sinusoidal.

1.5 CYCLE

A cycle may be defined as one complete set of positive and negative values of an alternating quantity repeating at equal intervals. Each complete cycle is spread over 360° electrical as shown in Fig. 1.5.

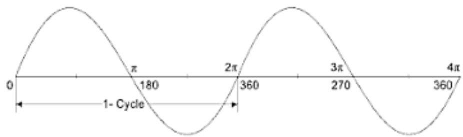


Fig. 1.5

1.6 PERIODIC TIME

The time taken by an alternating quantity in seconds to trace one complete cycle is called periodic time or time-period. It is usually denoted by symbol T.

1.7 FREQUENCY

The number of cycles per second is called frequency and is denoted by symbol f.

Thus, $$f = \frac{1}{T}$$

or, $$T = \frac{1}{f}$$

If the angular velocity w is expressed in radians per second, then

$$w = \frac{2\pi}{T}$$
$$= 2\pi f$$

1.8 PHASE DIFFERENCE

Let OP and OQ be the two vectors (more preferred to be called phasors) representing two alternating quantities of the same frequency at any instant. The angle ϕ between them is called the phase angle.

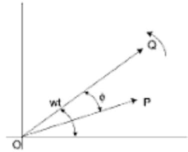


Fig. 1.6

The direction of rotation in counter clockwise direction is usually taken as positive. If OQ and OP represent voltage and current vectors, then

$$e = OQ \sin wt$$

and, $$i = OP \sin (wt - \phi)$$

where, ϕ is called the phase difference. In above phasor OQ is said to lead the phasor OP.

The 'phase' of an AC wave may be defined as its position with respect to a reference axis or reference wave.

Phase angle as the angle of lead or lag with respect to reference axis or

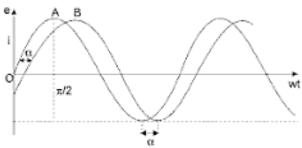


Fig. 1.7

with respect to another wave.

- A is α degree ahead of B.
- A attains its maxima α degrees before B or $\frac{\alpha}{2\pi}T$ second degrees or

$$\alpha = wt\left\{t = \frac{\alpha}{w}\right\} \text{ sec before } B.$$

before $$\begin{cases} \alpha = wt \\ t = \dfrac{\alpha}{w} = \dfrac{\alpha}{2\pi}T \text{ sec} \end{cases}$$

1.9 PHASOR NOTATION

Sinusoidal quantities can be represented by a function.

$$f(t) = V_m e^{j\omega t} = V_m e^{j\theta} = V_m \angle\theta$$

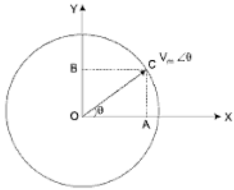


Fig. 1.8

This function has constant magnitude V_m and as wt moves through 0 ot 2π radians.

$$OA = V_m \cos\theta,\ \ OB = V_m \sin\theta$$

$$OC = (OA) + j(OB)\ \ \theta = \tan^{-1}\frac{OB}{OA}$$

by Euler theorem $e^{j\theta} = \cos\theta + j\sin\theta$

$$\boxed{V = V_m \angle\theta = V_m(\cos\theta + j\sin\theta)}$$

In rectangular form

$$\overline{OC} = \overline{OA} + j\,\overline{OB}$$

$$|OC| = \bar{x} + j\,\bar{y}\ \ \text{where } \theta = \tan^{-1} y/x$$

$$|OC| = \sqrt{x^2 + y^2}$$

$$V_1 = V_{m_1}\angle\theta_1,\quad V_2 = V_{m_2}\angle\theta_2$$

Then $V_1V_2 = V_{m_1}V_{m_2}\angle\theta_1 + \theta_2 = V_m[\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2)]$

$$V_1/V_2 = \frac{V_{m_1}}{V_{m_2}}\angle\theta_1 - \theta_2 = V_m[\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)]$$

$$V_1 + V_2 = V_{m_1}\angle\theta_1 + V_{m_2}\angle\theta_2$$

$$= V_{m_1}(\cos\theta_1 + j\sin\theta_1) + V_{m_2}(\cos\theta_2 + j\sin\theta_2)$$

$$\boxed{V_1 + V_2 = \left(V_{m_1}\cos\theta_1 + V_{m_2}\cos\theta_2\right) + j\left(V_{m_1}\sin\theta_1 + V_{m_2}\sin\theta_2\right)}$$

Phasor diagram:

Let $V_1 = V_{m_1} \sin\theta = V_{m_1}\angle O$ $\theta = wt$

$V_2 = V_{m_2} \sin(wt + \alpha) = V_{m_2}\angle\alpha$ (means V_2 leads, V_1 by angle $\alpha°$)

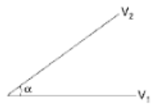


Fig. 1.9

1.10 MEASUREMENTS OF AC MAGNITUDE

So far we know that AC voltage alternates in polarity and AC current alternates in direction. We also know that AC can alternate in a variety of different ways, and by tracing the alternation over time we can plot it as a "waveform". We can measure the rate of alternation by measuring the time it takes for a wave to evolve before it repeats itself (the "period"), and express this as cycles per unit time, of "frequency". In music, frequency is the same as pitch, which is the essential property distinguishing one note from another.

However, we encounter a measurement problem if we try to express how large or small an AC quantity is. With DC, where quantities of voltage and current are generally stable, we have little trouble expressing how much voltage or current we have in any part of a circuit. But how do you grant a single measurement of magnitude to something that is constantly changing?

One way to express the intensity, or magnitude (also called the amplitude), of an AC quantity is to measure its peak height on a waveform graph. This is known as the peak or crest value of an AC waveform:

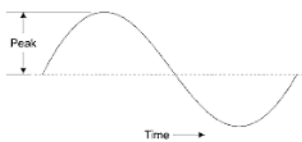


Fig. 1.10

Another way is to measure the total height between opposite peaks. This is known as the peak-to-peak (P-P) value of an AC waveform.

Unfortunately, either one of these expressions of waveform amplitude can be misleading when comparing two different types of waves. For example, a square wave peaking at 10 volts is obviously a greater amount of voltage for a greater amount of time than a triangle wave peaking at 10 volts. The effects of these two AC voltages powering a load would be quite different.

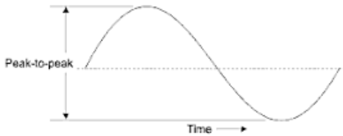


Fig. 1.11

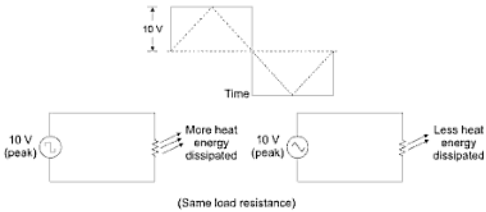


Fig. 1.12

One way of expressing the amplitude of different wave-shapes in a more equivalent fashion is to mathematically average the values of all the points on the graph of a waveform to a single, aggregate number. This amplitude measure is known as the average value of the waveform. If we average all the points on the waveform algebraically (that is, to consider their sign, either positive or negative), the average

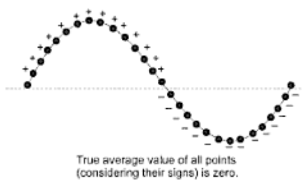


Fig. 1.13

value for most waveforms is technically zero, because all the positive points cancel all the negative points over a full cycle.

This, of course, will be true for any waveform having equal-area portions above and below the "zero" line of a plot. However, as a practical measure of a waveform's aggregate value, "average" is usually defined as the mathematical mean of all the points' absolute values over a cycle. In other words, we calculate the practical average value of the waveform by considering all points on the wave as positive quantities as if the waveform looked like this:

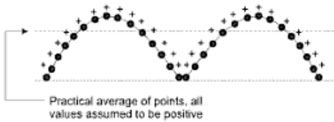


Fig. 1.14

Polarity-insensitive mechanical meter movements (meters designed to respond equally to the positive and negative half-cycles of an alternating voltage or current) register in proportion to the waveform's (practical) average value, because the inertia of the pointer against the tension of the spring naturally averages the force produced by the varying voltage/current values over time. Conversely polarity-sensitive meter movements vibrate uselessly if exposed to AC voltage or current, their needles oscillating rapidly about the zero mark, indicating the true (algebraic) average value of zero for a symmetrical waveform. When the "average" value of a waveform is referenced in this text, it will be assumed that the "practical" definition of average is intended unless otherwise specified.

Another method of deriving an aggregate value for waveform amplitude is based on the waveform's ability to do useful work when applied to a load resistance. Unfortunately, an AC measurement based on work performed by a waveform is not the same as that waveform's average value, because the power dissipated by a given load (work performed per unit time) is not directly proportional to the magnitude of either the voltage or current impressed upon it. Rather, power is proportional to the square of the voltage or current applied to a resistance ($P = E^2/R$, and $P = I^2R$). Although the mathematics of such an amplitude measurement might not be straightforward, the utility of it, is.

Current would produce the same amount of heat energy dissipation through an equal resistance:

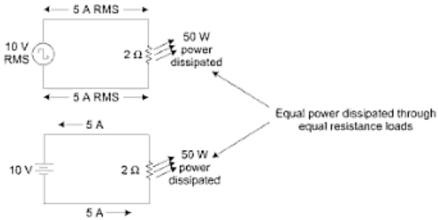


Fig. 1.15

In the two circuits above, we have the same amount of load resistance (2 Ω) dissipating the same amount of power in the form of heat (50 watts), one powered by AC and the other by DC. Because the AC voltage source pictured above is equivalent (in terms of power delivered to a load) to a 10 volt DC battery, we would call this a "10 volt" AC source. More specifically, we would denote its voltage value as being 10 volts RMS. The qualifier "RMS" stands for Root Mean Square, the algorithm used to obtain the DC equivalent value from point on a graph (essentially, the procedure consists of squaring all the positive and negative points on a waveform graph, averaging those squared values, then taking the square root of the average to obtain the final answer). Sometimes the alternative terms equivalent or DC equivalent are used instead of "RMS", but the quantity and principle are both the same.

RMS amplitude measurement is the best way to relate AC quantities to DC quantities, or other AC quantities of differing waveform shapes, when dealing with measurements of electric power. For other considerations, peak or peak-to-peak measurements may be the best to employ. For instance, when determining the proper size of wire (ampacity) to conduct electric power from a source to a load, RMS current measurement is the best to use, because the principal concern with current is overheating of the wire, which is a function of power dissipation caused by current through the resistance of the wire. However, when rating insulators for service in high-voltage AC applications, peak voltage measurements are the most appropriate, because the principal concern here is insulator "flashover" caused by brief spikes of voltage, irrespective of time.

Peak and peak-to-peak measurements are best performed with an oscilloscope, which can capture the crests of the waveform with a high degree of accuracy due to the fast action of the cathode-ray-tube in response to changes in voltage. For RMS measurements, analog meter movements (D' Arsonval, Weston, iron vane, electrodynamometer) will work so long as they have been calibrated in RMS figures. Because the mechanical inertia and dampening effects of an electromechanical meter movement makes the deflection of the needle naturally proportional to the average value of the AC, not the true RMS value, analog meters must be specifically calibrated (or mis-calibrated depending on how you look at it) to indicate voltage or current in RMS units. The accuracy of this calibration depends on an assumed waveshape, usually a sine wave.

Electronic meters specifically designed for RMS measurement are best for the task. Some instrument manufacturers have designed ingenious methods for determining the RMS value of any waveform. One such manufacturer produces "True-RMS" meters with a tiny resistive heating element powered by a voltage proportional to that being measured. The heating effect of that resistance element is measured thermally to give a true RMS value with no mathematical calculations whatsoever, just the laws of physics in action in fulfilment of the definition of RMS. The accuracy of this type of RMS measurement is independent of waveshape.

For "pure" waveforms, simple conversion coefficients exist for equating peak, peak-to-peak, average (practical, not algebraic), and RMS measurements to one another:

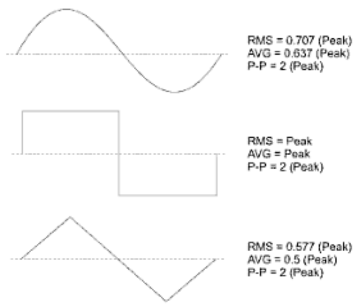


Fig. 1.16

In addition to RMS average, peak (crest) and peak-to-peak measures of an AC waveform, there are ratios expressing the proportionality between some of these fundamental measurements. The crest factor of an AC waveform, for instance, is the ratio of its peak (crest) value divided by its RMS value. The form factor of an AC waveform is the ratio of its RMS value divided by its average value. Square-shaped waveforms always have crest and form factors equal to 1, since the peak is the same as the RMS and average values. Sinusoidal waveforms have crest factors of 1.414 (the square root of 2) and form factors of 1.11. Triangle- and sawtooth-shaped waveforms have crest values of 1.732 (the square root of 3) and form factors of 2.

Bear in mind that the conversion constant shown here for peak, RMS and average amplitudes of sine waves, square waves, and triangle waves hold true only for pure forms of these waveshapes. The RMS and average values of distorted waveshapes are not related by same ratios.

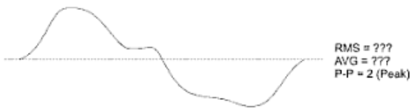


Fig. 1.17

This is a very important concept to understand when using an analog meter movement to measure AC voltage or current. An analog movement, calibrated to indicate sine-wave RMS amplitude, will only be accurate when measuring pure sine waves. If the waveform of the voltage or current being measured is anything but a pure sine wave, the indication given by the meter will not be the true RMS value of the waveform, because the degree of needle deflection in an analog meter movement is proportional to the average value of the waveform, not the RMS. RMS meter calibration is obtained by skewing the span of the meter so that it displays a small multiple of the average value, which will be equal to be the RMS value for a particular waveshape and a particular waveshape only.

Since, the sine-wave shape is most common in electrical measurements, it is waveshape assumed for analog meter calibration and the small multiple used in the calibration of the meter is 1.1107 (the form factor π/2 divided by the crest factor 1.414: the ratio of RMS divided by average for a sinusoidal waveform) Any waveshape other than a pure sine wave will have a different ratio of RMS and average values, and thus a meter calibrated for sine-wave voltage or current will not indicate true RMS when reading a non-sinusoidal wave. Bear in mind that this limitation applies only to simple, analog AC meters not employing "True-RMS" technology.

A sinusoidal wave continuously changes in magnitude and reverse it, direction at regular interval. So it is possible to state the value of AC as simple as DC. So for this, we take four different values for indication of AC. These are:

(i) instantaneous value
(ii) maximum or peak value
(iii) average or mean value
(iv) effective or rms value

1.11 EFFECTIVE, VIRTUAL OR ROOT-MEAN-SQUARE (RMS) VALUE OF AN ALTERNATING QUANTITY

The effective value of an alternating current is that value of direct current which when applied to a given circuit for a given time produces the same expenditure of energy as produced by the alternating current when flowing through the same circuit for the same period. It is also known as virtual or RMS value of alternating current. The effective value of an alternating voltage can also be defined likewise.

RMS value or effective value:

The effective value of an alternating quantity may be determined by the following two methods.

(a) Integral method
(b) Graphic method

For non-sinusoidal waves, graphic method is more suitable.

1.11.1 Integral Method

Let $f(t)$ be any function then the RMS value of function $f(t)$ is

$$f(t)_{\text{RMS}} = \left[\frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2} f(t)^2\, dt\right]^{1/2}$$

- The rms (or effective) value of an alternating current is given by that steady (DC) current which when flowing through a given ckt for a given time produces the same heat as produced by the AC when flowing through the same ckt for the same time.
- If a periodic current $i = I_m \sin \omega t$ flowing through a resistance R.
- Instantaneous power absorbed by R is p

$$p = i^2 R$$

- Average power absorbed over a complete cycle or time T is

$$\boxed{P_{av} = \frac{1}{T}\int_0^T i^2 R\, dT}$$

- $I_{eff}^2 \cancel{R} = \frac{1}{T}\int_0^T i^2 \cancel{R}\, dt$

$$I_{rms} = \left[\frac{1}{T}\int_0^T i^2 dt\right]^{1/2}$$

- For a sinusoidal wave

$$i = I_m \sin \omega t$$

$$I_{rms} = \left[\frac{1}{T}\int_0^T (i)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t . dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T I_m^2 \frac{(1-\cos 2\omega t)}{2} dt\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right)_0^T\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{T}{2} - 0\right)\right]^{1/2} \quad \left\{\sin 2\omega t = \sin 2.2\pi f t = \sin 2.\frac{2\pi}{T} t\right.$$

when $t = T$ $\{\sin 2\omega t = \sin 4\pi = 0$

$$= \frac{I_m}{\sqrt{2}}$$

- Ratio of *maximum* value to the RMS value is known as *crest or peak factor* or amplitude factor.
- Ratio of *effective value* to *average value* is known as *form factor*

$$\text{form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

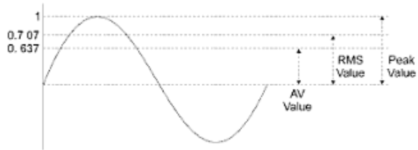


Fig. 1.18

1.11.2 Graphic Method

In Fig. 1.19(a) a positive half cycle of an unsymmetrical alternating current is shown. Divide the period T into n equal intervals of time $\frac{T}{n}$ seconds. Let the instantaneous middle values of current in the intervals be $i_1, i_2, i_3, ..., i_n$. If R be the resistance of the circuit through which varying current is passed, then:

Heat produced in:

$$\text{1st interval} = i_1^2 R \frac{T}{n} \text{ watts}$$

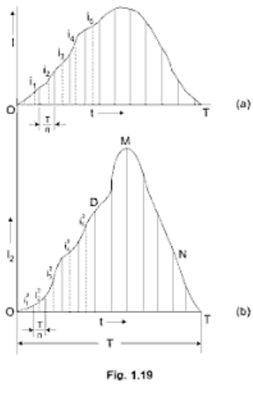


Fig. 1.19

$$\text{2nd interval} = i_2^2 R \frac{T}{n} \text{ watts}$$

$$\text{3rd interval} = i_3^2 R \frac{T}{n} \text{ watts}$$

....

....

$$n\text{th interval} = i_n^2 R \frac{T}{n} \text{ watts}$$

∴ Total heat produced in the interval T

$$= \frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n} RT \text{ watts} \qquad \text{(A)}$$

If I is the effective value of current, then total heat produced in time T

$$= I^2 RT \text{ watts} \qquad \text{(B)}$$

By definition the total amounts of heat produced as given by equations (A) and (B) should be equal.

$$\therefore \qquad I^2 RT = \frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n} RT$$

$$\text{or,} \qquad I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n}}$$

Alternative procedure:

If a curve is plotted in i^2 against time as shown in Fig. 1.19(b), then

$$\text{Area of 1st interval} = i_1^2 \frac{T}{n}$$

$$\text{Area of 2nd interval} = i_2^2 \frac{T}{n}$$

....

....

$$\text{Area of } n\text{th interval} = i_n^2 \frac{T}{n}$$

$$\text{Total area } ODMNT = \frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} . T$$

or, $$\frac{\text{Total area } ODMNT}{\text{Time interval } T} = \frac{i_1^2 + i_2^2 + ... + i_n^2}{n}$$

Hence, from equation

$$I = \sqrt{\frac{\text{Total area } ODMNT}{\text{Time interval}}}$$

Thus, effective value can be determined graphically by taking out the square root of the average of total area occupied by the square values of the alternating quantity in a given interval.

Similarly, the effective value of an alternating voltage is given by,

$$V = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}}$$

If the alternating voltage is sinusoidal, then effective value will be given by,

$$V = \frac{V_m}{\sqrt{2}}$$

where, V_m is the peak value of the sinusoidally varying alternating voltage.

1.12 AVERAGE VALUE

The average value of an alternating current is given by that direct current which transfers across any circuit, the same charge is transferred by the given alternating current.

If $f(t)$ is any function, then average value of $f(t)$ is

$$f(t)_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t)\, d(t)$$

1.12.1 Integral Method

In case of a symmetrical alternating quantity, the average value over a complete cycle is zero. So the average value is obtained by considering *half a cycle only*. If the wave is unsymmetrical, the average value can only be found out by considering the whole cycle.

For a sine wave, instantaneous value is given by,

$$i = I_m \sin \theta$$

Consider small interval $d\theta$ as shown in Fig. 1.20. If i is the average value of current in the interval, then area of elementary strip = $id\theta$.

Total area of half cycle

$$= \int^{\pi} i\,d\theta$$

Hence, the average value of current is given by two ways.

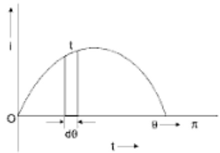


Fig. 1.20

(i)
$$I_{av} = \frac{\text{area of half cycle}}{\text{interval}}$$

$$= \frac{\int_0^{\pi} i\,d\theta}{\pi}$$

$$= \frac{\int_0^{\pi} I_m \sin\theta\,d\theta}{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{2I_m}{\pi}$$

$$= 0.637\,I_m$$

Similarly, for alternating sine voltage $E_{av} = \dfrac{2I_m}{\pi}$.

(ii)
$$I_{av} = \frac{1}{T}\int_0^{T/2} i\,dt$$

$$= \frac{1}{T/2}\int_0^{T/2} I_m \sin(wt).dt$$

$$= \frac{I_m}{T/2}\int_0^{T/2} \sin wt\,dt$$

$$= \frac{I_m}{T/2}\left[\frac{-\cos wt}{w}\right]_0^{T/2}$$

$$= \frac{-I_m.2}{Tw}\left[\cos\frac{wT}{2} - \cos 0\right]$$

$$w = 2\pi f = \frac{2\pi}{T}$$

$$= \frac{-2I_m}{2\pi}[\cos\pi - \cos 0]$$

$$\boxed{I_{av} = \frac{2I_m}{\pi} = 0.637\,I_m}$$

1.12.2 Graphical Method

For an unsymmetrical wave as shown in Fig. 1.19(a), area of curve

$$= (i_1 + i_2 + i_3 + \ldots + i_n).\frac{T}{n}$$

∴
$$I_{av} = \frac{(i_1 + i_2 + i_3 + \ldots + i_n).\frac{T}{n}}{T}$$

$$= \frac{(i_1 + i_2 + i_3 + \ldots + i_n)}{n}$$

1.13 FORM FACTOR

The form factor is defined as the ratio of the effective value to the average value of an alternating quantity.

For a sine wave,

$$\text{Form factor} = \frac{\text{Effective or RMS value}}{\text{Average value}}$$

$$= \frac{0.707 I_m}{0.637 I_m} = 1.11$$

1.14 PEAK OR CREST FACTOR

It is defined as the ratio of peak value of the effective value of an alternating wave.

For a sine wave,

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}}$$

$$= \frac{I_m}{0.707} = 1.414$$

1.15 OPERATOR *j*

- An alternating voltage or current is a phasor quantity, but since the instantaneous values are changing continuously, it must be represented by a rotating vector phasor.
- A phasor is a vector rotating at a constant angular velocity.
- *j* is defined as an operator which turns a phasor by 90° (CCW) without changing the magnitude of phasor

$$j = 1\angle 90°, \quad jr = r\angle 90°$$

1.16 CIRCUIT WITH PURE RESISTANCE ONLY

A pure resistance is that in which there is ohmic voltage drop only. Consider a circuit having a pure resistance R as shown is Fig. 1.21 below.

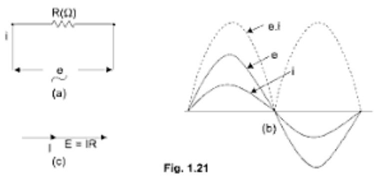


Fig. 1.21

Let the instantaneous value of the alternating voltage applied be,

$$e = E_m \sin wt$$

The instantaneous value of current,

$$i = \frac{e}{R} = \frac{E_m}{R} \sin wt$$

$$i = I_m \sin wt$$

where, $I_m = \frac{E_m}{R}$ = Maximum value of current.

The applied alternating voltage and the current are in phase as shown in Fig. 1.21(b).

If E and I are the effective values of voltage and current in the circuit, then

$$E = IR$$

The voltage diagram representing current and voltage vectors in phase is shown in Fig. 1.21(a).

Power:

Instantaneous power consumed in the circuit

$$= e.i$$

$$= E_m I_m \sin^2 wt$$

$$= E_m I_m \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{E_m I_m}{2} - \frac{E_m I_m}{2} \cos 2\,\omega t$$

This indicates that instantaneous power is not fixed but has one fixed component and other component is pulsating component of twice supply frequency.

For a complete cycle, the average value of $\frac{E_m I_m}{2} \cos 2\,\omega t$ is zero.

Hence, average power for the whole cycle is given by,

$$P = \frac{E_m I_m}{2}$$

$$= \frac{E_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}}$$

$$= E.I$$

where, E and I are effective values or rms values of voltage and current.

Hence in pure resistive circuit

(i) Current is in phase with the voltage

(ii) Current $I = \dfrac{E}{R}$ where I, E are RMS or effective values

(iii) Power in circuit is $P = VI = I^2R$

1.17 CIRCUIT WITH PURE INDUCTANCE ONLY

A pure inductive circuit possesses only inductance and no resistance or capacitance as shown in Fig. 1.22. When an alternating voltage is applied to it, a back emf of self inductance is induced in it. As there is no ohmic resistance drop, the applied voltage has to oppose the self induced emf only. So the applied voltage is equal and opposite to the back emf at all instants.

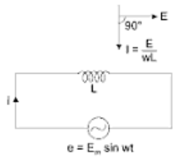


Fig. 1.22

Let the applied voltage

$$e = E_m \sin \omega t \qquad (1)$$

instantaneous value of self induced emf is e'

$$e' = -L\,\frac{di}{dt} = -e$$

$$di = \frac{1}{L}\, e\, dt$$

integrating both side, we get

$$\int di = \frac{1}{L}\int E_m \sin \omega t . dt$$

$$i = \frac{E_m}{\omega L}(-\cos \omega t)$$

$$i = \frac{E_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \begin{cases}\text{integration constant will}\\ \text{cancel out from both side}\end{cases}$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad (2) \quad \left\{ I_m = \frac{E_m}{\omega L}\right.$$

observing (1) and (2) we find that the current lags the applied voltage by 90° or $\dfrac{\pi}{2}$ radian.

$$\Rightarrow \text{ impedance} \qquad Z = \frac{E}{I} = \frac{\frac{E_m}{\sqrt{2}}\angle 0^\circ}{\frac{E_m}{\sqrt{2}}\angle -\frac{\pi}{2}}$$

$$Z = \frac{E_m}{I_m}\angle\frac{\pi}{2} = \omega L\angle\frac{\pi}{2}$$

The quantity ωL is called inductive reactance and is usually devoted by symbol X_L and units is ohm.

$$X_L = \omega L \text{ ohms}$$

where, L is in henry and ω is in rad/sec.

Wave diagram and Phasor diagram for Pure inductance

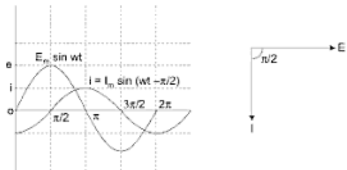


Fig. 1.23

Average Power→

$$P = \frac{1}{2\pi}\int_0^{2\pi} e\,i\,d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} E_m \sin wt . I_m \sin\left(wt - \frac{\pi}{2}\right) d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} -E_m I_m \sin wt.\cos wt.d(wt)$$

$$= -\frac{V_m I_m}{2\pi}\int_0^{2\pi}\frac{\sin 2wt}{2}.d(wt)$$

$$= 0$$

This shows power consumed in purely inductive circuit is zero.

Hence, the average power consumption in an inductive circuit is zero and is periodic with twice the supply frequency as expressed by equation (1). The stored energy in the inductive circuit in one quarter of a cycle is released in the next quarter.

1.18 CIRCUIT WITH PURE CAPACITANCE ONLY

When an alternating voltage is applied across a condenser, there is a regular flow of current through it, the condenser is charged and discharged alternately in each quarter of a cycle.

Consider a condenser having a capacity of C farads connected across an alternating voltage as shown in Fig. 1.24.

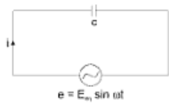
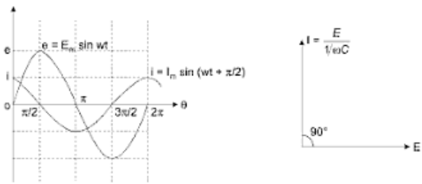


Fig. 1.24

Instantaneous voltage across the condenser,

$$e = E_m \sin \omega t$$



(a) Wave diagram (b) Phasor diagram

Fig. 1.25

If this voltage causes a current to flow through the condenser whose instantaneous value is i, then quantity of electricity stored in time dt is given by,

$$dq = idt$$

or,
$$i = \frac{dq}{dt}$$

Since,
$$q = C.e$$

$\therefore$
$$i = \frac{d}{dt}(C.e)$$

$$= C.\frac{de}{dt} = C\frac{d}{dt}(E_m \sin \omega t)$$

$$= C\omega\, E_m \cos \omega t$$

$$= C\omega\, E_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{E_m}{1/\omega C}.\sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing equations, we see that the current leads the voltage vector by 90° as shown in Fig. 1.25.

Maximum value of current is given by,

$$I_m = \frac{E_m}{1/C\omega}$$

The quantity $1/C\omega$ is called inductive capacitance and is usually denoted by X_c. Its unit is ohm.

$$\therefore \qquad X_c = \frac{1}{C\omega} \text{ ohms}$$

where, C = Capacity in farads

ω = angular velocity in rad/sec

Impedance
$$Z = \frac{E}{I} = \frac{E_m \angle 0/\sqrt{2}}{I_m \angle 90/\sqrt{2}}$$

$$Z = \frac{E_m}{I_m} \angle -90^\circ$$

Since
$$\frac{E_m}{I_m} = X_C = \frac{1}{\omega C}$$

$$\therefore \qquad Z = X_C \angle -90^\circ$$
$$= -j\, X_C\, \Omega$$

Average Power→

instantaneous power $P = vi$

$$P = V_m \sin \omega t . I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t . \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t \qquad (1)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\sin 2\omega t . d(\omega t)$$

$$= 0$$

This shows that the power consumed in purely capacitive circuit is zero.

A capacitor receives energy during the first quarter cycle of voltage and returns the same during the next quarter cycle.

1.19 CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

Consider circuit of Fig. 1.26.

Let R = Resistance in ohms in the circuit.

L = Inductance in henries

X_L = Inductive reactance

$= \omega L$

E = Effective value of applied emf

I = Effective value of current in circuit.

Voltage drop across resistance,

$E_R = R.I$ in phase with current vector as shown in vector diagram of Fig. 1.27.

Voltage across reactance,

$$E_L = I.\omega L = IX_L,\ 90° \text{ ahead of vector } I$$

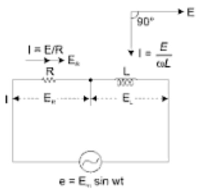


Fig. 1.26

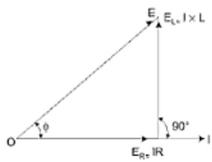


Fig. 1.27

$$Z = R + jX_L$$

$$= \sqrt{R^2 + X_L^2}\ \angle\tan^{-1}\frac{X_L}{R}$$

here $$\phi = \tan^{-1}\frac{X_L}{R}$$

and $$|Z| = \sqrt{R^2 + X_L^2}$$

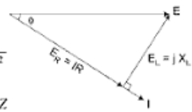
$$Z = |Z| \angle\phi$$

$$I = \frac{E}{Z} = \frac{E \angle\phi}{|Z| \angle\phi}$$

$$\boxed{I = \frac{E}{Z} \angle -\phi}$$

hence in R-L circuit current lags the applied voltage by angle $\phi = \tan^{-1} \frac{X_L}{R}$

The applied voltage is therefore given by,



$$\therefore \quad E = \sqrt{E_r^2 + E_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$\therefore \quad E = I\sqrt{R^2 + X_L^2} = IZ$$

$$\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{\omega L}{R}$$

or, $$I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ is called impedance.

Since, the power is consumed by the resistance only, so the power in the circuit is given by,

$$P = I^2 R = I.IR$$

$$= \frac{E}{\sqrt{R^2 + X_L^2}} . IR$$

or, $$P = E.I \frac{R}{\sqrt{R^2 + X_L^2}}$$

If ϕ is the angle between E and I, then

$$\cos\phi = \frac{E_R}{E} = \frac{IR}{I\sqrt{R^2 + X_L^2}} = \frac{R}{Z}$$

$\therefore \qquad P = E.I \cos \phi$

Cos ϕ is called the power factor of the circuit. Obviously the power factor is lagging in an inductive circuit. So instantaneous current across R-L is

$$i = I_m \sin(\omega t - \phi).$$

1.20 CIRCUIT WITH RESISTANCE AND CAPACITANCE IN SERIES

Consider circuit of Fig. 1.28.

Voltage drop across resistance,

$E_R = IR$ in phase with I as shown in vector diagram of a Fig. 1.29.

$$E_C = I.\frac{1}{C\omega} = IX_C, \text{ 90° lagging}$$

with respect to the current vector.

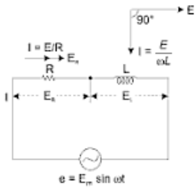


Fig. 1.28

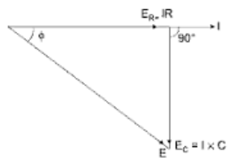


Fig. 1.29

The applied voltage is, therefore, given by,

$$E = \sqrt{E_R^2 + E_C^2}$$

$$= I\sqrt{R^2 + X_C^2} = IZ$$

Thus, ohm's law is applicable to AC circuit also after replacing the term resistance by impedance.

$$\text{Power} = EI \cos \phi$$

$$\text{Cos } \phi = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{Z} \qquad ...(1.53)$$

$$Z = R - j X_C$$

$$Z = \sqrt{R^2 + X_C^2} \ \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$Z = |Z| \angle -\phi \quad \text{here} \quad \phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$I = \frac{E}{Z} = \frac{E}{|Z| \angle -\phi}$$

$$I = \left|\frac{E}{Z}\right| \angle \phi \ = I_m \angle \phi$$

instantaneous value of current throw *R-C* is

$$i = I_m \sin(\omega t + \phi)$$

hence current leads the voltages by aug $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$.

1.21 SERIES R-L-C CIRCUIT

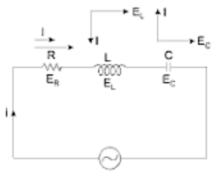


Fig. 1.29(a)

Problems on alternating current circuits can be attempted easily by using *j*-notation.

∴ Voltage across inductance = $+ jIX_L = E_L$

Voltage across capacitance = $- jIX_C = E_C$

Net voltage across them = $+ jI(X_L - X_C) = j(E_L - E_C)$

Resistance drop = $IR = E_R$.

∴ Applied voltage in *j*-notation is represented by,

$$E = IR + jI(X_L - X_C)$$

or, $$E = I\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in *j*-notation may be written as,

$$Z = R + j(X_L - X_C)$$

or, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{E}{Z} \qquad E = \frac{E_m}{\sqrt{2}}\angle 0^\circ$$

$$Z = R + j(X_L - X_C)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\angle\phi = |Z|\angle\phi$$

where $$\phi = \tan^{-1}\frac{X_L - X_C}{R}$$

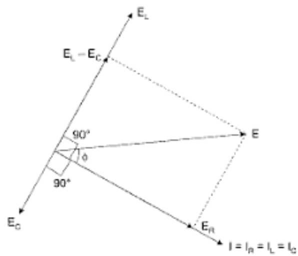
⇒ if $X_L > X_C$ then ϕ is +ve

if $X_L > X_C$ then ϕ is –ve

$$I = \frac{E_m}{\sqrt{2}}\angle 0^\circ \Big/ |Z|\angle\phi$$

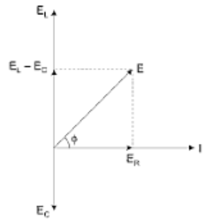
hence if $X_L > X_C$ then current lags the applied volt.

$$I = I'\angle -\phi$$



Phasor diagram of series

R-L-C Ckt taking *E* as a refrence phasor when $X_L > X_C$



Phasor diagram of a series R-L-C Ckt taking current *I* as a refrence phasor.

1.22 POWER IN AC CIRCUITS

- When the current is out of phase with the voltage the power indicated by the *product of the applied* voltage and the total current gives only what is known as *apparent power* and measured in volt-ampere.
- Power that is returned to the source by the reactive components in the circuit is called *reactive power* and is measured in *volt amp*.
- Power that actually used in the circuit (dissipated in resistance) is true or active power and is measured in watts or kW.

1.22.1 Active and Reactive Power

Form Fig. 1.30(a)

Impedance $\quad Z = R \pm jX = |Z| \angle\phi = |Z| \cos\phi + j|Z| \sin\phi$

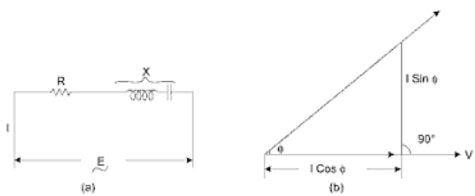


Fig. 1.30

Magnitude or amplitude of impedance,

$$|Z| = \sqrt{R^2 + X^2} \qquad \Rightarrow \quad R = |Z|\cos\phi, \quad X = |Z|\sin\phi$$

Power factor of the circuit,

$$\cos\phi = \frac{R}{Z}.$$

Current in the circuit $I = \dfrac{E}{Z}$.

This current has two components $I\cos\phi$ and $I\sin\phi$. The component $I\cos\phi$ is called in phase or **wattfull** component and $I\sin\phi$ is perpendicular to E and is called **wattless** component, as shown in Fig. 1.30(b). Then

Active (Real) Power = Voltage × Current × cos ϕ watts

Since, the angle between the voltage and the wattless component of current is 90°, hence the power absorbed by this component is zero. The power is only absorbed by the wattful component.

The total power EI in volt amperes supplied to a circuit consists of two components:

(a) Active power = $EI\cos\phi$ watts

(b) Reactive power = $EI\sin\phi$ volt amperes reactive or simply VAR.

The above components can be shown in vector from in Fig. 1.31(a).

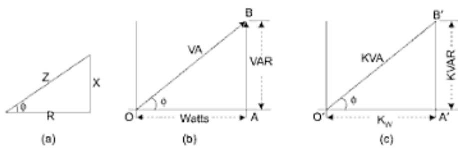


Fig. 1.31

From Fig. 1.31(b)

OA = Active power = $EI\cos\phi$ presented by watts

AB = Reactive power = $EI\sin\phi$ expressed by VAR

OB = Total power = EI expressed by VA

Obviously $$\text{VA} = \sqrt{\text{Watts}^2 + \text{VAR}^2} \qquad ...(1.4)$$

For practical purpose, it is more useful to express these quantities in kilos. Hence, dividing by 1000 throughout we obtain

$$\text{KVA} = \sqrt{\text{KW}^2 + \text{KVAR}^2} \quad ...(1.5)$$

In Fig. 1.31(c)

$$O'A' = \text{Active power} = \frac{EI\cos\phi}{1000} \text{ expressed by KW.}$$

$$A'B' = \text{Reactive power} = \frac{EI\sin\phi}{1000} \text{ expressed by KVAR.}$$

$$O'B' = \text{Total power} = \frac{EI}{1000} \text{ expressed by KVA.}$$

$$Z = R + jX$$

$$I^2Z = I^2R + j(I^2X)$$

$$(\text{Apparent power}) = (\text{True power}) + j(\text{reactive power})$$

$$\Rightarrow \quad \cos\theta = \frac{\text{True power}}{\text{Apparent power}} = \frac{I_Z^2 R}{I_Z^2 Z} = \frac{R}{Z} = \text{power factor}$$

⇒ The apparent power in an AC CKT has been described as the power the source "sees". As far the source is concerned the apparent power is the power that must be provided to the circuit.

⇒ True power is the power actually used in the circuit.

⇒ The difference between apparent power and true power is wasted because, in reality, only true power is consumed.

Ideal situation for a system is that apparent power and true power to be equal means P.F = 1. This situation can occur if net reactance is zero.

1.23 TO FIND ACTIVE AND REACTIVE POWER IN J-FORM

Consider vectors OA and OB as shown in Fig. 1.32.

Let $V = a_1 + jb_1 = OA$

and, $I = a_2 + jb_2 = OB$

$$\therefore \quad OA = \sqrt{a_1^2 + b_1^2}$$

$$OB = \sqrt{a_2^2 + b_2^2}$$

$$\cos\theta_1 = \frac{a_1}{\sqrt{a_1^2 + b_1^2}}, \sin\theta_1 = \frac{b_1}{\sqrt{a_1^2 + b_1^2}}$$

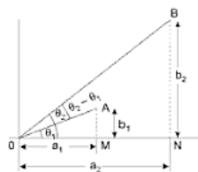


Fig. 1.32

$$\cos\theta_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}, \sin\theta_2 = \frac{b_2}{\sqrt{a_2^2 + b_2^2}}$$

Active power:

$$= OA.OB \cos AOB = \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} \,.\, \cos(\theta_2 - \theta_1)$$

$$= \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} \,[\cos\theta_2.\cos\theta_1 + \sin\theta_2.\sin\theta_1]$$

$$= a_1^2 + b_1^2 \sqrt{a_2^2 + b_2^2} \left\{ \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \cdot \frac{a_1}{\sqrt{a_1^2 + b_1^2}} + \frac{b_2}{\sqrt{a_2^2 + b_2^2}} \cdot \frac{b_1}{\sqrt{a_1^2 + b_1^2}} \right\}$$

$$= a_1 a_2 + b_1 b_2$$

Reactive power:

$$= OA.OB \sin AOB = \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} \,.\sin(\theta_2 - \theta_1)$$

$$= \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} \,[\sin\theta_2.\cos\theta_1 - \cos\theta_2.\sin\theta_1]$$

$$= \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} \left\{ \frac{b_2}{\sqrt{a_2^2 + b_2^2}} \cdot \frac{a_1}{\sqrt{a_1^2 + b_1^2}} - \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \cdot \frac{b_1}{\sqrt{a_1^2 + b_1^2}} \right\}$$

$$= a_1 b_2 - a_1 b_2$$

Note: $\quad$ V.I. $= (a_1 + jb_1)(a_2 + jb_2)$

$$= (a_1 a_2 - b_1 b_1) + j(a_2 b_1 + a_1 b_2)$$

If we write $V \times$ Conjugate of I

$$= (a_1 + jb_1)(a_2 - jb_2) = (a_1 a_2 + b_1 b_2) + j(a_1 b_2 - b_1 a_2)$$

$$= \underset{\text{(Active power)}}{a_1 a_2 + b_1 b_2} \quad + \quad \underset{\text{(Reactive power)}}{j(a_1 b_2 - b_1 a_2)}$$

Note 1: Hence, the active and reactive powers would be given by the real and *j* ports of the vector product of voltage with the conjugate of the current vector.

Note 2: Active power can also be expressed by the sum of the algebraic product of the real parts of the current and the voltage and the algebraic product of the *j* parts of the current and voltage.

Alternate approach→

Let E and I are the phasors given by

$$\bar{E} = E \angle\theta_1$$

and $\bar{I} = I \angle \pm \theta_2$ $\begin{cases} + \text{ sign for leading current} \\ - \text{ sign for lagging current} \end{cases}$

there complex power is given by S

$$S = \bar{\bar{E}} \times \bar{I}^* \qquad \begin{cases} \text{if } \bar{I} = I \angle \pm \theta_2 \\ \quad \bar{I}^* = I \angle \mp \theta_2 \end{cases}$$

$$= E \angle \theta_1 . I \angle \mp \theta_2$$

$$= EI \angle \theta_1 \mp \theta_2$$

$$S = EI \cos(\theta_1 \mp \theta_2) + j\, EI \sin(\theta_1 \mp \theta_2)$$

$$\boxed{S = P + j\theta}$$

if V is the refrence phasor $\theta_1 = 0$

$$S = EI \cos\theta_2 + j\, EI \sin(\mp \theta_2)$$

$$S = EI \cos\theta_2 \mp j\, EI \sin\theta_2$$

$$\boxed{S = P \mp j\theta} \qquad \begin{cases} -\text{ve for leading P.f load} \\ S = P - j\theta \\ +\text{ve for lagging P.f load} \\ S = P + j\theta \end{cases}$$

P = active power

θ = reactive power

1.24 PARALLEL AC CIRCUITS

Parallel ckt can be solved by the following methods (i) Admittance method (ii) Vector method (iii) j method or symbolic method.

1.24.1 Admittance Method

The reciprocal of impedance Z is called admittance and is denoted by the symbol Y.

$$\therefore \qquad Y = \frac{1}{Z} \qquad ...(1.61)$$

If Z_1, Z_2, Z_3, ... are the impedances connected in parallel, then equivalent impedance of their combination is given by,

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \qquad ...(1.61)$$

or, $$Y = Y_1 + Y_2 + Y_2 + \ldots \quad \ldots(1.63)$$

The impedance Z has two components resistance R and reactance X. Admittance has also two components, the conductance 'g' and suceptance 'b'. The impedance and admittance triangles are similar as shown in Fig. 1.33.

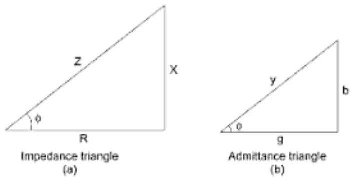


Fig. 1.33

From Fig. 1.33(b), conductance is given by,

$$g = Y \cos \phi$$

Since, $$Y = \frac{1}{Z}$$

and, $$\cos \phi = \frac{R}{Z} \text{ from Fig. 1.33(a)}$$

$\therefore$ $$g = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2}$$

$$= \frac{R}{R^2 + X^2} \quad \ldots(1.64)$$

Similarly, susceptance is given by,

$$b = Y \sin \phi$$

$$= \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2}$$

$$= \frac{X}{R^2 + X^2} \quad \ldots(1.65)$$

If $y_1, y_2, y_3, \ldots$ are equal to $g_1 + jb_1, g_2 + jb_2, g_3 + jb_3 \ldots$ then,

$$g = g_1 + g_2 + g_3 + \ldots \text{ mho} \quad \ldots(1.66)$$

$$b = b_1 + b_2 + b_3 + \ldots \text{ mho} \quad \ldots(1.67)$$

$$y = g + jb \quad \ldots(1.68)$$

$$= \sqrt{g^2} + b^2 \quad \ldots(1.69)$$

Total current

$$I = \frac{E}{Z} = E.Y \quad \ldots(1.70)$$

Power factor angle,

$$\phi = \tan^{-1} \frac{b}{g} \quad \ldots(1.71)$$

Power factor will be lagging if b is + *ve*

Power factor will be leading if b is – *ve*

Note: Inductive suceptance b is assigned + ve sign and capacitive suceptance –*ve* sign.

1.24.2 Vector-method

Consider a parallel circuit shown in Fig. 1.34(a)

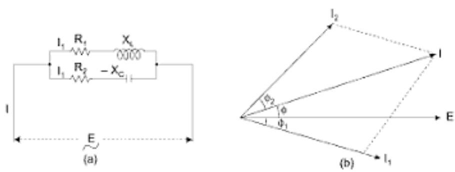


Fig. 1.34

Branch I.

$$\text{Impedance } Z_1 = \sqrt{R_1^2 + X_L^2} \quad \ldots(1.72)$$

$$\therefore I_1 = \frac{E}{Z_1} \quad \ldots(1.73)$$

$$\phi_1 = \tan^{-1} \frac{X_L}{R_1} \text{ lagging} \quad \ldots(1.74)$$

Take E as reference vector. Draw I_1 lagging at an angle ϕ_1 with E as shown in vector diagram of Fig. 1.34(b).

Branch II.

Impedance $$Z_2 = \sqrt{R_2^2 + (-X_c)^2} = \sqrt{R_2^2 + X_c^2} \qquad ...(1.75)$$

$\therefore$ $$I_2 = \frac{E}{Z_2} \qquad ...(1.72)$$

$$\phi_2 = \tan^{-1}\frac{X_c}{R_2} \text{ leading} \qquad ...(1.77)$$

Draw I_2 leading E by ϕ_2. The resultant of I_1 and I_2 will give total current I and the angle between E and I will give the *p.f.* angle.

Thus, a parallel circuit can be solved easily in this way.

1.24.3 *j*-Method

Consider a parallel CKT of Fig. 1.34, we can express the various impedances in *j* form as under.

$$Z_1 = R_1 + j\,X_L \times L$$

$$Z_2 = R_2 - j\,X_C$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R_1 + jx_L} + \frac{1}{R_2 - jx_c}$$

$$Z = \frac{(R_1 + jx_L)(R_2 - jx_c)}{(R_1 + R_2) + j(X_L - X_c)}$$

$$Z = \frac{(R_1 + R_2\; X_L X_c)}{(R_1 + R_2)^2 + (X_L - X_c)^2} + \frac{j\left[(X_L R_2 - X_c R_1)(R_1 + R_2) - (X_L - X_c)\right]}{(R_1 + R_2)^2 + (X_L - X_c)^2}$$

$$Z = R + jX$$

$\therefore$ Total current drawn $= \dfrac{E}{Z}$

Power factor $\cos\phi = \dfrac{R}{Z}$

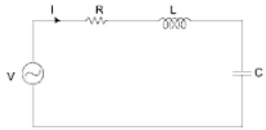
Power factor be lagging if X is +ve
leading if X is –ve

1.25 RESONANCE

An AC ckt is said to be in resonance when the applied voltage and the resulting current are in phase.

Thus, at resonance the equivalent complex impedance of the CKT consist of only resistance.

1.26 SERIES RESONANCE OR VOLTAGE RESONANCE



$$Z = R + j\left(wL - \frac{1}{wc}\right)$$

Fig. 1.35

$\Rightarrow$
$$I = \frac{V}{R + j\left(wL - \frac{1}{wc}\right)}$$

$\Rightarrow$ V and I will be in phase when reactance is zero.

$\Rightarrow$ at resonance the net reactance is zero. If the resonant frequency is f_o.

$$w_o = 2\pi f_o$$

$$w_o L = \frac{1}{w_o c}$$

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

$\Rightarrow$ Series CKT at resonance termed as acceptor CKT, since, it accepts frequencies close to resonant frequencies when impedance is minimum.

$\Rightarrow$ at low frequency $X_C > X_L$ and the CKT is capacitive $\{f_o > f\}$ and power factor is leading

$\Rightarrow$ at high frequency $X_L > X_C$ and the CKT is inductive $\{f_o < f\}$ and power factor is lagging

$\Rightarrow$ as f goes on increasing impedance also goes on increasing

$\Rightarrow$ if the resistance is zero, at resonance the CKT acts like a short circuit.

voltage across capacitance

$$V_C = \frac{I}{jwc} = \frac{V}{Z\,(jwc)} = \frac{V}{(jwc)\left[R + j\left(wL - \frac{1}{wc}\right)\right]}$$

$$|V_C| = \frac{V}{wc\left[R^2 + \left(wL - \frac{1}{wc}\right)^2\right]^{\frac{1}{2}}}$$

Frequency f_c at which V_C is maximum can be obtained $\frac{dV_C}{dw} = 0$

$$\boxed{f_c = \frac{1}{2\pi}\left[\frac{1}{LC} - \frac{R^2}{2L^2}\right]^{\frac{1}{2}}}$$

$$V_L = I(jwL) = \frac{wLV}{\left\{R^2 + \left(wL - \frac{1}{wc}\right)^2\right\}^{\frac{1}{2}}}$$

$$\boxed{f_L = \frac{1}{2\pi\left[LC - \frac{R^2C^2}{2}\right]^{\frac{1}{2}}}}$$ frequency at which V_L is max

⇒ it has been found that at resonance the values of V_L and V_C may be higher even then the supply voltage at resonance $V_{L_o} = V_{C_o}$.

Phasor diagram at resonance →

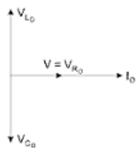


Fig. 1.36

1.27 BAND WIDTH

Band width of a series CKT is defined as the range of frequency for which the power delivered to the resistance is greater than or equal to half the power delivered at resonance.

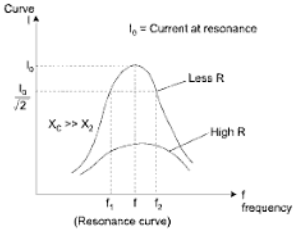


Fig. 1.37

Curve between current and frequency is known as resonance curve.

$$\text{Band width} = f_2 - f_1$$
$$= w_2 - w_1$$

w_1 and w_2 are the angular frequencies at which the power delivered is half the power delivered at resonance. These are also known as half power frequencies.

⇒ At resonance $Z = \frac{V}{I} = R$

At half power point $Z = \frac{V}{\frac{I}{\sqrt{2}}} = \frac{V}{I}\sqrt{2} = R\sqrt{2}$

$$Z = \sqrt{R^2 + X^2}$$

$$R\sqrt{2} = \sqrt{R^2 + X^2} \quad \text{so at this point } X = R$$

⇒ Lower half power frequency w_1 $X_C > X_L$

$$X_C - X_L = R$$

$$\frac{1}{w_1 c} - w_1 L = R$$

$$w_1^2 + \frac{R}{L} w_1 - \frac{1}{LC} = 0$$

$$w_1 = -\frac{R}{2L} \pm \left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]^{\frac{1}{2}}$$

$$\boxed{w_1 = -\propto \pm [(\propto^2 + w_0^2)]^{\frac{1}{2}}} \qquad \left.\propto = \frac{R}{2L},\ w_o = \frac{1}{\sqrt{LC}}\right\}$$

–ve frequency is meaningless so we take only +ve frequency.

At upper half frequency w_1 $\quad X_L - X_C = R$

$$w_2 L - \frac{1}{w_2 c} = R$$

$$w_2^2 - \frac{R}{L} w_2 - \frac{1}{LC} = 0$$

$$w_2 = \frac{R}{2L} \pm \left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]^{\frac{1}{2}}$$

$$\boxed{w_2 = \propto \pm (\propto^2 + w_0^2)^{\frac{1}{2}}} \quad \text{take +ve value}$$

Band width = $\boldsymbol{w_2 - w_1 = 2\propto = \frac{R}{L}}$

and, $\qquad w_1 \cdot w_2 = w_0^2$

1.28 QUALITY FACTOR AND SELECTIVITY

Ratio of resonant *frequency* to *band width* is an indication of the degree of selectivity of the CKT and this is known as Quality factor, Q.

$$\frac{1}{\text{selectivity}} = Q = \frac{w_0}{w_2 - w_1} = \frac{w_0 \cdot L}{R}$$

or,

$$Q = \frac{w_0^2}{w_0(w_2 - w_1)} = \frac{\frac{1}{LC}}{w_0\left(\frac{R}{L}\right)} = \frac{1}{w_0 cR}$$

Higher values of $Q\left(\frac{w_o L}{R}\right)$ Resonance curve is very narrow and sharp (Ω)

⇒ Sharpness of the curve depends on the parameters R and L. By changing C, the resonance can be made to occur at different values of frequencies.

⇒ $$Q = \frac{w_o L}{R} = \frac{(2\pi f_0)\left(\frac{1}{2} L I_0^2\right)}{\frac{1}{2} I_0^2 R}$$

$$\begin{cases} V_L = \frac{V}{R} X_L \\ \quad = \frac{V}{R} \cdot w_o L \\ V_L = QV \end{cases}$$

$$Q = \frac{\frac{1}{2}(L I_0^2)}{\frac{(I_0^2 R)}{2 f_0}} \cdot 2\pi = 2\pi \cdot \frac{\text{total stored energy}}{\text{energy dissipated/cycle}}$$

$$Q = \frac{w_o L}{R} = \frac{2\pi \cdot L}{2\pi\sqrt{LC} \cdot R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

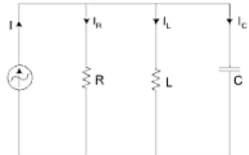
$$\text{Selectivity} = \frac{w_o}{\Delta w} = \frac{w_o L}{R}$$

⇒ A CKT with a flat frequency response curve (high R) will be more responsive and therefore less selective at frequencies in the neighbourhood of the resonant frequency.

1.29 PARALLEL RESONANCE

A parallel combination of R, L and C or (R, L) and C branches connected to a source will produce a parallel resonance (anti-resonance) when the resultant current through the combination is in phase with applied voltage at resonance power factor is unity for this.

1.30 DERIVATION FOR RESONANCE FREQUENCY



Parallel *R*, *L*, *C* Circuit

$$y = y_R + y_L + y_C$$

$$= \frac{1}{R} + \frac{1}{jW_L} + jW_C$$

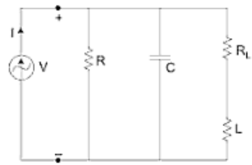
$$= \frac{1}{R} + j\left(W_C - \frac{1}{W_L}\right)$$

$$y = G + jB$$

at resonance imaginary

Part *B* is zero

$$\Rightarrow \quad W_oC = \frac{1}{W_oL}$$



Parallel (R_L, *L*) and *C*

$$y = \frac{1}{R} + jWC + \frac{1}{R_L + jWL}$$

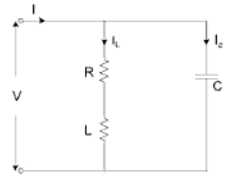
$$= \frac{1}{R} + jWC + \frac{R_L - jWL}{R_L^2 + W^2L^2}$$

$$= \frac{1}{R} + \frac{R_L}{R_L^2 + W^2L^2} + j\left(WC - \frac{WL}{R_L^2 + W^2L^2}\right)$$

$$Y = G + jB$$

at resonance $B = 0$, $W = W_o$

$$W_oC - \frac{W_oL}{R_L^2 + W^2L^2} = 0$$



Practical parallel circuit (tank circuit)

A coil of inductance *L* and effective series resistance *R* is connected in parallel with a capacitor.

$$Y = \frac{1}{Z} = \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$Y = -\frac{1}{jX_C} + \frac{1}{R + jX_L}$$

$$Y = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R - jX_L}{R^2 + X_L^2} + jWC$$

$\Rightarrow \quad W_o = \sqrt{\frac{1}{LC}}$ rad/sec $\Rightarrow \quad f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ Hz admittance at resonance is $y_o = \frac{1}{R}$. Thus, the is minimum at resonance. ⇒ A CKT consisting of parallel R, L, and C is called a second order parallel resonant circuit ⇒ Parallel LC combination is known as tank circuit.	$W_o = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$ $f_o = \frac{1}{2H}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$	$Y = \frac{R}{R^2 + X_L^2} + j\left(WC - \frac{X_L}{R^2 + X_L^2}\right)$ $\mathbf{Y = G + jB}$ at resonance $B = 0$, $W = W_o$ $W_oC - \frac{W_oL}{R^2 + W_o^2L^2} = 0$ $W_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L_2}}$ $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L_2}}$ if R is small— $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

1.31 IMPEDANCE AT RESONANCE

$$Y = \frac{R}{R^2 + w^2L^2}$$

$$Z = \frac{R^2 + w^2L^2}{R}$$

At resonance $w = w_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

At resonance Z_d = Resistive part

$Z_d = R_d$ is dynamic resistance

$$Z_d = R + \frac{L^2}{R}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)$$

$$Z_d = \cancel{R} + \frac{L}{RC} - \cancel{R} \Rightarrow Z_d = \frac{L}{RC}$$

$$\boxed{Z_d = \frac{L}{RC}}$$

Z_d is called dynamic impedance, this is pure resistive. It is seen lower the R higher the Z_d. Hence the value of impedance at resonance is maximum and the resultant current is minimum. *A* parallel resonant circuit is also called a rejector circuit since the current at resonance is minimum or tank circuit almost rejects the current at resonance.

I_o = Current at resonance $= \frac{V}{Z_d} = \frac{VCR}{L}$, it $R = 0$ ckt will draw no current at resonance. The supply current is zero and large current circulates in parallel ckt at resonance.

1.32 CURRENT MAGNIFICATION

Current drawn from supply at resonance is $I = \frac{V}{R_d}$

or, $$I = \frac{VCR}{L}$$

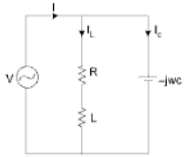


Fig. 1.38

So circulating current is Vw_oC.

$$Q = \frac{\text{Circulating current}}{\text{Current drawn from supply}} = \frac{Vw_oC}{VC.\frac{R}{L}} = \frac{Lw_o}{R}$$

⇒ Parallel tuned circuit exhibits a current amplification of Q, whereas series ckt exhibits voltage amplification of Q.

1.33 SELECTIVITY AND BAND WIDTH

At half power frequency w_1 and w_2, ckt impedance is $\frac{R_d}{\sqrt{2}}$

{At reasonance $V = I.\ R_d$

{At half power $V = \frac{I}{\sqrt{2}}.\ R_d \left\{\text{so } Z = \frac{R_d}{\sqrt{2}}\right\}$

Band width = $w_2 - w_1 = \frac{w_o}{Q}$

$$Q = \frac{w_oL}{R} = \frac{1}{w_oCR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Comparison between Parallel and Series Resonance

(1) Net susceptance is zero while in series resonance reactance (net) is zero
(2) Admittance is equal to admittance at parallel resonance.
(3) Reactive or wattless component ($I \sin \theta - I_L$) is zero, hence circuit power factor is unity.
(4) Impedance = $\frac{L}{CR}$ while in series resonance $Z = R$.

(5) Line current is minimum and equal to $\frac{V}{Zd}$ while at series resonant it is maximum and $I = \frac{V}{R}$ so series resonant ckt is acceptor while parellel ckt is rejector.

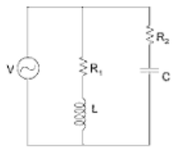
(6) $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz and in series ckt $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

1.34 CURRENT AT RESONANCE

The current flowing in the circuit at the time of parallel resonance is minimum. For circuit given below, equivalent conductance is,

$$g = g_1 + g_2$$

$$= \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} \quad ...(1.82)$$



If $R_1 = R_2 = O$, then

$$g = O$$

Also susceptance,

$$b = b_1 + b_2$$

$$= \frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} \quad ...(1.83)$$

$$= O \text{ at resonance}$$

∴ Equivalent impedance,

$$Z = \frac{1}{Y} - \frac{1}{g^2 + 2} = \frac{1}{O} = \infty \quad ...(1.84)$$

$$\text{Current} = \frac{E}{Z} = \frac{E}{\infty} = O \quad ...(1.85)$$

Such a circuit is called a **rejector circuit.**

In actual practice, the inductance coil may have a small resistance say $R_1 = R$ and the resistance in the capacitive branch R_2 be assumed zero, then

Total conductance of $g = \frac{R}{R^2 + X_L^2} + O$

If R is small enough, whose squares may be neglected, then

$$g = \frac{R}{X_L^2} \qquad ...(1.86)$$

SOLVED EXAMPLES

Example 1: A two element series circuit is connected across an AC source $V = 300 \cos (314t + 20°)$ volts. The current is drawn $15 \cos (314t - 10°)$ Amp. Determine circuit impedance magnitude and phase angle. What is the average power drawn? (U.P. Tech 2003-04)

Solution:

Given, $V = 300 \cos (314t + 20°)$ $\quad [\cos \theta = \sin (\theta + 90°)]$

$$V = 300 \sin (314t + 110°)$$

In polar form $V = \frac{300}{\sqrt{2}} \angle 110°$

$$i = 15 \cos (314t - 10°)$$
$$= 15 \sin (314t + 80°)$$

$$I = \frac{15}{\sqrt{2}} \angle +80°$$

$$Z \text{ (circuit impedance)} = \frac{\frac{300}{\sqrt{2}} \angle 100°}{\frac{15}{\sqrt{2}} \angle +80°}$$

$$Z = \boxed{20 \angle 30°}$$

Hence, the angle between voltage and current is 30° and current lags $\left(I = \frac{V}{Z}\right)$ the voltage by 30°. **Phase angle = 30°**

$$P_{av} = \frac{V_m I_m}{2} \cos \phi \text{ (in } R\text{-}L \text{ ckt)}$$

$$= \frac{1}{2} \times 15 \times 300 \times \cos 30°$$

$$= 1949.85 \text{ watt.}$$

Example 2: A 120 V, 100 W lamp is to be connected to a 220 V, 50 Hz AC supply. What value of pure inductance should be connected in series in order to run the lamp on rated voltage? (2003-04)

Solution:

$$\Rightarrow \quad Z = R + jX_L$$

$$\Rightarrow \quad V_{\text{supply}} = V + jV_L = \sqrt{V^2 + V_L^2}$$

$$\Rightarrow \quad 220 = \sqrt{120^2 + V_L^2}$$

$$\Rightarrow \quad V_L = 184.39$$

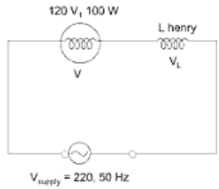


Fig. 1.39

$\Rightarrow$ Current through the lamp and inductance is same. Current through lamp

$$i = \frac{P}{V} = \frac{100}{120}$$

$$\Rightarrow \quad \frac{100}{120} = \frac{V_L}{X_L}$$

$$\Rightarrow \quad X_L = \frac{120 \times V_L}{100} = \frac{120}{100} \times 184.39 = 221.269 \text{ ohm.}$$

$$\Rightarrow \quad X_L = 2\pi f L$$

$$L = \frac{X_2}{2\pi f} = \frac{221.269}{2 \times 3.14 \times 50}$$

$$\boxed{L = 0.7046 \text{ henry}}$$

Example 3: For the circuit shown in figure, find the current and power drawn from the source. (2004-05)

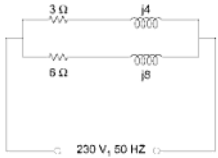


Fig. 1.40

Solution:

Let

$$Z_1 = 3 + j4 = 5 \angle 53.13\ \Omega$$

$$Z_2 = 6 + j8 = 10 \angle 53.13\ \Omega$$

$$Z_1 + Z_2 = 9 + j12 = 15 \angle 53.13\ \Omega$$

Both Z_1 and Z_2 are parallel hence net impedance of the circuit is Z

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{5 \times 10 \angle 106.26}{15 \angle 53.13}$$

$$\boxed{Z = 3.33 \angle 53.13}$$

$\Rightarrow$ Current drawn from the ckt is $I = \dfrac{V}{Z}$

$$I = \frac{V \angle 0}{Z \angle \theta} = \frac{230}{3.33 \angle 53.13}$$

$$\boxed{I = 69 \angle -53.13 \text{ Amp}}$$

Hence, net current lags the net voltage by 53.13° and circuit is inductive in nature.

$\Rightarrow$ Power drawn from source $= VI \cos \phi$

$$= 230 \times 69 \times \cos (53.13)$$

$$= 9.522 \text{ kw}$$ **Ans.**

Example 4: A coil connected to 100 V DC supply draws 10 Amp and the same coil connected 100V, AC voltage of frequency 50 Hz draws 5 Amp. Calculate the parameters of the coil and power factor. [2004-05]

Solution:

⇒ Coil means a resistance and inductance both.

Let impedance of a coil $Z = R + jX_L$

⇒ When DC supply is connected to coil inductance behave like a short circuit ($X_L = 2\pi fL = 2\pi \times 0 \times L = 0\ \Omega$)

So resistance of coil $R = \frac{V_{dc}}{I} = \frac{100}{10} =$ **10 ohm.**

⇒ When AC is applied across the same coil.

Given $V = 100$ volt of 50 Hz frequency.

$$I = 5 \text{ amp.}$$

⇒ $$V = IZ$$

⇒ $$Z = \frac{V}{I} = \frac{100}{5} = 20\ \Omega$$

⇒ $$Z^2 = R^2 + X_L^2$$

⇒ $$X_L^2 = Z^2 - R^2 = 20^2 - 10^2$$

$$X_L = \sqrt{300} = 17.32\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{17.32}{2 \times 3.14 \times 50} = 0.05 \text{ henry}$$

⇒ Power factor of coil $= \frac{R}{Z} = \frac{10}{20} = 0.5$ lagging **Ans.**

Example 5: Discuss the effects of varying the frequency upon the current drawn and the power factor in a RLC series circuit, a series RLC circuit with $R = 10\ \Omega$, $L = 0.02$ Hz, and $C = 2\mu f$ is connected to 100 V variable frequency source. Find the frequency for which the current is maximum. (2004-05)

Solution:

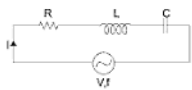


Fig. 1.41

Impedance $Z = R + jX_L - jX_C$

$$Z = R + j(X_L - X_C)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \;\angle \tan^{-1} \frac{X_L - X_C}{R} = |Z| \angle\theta$$

$$I = \frac{V}{|Z| \angle\theta}$$

$\Rightarrow$ $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ and P.F. $\cos\theta = \cos\tan^{-1} \frac{(X_L - X_C)}{R}$

(1) when $X_L = X_C$ source frequency f = resonant freq (f_r)

$\Rightarrow$ $|Z| = R$ so current is maximum and power factor is unity.

(2) now if we increase the frequency from resonance frequency $f > f_r$. Then $X_C = \frac{1}{2\pi fC}$ will decrease and X_L increases. Impedance increases hence current will decrease and power factor decreases and becomes lagging.

(3) If frequency decreases below resonance frequency ($f < f_r$), then X_L decrease and X_C increases but net impedance will increase, so current will decrease and power factor will also decrease.

$\cos\phi = \frac{R}{Z}$ but it becomes capacitive.

$\Rightarrow$ Current is maximum at resonance so at resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

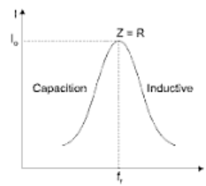


Fig. 1.42

$$f_r = \frac{1}{2\pi\sqrt{2\times 10^{-6} \times 2\times 10^{-2}}} = 795.5 \text{ Hz}$$

Example 6: A load having impedance of (1 + j1) Ω is connected to an AC voltage represented as $V = 20\sqrt{2}\cos(wt + 10°)$ volt. Find the current in load expressed in the form of $i = I_m \sin(wt + \phi)\, A$. Find the real and apparent power. (2004-05)

Solution:

Load impedance $Z = 1 + j1 = \sqrt{2}\angle 45°$

Voltage across the load $V = 20\sqrt{2}\cos(wt + 10°)$

$$= 20\sqrt{2}\sin(wt + 100°)$$

$$V = \frac{20\sqrt{2}}{\sqrt{2}}\angle 100° = 20\angle 100°$$

Current through load $I = \frac{V}{Z} = \frac{20\sqrt{100}}{\sqrt{2}\angle 45°}$

$\boxed{I = 14.144\angle 55°}$ is rms value of current

$$i = I_m \sin(wt + \phi),\ I_m = \left(\frac{14.144}{5}\right)\sqrt{2}$$

$$i = \sqrt{2}\,.\,I\sin(wt + 55°)$$

$$i = 20\sin(wt + 55)$$

(ii) Real power = $V_{rms}.I_{rms}\cos\phi$

$$= \frac{1}{2}V_m I_m \cos\phi$$

$$= \frac{1}{2}\pi\, 20\sqrt{2}\,.20.\cos 45$$

$\boxed{P = 200 \text{ watt}}$

(iii) Apparent power $= \frac{1}{2} V_m I_m$

$$= \frac{1}{2} \times 20\sqrt{2} \times 20$$

$$= 282.84 \text{ VAR}$$

Example 7: An emf given by $100 \sin\left(314t - \frac{\pi}{4}\right)$ volts in applied to a circuit and the current is 20 sin (314t – 1.5808) ampere. Find (i) frequency (ii) circuit elements. [2005-06]

Solution:

(i) Let instantaneous emf be e

$$e = 100 \sin\left(314t - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad wt = 314t$$

$$\Rightarrow \quad 2\pi f = 314 \quad \Rightarrow \quad f = 50 \text{ Hz}$$

(ii)

$$E = \frac{100}{\sqrt{2}} \angle -\frac{\pi}{4}$$

$$i = 20 \sin (314t - 1.5808)$$

$$= 20 \sin\left(314t - \frac{1.5808 \times 100}{3.14}\right)$$

$$i = 20 \sin\left(314t - \frac{\pi}{2}\right)$$

$$I = \frac{20}{\sqrt{2}} \angle -\frac{\pi}{2}$$

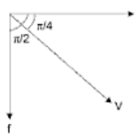


Fig. 1.43

Circuit impedance $Z = \dfrac{V \angle -\frac{\pi}{4}}{I \angle -\frac{\pi}{2}}$

$$Z = \frac{\frac{100}{\sqrt{2}}}{\frac{20}{\sqrt{2}}} \angle 45°$$

$$\boxed{Z = 5\angle 45°}$$

⇒ Current lags the voltage by $\frac{\pi}{4}$. So circuit elements are R and L.

$$Z = R + jX_L = 5 \cos 45 + j5 \sin 45°$$

$$R = 5 \cos 45° = \frac{5}{\sqrt{2}}\ \Omega$$

$$X_L = 5 \sin 45° = \frac{5}{\sqrt{2}}\ \Omega$$

⇒ $$X_L = 2\pi f L$$

⇒ $$L = \frac{X_L}{2\pi f} = \frac{5}{\sqrt{2} \times 2 \times \pi \times 50} = 0.01126 \text{ henry.}$$

Example 8: A choke coil takes a current of 2 amperes lagging 60° behind the applied voltage of 200 volts at 50 Hz. Calculate inductance, resistance and impedance of the coil. Also determine the power consumed when it is connected across 100 V, 25 Hz supply. (2005-06)

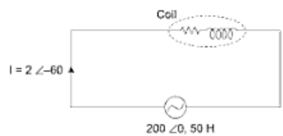


Fig. 1.44

Solution: Coil having a resistance and inductance both

⇒ Impedance of the coil $= \frac{V}{I} = \frac{200}{2\angle -60°}$

$$Z = 100 \angle -60° = R + jX_L \quad \text{(ckt is inductive)}$$

$100 \cos 60 + j100 \sin 60 = R + jX_L$

$$\Rightarrow \quad R = 100 \cos 60 = 100 \times \frac{1}{2} = 50\ \Omega$$

$$X_L = 100 \sin 60 = 100\ \frac{\sqrt{3}}{2} = 86.6\ \Omega$$

$\Rightarrow$ Frequency of supply = 50 Hz

$$L = \frac{X_L}{2\pi f} = \frac{86.6}{314} = 2.758 \times 10^{-1}\ \text{H}$$

(ii) Now the choke coil is connected to 100 V, 25 Hz supply.
R and L will be same as above.

Now, $X_L = 2\pi fL = 2 \times 3.14 \times 25 \times 2.758 \times 10^{-1}$

$= 43.3\ \Omega$

Now, $Z = \sqrt{R^2 + X_L^2} = \sqrt{50^2 + 43.3^2} = 66.1\ \angle \tan^{-1} \frac{43.3}{50}$

$= 66.1 \angle 40.89$

and current from the coil $= \frac{V}{Z}$

$$I = \frac{100}{66.1 \angle 40.89} = 1.5\ \angle -40.89\ \text{Amp}$$

Power consumed $= VI \cos \varphi = 100 \times 1.5 \times 0.75 = 112.5$ W

or, $I^2 R = (1.5)^2 \times 50 = 112.5$ W

Example 9: Two coils of 5 Ω and 10 Ω and inductances 0.04 H and 0.05 H respectively are connecting in parallel across a 200 V, 50 Hz supply. Calculate:

(i) Conductance, susceptance and admittance of each coil.
(ii) Total current drawn by the circuit and its power factor.
(iii) Power absorbed by the circuit.

(iv) The value of resistance and inductance of single coil which will take the same current and power as taken by the original circuit.

[2005-06]

Solution: Given

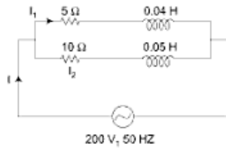


Fig. 1.45

(i)

$$Z_1 = R_1 + jX_{L_1}$$

$$Z_1 = 5 + j12.56 = 13.52 \angle 68.29$$

$$Z_2 = R_2 + jX_{L_2}$$

$$= 10 + j15.7 = 18.62 \angle 57.51$$

$$X_{L_1} = 2\pi f L_1 = 2 \times 3.14 \times 50 \times 0.04 = 12.56\ \Omega$$

$$X_{L_2} = 2\pi f L_2 = 2 \times 3.14 \times 50 \times 0.05 = 15.7\ \Omega$$

Admittance of coil (1) is $y_1 = G_1 + jB_1 = \dfrac{1}{Z_1}$

$$\Rightarrow \quad \frac{1}{Z_1} = \frac{1}{13.52 \angle 68.29} = 0.074 \angle -68.29$$

$$\Rightarrow \quad Y_1 = 0.074 \angle -68.29$$

$$Y_1 = 0.0274 - j0.069$$

$$\Rightarrow \quad G_1 = 0.0274 \text{ and susceptance } B_1 = 0.069$$

Admittance of coil (2) is $Y_2 = G_2 + jB_2$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{18.62 \angle 57.51} = 0.0537 \angle -57.51$$

$$Y_2 = 0.029 - j0.0453$$

So conductance $G_2 = 0.029$, susceptance $B_2 = 0.0453$

(ii) Total admittance of ckt is $Y = Y_1 + Y_2$

$$Y = 0.0274 - j0.069 + 0.029 - j0.0453$$

$$= 0.0564 - j0.1143$$

$$Y = 0.1275 \angle -63.74$$

Current drawn by the circuit is $I = VY$

$$I = 200 \times 0.1275 \angle -63.74$$

$$\boxed{I = 25.5 \angle -63.74 \text{ Amp}}$$

Hence, current lags the supply voltage by 63.74°.

So power factor = cos (63.74) = 0.443

(iii) Power absorbed by the circuit $P = VI \cos \phi$

$$P = 200 \times 25.5 \cos (63.74)$$

$$= 2.256 \text{ kW} \quad \textbf{Ans.}$$

(iv) Current taken by original circuit is $I = 25.5 \angle -63.74$ Amp

$$V = 200 \text{ V}$$

Impedance of coil $Z = R + jX_L = \dfrac{V}{I} = \dfrac{200}{25.5 \angle -63.74}$

$$Z = 7.843 \angle 63.74$$

$$= 3.47 + j7.034$$

So $R = 3.47\ \Omega$ and $X_2 = 7.034\ \Omega$,

$$\text{Power} = I^2 R$$

$$= (25.5)^2 \times 3.47$$

$$= 2.256 \text{ kW}$$

Example 10: An AC voltage $e(t) = 141.4 \sin 120t$ is applied to a series RC circuit. The current through the circuit is obtained as

$$i(t) = 14.14 \sin 120t + 7.07 \cos (120t + 30°).$$

(2004-05)

Determine:

(i) Value of resistance and capacitance

(ii) Power factor

(iii) Power delivered by the source.

Solution:

$$e(t) = 141.4 \sin 120t \rightarrow \text{time domain}$$

$$E = \frac{141.4}{\sqrt{2}} \angle 0^\circ \rightarrow \text{polar form}$$

Given, $i(t) = 14.14 \sin 120t + 7.07 \cos (120t + 30^\circ)$ | $\cos\theta = \sin(90^\circ + \theta)$

$$= 14.14 \angle 0 + 7.07 \sin (120t + 120^\circ)$$

$$= 14.14 \angle 0 + 7.07 \angle 120^\circ$$

$$= 14.14 + 7.07 \cos 120 + j7.07 \sin 120$$

$$= 14.14 - 3.535 + j6.123$$

$$i(t) = 10.61 + j6.123 = 12.25 \angle 29.989$$

$$\boxed{i(t) = 12.25 \sin (120t + 30^\circ)}$$

$$I = \frac{12.25}{\sqrt{2}} \angle 30^\circ$$

(i) $$Z = \frac{V}{I} = \frac{\frac{141.4}{\sqrt{2}}}{\frac{12.25}{\sqrt{2}}} \angle 30 = 11.54 \angle -30^\circ$$

$$Z = 9.996 - j5.77 = R - jX_C$$

$$\Rightarrow \quad R = 9.996\ \Omega,\ X_C = 5.77\ \Omega$$

(ii) Power factor $= \cos\phi = \cos 30^\circ$

$$= 0.866$$

(iii) Power delivered by the source $= V_{rms} I_{rms} \cos\phi$

$$= \frac{V_m I_m}{2} \cos\phi$$

$$P = \frac{(141.4)(12.25)}{2} \cos 30°$$

$$\boxed{P = 750 \text{ watt}}$$

Example 11: A non-inductive resistance of 10 ohms is connected in series with an inductive coil across 200 V, 50 Hz ac supply. The current drawn by the series combination is 10 amperes. The resistance of the coil is 2 ohms.

Determine:

(i) Inductance of the coil

(ii) Power factor

(iii) Voltage across the coil

Solution: Given→

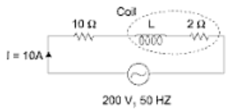


Fig. 1.46

(i) $$Z = 10 + 2 + jX_L = 12 + jX_L = \sqrt{12^2 + X_L^2}$$

$$I = \frac{V}{Z}$$

$$\Rightarrow \quad Z = \frac{V}{I} = \frac{200}{10} = 20$$

$$\Rightarrow \quad Z^2 = 12^2 + X_L^2$$

$$\Rightarrow \quad (20)^2 = (12)^2 + X_L^2$$

$$\Rightarrow \quad X_L = \sqrt{256} = 16 \text{ ohm}$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{16}{2 \times 3.14 \times 50} = 50.9 \text{ mili henry}$$

(ii) Power factor = $\frac{R}{Z} = \frac{12}{20} = 0.6$

(iii) Voltage across the coil is V_L, then

$$V_L = I(2 + jX_L)$$

$$V_L = 10(2 + j16) = 161.245 \angle 82.87 \text{ volt}$$

Example 12: For the given figure shown

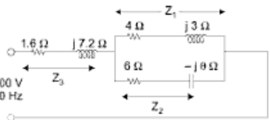


Fig. 1.47

(i) Admittance of each parallel branch

(ii) Total circuit impedance

(iii) Supply current and power factor

(iv) Total power supplied by the source. (2005-06)

Solution:

$$Z_3 = 1.6 + j7.2 = 7.375 \angle 77.47$$

$$Z_1 = 4 + j3 = 5 \angle 36.86$$

$$Z_2 = 6 - j8 = 10 \angle -53.13$$

(i) Admittance of each parallel branch is Y_1 and Y_2, then

$$Y_1 = \frac{1}{Z_1} = \frac{1}{7.375 \angle 77.47} = 0.1356 \angle -77.47 \, \Omega$$

$$\boxed{Y_1 = 0.029 - j7.18}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{5 \angle 36.86} = 0.2 \angle -36.86 \, \Omega$$

$$Y_2 = 0.2\,[\cos(-36.86)^2 + j\sin(-36.86)]$$
$$= 0.16 - j0.119$$

(ii) Total circuit impedance is $Z = (Z_1 \parallel Z_2) + Z_3$

$$Z = Z_3 + \frac{Z_2 Z_1}{Z_2 + Z_1}$$

$$= 1.6 + j7.2 + \frac{5\angle 36.86 \cdot 10\angle -53.13}{(4 + j3) + (6 - j8)}$$

$$= 1.6 + j7.2 + \frac{50\angle -16.27}{10 - j5}$$

$$= 1.6 + j7.2 + \frac{50\angle -16.27}{11.18\angle -26.56}$$

$$= 1.6 + j7.2 + 4.47\ \angle 10.29$$
$$= 1.6 + \text{j}\ 7.2 + 4.398 + j0.798$$

$$\boxed{Z = 5.998 + j11.598 = 13.06\ \angle 62.65}$$ **Ans.**

(iii) Supply current $I = \frac{V}{Z} = \frac{100}{13.06\ \angle 62.65}$

$$\boxed{I = 7.65\ \angle -62.65 \text{ Amp}}$$

Power factor = cos 62.65 = 0.459

(iv) Power supplied by source = $VI \cos \phi$

$$P = 100 \times 7.65 \cos(62.65)$$
$$= 351.13 \text{ watt}$$

or, $$P = I^2R = (7.65)^2 \times 5.998$$
$$= 351.02 \text{ watt}$$

Example 13: For the circuit shown below, determine:

(i) Resonant frequency

(ii) Total impedance at resonance

(iii) Band width

(iv) Quality factor.

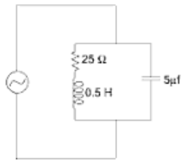


Fig. 1.48

Solution:

(i) For a parallel ckt resonant frequency f_r is given by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\times 3.14}\sqrt{\frac{1}{0.5\times 5\times 10^{-6}} - \frac{(25)^2}{(0.5)^2}}$$

$$= 100.39 \text{ Hz}$$

(ii) Total impedance at resonance is $Z = \frac{L}{RC}$

$$Z = \frac{0.5}{25\times 5\times 10^{-6}} = 4 \text{ k}\Omega$$

(iii) Band width $(f_2 - f_1) = \frac{R}{2\pi L} = \frac{250}{2\pi\, 0.5} = 7.96$ Hz

(iv) Quality factor$(Q) = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{25}\sqrt{\frac{0.5}{5\times 10^{-6}}}$$

$$\boxed{Q = 12.65}$$

Example 14: Draw the phasor diagram showing the following voltage and find the RMS value of resultant voltage.

$$V_1 = 100 \sin 500t,\ V_2 = 200 \sin\left(500t + \frac{\pi}{3}\right)$$

$$V_3 = -50 \cos(500t),\ V_4 = 150 \sin\left(500t - \frac{\pi}{4}\right)$$

Solution:

$\Rightarrow$ If $V = V_m \sin(wt + \theta)$ can be represented in a polar for $V = \frac{V_m}{\sqrt{2}} \underline{|\theta}$ and shown in X-Y plane

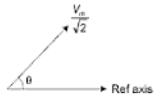


Fig. 1.49

Similarly $\rightarrow$ $V_1 = 100 \sin 500t$

$\Rightarrow$ $$V_1 = \frac{100}{\sqrt{2}} \angle 0°$$

$\Rightarrow$ $$V_2 = 200 \sin\left(500t + \frac{\pi}{3}\right) \Rightarrow V_2 = \frac{200}{\sqrt{2}} \angle \frac{\pi}{3}$$

$\Rightarrow$ $$V_3 = -50 \cos(500t)$$

$$= -50 \sin\left(500t + \frac{\pi}{2}\right) \Rightarrow V_3 = \frac{-50}{\sqrt{2}} \angle \frac{\pi}{2}$$

$\Rightarrow$ $$V_4 = 150 \sin\left(500t - \frac{\pi}{4}\right) \Rightarrow V_4 = \frac{150}{\sqrt{2}} \angle \frac{-\pi}{4}$$

Phasor diagram

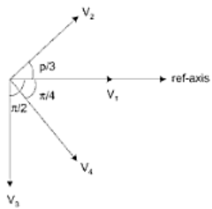


Fig. 1.50

Resultant voltage

$$V = V_1 + V_2 + V_3 + V_4$$

$$V = \frac{100}{\sqrt{2}}\angle 0 + \frac{200}{\sqrt{2}}\angle\frac{\pi}{3} + \left(\frac{-50}{\sqrt{2}}\right)\angle\frac{\pi}{2} + \frac{150}{\sqrt{2}}\angle\frac{-\pi}{4}$$

$$= \frac{1}{\sqrt{2}}\left[100 + 200\cos\frac{\pi}{3} + j200\sin\frac{\pi}{3} - j50 + 150\cos\left(\frac{-\pi}{4}\right) + j150\sin\left(\frac{-\pi}{4}\right)\right]$$

$$= \frac{1}{\sqrt{2}}[100 + 100 + j173.20 - j50 + 106.06 - j106.06]$$

$$= \frac{1}{\sqrt{2}}[306.06 + j17.14]$$

$$V = \frac{306.54}{\sqrt{2}}\angle 32.05°$$

RMS value of resultant voltage $= \frac{306.54}{\sqrt{2}} = 216.756$ volt

and resultant voltage leads from reference axis by 32.05°.

Instantaneous voltage $V = V_m \sin(wt + \phi)$

$$\boxed{V = 306.54\sin(500t + 32.05)}$$

Example 15: A series R-L-C circuit has $R = 10\ \Omega$, $L = 0.1$ H and $C = 8\ \mu F$. Determine,

(i) Resonant frequency
(ii) Q-factor of circuit at resonance
(iii) The half power frequencies

Solution: Given: $R = 10\ \Omega$, $L = 0.1$ H, $C = 8 \times 10^{-6}$ F

(i) For a series R-L-C circuit resonant frequency f_r is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\times 3.14\sqrt{0.1\times 8\times 10^{-6}}} = 177.94 \text{ Hz.}$$

(ii) Q–factor at resonant $= \frac{w_r L}{R}$

$$= \frac{2\pi f_r L}{R} = \frac{2\times 3.14\times 177.94\times 0.1}{10}$$

$$= 11.17$$

(iii) $B.W = f_2 - f_1$ $\{f_1$ and f_2 are half power frequencies.

$$\Rightarrow \quad f_1 = f_r - \frac{B.W}{2}$$

$$= f_r - \frac{R}{4\pi L} = 177.94 - \frac{10}{4\times 3.14\times 0.1} = 169.99 \text{ Hz}$$

$$\Rightarrow \quad f_2 = f_r + \frac{B.W}{2} = 177.94 + 7.95 = 185.89 \text{ Hz.}$$

Example 16: An alternating current of frequency 50 Hz, has a maximum of 100 A. Calculate (a) its value $\frac{1}{600}$ second after the instant the current is zero and its value decreasing thereafter (b) How many seconds after the instant the current is zero (increasing therefore words)? Will the current attain the value of 86.6 A? (Elect. Tech. Allah. Univ. 1991).

Solution: The equation of the alternating current (assumed sinusoidal) with respect to the origin of Fig. 1.51.

$$i = 100 \sin 2\pi \times 50t = 100 \sin 100\pi t.$$

(a) It should be noted that, in this case, time is being measured from point A and not from O.

If the above equation is to be utilized, then, this time must be referred to point O. For this purpose, half period i.e., $\frac{1}{100}$ sec. has to be added to

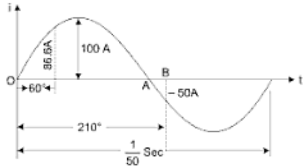


Fig. 1.51

$\frac{1}{600}$ sec. The given time as referred to point O becomes = $\frac{1}{100} + \frac{1}{600}$

$= \frac{7}{600}$ sec.

$$\therefore \quad i = 100 \sin 100 \times 180 \times \frac{7}{600} = 100 \sin 210°.$$

$$= 100 \times \left(-\frac{1}{2}\right) = -50 \text{ A} \quad \text{...Point B.}$$

(b) In this case the reference point is O

$$\therefore \quad 86.6 = 100 \sin 100 \times 180t \text{ or } \sin 18{,}000t = 0.866$$

$$\text{or} \quad 18{,}000t = \sin^{-1}(0.866) = 60°$$

$$\therefore \quad t = \frac{60}{18000} = \frac{1}{300} \text{ second.}$$

Example 17: An alternating voltage $e = 200 \sin 314t$ is applied to a device which offers an ohmic resistance of 20 Ω to the flow current in one direction, while preventing the flow of current in opposite direction. Calculate RMS value, average value and form factor for the current over one cycle.

(Elect. Engg. Nagpur Univ. 1992).

Solution: Comparing the given voltage equation with the standard form of alternating voltage equation, we find that

$$V_m = 200 \text{ V}, R = 20\ \Omega, I_m = \frac{200}{20} = 10 \text{ A}.$$

For such a half-wave rectified current, RMS value = $\frac{Im}{2} = \frac{10}{2} = 5$ A.

$$\text{Average current} = \frac{Im}{\pi} = \frac{10}{\pi}$$

$$= 3.18 \text{ A;}$$

$$\text{Form factor} = \frac{5}{3.18} = 1.57.$$

Example 18: A 50 Hz sinusoidal voltage wave shape has maximum value of 350 V. Calculate its instantaneous value:

(a) 0.005 second after the wave passes through zero in the positive direction

(b) 0.008 sec after the wave passes through zero in the negative direction.

Solution:

(a) $e = E_m \sin wt$

$t = 0.005$ sec, $E_m = 350$ V, $f = 50$ Hz

$$e = 350 \sin 2 \times \pi \times 50 \times t$$

$$= 350 \sin (100 \times 180 \times 0.005)$$

$$\boxed{e = 350 \text{ volt}}$$

So instantaneous value at $t = 0.005$ sec is the maximum value.

(b) After the wave passes zero in negative direction.

Sine wave will pass zero in negative direction after half cycle or $\frac{T}{2}$ sec.

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}, \frac{T}{2} = 0.01 \text{ sec}$$

So total time $t = \frac{0.02}{2} + 0.008 = \mathbf{0.018\ sec}$

Now, $e = 350 \sin (2\pi ft)$

$$= 350 \sin (2 \times 180 \times 50 \times 0.018)$$

$$= -205.72 \text{ Volt} \quad \textbf{Ans.}$$

Example 19: A sinusoidal alternating current of frequency 25 Hz has a maximum value of 100 A. How long will it take for the current to attain values of 20, and 100 A?

Solution:

For AC current $i = I_m \sin (wt)$

Given $$I_m = 100 \text{ A}, f = 25 \text{ Hz}$$

$$i = 100 \sin 50\pi t$$

(a) When current attain value of 20 amp, means instantaneous value $i = 20$ amp.

$$20 = 100 \sin 50\pi t$$

$$\sin 50\pi t = 0.2$$

$$50\pi t = \sin^{-1} 0.2 = 11.5°$$

$$t = \frac{11.5°}{50\pi} = \frac{11.5°}{50 \times 180°} = 0.00128 \text{ sec}$$

(b) When instantaneous current $i = 100$ amp

$$i = 100 \sin 50\pi t$$

$$100 = 100 \sin 50\pi t$$

$$\Rightarrow \quad 50\pi t = \sin^{-1} = 90°$$

$$\Rightarrow \quad t = \frac{90°}{50 \times 180°} = 0.01 \text{ sec.}$$

Example 20: A sinusoidal current wave is given by $i = 50 \sin (100\pi t)$. Determine (i) the greatest rate of change of current (ii) the average value (iii) root mean square value (iv) the time interval between a maximum value and the next zero value.

Solution:

(i) Given, $i = 50 \sin (100\pi t)$

Rate of change of current = $\frac{di}{dt}$

$$i' = \frac{di}{dt} = 50 \times 100\pi \cos (100\pi t)$$

$$i' = I'_m \cos (100\pi t)$$

So greatest rate of change of current is I'_m

$$I'_m = 5000\pi = 15{,}715 \text{ amp/sec}$$

(ii) $$i = 50 \sin (100\pi t)$$

$$i_{av} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} I_m \sin (wt) dt$$

$\Rightarrow$ $$t_1 = 0,\ t_2 = \frac{T}{2}$$

$$i_{av} = \frac{2}{T} \int_0^{T/2} I_m \sin (wt) dt$$

$$= \frac{2}{wT} I_m [-\cos wt]_0^{T/2} = \frac{-2I_m}{wT} \cdot \left[\cos \frac{2\pi}{T} \cdot \frac{T}{2} - \cos 0\right]$$

$$= \frac{2I_m}{\frac{2\pi}{T} \cdot T} \cdot 2 = \frac{2I_m}{\pi}$$

$$i_{av} = 0.637\ I_m = 0.637 \times 50 = 31.85 \text{ amp}$$

(iii) RMS value for $i = I_m \sin wt$

$$I_{\text{RMS}} = \left[\frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} (I_m \sin wt)^2\, dt\right]^{1/2} \qquad t_1 = 0,\ t_2 = T$$

$$I_{\text{RMS}} = \left[\frac{1}{T}\int_0^T I_m^2 \sin^2 wt\, dt\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\int_0^T \frac{(1-\cos 2wt)}{2} dt\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{t}{2} - \frac{\sin 2wt}{4w}\right)_0^T\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{T}{2}\right)\right]^{1/2}$$

$$\boxed{I_m = \frac{I_m}{\sqrt{2}}} = 0.707\, I_m$$

$$I_{\text{rms}} = 50 \times 0.707$$

$$= 35.35 \text{ amp}$$

(iv) $$i = I_m \sin 100\pi t$$

The maximum value will attain when $i = I_m$

$$\sin 100\pi t = 1$$

$$100\pi t = 90°$$

$$t_1 = \frac{90}{100 \times 180} = 0.005 \text{ sec}$$

⇒ After a maximum value the zero value will come after 1st half cycle.

$$t_2 = \frac{T}{2} \text{ sec} = \frac{1}{2f} = \frac{1}{2 \times 50} = 0.01 \text{ sec}$$

⇒ So the time interval between a maximum value and the next zero value is 0.05 sec to 0.01 sec.

Example 21: The voltage across and current through a circuit are given by $V = 250 \sin(314t - 10°)$ volt and $i = 10 \sin(314t + 50°)$ A. Calculate, the impedance, resistance, reactance and power factor of the circuit.

Solution: given $V = 250 \sin(314t - 10°)$ volt

$i = 10 \sin(314t + 50°)$ amp

above voltage are in time domain we can write in polar form

$$V = \frac{250}{\sqrt{2}} \angle -10°$$

$$I = \frac{10}{\sqrt{2}} \angle 50°$$

$\Rightarrow$ Impedance of ckt $= Z = \dfrac{V}{I}$

$$Z = \frac{\frac{250}{\sqrt{2}} \angle -10°}{\frac{10}{\sqrt{2}} \angle -50°}$$

$$\boxed{Z = 25 \angle -60°}$$

From this it is clear that current leads the voltage by 60°.

So power factor = cos 60° = 0.5

$\Rightarrow$ $Z = R - jX_C = 25[\cos 60 + j \sin(-60)]$

$$R - jX_C = 12.5 - j21.65$$

Comparing real and imaginary part

$R = 12.5\ \Omega,\ X_C = 21.65\ \Omega$ **Ans.**

Example 22:

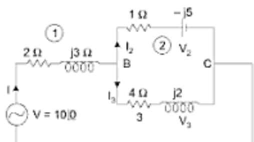


Fig. 1.52

Find (a) Total impedance

(b) Current drawn from supply

(c) Voltage across each branch (1), (2), (3)

(d) Current in each branch

(e) Power factor

(f) Apparent, active and reactive power

(g) Draw the phasor diagram.

Solution: Let
$$Z_1 = 2 + j3 = \sqrt{13}\ \angle \tan^{-1}\frac{3}{2}$$

$$Z_2 = 1 - j5 = \sqrt{26}\ \angle \tan^{-1}\frac{-5}{1}$$

$$Z_3 = 4 + j2 = \sqrt{20}\ \angle \tan^{-1}\frac{1}{2}$$

(a) Total impedance $Z = Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$$Z = (2 + j3) + \frac{(1 - j5)(4 + j2)}{(1 - j5) + (4 + j2)}$$

$$= (2 + j3) + (3.65 - j1.41)$$

$$Z = 5.65 + j1.59 = 5.87\ \angle 15.7^\circ \text{ ohm}$$

(b)
$$I = \frac{V}{Z} = \frac{10}{5.87\ \angle 15.7} = 1.70\ \angle -15.7^\circ \text{ amp}$$

means total current I lags the applied voltage $V = 10\ \angle 0$ by **15.7°**

(c)
$$V_{BC} = I.Z_{BC} \qquad \left\{ Z_{BC} = \frac{Z_3 Z_2}{Z_2 + Z_3} \right\}$$

$$V_{BC} = 1.70\ \angle -15.7\ (3.65 - j1.41)$$

$$= 5.32 - j4 = 6.65\ \angle -36.8^\circ$$

⇒ This indicates voltage across BC lags the current I by $(36.8 - 15.7^\circ) = 21.1^\circ$.

(d) $$\sqrt{2} = \frac{V_{BC}}{Z_2} = \frac{6.65 \angle -36.8^\circ}{1 - j5} = 1.30 \angle 41.9^\circ$$

current I_2 leads the V_{BC} by **(41.9° + 36.8°)**

$$I_m = \frac{V_{BC}}{Z_3} = \frac{6.65 \angle -36.8^\circ}{4 + j2} = 1.49 \angle -63.4^\circ$$

(e) Power factor $\cos\phi = \cos(15.7) = 0.963$ lagging

or, $$\cos\phi = \frac{R}{Z} = \frac{5.65}{5.87} = 0.963$$

(f) Apparent power $s = VI$

$= 10 \times 1.70 = 17.0$ VA

True power $= I^2R = 1.7 \times 1.7 \times 5.65 = 16.32$ W.

$= VI\cos\theta = 10 \times 1.7 \times 0.963 = 16.34$ W.

Reactive power $= I^2X = 1.7 \times 1.7 \times 1.59 = 4.59$ vars

$= VI\sin\theta = 10 \times 1.7 \times \sin(15.7^\circ) = 4.6$ vars.

(g) Phasor diagram

Let $V = 10 \angle 0$ is a reference.

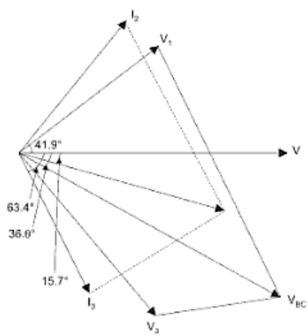


Fig. 1.53

Example 23: An alternating current of frequency 60 Hz, has a maximum value of 120 A. Write down the equation for its instantaneous value. Reknocking time from the instant the current is zero and is becoming positive Find

(a) The instantaneous value after 1/360 second and
(b) The time taken to reach 96 A for the first time.

Solution:

The instantaneous current equation is

$\therefore \quad i = 120 \sin 2\pi ft = 120 \sin 120\,\pi t.$

Now, when $t = 1/360$ second, then

(a) $i = 120 \sin (120 \times \pi \times 1/360)$ ... angle in radians

$= 120 \sin (120 \times 180 \times 1/360)$ ... angle in degree

$= 120 \sin 60° = 103.9$ A.

(b) $96 = 120 \times \sin 2 \times 180 \times 60 \times t$ angle in degree

or, $\sin (360 \times 60 \times t) = 96/120 = 0.8$

$\therefore \quad 360 \times 60 \times t = \sin^{-1} 0.8 = 53°$ (approx.)

$\therefore \quad t = 0/2\pi f = 53/360 \times 60 = 0.00245$ second.

Example 24: An alternating current varying sinusoidally with a frequency of 50 Hz, has an RMS value of 20A. Write down the equation for the instantaneous value and find this value.

(a) 0.0025 second (b) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A? [Elect. Sc. I Allah. Univ. 1992]

Solution:

$$I_m = 20\sqrt{2} = 28.2 \text{ A}, \; W = 2\pi \times 50 = 100\pi \text{ red/sec.}$$

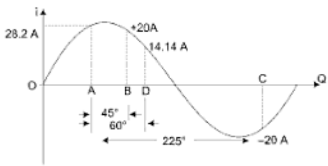


Fig. 1.54

The equation of the sinusoidal current wave with reference to point O Fig. (1.54) as Zero time point as $i = 28.2 \sin 100\pi t$ ampere.

Since, time values are given from point A where voltage has positive and max. value, the equation may itself be referred to point A. In the case, the equation becomes:

$$i = 28.2 \cos 100\pi t$$

(i) When, $t = 0.0025$ second

$$i = 28.2 \cos 100\pi \times 0.0025 \text{ angle in rad.}$$
$$= 28.2 \cos 100 \times 180 \times 0.0025 \text{ angle in degree}$$
$$= 28.2 \cos 45° = 20\,A \text{ ... Point } B.$$

(ii) When, $t = 0.0125$ second.

$$i = 28.2 \cos 100 \times 180 \times 0.0125$$
$$= 28.2 \cos 2250 = 28.2\,(-1/\sqrt{2}\,)$$
$$= -20\,A \text{ ... point } C.$$

(iii) Here, $i = 14.14\,A$.

$$\therefore \quad 14.14 = 28.2 \cos 100 \times 180\,t \quad \therefore \cos 100 \times 180\,t = \frac{1}{2}$$

or $100 \times 180\,t = \cos^{-1}(0.5) = 60°$, $t = 1/300$ sec. ... point D.

Example 25: For the trapezoidal current waveform of Fig. 1.55 determine the effective value. [Elect. Tech. Vikram. Univ. Ujjain. Nagpur Univ. 1999]

Solution: For $O < t < 3T/20$, equation of the current can be found from the relation.

$$\frac{i}{t} = \frac{I_m}{3T/20} \text{ or } i = \frac{20I_m}{3T}.t \qquad \{\text{From } O \text{ to } A \text{ is equation at } Y = mx\}$$

$$\left\{ i = \left(\frac{I_m}{3T/20}\right).t \right.$$

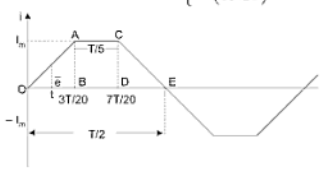


Fig. 1.55

When $3T/20 < t < 7T/20$, equation of the current is given by $i = I_m$ keeping in mind the fact that $\Delta\, OAB$ is identical with $\Delta\, CDE$.

$$\text{RMS value of current} = \sqrt{\frac{1}{T/2}\left[2\int_0^{3T/20} i^2\, dt + \int_{3T/20}^{7T/20} I_m^2\, dt\right]}$$

$$= \sqrt{\frac{2}{T}\left[2\left(\frac{20I_m}{3T}\right)^2 \int_0^{3T/20} t^2\, dt + I_m^2 \int_{3T/20}^{7T/20} dt\right]}$$

$$= \frac{3}{5}\, I_m^2,\ \ I = \sqrt{3/5}\, I_m = 0.775\, I_m.$$

Incidently, the average value is given by

$$I_{ac} = \frac{2}{T}\left\{2\int_0^{3T/20} i dt + \int_{3T/20}^{7T/20} I_m dt\right\} = \frac{2}{T}\left\{2\int_0^{3T/20}\left(\frac{20I_m}{3T}\right) + dt + I_m \int_{3T/20}^{7T/20} dt\right\}$$

$$= \frac{2}{T}\left\{2\left(\frac{20I_m}{3T}\right)\left|\frac{t^2}{2}\right|_0^{3T/20} + I_m\, |t\,|_{3t/20}^{7T/20}\right\}$$

$$= \frac{7}{10}\, I_m.$$

Example 26: What is the significance of the RMS and average values of a wave? Determine the RMS and average values of the waveform shown in Fig. 1.56. (Elect. Tech. Indore Univ.)

Solution: The slope of the curve AB is $BC/AC = 20/T$. Next, consider the function y at any time t. It is seen that $DE/AE = BC/AC = 10T$

or, $(y - 10)/t = 10/T$

or, $y = 10 + (10/T)t$

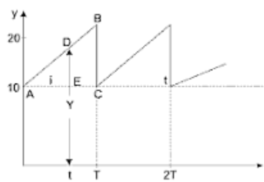


Fig. 1.56

This gives us the equation for the function for one cycle.

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right)dt$$

$$= \frac{1}{T}\int_0^T \left[10dt + \frac{10}{T}t.dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = 15$$

Mean square value $= \frac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$

$$= \frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt$$

$$= \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

or, RMS value $= 10\sqrt{7/3} = 15.2$

Example 27: The half cycle of an alternating signal is as follows it increases uniformly from zero at $O°$ to f_m at $\propto°$, remains constant from $\propto°$ to $(180 - \propto)°$ decreases uniformly from f_m at $(180 - \propto)°$ to zero at 180°. Calculate the average and effective values of the signal.

[Elect. Science – I Allah. Univ. 1992]

Solution: For finding the average value, we would find the total area of the trapezium and divide it by π (Fig. 1.57)

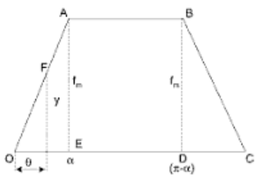


Fig. 1.57

$$\text{Area} = 2 \times \Delta OAE + \text{rectangle } ABDE$$
$$= 2 \times (1/2) \times f_m \times \propto + (\pi - 2\propto)f_m$$
$$= (\pi - \propto)f_m$$

$\therefore$ Average value $= (\pi - \propto)f_m/\pi$

RMS value from similar triangles, we get $\frac{y}{\theta} = \frac{F_m}{\propto}$ or $y^2 = \frac{F_m^2}{\propto^2}\ \theta^2$.

This gives the equation of the signal over the two triangles *OAE* and *DBC*. the signal remains constant over the angle $\propto$ to $(\pi - \propto)$ i.e., over an angular distance of $(\pi - \propto) - \propto = (\pi - 2\propto)$

$$\text{Sum of the squares} = \frac{2F_m^2}{\propto^2}\int_{\theta}^{\propto} \theta^2 d\theta + f_m^2(\pi - 2\propto) = f_m^2\left(\pi - \frac{4\alpha}{3}\right)$$

$$\text{The mean value of the squares is} = \frac{1}{\pi}f_m^2\left(\pi - \frac{4\alpha}{3}\right) = f_m^2\left(1 - \frac{4\alpha}{3\pi}\right)$$

$$\text{RMS value} = f_m\sqrt{\left(1 - \frac{4\alpha}{3\pi}\right)}.$$

Example 28: Find the average value, effective value and form factor of a symmetrical alternating current wave, whose half cycle is shown in Fig. 1.58. The current rises from 0 to 10 A in one-third period, it remains constant for the middle one-third period and again falls to zero at a uniform rate in the last one-third period of half cycle.

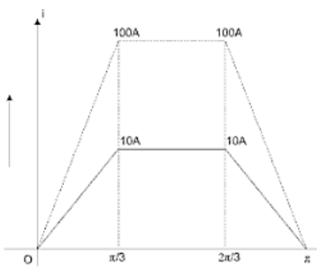


Fig. 1.58

Solution: Average value of current

$$I_{av} = \frac{\text{area of trapezuim}}{\text{interval}}$$

$$= \frac{\frac{1}{2}\times 10\times\frac{\pi}{3}+10\times\frac{\pi}{3}+\frac{1}{2}\times 10\times\frac{\pi}{3}}{\pi}$$

= 6.67 A Ans.

Draw the dotted trapezium by plotting square values of current, then effective value of current,

$$I = \sqrt{\frac{\text{area of the dotted trapezium}}{\text{interval}}}$$

$$= \sqrt{\frac{\frac{1}{2}\times 100\times\frac{\pi}{3}+100\times\frac{\pi}{3}+\frac{1}{2}\times 100\times\frac{\pi}{3}}{\pi}}$$

= 8.15 A Ans.

$\therefore$ Form factor $= \frac{8.15}{6.67}$

= 1.222 Ans.

Example 29: Determine average value, effective value and form factor of a sinusoidally varying alternating current whose half wave is rectified in each cycle.

Solution: Average value of current is given by,

$$I_{av} = \frac{\text{area of reactified wave}}{\text{interval}}$$

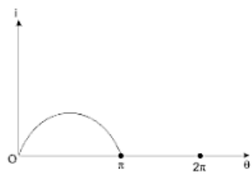


Fig. 1.59

$$= \frac{\int_0^{\pi} i\,d\theta}{2\pi}$$

$$= \frac{\int_0^{\pi} I_m \sin\theta\,d\theta}{2\pi} = \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi} \qquad ...(1.24)$$

Effective value of current,

$$I = \sqrt{\frac{\int_0^{\pi} i^2\,d\theta}{2\pi}} = \sqrt{\frac{\int_0^{\pi} I_{m^2} \sin^2\theta\,d\theta}{2\pi}}$$

$$= \frac{I_m}{\sqrt{2\pi}} \sqrt{\int_0^{\pi} \frac{(1-\cos 2\theta)}{2}\,d\theta}$$

$$= \frac{I_m}{\sqrt{2\pi}} \sqrt{\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{I_m}{\sqrt{2\pi}} = \sqrt{\frac{\pi}{2}} = \frac{I_m}{2}$$

$\therefore$ Form factor $= \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2}$ **= 1.57 Ans.**

Example 30: Three coils of resistances 20, 30 and 40 Ω and inductance 0.5, 0.3 and 0.2 H, respectively are connected in series across a 230 V, 50 c/s supply. Calculate the total current, power factor and the power consumed in the circuit.

Solution:

Total resistance $R = 20 + 30 + 40 = 90\ \Omega$

Total resistance $L = 0.5 + 0.3 + 0.2 = 1.0\ \Omega$

$\therefore$ $X_L = 2\pi fL = 2\pi \times 50 \times 1.0 = 314\ \Omega$

Impedance $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{90^2 + 314^2} = 327\ \Omega$$

$\therefore$ $$\text{Current } I = \frac{E}{Z} = \frac{230}{327} = \mathbf{0.704\ A.}$$

$$\text{Power factor } \cos\phi = \frac{R}{Z} = \frac{90}{327}.$$

$$= \mathbf{0.275\ lagging.}$$

$$\text{Power consumed} = EI\cos\phi.$$

$$= 230 \times 0.704 \times 0.275.$$

$$= \mathbf{44.5\ watts.}$$

Example 31: A resistance of 100 Ω and a capacitance of 40 μF are connected in series across a 400 V supply of 50 c/s. Find the current, power factor and the power consumed in the circuit. Draw the vector diagram.

Solution:

$$R = 100\ \Omega$$

$$X_C = \frac{1}{2\pi f.c.} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}} = 79.5\ \Omega$$

$$\text{Impedance } Z = \sqrt{100^2 + 79.5^2} = 127.8\ \Omega$$

$\therefore$ $$\text{Current} = \frac{400}{127.8} = \mathbf{3.13\ A.}$$

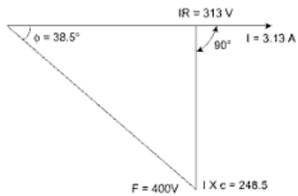


Fig. 1.60

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

$$= \frac{100}{127.8} = \mathbf{0.783\ leading.}$$

Power consumed $= EI\cos\phi.$

$= 400 \times 3.13 \times 0.783.$

= 980 watts.

Now, $IR = 3.13 \times 100 = 313$ V

$IX_C = 313 \times 79.5 = 248.5$ V

Take I as the reference vector. Draw $IR = 313$ V to a suitable scale in phase with I as shown in phase with I as shown in Fig. 1.60 Draw $IX_C = 248.5$ V, 90° lagging with respect to current vector. The vector sum of IR and IX will represent the applied voltage $E = 400$ V. On measurement, the angle ϕ between I and E is found 398.5°. Thus, the complete vector diagram is shown in Fig. 1.60.

Example 32: A series circuit consisting of a resistance of 100 Ω, inductance of 0.2 H and a capacitance of 20 μF is connected across a 240 V, 50 c/s supply. Determine:

(a) Total impedance.

(b) Total current.

(c) Voltage across each component.

(d) Power factor

(e) Power consumption in the circuit

(f) Frequency at which resonance will occur.

Draw the complete vector diagram.

Solution:

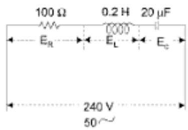


Fig. 1.61

(a) $R = 100\ \Omega$

$X_L = 2\pi fL = 2\pi \times 50 \times 0.2$

$= 62.8\ \Omega.$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}}$$

$= 159\ \Omega.$

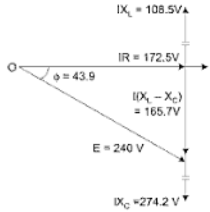


Fig. 1.62

$$\text{Net reactance } X = X_L - X_C.$$
$$= 62.8 - 159.$$
$$= -96.2\ \Omega$$
$$\text{Impedance } Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{100^2 + (-96.2)^2}$$
$$= \mathbf{139\ \Omega.} \quad \textbf{Ans.}$$

$$b \text{ total current } I = \frac{240}{139} = \mathbf{1.725\ A} \quad \textbf{Ans.}$$

(c) Voltage across resistance,

$$E_R = 1.725 \times 100 = 172.5 \text{ V}$$

Voltage across inductance,

$$E_L = 1.725 \times 62.8 = 108.5 \text{ V}.$$

Voltage across capacitance,

$$E_C = 1.725 \times 159 = \mathbf{274.2\ V.} \quad \textbf{Ans.}$$

(d) Power factor $\cos\phi = \frac{R}{Z} = \frac{100}{139}$ **= 0.72 leading Ans.**

The power factor is leading as capacitive reactance in greater than the inductive reactance in the circuit.

(e) Power consumption in the circuit,

$$= EI\cos\phi$$

$$= 240 \times 1.725 \times 0.72$$

$$\mathbf{= 298\ watts.}\quad \textbf{Ans.}$$

(f) Resonance will occur, when

$$X_L = X_C$$

or, $$2\pi fL = \frac{1}{2\pi f.c.}$$

$\therefore$ $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{.2 \times 20 \times 10^{-6}}} = \mathbf{39.8\ c/s.}\quad \textbf{Ans.}$$

Example 33: A circuit consisting of resistance of 10 Ω in series with an X_L = 15 Ω is connected in parallel with another circuit consisting of resistance of 12 Ω and capacitive reactance of 20 Ω combination is connected across a 230 V, 50 Hz supply. Find (a) Total current taken from supply (b) Power factor of circuit.

Solution:

(a) The given circuit is shown in Fig. 1.63.

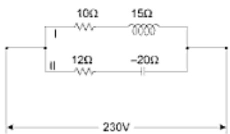


Fig. 1.63

Branch I:

$$\text{Conductance } g_1 = \frac{R_1}{Z_1^2}$$

$$= \frac{10}{10^2 + 15^2} = 0.0307\ \Omega$$

$$\text{Susceptance } b_1 = \frac{-15}{10^2 + 15^2} = -0.0461 \text{ ℧}$$

Branch II:

$$\text{Conductance } g_2 = \frac{R_2}{Z_2^2} = \frac{12}{12^2 + 20^2} = 0.022 \text{ ℧}$$

Since, branch II has capacitive susceptance, so it will be assigned –ve sign.

$$\therefore \quad \text{Susceptance } b_2 = \frac{+20}{12^2 + 20^2} = +0.0368 \text{ ℧}$$

Combined circuit:

$$\text{Total conductance } g = 0.0307 + 0.022 = 0.0527 \text{ ℧}$$

$$\text{Total conductance } b = b_2 - b_1 = 0.0368 - 0.0461 = -0.0093 \text{ ℧}$$

$$\therefore \quad \text{Total admittance } Y = \sqrt{g^2 + b^2}$$

$$= \sqrt{0.0527^2 + 0.0093^2}$$

$$= 0.0535 \ \Omega$$

∴ Current taken from supply,

$$I = E.Y = 230 \times 0.0535 = \mathbf{12.3\ A} \quad \textbf{Ans.}$$

(b) Power factor

$$\cos\phi = \frac{g}{Y}$$

$$= \frac{0.0527}{0.0535} = \mathbf{0.985\ lagging} \quad \textbf{Ans.}$$

Example 34: In a parallel circuit, branch I consists of a resistance of 20 Ω in series with an inductive reactance of 15 Ω and branch II has a perfect condenser of 50 Ω reactance. The combination is connected across 200 V, 60 c/s supply. Calculate:

(a) Current taken by each branch.

(b) Total current taken.

(c) P./F. of the combination.

Draw vector diagram.

Solution:

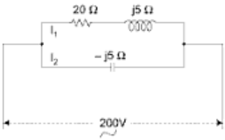


Fig. 1.64

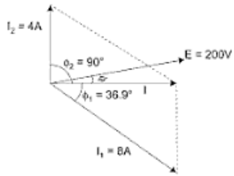


Fig. 1.65

(a) **Branch I:**

$$Z_1 = \sqrt{20^2 + 15^2} = 25\ \Omega$$

$\therefore$
$$I_1 = \frac{200}{25} = 8\text{ A}$$

$$\phi_1 = \tan^{-1}\frac{15}{20} = 36.9° \text{ lagging.}$$

Branch II:

$$Z_a = \sqrt{0 + (-50)^2} = 50\ \Omega.$$

$\therefore$
$$I_2 = \frac{200}{50} = 4\text{ A.}$$

$$\phi_2 = \tan^{-1}\frac{50}{0} = \tan^{-1}\infty$$
$$= 90° \text{ leading.}$$

(b) **Combined circuit:**

Total current I is the vector sum of the two branch currents I_1 and I_2.

Resolving the currents along E (i.e., in their active components).

$$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$
$$= 8\cos 36.9° + 4\cos 90°$$
$$= 8 \times 0.8 = 6.4\text{ A.}$$

Resolving the currents perpendicular to E (i.e., in their reactive components).

$$I \sin \phi = I_2 \sin \phi_2 - I_1 \sin \phi_1.$$
$$= 4 \sin 90° - 8 \sin 36.9°.$$
$$= 4 - 8 \times 0.6 = -0.8 \text{ A}.$$

$\therefore$ $$I = \sqrt{6.4^2 + (-0.8)^2} = \mathbf{6.45\ A.} \quad \textbf{Ans.}$$

(c) Power factor of the combination,

$$\cos \phi = \frac{I \cos \phi}{I} = \frac{6.4}{6.45} = \mathbf{0.991\ lagging.} \quad \textbf{Ans.}$$

Example 35: Solve Ex. 33 by j-method.

Solution:

$$\frac{1}{z} = \frac{1}{10 + j15} + \frac{1}{12 - j20}$$

$$= \frac{(12 - j20) + (10 + j15)}{(10 + j15)(12 - j20)}$$

$\therefore$ $$Z = \frac{(120 + 300) + j(180 - 200)}{22 - j5}$$

$$= \frac{(420 - j20)}{(22 - j5)} \cdot \frac{22 + j5}{22 + j5}$$

$$= \frac{(420 \times 22 + 20 \times 5) + j(420 \times 5 - 20 \times 22)}{22^2 + 5^2}$$

$$= \frac{9340 + j1660}{509} = 18.35 + j\,3.26$$

$$= \sqrt{18.35^2 = 3.26^2} = 18.65.$$

(a) $\therefore$ $$I = \frac{230}{18.65} = 12.3 \text{ A}.$$

(b) $$\cos \phi = \frac{18.35}{18.65} = \mathbf{0.985\ lagging.} \quad \textbf{Ans.}$$

Series-parallel circuits:

Consider a series parallel circuit as shown in Fig. 1.66.

In this case the parallel circuit is first reduced to its equivalent series circuit and then combined with the rest of the circuit.

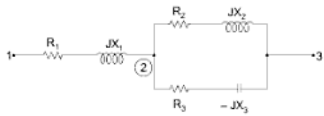


Fig. 1.66

Example 36: A series parallel circuit is shown in Fig. 1.67. An alternating voltage of 200 V, 50 c/s is applied across the terminals 1 and 3. Calculate:

(a) Symbolic expression of the total impedance of the circuit
(b) Brach currents and total current.
(c) P.F. of the circuit.
(d) Voltages across the series branch and the parallel combination.
(e) Total power absorbed in the circuit.

Draw the complete vector diagram.

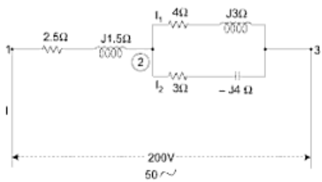


Fig. 1.67

$$\text{Given, } \dot{Z}_1 = 2.5 + j\,1.5$$

$$\dot{Z}_2 = 4 + j\,3$$

$$\dot{Z}_3 = 3 - j\,4$$

equivalent impedance of the parallel circuit,

$$\frac{1}{\dot{Z}_{23}} = \frac{1}{\dot{Z}_2} + \frac{1}{\dot{Z}_3}$$

$$= \frac{1}{4+j3} + \frac{1}{3-j4} = \frac{(3-j4)+(4+j3)}{(4+j3)(3-j4)}$$

$$= \frac{7-j1}{24-j7}$$

$$= \frac{24-j7}{7-j1}$$

$$= \frac{(24-j7)}{7-j1} \times \frac{7+j1}{7+j1}$$

$$= \frac{175-j25}{7^2+1^2} = \frac{175-j25}{50}$$

$= 3.5 - j\,0.5.$

Symbolic expression of the total impedance,

$$\dot{Z}_{13} = \dot{Z}_1 + \dot{Z}_{23}$$
$$= (2.5 + j\,1.5) + (3.5 - j\,0.5)$$
$$\mathbf{= 6 + j\,1.} \quad \textbf{Ans.}$$

Taking the voltage as reference vector,

$$= \dot{E} = 200 + j\,0.$$

Total current $\dot{I} = \frac{\dot{E}}{\dot{Z}_{13}} - \frac{200+j0}{6+j1}$

$$= 200\,(6 - j\,1) = 32.4 - 5.4 = 6^2 + 1$$

$\therefore$ $I = \sqrt{32.4^2 + 5.4^2}$ **= 328 A Ans.**

and, $\phi = \tan^{-1}\left(-\frac{5.4}{32.4}\right) = \tan^{-1}(-0.1665) = -9.5°$

Voltage across the series branch,

$$\dot{E} = \dot{I}\,.\,\dot{Z}_1 = (32.4 - j\,5.4)\,(2.5 + j\,1.5)$$
$$= 89.1 + j\,35.1$$

$$\phi_{12} = \tan^{-1}\left(\frac{35.1}{89.1}\right) = \tan^{-1} 0.394 = 21.5°$$

$\therefore$
$$\dot{E}_{23} = \dot{E}_{13} - \dot{E}_{12}$$
$$= 200 + j\,0 - (89.1 + j\,35.1)$$
$$= 110.9 - j\,35.1$$

$$\phi_{23} = \tan^{-1}\left(-\frac{35.1}{110.9}\right) = \tan^{-1}(-0.317) = -17.6°$$

$\therefore$
$$E_{23} = \sqrt{110.9^2 + 35.1^2}$$
$$= 116 \text{ V.}$$

Current in upper parallel branch,

$$\dot{I}_1 = \frac{\dot{E}_{23}}{\dot{Z}_2} = \frac{110.9 - j35.1}{4 + j3}$$

$$= \frac{(110.9 - j35.1)\,(4 - j3)}{4^2 + 3^2} = \frac{338.3 - j473.1}{25}$$

$$= 13.55 - j\,18.9$$

$\therefore$
$$I_1 = \sqrt{13.5^2 + 18.9^2} = \mathbf{23.2\ A} \quad \textbf{Ans.}$$

and,
$$\phi_1 = \tan^{-1}\left(-\frac{18.9}{13.55}\right) = \tan^{-1}(-1.395) = -54.4°.$$

Current in lower parallel branch,

$$I_2 = \frac{\dot{E}_{23}}{\dot{Z}_3} = \frac{110.9 - j35.1}{3 - j4} \times \frac{3 + j4}{3 + j4}$$

$$= \frac{473.1 + j338.3}{25} = 18.9 + j\,13.55.$$

$\therefore$ $$I_2 = \sqrt{18.9^2 + 13.55^2} = \mathbf{23.2\ A.}\quad \textbf{Ans.}$$

$$\phi_2 = \tan^{-1}\frac{13.55}{18.9} = \tan^{-1}0.716 = 35.6°$$

(c) Power factor of the complete circuit

$$= \cos\phi = \cos(9.5) = \mathbf{0.986\ lagging.}\quad \textbf{Ans.}$$

(d) Voltages across the series branch,

$$= E_{12} = \sqrt{89.1^2 + 35.1^2}$$

$$= \mathbf{95.6\ V.}\quad \textbf{Ans.}$$

Voltage across the parallel combination,

$$E_{23} = \mathbf{116\ V.}\quad \textbf{Ans.}$$

(e) Total power absorbed in the circuit

$$P = EI\cos\phi$$

$$= 200 \times 32.8 \times 0.986.$$

$$= \mathbf{6480\ watts.}\quad \textbf{Ans.}$$

Total power can also be found by vector product of voltage and conjugate of current. Real part will give active power and imaginary reactive power.

$\therefore$ $$P = (200 + j\,0)\,(32.4 + j\,5.4)$$

$$= 6480 + j\,1080.$$

$\therefore$ $$P = \mathbf{6480\ watts.}$$

Vector diagram: The complete vector diagram is shown in Fig. 1.68 and is explained in the following points:

(i) Draw OA representing the applied voltage $E = 200$ V to a suitable scale.

(ii) Assume OA as reference vector.

(iii) Draw OC representing $I_1 = 23.2$ A lagging at an angle of $\phi_1 = -54.4°$.

(iv) Draw OF representing $I_2 = 23.2$ A leading at angle of $\phi_2 = 35.6°$.

(v) Vector sum of OG and OF is OH equal to the resultant current $I = 32.8$ A lagging at an angle $\phi = -9.5°$.

(vi) Voltage drop in series resistance of 2.5 Ω is equal to $OK = 32.8 \times 2.5 = 82$ V in phase with I. Voltage across series reactance of 1.5 Ω is equal to $KB = 32.8 \times 1.5 = 49.2$ V. The resultant OK and KB is equal to OB. Then $OB = E_{12} = 95.6$ V at an angle of $\phi_{12} = 21.5°$ from the reference vector OA.

(vii) Voltage drop in resistance of upper parallel branch is $OE = 23.2 \times 4 = 92.8$ V in phase with I_1. Voltage across 3 Ω reactance is $EC = 23.2 \times 3 = 69.6$ V, 90° ahead of I_1. Resultant of OE and EC is $OC = E_{23} = 116$ V at an angle of $\phi_{23} = -17.6°$ from the reference vector.

(viii) Voltage drop in resistance of lower parallel branch is $OD = 23.2 \times 3 = 69.6$ V in phase with I_2. Voltage across 4 Ω reactance is $DC = 23.2 \times 4 = 92.8$ V, 90° lagging with respect to I_2. Resultant of OD and DC is OC.

(ix) The vector sum OC and OB is equal to $OA = E = 200$ V.

Many problems can also be solved vectorially.

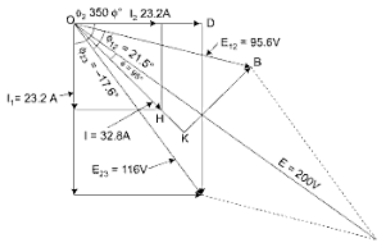


Fig. 1.68

Example 37: A sinusoidal alternating voltage of 110 V is applied across a moving coil ammeter, a hot-wire ammeter and a half-wave rectifier, all connected in series. The rectifier offers a resistance of 25 Ʊ in one direction and infinite resistance in opposite direction. Calculate (i) the readings on ammeters (ii) the form factor and peak factor of the current wave.

[Elect. Engg-I, Nagpur Univ. 1992]

Solution: For solving this question, it should be noted that

(a) Moving-coil ammeter, due to the inertia of its moving system, registers the average current for the whole cycle.

(b) The reading of hot-wire ammeter is proportional to the average heating effect over the whole cycle. It should further be noted that in AC circuits, the given voltage and current values, unless indicated otherwise, always refer to RMS values.

$$E_m = 110/0.707 = 155.5 \text{ V (approx.)}, \; I_m = \frac{155.5}{25} = 6.22 \text{ A}$$

average values of current for +ve half cycle

$$= 0.637 \times 6.22 = 3.96 \text{ A}$$

Value of current in the –ve half cycle in zero but as said earlier due to inertia of the coil M.C. ammeter reads the average value for the whole cycle.

(i) M.C. ammeter reading = 3.96/2 = 1.98 A. Let R be the resistance of hot-wire ammeter. Average heating effect over the positive half cycle is $\frac{1}{2} I^2 . R$ watts. But as there is no generation of heat in the negative half cycle. the average heating effect over the whole cycle is $\frac{1}{4} I_m^2 R$ watt.

Let I be the DC current which produces the same heating effect, then

$$I^2 R = \frac{1}{4} I_m^2 R \quad \therefore I = I_m/2 = 6.22/2 = 3.11 \text{ A}$$

Hence, hot-wire ammeter will read 3.11 A

(ii) $$\text{Form factor} = \frac{\text{RMS value}}{\text{average value}} = \frac{3.11}{1.98} = 1.57$$

$$\text{Peak factor} = \frac{\text{max value}}{\text{RMS value}} = \frac{6.22}{3.11} = 2.$$

Example 38: A voltage $V = 100 \sin (314\, t + 5°)\, V$ is applied to a circuit. The resulting current is $i = 5 \sin (314\, t - 40°)\, A$. Determine power factor and the average power delivered to the circuit.

Solution:

The phase difference between current and voltage phasors is given by

$$\phi = \phi_i - \phi_v$$
$$= -40° - 5 = -45°$$

Therefore, the power factor

$$\cos \phi = \cos (-45°) = 0.707 \text{ (lagging)}$$

The power factor is lagging because the current lags behind the voltage. Power delivered to the circuit

$$P = VI \cos \phi$$
$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos \phi$$

$$= \frac{100}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos 45° = 176.78 \text{ W}$$

Example 39: A 100 V, 60 W lamp is to be operated on a 250 V 50 Hz supply. Calculate the value of (a) non-inductive resistor, (b) pure inductance, lamp in order that it may be used at its rated voltage. What would be required to place in series with the lamp in order that it may be used as its rated voltage.

Solution:

Current taken by the lamp

$$I = \frac{P}{V_1} = \frac{60}{100} = 0.6 \text{ A}$$

If R_1 is the resistance of the lamp,

$$P = I^2 R_1$$

$$R_1 = \frac{P}{I^2} = \frac{60}{(0.6)^2} = 166.66\ \Omega$$

(a) Non-inductive resistor R

When a non-inductive resistance R is placed in series with the lamp, the total resistance of the circuit becomes R_T (say), where

$$R_T = R_1 + R = 166.66 + R$$

Since, the circuit is purely resistive

$$V = R_T I$$

$$250 = (166.66 + R) \times 0.6$$

$$R = \frac{250}{0.6} - 166.66 = 250\ \Omega$$

(b) Pure inductance L

When a pure inductance L placed in series with the lamp the total impedance of the circuit is given by

$$Z_2 = R_1 + jX_L = 166.66 + j\, 2\pi \times 50L$$

By Ohm's law

$$V = Z_2 I$$

$$250 = (166.66 + j\, 2\pi \times 50L) \times 0.6$$

$$\frac{250}{0.6} = \sqrt{(166.66)^2 + (2\pi \times 50L)^2}$$

$$\left(\frac{250}{0.6}\right)^2 - (166.66)^2 = (2\pi \times 50L)^2$$

$$\sqrt{(173611 - 27775)} = 2\pi \times 50L$$

$$L = \mathbf{1.2155\ H}$$

Example 40: Three voltages represented by

$$e_1 = 20 \sin wt;\ e_2 = 30 \sin (wt - \pi/4)$$

$$e_3 = 40 \cos (wt + \pi/6)$$

act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors. [Electro, Technic Madras Univ. 1991]

Solution:

First let us draw the three vectors representing the max. value of the given alternating voltages.

$e_1 = 20$ sin of wt – here phase angle with X axis is zero, hence the vector will be drawn parallel to the X-axis

$e_2 = 30 \sin (wt - \pi/4)$. Its vector will be below OX by 45°

$e_3 = 40 \cos (wt + \pi/6) = 40 \sin (90° + wt + \pi/6)$

$= 40 \sin (wt + 120°)$. Its vector will be 120° with respect to OX in counter clockwise direct.

These vectors are shown in Fig. 1.69(a) Resolving them into X-and Y components, we get

$$X\text{-component} = 20 + 30 \cos 45° - 40 \cos 60° = 21.2 \text{ V.}$$

$$Y\text{-component} = 40 \sin 60° - 30 \sin 45° = 13.4 \text{ V.}$$

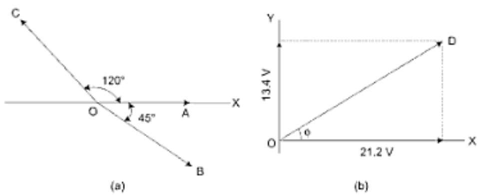


Fig. 1.69

As seen from Fig. (b). The max. value of the resultant voltage as $OD = \sqrt{21.2^2 + 13.4^2} = 25.1$ V

The phase angle of the resultant voltage $\tan\phi = \dfrac{\text{Y component}}{\text{X component}}$

$$= \frac{13.4}{27.2} = 0.632$$

$$\therefore \qquad \phi = \tan^{-1} 0.632 = 32.3°$$

$$= 0.564 \text{ radian}$$

The equation $e = 25.1 \sin(wt + 0.564)$

Example 41: Two currents i_1 and i_2 are given by the expressions $i_1 = 10 \sin (314t + \pi/4)$ Amp and $i_2 = 8 \sin (314t - \pi/3)$ amp Find (a) $i_1 + i_2$ and (b) $i_1 - i_2$ express the answer in the form $i = I_m \sin (314t \pm \phi)$.

Solution:

(a) The current vectors representing maximum value of the two currents are shown in Fig. (a). Resolving the current into their X-and Y components, we get

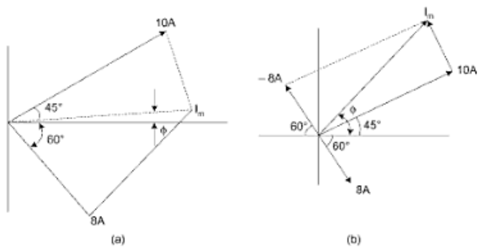


Fig. 1.70

$$X \text{ components} = 10 \cos 45° + 8 \cos 60° = 10/\sqrt{2} + 8/2 = 11.07 \text{ A}$$

$$Y \text{ component} = 10 \sin 45° - 8 \sin 60° = 0.14 \text{ A.}$$

$$\therefore \qquad I_m = \sqrt{(11.07)^2 + (0.14)^2} = 11.08 \text{ A.}$$

$$\tan\phi = (0.14/11.07) = 0.01265 \quad \therefore \phi = 44°$$

Hence, the equation for the resultant current is $i = 11.08 \sin (314t + 44°)$ amperes.

(b) X components = 10 cos 45° – 8 cos 60 = 3.07 A.

Y component = 10 sin 45° + 8 sin 60 = 14 A.

$$I_m = \sqrt{(3.07)^2 + (14)^2} = 14.33 \text{ A.}$$

$$\phi = \tan^{-1} (14/3.07) = 77° \, 38'$$

Hence, the equation of the resultant current is

$$i = 14.33 \sin (314 + 77° \, 38') \text{ amperes.}$$

Example 42: Three sinusoidally alternating currents of RMS values 5, 7, 5 and 10 A are having same frequency of 50 Hz. with phase angles of 30°, – 60° and 45°.

(i) Find their average values.

(ii) Write equation for their instantaneous values.

(iii) Draw wave forms and phasor diagrams taking first current as the reference.

(iv) Find the instantaneous values at 100 m sec from the original reference.

[Nagpur Univ. Nov. 1996]

Solution:

(i) Average value of alternating quantity in case of sinusoidal nature of variation = (RMS values)/1.11

Average value of 1st current = 5/1.11 = 4.50 A

Average value of 2nd current = 7.5/1.11 = 6.76 A

Average value of 3rd current = 10/1.11 = 9.00 A

(ii) Instantaneous values

$$i_1(t) = 5\sqrt{2} \sin (314t + 30°)$$

$$i_2(t) = 7.5\sqrt{2} \sin (314t - 60°)$$

$$i_3(t) = 10\sqrt{2} \sin (314t + 45°)$$

(iii) First current is to be taken as a reference, none form the expression second current lags, behind the first current by 90°. Third current leads the first current by 15° wave form with this description are drawn in Fig. 1.71 (a) and the phasor diagrams in Fig. 1.71 (b).

(iv) A 50 Hz AC quantity completes a cycle in 20 m sec. In 100 m sec, it completes five cycles original reference is the starting point required for this purpose. Hence, at 100 m sec from the reference.

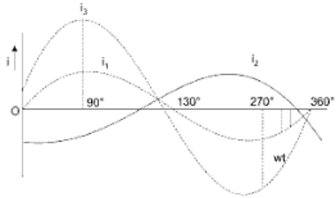


Fig. 1.71(a)

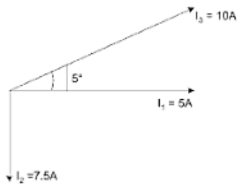


Fig. 1.71(b)

(v) Instantaneous value of $i_1(t) = 5\sqrt{2}\ \sin 30° = 3.53$ A

Instantaneous value of $i_2(t) = 7.5\sqrt{2}\ \sin(-60°) = -9.816$ A

Instantaneous value of $i_3(t) = 10\sqrt{2}\ \sin(45°) = 10$ A

Example 43: A 60 Hz voltage of 230 V effective value is impressed on an inductance of 0.265 H.

(i) Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at $t = 0$.

(ii) Show the voltage and current on a phasor diagram. (iii) Find the maximum energy stored in the inductance.

[Elect. Engg. Bhagalpur, Univ.]

Solution:

$$V_{max.} = \sqrt{2V} = \sqrt{2} \times 230 \text{ V}, \; f = 60 \text{ Hz}$$

$$w = 2\pi f = 2\pi \times 60 = 377 \text{ rad/s}, \; X_L = wL$$
$$X_L = 377 \times 0.265 = 100\ \Omega$$

(i) The time equation for voltage is $V(t) = 230\sqrt{2}\ \sin 377t$

$$I_{max} = V_{max.}/X_L = 230\sqrt{2}/100 = 2.3\sqrt{2},\ \phi = 90° \text{ (lag.)}$$

$\therefore$ Current equation is $i(t) = 2.3\sqrt{2}\ \sin(377t - \pi/2)$ or

$$= 2.3\sqrt{2}\ \cos 377t.$$

(ii) It is shown in Fig. 1.56

(iii)
$$E_{max.} = \frac{1}{2} L I^2_{max}$$
$$= \frac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2$$
$$= \mathbf{1.4\ J.}$$

Example 44: A resultant current wave is made up of two components. 5 A DC component and a 50 Hz AC component. Which is of sinusoidal wave form and which has a maximum value of 5 A.

(i) Draw a sketch of the resultant wave.

(ii) Write an analytical expression for the current wave, reckoning $t = 0$ at a point where the AC component is at zero value and when di/dt is +ve.

(iii) What is the average value of the resultant current over a cycle?

(iv) What is the effective or RMS value after resultant current?

Solution:

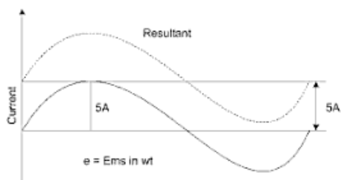


Fig. 1.72

(i) The two current components and resultants current wave have been shown in Fig. 1.72.

(ii) Obviously, the instantaneous value of the resultant current is given by the

$$i = (5 + 5 \sin wt) = (5 + 5 \sin \theta)$$

(iii) Over one complete cycle, the average value of the alternating current is zero. Hence, the average value of the resultant current is equal to the value of DC components i.e., 5 A.

(iv) Mean value of i^2 ovever complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi}\int_0^{2\pi} (5 + 5 \sin \theta)^2 d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (25 + 50 \sin \theta + 25 \sin^2 \theta) d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[25 + 50 \sin \theta + 25\left(\frac{1 - \cos 2\theta}{2}\right)\right] d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (37.5 + 50 \sin \theta - 12.5 \cos 2\theta) d\theta$$

$$= \frac{1}{2\pi}\left|37.5\theta - 50 \cos \theta \frac{12.5}{2} \sin 2\theta\right|_0^{2\pi}$$

$$= \frac{75\pi}{2\pi} = 37.5$$

$\therefore$ RMS value $I = \sqrt{37.5}$ **= 6.12 A.**

Alternate →

Let the effective value of resultant current is I.

Instantaneous current $i = 5 + 5 \sin \omega t$

$$\Rightarrow \quad I^2 R = 5^2 R + \left(\frac{5}{\sqrt{2}}\right)^2 R$$

$$I = \sqrt{5^2 + \frac{25}{2}} = \sqrt{37.5}$$

$$I = 6.12 \text{ amp}$$

Example 44a: If the current in a 20Ω resistor is given by $i = 4 + 5 \sin \omega t - 3 \cos 3\,\omega t$. Determine the power consumed by the resistor.

Solution:

$$P = P_0 + P_1 + P_2$$

$$= 4^2 \times 20 + 20 \times \left(\frac{5}{\sqrt{2}}\right)^2 + 20 \times \left(\frac{3}{\sqrt{2}}\right)^2$$

$$= (16 + 12.5 + 4.5) \times 20 = 660 \text{ watt.}$$

effective value of current = $\sqrt{\frac{660}{20}} = \sqrt{33} = 5.7$ Amp

Example 45: A large coil of inductance 1.405 H and resistance of 40 Ω is connected in series with a capacitor of capacitance 20 μF. Calculate the frequency at which the circuit resonates. If a voltage of 100 V is applied to the circuit at resonant condition, calculate the current drawn from the supply and the voltage across the coil and the capacitor, quality factor, band width.

Solution:

$$R = 40\ \Omega,\ L = 1.405\text{ H},\ C = 20 \times 10^{-6}\ F$$

resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1.405 \times 20 \times 10^{-6}}} = 30$ Hz.

At resonance current $I_0 = \frac{V}{R} = \frac{100}{40} = 2.5$ A

At resonance impedance $Z_0 = R + j\,X_{L_0}$

$$Z_0 = \sqrt{R^2 + X_{L_0}^2} = \sqrt{40^2 + 264.8^2} = 267.8\ \Omega$$

⇒ Voltage across the coil at resonance is X_{L_0}

$$V_{L_0} = I_0\,Z_0 = 2.5 \times 267.8\ \Omega = 669.5 \text{ volt}$$

⇒ Capacitive reactance at resonance is X_{C_0}

$$X_{C_0} = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \times 30 \times 20 \times 10^{-6}} = 265.2\ \Omega$$

⇒ Voltage across the capacitor

$$V_{C_0} = X_{C_0} I_0 = 265.2 \times 2.5 = 663 \text{ V}$$

⇒ Quality factor $Q_0 = \frac{W_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 30 \times 1.405}{40} = \mathbf{6.6175}$ **Ans.**

⇒ Band width = $\frac{R}{L} = \frac{40}{1.405} = \mathbf{28.469}$

Example 46: A current of 120 – *j* 50 flows through a circuit when the applied voltage is 8 + *j* 2, determine (i) impedance (ii) power factor (iii) power consumed and reactive power.

Solution:

$$V = (8 + j\,2)V = 8.25 \angle 14° \text{ V}$$

$$I = (120 - j\,50)A = 130 \angle -22.62° \text{ A}$$

(i) $$Z = \frac{V}{I} = \frac{8.25 \angle 14°}{130 \angle -22.62°} = 0.0635 \angle 36.62° \ \Omega$$

∴ $Z = 0.0635\ \Omega$

(ii) $\phi = 36.62°$ lag.

∴ p.f. $= \cos\phi = \cos 36.62° = 0.803$ lag

(iii) Complex VA, S = Phasor voltage × conjugate of phasor current

or $p + jQ = 8.25 \angle 14° \times 130 \angle 22.62° = 1072.5 \angle 36.62°\ VA$

$= 1072.5\,(\cos 36.62° + j \sin 36.62°)$

$= (860.8 + j\,639.75)VA$

∴ Power consumed, $P = 860.8$ W

Reactive power, $Q = 639.75\ VAR$.

Example 47: In an $R - L$ series circuit $R = 10\ \Omega$ and $X_L = 8.66\ \Omega$ if current in the circuit is $(5 - j\,10)A$, find (i) the applied voltage (ii) power factor and (iii) active power and reactive power.

Solution:

$$Z = R + jX_L = (10 + j\,8.66)\ \Omega = 13.23 \angle 40.9°\ \Omega$$

$$I = (5 - j\,10)A = 11.18 \angle -63.43° \text{ A}$$

(i) $V = IZ = 11.18 \angle -63.43° \times 13.23 \angle 40.9° = 148 \angle -22.53°$ V

∴ $V = 148$ Volts.

(ii) $\phi = 63.43° - 22.53° = 40.9°$

p.f. $= \cos\phi = \cos 40.9° = 0.756$ lag.

(iii) S = phasor voltage × conjugate of phasor current.

or $P + jQ = 148 \angle -22.53° \times 11.18 \angle 63.43° = 1654.64 \angle 40.9°\ VA$.

$= (1250.66 + j\,1083.36)VA$

∴ Active power, $P = 1250.66$ W

Reactive power $Q = 1083.36$ VAR.

Example 48: An inductive coil is connected in parallel with a pure resistor of 30 Ω and this parallel circuit is connected to a 50 Hz supply. The total current taken from the circuit is 8 A while the current in the resistor is 4 A and that in inductive coil in 6 A. Calculate (i) Resistance and inductance of the coil (ii) Power factor of the circuit and (iii) Power taken by the circuit.

Solution: The second branch has a pure resistance ($Z_2 = 30\ \Omega$) so that current I_2 (= 4A) will be in phase with the applied voltage. The first branch has an impedance of Z_1 and current I_1(= 6 A) through it will lag behind the applied voltage by ϕ_1^0 the line current I(= 8A) is the phasor sum of I_1 and I_2 as shown in the phasor diagram in Fig. 1.73.

(i) Supply voltage, $V = I_2Z_2 = 4 \times 30 = 120$ V

Coil impedance, $Z_1 = V/I_1 = 120/6 = 20\ \Omega$

Referring to the phasor diagram in Fig. 1.72 we have,

$$I^2 = I_1^2 + I_2^2 + 2\,I_1I_2 \cos \phi_1$$

or, $$8^2 = 6^2 + 4^2 + 2 \times 6 \times 4 \times \cos \phi_1$$

$\therefore$ $$\cos \phi_1 = \frac{8^2 - 6^2 - 4^2}{2 \times 6 \times 4} = 0.25 \text{ and } \sin \phi_1 = 0.968$$

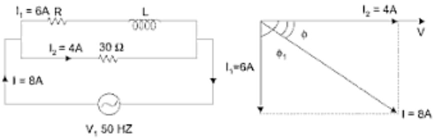


Fig. 1.73

$\therefore$ Coil resistance, $R = Z_1 \cos \phi_1 = 20 \times 0.25 = 5\ \Omega$

Coil reactance, $X_L = Z_1 \sin \phi_1 = 20 \times 0.968 = 19.36\ \Omega$

Coil reactance, $L = \dfrac{\propto L}{2\pi f} = \dfrac{19.36}{2\pi \times 50} = 0.0616$ H.

(ii) Resolving the currents along x-axis (see Fig. 18.8)

$$I \cos \phi = I_2 + I_1 \cos \phi_1$$

$\therefore$ circuit P.F. $\cos \phi = \dfrac{I_2 + I_1 \cos \theta_1}{I} = \dfrac{4 + 6 \times 0.25}{8}$

= 0.687 lag.

(iii) Power consumed, $P = VI \cos \phi = 120 \times 8 \times 0.687$

= 660 W.

Example 49: An iron cored coil A connected to a 100 V, 50 Hz supply is found to take a current of 5 A and to dissipate a power of 200 W. When the same supply is connected to coil B, the current is 8 A and the power dissipated is 450 W. Calculate the current and power taken when the two coils are connected in series across the same 100 V, 50 Hz supply.

Solution:

Coil A

$$P_A = I_A^2\,R_A$$

$$200\;(5)^2\,R_A,\; R_A = \frac{200}{25} = 8\;\Omega$$

$$Z_A = \frac{V_A}{I_A} = \frac{100}{5} = 20\;\Omega$$

Inductive reactance

$$X_A = \sqrt{Z_A^2 - R_A^2}$$

$$= \sqrt{20^2 - 8^2} = 18.33\;\Omega$$

$$Z_A = R_A + j\,X_A = 8 + j\,18.33\;\Omega$$

Coil B

$$P_B = I_B^2 R_B$$

$$450 = (8)^2 R_B,\; R_B = \frac{450}{64} = 7.03\;\Omega$$

$$Z_B = \frac{V_B}{I_B} = \frac{100}{8} = 12.5\;\Omega$$

Inductive reactance

$$X_B = \sqrt{Z_B^2 - R_B^2}$$

$$= \sqrt{12.5^2 - 7.03^2} = 10.36\;\Omega$$

$$Z_B = R_B + j\,X_B = 7.03 + j\,10.36$$

When the two coils are connected in series, the total impedance

$$Z = Z_A + Z_B = 8 + j\,18.33 + 7.03 + j\,10.36$$

$$= 15.03 + j\,28.69 = 32.388\;\angle 62.35°\;\Omega$$

Current taken by the combination

$$I = \frac{V}{Z} = \frac{100\angle 0°}{32.388\angle 62.35°} = 3.087\;\angle -62.35°\text{ A}$$

Power dissipated by the combination

$$P = VI\cos\phi = 100 \times 3.087\cos 62.35° = 143.26\text{ W}$$

or, $$P = I^2R = (3.087)^2 \times 15.03 = 143.26\text{ W}$$

Example 50: Two circuits having the same numerical ohmic impedance are joined in parallel. The power factor of one circuit is 0.8 and the other 0.6. What is the power factor of the combination?

Solution:

Let Z be the impedance of each circuit

$$Z_1 = Z \angle \cos^{-1} 0.8 = Z \angle 36.87° = Z\,(0.8 + j\,0.6)$$

$$Z_2 = Z \angle \cos^{-1} 0.6 = Z \angle 53.13° = Z\,(0.6 + j\,0.8)$$

Since, the two impedances are connected in parallel, the equivalent impedance of the combination is given by

$$Z_p = Z_1 \,||\, Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(Z\angle 36.87°)\,(Z\angle 53.13°)}{Z\,(0.8 + j\,0.6 + 0.6 + j\,0.8)} = \frac{Z^2 \angle 90°}{Z\,(1.4 + j1.4)}$$

$$= \frac{Z^2 \angle 90°}{Z(1.98 \angle 45°)} = \frac{Z \angle 45°}{1.98}$$

∴ The power factor of the combination is

$$\cos\phi = \cos 45° = 0.707$$

Example 51: A voltage of 200 ∠ 30° V is applied to two circuits connected in parallel. The currents in the respective branches are 20 ∠ 60° A and 40 ∠ –30° A. Find the kVA and kW in each branch circuit and in the main circuit.

Solution:

$$V = 200 \angle 30° \text{ V} = 173.2 + j\,100 \text{ V}$$

$$I_1 = 20 \angle 60° \text{ V} = 10 + j\,17.32 \text{ A}$$

$$I_2 = 40 \angle -30° \text{ V} = 34.64 - j\,20 \text{ V}$$

$$S_1 = VI_1^* = (200 \angle 30°)\left(20 \angle -60°\right)$$

$$= 4000 \angle -30° \text{ VA} = 3464 - j\,2000$$

$$S_1 = 4000 \text{ VA} = 4 \text{ kVA}$$

$$P_1 = \text{real part of } VI_1^* = 3464 \text{ W} = 3.464 \text{ kW.}$$

$$S_2 = VI_2^* = (200 \angle 30°)\,(40 \angle +30°)$$

$$= 8000 \angle 60° \text{ VA} = 4000 + j\,6928$$

$$S_2 = 8000 \text{ VA} = 8 \text{ kVA}$$

$$P_2 = Re[VI^*] = 4000 \text{ W} = 4 \text{ kW}$$

Total current of the circuit

$$I = I_1 + I_2 = 10 + j\,17.32 + 34.64 - j\,20$$

$$= 44.46 - j\,2.68 = 44.72 \angle -3.43° \text{ A}$$

$$S = VI^* = (200 \angle 30°)\,(44.72 \angle +3.43°)$$

$$= 8944 \angle 33.43° \text{ VA} = 7464 + j\,4927 = P + jQ$$

$$S = 8944 \text{ VA} = 8.944 \text{ kVA}$$

$$P = 7464 \text{ W} = 7.464 \text{ kW}.$$

Example 52: Three impedances $(6 + j5)\ \Omega$, $(8 - j6)\ \Omega$ and $(8 + j10)\ \Omega$ are connected in parallel. Calculate the current in each branch when the total current is 20 A.

Solution:

Let the total current I be taken as reference phasor.

$$I = 20 \angle 0° = 20 + j\,0 \text{ A}.$$

$$Z_1 = 8 + j\,5 = 7.81 \angle 39.8°\ \Omega$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{6 + j5} = \frac{6 - j5}{(6 + j5)(6 - j5)}$$

$$= \frac{6 - j5}{6^2 + 5^2} = 0.09836 - j\,0.08196 \text{ S}$$

$$Z_2 = 8 - j\,6 = 10 \angle -36.87°\ \Omega$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{8 - j6} = \frac{8 - j6}{(8 - j6)(8 - j6)} = \frac{8 - j6}{8^2 + 6^2}$$

$$= 0.08 + j\,0.06 \text{ S}$$

$$Z_3 = 8 + j10 = 12.8 \angle 51.34°\ \Omega$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{8 + j10} = \frac{8 - j10}{(8 + j10)(8 - j10)} = \frac{8 - j10}{8^2 + 10^2}$$

$$= 0.04878 - j\,0.06097 \text{ S}$$

Total admittance of the circuit

$$Y = Y_1 + Y_2 + Y_3$$

$$= 0.09836 - j\,0.08196 + 0.08 - j\,0.06 + 0.04878 - j\,0.06097$$

$$= 0.22714 - j\,0.08293 = 0.2418 \angle -20.06°\ S$$

Total circuit voltage

$$V = IZ = \frac{I}{Y} = \frac{20 < 0°}{0.2418 \angle -20.06°} = 82.71 \angle 20.06° \text{ V}$$

$$I_1 = \frac{V}{Z_1} = \frac{82.71 \angle 20.06°}{7.81 \angle 39.8°}$$

$$= 10.59 \angle -19.74° \text{ A} = 9.967 - j\,3.576 \text{ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{82.71 \angle 20.06°}{10 \angle -36.87°}$$

$$= 8.271 \angle 56.93° \text{ A} = 4.513 + j\,6.930 \text{ A}$$

$$I_3 = \frac{V}{Z_3} = \frac{82.71 \angle 20.06°}{12.8 \angle -51.34°}$$

$$= 6.46 \angle -31.28° \text{ A} = 5.52 - j\,3.35 \text{ A}$$

$$I_1 + I_2 + I_3 = 20 + j\,0 = I$$

Example 53: A single phase circuit consists of three parallel branches, the admittance of the branches are

$$Y_1 = 0.4 + j\,0.6$$

$$Y_2 = 0.1 + j\,0.4$$

$$Y_3 = 0.06 + j\,0.23$$

Determine the total admittance and impedance of the circuit.

Solution: Since, the admittances are in parallel

$$Y = Y_1 + Y_2 + Y_3$$

$$= (0.4 + j\,0.6) + (0.1 + j\,0.42) + (0.06 + j\,0.23)$$

$$= (0.4 + 0.1 + 0.06) + j\,(0.6 + 0.42 + 0.23)$$

$$= 0.56 + j\,1.25 = 1.369 \angle 65.86$$

$$\text{Impedance} = Z = \frac{1}{Y} = \frac{1}{1.369 \angle 65.86} = 0.73 \angle -65.86$$

$$Z = 0.298 - j\,0.666$$

Example 54: Four loads are connected across a 230 V, 50 Hz line:

(a) lights 10 kVA at unity power factor,

(b) a motor 4 kW at power factor 0.8 lagging

(c) a rectifier 3.6 kW at power factor 0.6 leading

(d) a capacitor 8 kVA

Determine the total kW, the total kVAr, the total kVA, the overall power factor, and the total line current.

Solution:

By convention, Q_L is taken positive and Q_C negative.

Load a

$$\cos \phi_a = 1, \sin \phi_a = 0$$

$$S_a = S_a \angle \phi_a = S_a (\cos \phi_a + j \sin \phi_a)$$

$$= 10 (1 + j\,0) = (10 + j\,0) \text{ kVA}$$

Load b

$$\cos \phi_b = 0.8, \ \sin \phi_b = 0.6$$

$$P_b = \text{Z}\, j \sin \phi_c)$$

$$= 6 (0.6 + j\,0.8) = (3.6 - j\,4.8) \text{ kVA}$$

Laod c

$$\cos \phi_C = 0.6, \sin \phi_c = 0.8$$

$$P_c = S_c \cos \phi_c$$

$$3.6 = S_c \times 0.6, S_c = \frac{3.6}{0.6} = 6 \text{ kVA}$$

$$S_c = S_c \angle - \phi_c = S_c (\cos \phi_c - j \sin \phi_c)$$

$$= 6(0.6 - 0.8) = (3.6 - j4.8) \text{ kVA}$$

Load d

For a capacitor, the current leads the voltage by 90°. Therefore, the power factor cos ϕ is leading.

$$\cos \phi_d = \cos 90° = 0 \text{ (leading)}$$

$$\sin \phi_d = \sin 90° = 1$$

$$P_d = S_d \cos \phi_d$$

$$P_d = 8 \times 0, P_d = 0 \text{ kW}$$

$$S_d = S_d \angle - 90° = 8 \angle - 90° = (\cos 90° - j \sin 90°)$$

$$= 8 (0 - j\,1) = 0 - j\,8 \text{ kVA}$$

Total complex power

$$S = S_a + S_b + S_c + S_d$$

$$= 10 + j\,0 + 4 + j\,3 + 3.6 - j\,4.8 + 0 - j\,8$$

$$= 17.6 - j\,9.8 = 20.144 \angle -29.1° \text{ kVA}$$

Also, $S = P + j\,Q = S \angle \phi$

$S = 20.144$ kVA

$P = 17.6$ kW

$Q = 9.8$ kVAr (leading)

Overall power factor

$$\cos\phi = \cos(-29.1°) = 0.8737 \text{ (leading)}$$

Total current

$$I = \frac{\text{kVA} \times 1000}{V} = \frac{20.144 \times 1000}{230} = 87.58 \text{ A}$$

Example 55: In the network shown in Fig. 1.74, determine (a) the total impedance, (b) the total current (c) the current in each branch, (d) the overall power factor, (e) volt-amperes, (f) active power, and (g) reactive volt-amperes.

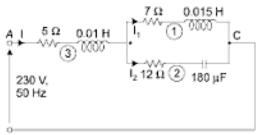


Fig. 1.74

Solution:

(a) **Branch 1:**

$$R_1 = 7\ \Omega\ L_1 = 0.015 \text{ H}$$

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.015 = 4.71\ \Omega$$

$$Z_1 = R_1 + j\,X_{L_1} = 7 + j\,4.71 = 8.437 \angle 33.93°\ \Omega$$

Branch 2:

$$R_2 = 12\ \Omega\ C_2 = 180\ \mu F = 180 \times 10^{-6}\ F$$

$$X_{C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times 180 \times 10^{-6}} = 17.68\ \Omega$$

$$Z_2 = R_2 - j\,X_{C_2} = 12 - j\,17.68 = 21.37 \angle -55.83°\ \Omega$$

Since, Z_1 and Z_2 are connected in parallel, their equivalent impedance Z_p is given by

$$Z_p = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(8.437 \angle 33.93^\circ)(21.37 \angle -55.83^\circ)}{(7 + j4.71) + (12 - j17.68)}$$

$$= \frac{180.3 \angle -21.9^\circ}{19 - j12.97} = \frac{180.3 \angle -21.9^\circ}{23 \angle -34.3^\circ}$$

$$= 7.839 \angle + 12.4^\circ = 7.656 + j\,1.68\ \Omega$$

Branch 3:

$$R_3 = 5\ \Omega,\ L_3 = 0.01\ \text{H}$$

$$X_{L_3} = 2\pi f L_3 = 2\pi \times 50 \times 0.01 = 3.14\ \Omega$$

$$Z_3 = R_3 + jX_{L_3} = 5 + j\,3.14 = 5.9 \angle 32.13^\circ\ \Omega$$

Since, Z_3 and Z_p are connected in series, the total impedance of the circuit is

$$Z = Z_3 + Z_p = 5 + j\,3.14 + 7.656 + j\,1.68 = 12.656 + j\,4.82 = 13.54\ \angle 20.85^\circ\ \Omega$$

(b) Let the supply voltage V be taken as reference phasor.

$$V = 230 \angle 0^\circ\ \text{V} = 230 + j\,0\ \text{V}.$$

By Ohm's law total circuit current is

$$I = \frac{V}{Z} = \frac{230 \angle 0^\circ}{13.54 \angle 20.85^\circ} = 16.99 \angle -20.85^\circ\ \text{A}$$

$$= 15.87 - j\,6.046\ \text{A}$$

(c) Voltage drop across Z_3 is

$$V_{AB} = IZ_3 = (16.99 \angle -20.85^\circ)(5.9 \angle 32.13^\circ)$$
$$= 100.24 \angle 11.28^\circ = 98.3 + j\,19.6$$

By KVL

$$V = V_{AB} + V_{BC}$$

$$\therefore \quad V_{BC} = V - V_{AB}$$

$$= 230 + j\,0 - (98.3 + j\,19.6) = 131.7 - j\,19.6 = 133.15 \angle -8.46^\circ\ \text{V}$$

Alternatively,

$$V_p = IZ_p = (16.99 \angle -20.85°)(7.839 \angle 12.4°)$$
$$= 133.18 \angle -8.45° \text{ V}$$
$$= 131.73 - j\,19.57 \text{ V}$$

Since, Z_p is the parallel combination of Z_1 and Z_2

$$V_{BC} = V_p = V_1 = V_2 = 133.15 \angle -8.46° \text{ V}$$

By Ohm's law

$$I_1 = \frac{V_1}{Z_1} = \frac{133.15 \angle -8.46°}{8.437 \angle 33.93°} = 15.78 \angle -42.39° \text{ A}$$

or, $$I = 11.65 - j\,10.64 \text{ A}$$

$$I_2 = \frac{V_2}{Z_2} = \frac{133.15 \angle -8.46°}{21.37 \angle -55.83°}$$

$$= 6.23 \angle 47.37° \text{ A} = 4.22 + j\,4.58 \text{ A}$$

Check $I_1 + I_2 = 11.65 - j10.64 + 4.22 + j\,4.58 = 15.87 - j\,6.06 \text{ A} = I$

(d) Total current

$$I = 16.99 \angle -20.85° \text{ A}$$

Example 56: A coil of resistance 50 Ω and inductance 0.318 H is connected in parallel with a circuit comprising a 75 Ω resistor in series with a 159 μF capacitor. The circuit is connected to 240 V, 50 Hz supply. Calculate.

(i) Supply current

(ii) Phase angle between supply current and applied voltage. Find also the resistance and reactance of series circuit which will take same current at the same p.f. as the parallel circuit.

Solution: $$X_L = 2\,\pi f_L = 2\,\pi \times 50 \times 0.318 = 100\ \Omega$$

$$Z_1 = \sqrt{R_L^2 + X_L^2} = \sqrt{50^2 + 100^2} = 112\ \Omega$$

$$\phi_1 = \cos^{-1} R_1/Z_1 = \cos^{-1} 50/112 = 63.5° \text{ lag.}$$

$$I_1 = V/Z_1 = 240/112 = 2.15 \text{ A}$$ ...lags V by 63.5°

$$X_C = \frac{1}{2\pi f_C} = \frac{10^6}{2\pi \times 50 \times 159} = 20\ \Omega$$

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{75^2 + 20^2} = 77.7\ \Omega$$

$$\phi_2 = \cos^{-1} R_2/Z_2 = \cos^{-1} 75/77.7 = 15° \text{ load.}$$

$I_2 = V/Z_2 = 240/77.7 = 3.09\ A$ ... lead V by 15°

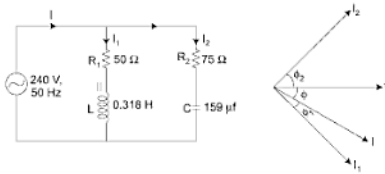


Fig. 1.75

Referring to the phasor diagram shown in Fig. 1.75.

$$\text{Total } X\text{-comp.} = I_1 \cos \phi_1 + I_2 \cos \phi_2$$
$$= 2.15 \cos 63.5° + 3.09 \cos 15° = 3.94 \text{ A}$$
$$\text{Total } Y\text{-comp} = -I_1 \sin \phi_1 + I_2 \sin \phi_2$$
$$= -2.15 \sin 63.5° + 3.09 \sin 15° = -1.13 \text{ A.}$$

(i) Supply current, $I = \sqrt{(3.94)^2 + (-1.13)^2} = 4.1$ A

(ii) Phase angle. $\phi = \tan^{-1} -1.13/3.94 = -16°$

Since, voltage in the reference phasor, negative angle means that supply current lags the applied voltage, i.e., circuit is inductive.

Circuit impedance, $Z = V/I = 240/4.1 = 58.5\ \Omega$

Circuit resistance, $R = Z \cos \phi = 58.5 \cos (-16°) = 56.2\ \Omega$

Circuit reactance, $X_L = Z \sin \phi = 58.5 \sin (-16°) = 16.12\ \Omega$

Thus, the parallel circuit is equivalent to 56.2 Ω resistor in series with 16.12 Ω inductive reactance.

Example 57: A voltage of 230 ∠30° V is applied to two circuits connected in parallel. The current in the branches are 20 ∠60° A and 40 ∠– 30° A. Find (i) The total impedance of the circuit (ii) Power taken.

Solution: $I_1 = 20 \angle 60°\ A = 20 (\cos 60° + j \sin 60°) = (10 + j\ 17.3)\ A$

$I_2 = 40 \angle{-30°}\ A = 40 (\cos 30° - j \sin 30°) = (34.6 - j\ 20)\ A$

$= (44.6 - j\ 2.7) = 44.7 \angle -3.46°\ A$

Fig. 1.76

(i) $\therefore Z = \dfrac{V}{I} = \dfrac{230\angle 30^\circ}{44.7\angle -3.46^\circ} = 5.14\angle 33.46^\circ\ \Omega$

(ii) $P = V.I\cos\phi = 230 \times 44.7 \times \cos 33.46^\circ = 8577$ W.

Example 58: A parallel circuit consists of a 2.5 μF capacitor and a coil whose resistance and inductance are 15 Ω and 260 mH, respectively. Determine (i) the resonant frequency (ii) *Q*-factor of the circuit at resonance (iii) dynamic impedance of the circuit.

Solution:

(i) Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{10^6}{0.260 \times 2.5}\ \frac{-(15)^2}{(0.260)^2}} = 197 \text{ Hz}$$

(ii) *Q*-factor $= \dfrac{2\pi f_r L}{R} = \dfrac{2\pi \times 197 \times 0.260}{15} = 21.45$

(iii) $Z_r = \dfrac{L}{CR} = \dfrac{0.260}{2.5 \times 10^{-6} \times 15} = 6933\ \Omega$

Example 59: A square coil of 10 cm side and 100 turns is rotated at a uniform speed of 1000 revolution per minute, about an axis at right angles to a uniform magnetic field of 0.5 *Wb*/m^2. Calculate the instantaneous value of the induced electromotive force. When the plane of the coil in (i) at right angles to the field (ii) in the plane of the field. (Electromagnetic Theory, AMIE, Sec *B*, 1992)

Solution: Let the magnetic field lie in the vertical plane and the coil in the horizontal plane. Also let the angle Q be measured from X-axis.

Maximum value of the induced emf. $E_m = 2\pi fNB_m$ A volt instantaneous value of the induced e.m.f. $e = E_m \sin Q$.

Now $t = 100/60 = (50/3)$ r.ps., $N = 100$, $B_m = 0.5$ Wb/m^2, $A = 10^{-2}$ m^2

(i) In this case, $\theta = 90°$

(ii) Here, $\theta = 90°$, $\therefore e = E_m \sin 90° = E_m$

Substituting the value (given), we get,

$$e = 2\pi \times (50/3) \times 100 \times 0.5 \times 10^{-2}$$

$$e = \mathbf{52.3\ V.}\quad \textbf{Ans.}$$

Example 60: The maximum values of the alternating voltage and current are 400 V and 20 A, respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A, respectively at $t = 0$ both increasing positively.

(i) Write down the expression for voltage and current at time t.

(ii) Determine the power consumed in the circuit. (Elect. Engg. Pune Univ.)

Solution:

(i) In general, the expression for an AC voltage is $V = V_m \sin(wt + \phi)$ where ϕ its the phase difference with respect to the point where $t = 0$.

Now, $V = 283$; $Vm = 400\ V$, substituting $t = 0$ in the above equation, we get

$$283 = 400(\sin w \times 0 + \phi) \quad \therefore \sin\phi = 283/400$$

$$= 0.707, \quad \therefore \phi = 45° \text{ or } \pi/4 \text{ radians.}$$

Hence, general expression for voltage is

$$V = 400(\sin 2\pi \times 50 \times t + \pi/4)$$

Similarly, at $t = 0$

$$10 = 20 \sin(w \times 0 + \phi)$$

$$\therefore \quad \sin\phi = 0.5 \quad \therefore \phi = 30° \text{ or } \pi/6 \text{ radians.}$$

Hence, the general expression for the current is

$$i = 20(\sin 100\pi t + 30°) = 20 \sin(100\pi t + \pi/6)$$

(ii) $P = VI \cos\theta$ where V and I are RMS values and θ is the phase difference between the voltage and current

Now, $$V = V_m/\sqrt{2} = 400/\sqrt{2};\ I = 20/\sqrt{2};\ \theta = 45° - 30°$$

$$\theta = 15° \text{ (see Fig. 1.77)}$$

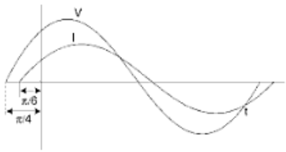


Fig. 1.77

∴ $P = (400/\sqrt{2}) \times (20/\sqrt{2}) \times \cos 15°$

$P = 3864\ \Omega$.

EXERCISES

1. What is meant by an alternating quantity? Explain how a sine wave is produced.
2. Define: cycle, periodic time and frequency.
3. What is understood by "phase difference" between two alternating quantities? Explain the term lagging current and leading current with the aid of suitable curves.
4. Define RMS value of an alternating current.

   Derive RMS value in case of a:

   (a) Sinusoidal wave
   (b) Rectangular wave
   (c) Triangular wave
   (d) Semicircular wave
   (e) Trapezoidal wave
   (g) Stepped wave.
5. Define average value of an alternating current.

   Derive average value in case of a:

   (a) Sinusoidal wave
   (b) Rectangular wave
   (c) Triangular wave
   (d) Semicircular wave
   (e) Trapezoidal wave
   (g) Stepped wave.

   Also define form factor and find its value in case of all the above waves.

6. Define peak or crest factor and state its practical utility.
7. Determine average value, effective value and form factor of a triangular wave whose half wave is suppressed in each cycle.
8. Two waves represented by $e_1 = 3 \sin wt.$ and $e_2 = 4 \sin \left(wt - \frac{\pi}{3}\right)$ are acting in a circuit. Find an expression of their resultant and check the result by a graphical construction. Also find the peak and RMS values of the resultant.

   [$\sqrt{37}$ sin ($wt.$ – 0.605); $\sqrt{37}$, 4.3]
9. An alternating current is given in amperes by the expression,

   $$i = 50 \sin 44 \phi t.$$

Find (a) frequency.

   (b) w in radians per second

   (c) maximum value of the current

   (d) effective value of the current

   [(a) 70 c/s, (b) 440, (c) 50 A and (d) 35.35 A]
10. Show that in a purely resistive circuit, the current and voltage wave are in phase. Draw curves of the applied voltage, current and power. Prove that the average power in the circuit is equal to the product of effective values of voltage and current.
11. Show that in a purely inductive circuit, current lags the applied voltage by 90°. Prove that the average power absorbed in such a circuit is zero.
12. Show that in a purely capacitive circuit, current leads the applied voltage by 90°. Show that the power wave fluctuates with double the frequency and that the average power absorbed is zero in such a circuit.
13. What is meant by:

    (a) inductive reactance.

    (b) capacitive reactance.

    (c) impedance.

    Specify the unit of each one of them. Also express them in their symbolic form.
14. Calculate the reactance of a condenser of capacitance 100 μF and inducing coil of 20 mH at (a) 25 c/s. (b) 50 c/s and (c) 100 c/s

    [(a) 63.6 Ω, 63.14 Ω (b) 31.8 Ω, 6.28 Ω (c) 15.9 Ω, 12.56 Ω]
15. A set of 200 V lamps take 2 A. Find inductance of a choke coil to be connected in series with them so that they may work satisfactorily on a 250 V, 50 cycle supply mains. Draw vector diagram. [0.238 H]

16. A current of 10 A flows in a circuit lagging behind the applied voltage of 100 V by 30°. Determine the resistance, reactance and impedance of the circuit. [8.66 Ω, 5 Ω, & 10 Ω]

17. A non-inductive resistance in series with an ideal inductor is connected to a 125 V, 50 c/s supply. The current in the circuit is 2.2 A and the power loss in the resistor is 96.8 W calculate the resistance and the inductance. Draw the vector diagram for the circuit.

    [20 Ω, 60 μ*F*]

18. A circuit consists of resistance of 20 Ω, an induct are of 0.1 *H* and a capacitance of 250 μ*F*, all connected in series. The combination is connected across 200 V, 50 c/s mains. Calculate (a) Current (b) p.f. (c) power, (d) voltages across the coil and condenser. Draw vector diagram.

    [(a) 7.32 A, (b) 0.73 lagging, (c) 1070, (d) 273 V & 93.2 V.]

19. State the condition of resonance in a series *R-L-C* circuit. Define resonance frequency and express it in terms of *L* and *C*.

20. An alternating voltage of 1 volt having a frequency of 100 kilo-cycles is applied across the combination of a coil and a condenser in series. The coil has a resistance of 250 Ω and inductance of 12.5 *mH*. Determine the capacitance of a condenser which will produce resonance. Find also the power dissipated in the circuit and the voltage across the coil. Draw the vector diagram. [20.6×10^{-5} μF, 0.004 W. 31.4 V]

21. Define conductance, susceptance and admittance, and express them in terms of *R*, *X* and *Z*.

    A coil of resistance 15 Ω and inductance 0.05 *H* is connected in parallel with a non-inductance resistor of 20 Ω. Find (a) the current in each branch circuit (b) the total current supplied and (c) the phase angle of combination when a voltage of 200 V at 50 c/s is applied. Draw the vector diagram. [(a) 9.2 A, 10 A (b) 17.6 A, (c) 22° lagging]

22. Two circuits, the impedances of which are given by $Z_1 = 10 + j\,15\ \Omega$ and $Z_2 = 6 - j\,8\ \Omega$ are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch. Draw vector diagram.

    [737 W, 1400 W]

23. A small 1 – phase 240 V induction motor is tested in parallel with a 160 Ω resistor. The motor takes 2.0 A and the total current is 3 A. Find the power and power factor of (a) the whole circuit (b) the motor. Draw vector diagram. [(a) 580 W, 0.807 lag (b) 220 W, 0.46 lag.]

24. Find the total current taken and the power factor of the circuit given in Fig. 1.178. Draw also the vector diagram.

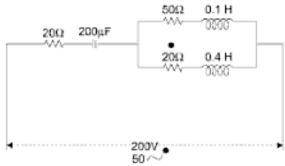


Fig. 1.78

[3.85 *A*, 0.95 lagging]

25. Explain the phenomenon of "Parallel Resonance".

    A coil of resistance 15 Ω and inductance of 0.5 *H* is connected in series with a condenser. On applying a sinusoidal voltage the current is maximum when the frequency is 50 c/s. A second condenser is connected in parallel with the circuit. What capacitance it must have so that the combination acts like a non-inductive circuit at 100 c/s?

    Calculate also the total current supplied in each case if the applied voltage is 240 V. [6.74 μ*F*, 16 A & 0.0648 A]

26. Define effective, equivalent or dynamic impedance of a rejecter circuit.

    Find the current in a parallel circuit at resonance after making practical assumptions.

27. Define *Q*-factor and determine its value in:

    (a) Series resonant circuit

    (b) Parallel resonant circuit

2

Magnetic Circuits

2.1 MAGNETIC CIRCUIT

The path followed by a magnetic line of force is called magnetic circuit. Every line of force follows a closed path i.e., it comes back to the originating point. A simple magnetic circuit is shown in Fig. 2.1. Consider a ring-shaped specimen of iron wound with a suitable number of turns of insulated wire. When a current flows through the coil, lines of force are produced in the specimen. In this case, the magnetic circuits are circular paths of lines of force as shown by dotted lines.

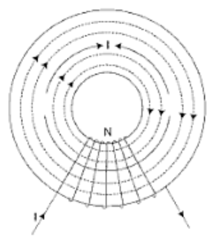


Fig. 2.1

Let N = Number of turns wound over the ring.

I = Current in amperes.

l = Length of the mean path of lines of force.

Field intensity H at the centre of the solenoid is given by,

$$H = \frac{NI}{l}$$

If μ is the permeability of the ring material, then flux density B in the ring is given by,

$$B = \mu H = \mu_0 \mu_r \frac{NI}{l} \text{ web/met}^2$$

If A is the area of cross-section of the iron ring in meter^2, then flux produced is given by,

$$\phi = B.A.$$

$$= \mu_0 \mu_r \frac{NI\,A}{l} \quad \text{...(2.1)}$$

Here, NI is called magnetomotive force of magnetic circuit, $l/\mu_0\,\mu_r A$ is called reluctance.

$$\phi = \frac{NI}{l/\mu_0\mu_r A} = \frac{\text{mmf}}{\text{Reluctance}}$$

Thus, $$\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} \quad \text{...(2.2)}$$

This is sometimes called "Ohm's law" of magnetic circuit. It resembles the expression for electric circuit,

$$\text{Current } I = \frac{\text{E.M.F.}}{\text{Resistance}} \quad \text{...(2.3)}$$

2.2 CIRCUITAL LAWS

There are two circuital laws of the magnetic field.

First law—(Ampere's law) states that the total magnetomotive force acting round a closed magnetic circuit is equal to the algebraic sum of currents enclosed by the path.

So, $$\text{M.M.F.} = \Sigma I \quad \text{...(2.4)}$$

Where, $\Sigma I = NI$ in M.K.S. rationalised system

$= 0.4\pi\, NI$ Gilberts in c.g.s. system.

Second law—(Faraday's laws of electromagnetic induction) states that the emf induced around a closed path is equal to the negative rate of change of the magnetic flux linkages.

So, $$e = -N\frac{d\phi}{dt} \times 10^{-8} \text{ volts.} \qquad ...(2.5)$$

where, N = number of turns of the coil

ϕ = interlinking flux in lines.

2.3 DEFINITIONS

2.3.1 Magnetic Flux (ϕ)

It is defined as the total number of lines of force emitted from a source.

In c.g.s. system, its unit is Maxwell or lines and in M.K.S. system, its unit is Weber. (1 weber = 10^8 lines).

2.3.2 Magnetomotive Force (M.M.F.)

It is the force which drives the magnetic flux through a magnetic circuit.

It is also defined as the work done in carrying a unit magnetic pole once round a closed path of lines of force.

In c.g.s. system, M.M.F. = $0.4\pi\, NI$ gilberts ...(2.6)

In M.K.S. rationalized system M.M.F. = NI Amp. Turns.

In M.K.S. unrationalized system M.M.F. = $4\pi\, NI$

Ampere Turns. ...(2.8)

I may be expressed as A.

N may be expressed as T.

Then, $NI = AT$ i.e., Amp. Turns. ...(2.9)

2.3.3 Reluctance (R)

It is the obstruction offered by the magnetic circuit to the flow of magnetic flux. It is analogous to resistance in an electric circuit:

$$R = \frac{l}{\mu . a} \qquad ...(2.10)$$

In c.g.s. system:

l is in cm, a is in cm^2, and $\mu = \mu_r$

If, $l = 1$ cm

$\mu = 1$ in vacuum in c.g.s. units

$a = 1 \text{ cm}^2$

Thus, the unit reluctance is the reluctance of a centimetre cube in vacuum. In c.g.s. system no name is given to this unit.

In m.k.s. system:

l is in metre, a is in metres^2 and $\mu = \mu_0 \mu_r$

The value of $\mu_0 = 4\pi \times 10^{-7}$ in r.m.k.s. units and 10^{-7} in m.k.s. unrationalized and the unit of reluctance is henry.

2.3.4 Permeance (*P*)

It is the reciprocal of reluctance and is analogous to conductance in an electric circuit.

$$P = \frac{1}{R} = \frac{\mu a}{l} \quad \text{...(2.11)}$$

Permeance may be defined as the property with which magnetic flux can flow in a magnetic circuit.

In m.k.s. system, its unit is henry.

2.3.5 Reluctivity

The reciprocal of permeability is called reluctivity and is usually denoted by v

$$\therefore \quad v = \frac{1}{\mu} \quad \text{...(2.12)}$$

$$\text{Reluctance} = \frac{l}{\mu . a} = \frac{vl}{a}$$

The reluctivity corresponds to resistivity in an electric circuit.

2.4 SIMILARITY OF MAGNETIC & ELECTRIC CIRCUITS

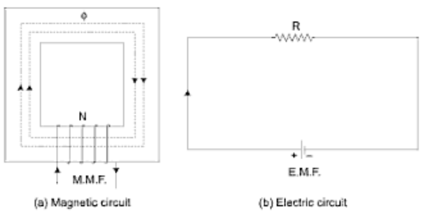


Fig. 2.2

Magnetic Circuit		*Electric Circuit*
(i) Flux	Corresponds to	Current
(ii) M.M.F.	,,	E.M.F.
(iii) Reluctance	,,	Resistance
(iv) Permeance	,,	Conductance
(v) Flux density	,,	Current density
(lines/sq. cm^2)	,,	($Amps/cm^2$)
(vi) Permeability (μ)	,,	Conductivity (σ)
(vii) Reluctivity (v)	,,	Resistivity (ρ)

2.5 DISSIMILARITY BETWEEN MAGNETIC & ELECTRIC CIRCUITS

(i) *Flux*—No flow of flux in a magnetic circuit.	Current—It actually flows in an electric circuit.
(ii) *Permeability*—(a) It depends upon the total flux. (b) It does not vary too much and so no material could be said as insulator to magnetic flux.	Conductivity—(a) It does not depend upon the current strength. (b) It varies within very large limits and so materials are termed as conductors or insulators.
(iii) *Energy*—It is expended in establishing flux but not for maintaining it.	Energy—It is expended so long as the current flows.

2.6 RELUCTANCES IN SERIES

A composite magnetic circuit is said to have reluctances in series, if the same flux passes through the different branches.

Let l_a, l_b and l_c be the lengths, a_a, a_b and a_c be the cross-sectional areas and μ_a, μ_b and μ_c be the permeabilities of the limbs *A*, *B* and *C*, respectively as shown in Fig. 2.3.

If R_a, R_b and R_c are the reluctances of the three limbs,

Then,

$$R_a = \frac{l_a}{\mu_a . a_a}$$

$$R_b = \frac{l_b}{\mu_b . a_b}$$

and,

$$R_c = \frac{l_c}{\mu_c . a_c}$$

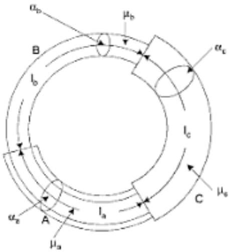


Fig. 2.3

If M_a, M_b and M_c be the M.M.F.'s of the three branches and ϕ be the flux passing through them, then from eqn. (2.4)

$$M_a = \phi . R_a = \phi . \frac{l_a}{\mu_a . a_a} \qquad ...(2.13)$$

$$M_b = \phi . R_b = \phi . \frac{l_b}{\mu_b . a_b} \qquad ...(2.14)$$

$$M_c = \phi . R_c = \phi . \frac{l_c}{\mu_c . a_c} \qquad ...(2.15)$$

Total M.M.F. = $M_a + M_b + M_c$ ...(2.16)

If the equivalent reluctance is R, then

Total M.M.F. = $\phi . R$ ...(2.17)

Comparing eqns. (2.17) and (2.18)

$$\phi R = .\phi . R_a + \phi R_b + \phi R_c$$

or,
$$R = R_a + R_b + R_c \qquad ...(2.18)$$

$$= \frac{l_a}{\mu_a . a_a} + \frac{l_b}{\mu_b . a_b} + \frac{l_c}{\mu_c . a_c} \qquad ...(2.19)$$

Thus, when the reluctances are connected in series, the equivalent reluctance is the sum of the reluctances of individual branches.

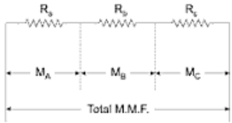


Fig. 2.4

So the circuit of Fig. 2.3, may be represented by an equivalent circuit of Fig. 2.4.

So the total flux is given by

$$\phi = \frac{\text{M.M.F.}}{R_a + R_b + R_c} \quad \ldots(2.20)$$

$$= \frac{\text{M.M.F.}}{R.}$$

In c.g.s. system, flux will be

$$\phi = \frac{0.4\pi NI}{\Sigma \frac{l}{\mu . a}} \quad \ldots(2.21)$$

2.7 OHM'S LAW OF MAGNETIC CIRCUIT

2.7.1 In C.G.S. System

M.M.F = $0.4\pi NI$ gilbert.

where, N = No. of turns.

I = Current in amps.

$$R = \frac{l}{\mu . a}$$

where, l = length in cm.

a = area in cm^2.

$\mu = \mu_0 . \mu_r$.

$\mu_0 = 1$ = Absolute permeability of free space in c.g.s. unit.

μ_r = Relative permeability of the material.

Thus, $\mu = \mu_r$ in c.g.s. system.

ϕ = flux in lines.

$$\therefore \quad \phi = \frac{0.4\pi NI}{l/\mu.a}$$

$NI = AT$ = Ampere Turns.

$$\therefore \quad \phi = \frac{0.4\pi AT}{l}.\mu_a$$

Since,
$$H = \frac{\phi}{\mu.a}$$

So,
$$AT = \frac{1}{0.4\pi}.\frac{\phi}{\mu.a}.l$$

$$= 0.796\ H.l. \qquad \text{...(2.22)}$$

Thus, ampere-turns required.

= 0.796 × Magnetising force × length of magnetic path.

2.7.2 In M.K.S. Rationalised System

$$\text{M.M.F.} = NI \text{ ampere-turns} \qquad \text{...(2.23)}$$

Where, N = No. of turns.

I = Current in ampere.

$$R = \frac{l}{\mu.a} \text{ henry.}$$

2.8 RELUCTANCES IN PARALLEL

When the flux established in a single coil passes through several parallel branches of magnetic circuit, the reluctances are said to be in parallel.

In a simple magnetic circuit of Fig. 2.5(a), the flux ϕ divides into two parallel branches, so

$$\phi = \phi_1 + \phi_2 \qquad \text{...(2.24)}$$

Let M = M.M.F. of the coil on central limb between the points A and B.

R_1 = Reluctance of the circuit 1.

R_2 = Reluctance of the circuit 2.

R = Equivalent reluctance of both the paths, then

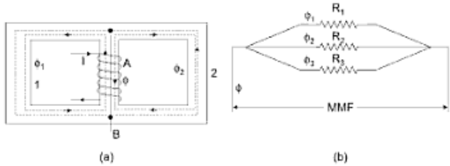


Fig. 2.5

$$\phi = \frac{M.M.F.}{R} = \frac{M}{R}.$$

$$\phi_1 = \frac{M}{R_1}.$$

$$\phi_2 = \frac{M}{R_2}.$$

Substituting the value of ϕ, ϕ_1 and ϕ_2 in equation ...(2.25)

$$\frac{M}{R} = \frac{M}{R_1} + \frac{M}{R_2}$$

or,
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad ...(2.26)$$

Hence, in general if R_1, R_2, R_3 ... are the reluctances in parallel, then total equivalent reluctance R is given by,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad ...(2.27)$$

Thus, reluctances in parallel may be represented by an equivalent electric circuit shown in Fig. 2.5(b).

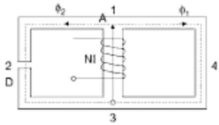
If P is the total permeance of the circuit and P_1, P_2, P_3, ... be the permeances of the parallel branches, so that

$$P = \frac{1}{R},\ P_1 = \frac{1}{R_1},\ P_2 = \frac{1}{R_2},\ P_3 = \frac{1}{R_3},$$

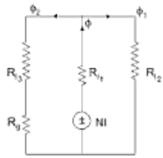
Then, $$P = P_1 + P_2 + P_3 + \dots \dots \qquad \dots(2.28)$$

Hence, the permeances in parallel are added up to obtain the total permeance.

2.9 ELECTRIC CKT CORRESPONDS TO MAGNETIC CIRCUIT (SERIES AND PARALLEL)



Magnetic CKT



Equivalent electric CKT

Fig. 2.6

R_{l1} = reluctance for path (1) to (3) = $l_{1-3}/\mu_0 \mu_r A$

R_{l2} = reluctance for path (1) → (2) → (4)

R_{l3} = reluctance for iron path (1) → (2) → (3)

R_g = reluctance for air gap

Net mmf required to establish a flux ϕ is

$$(\text{mmf}) = \phi[\text{net reluctance}] \qquad \{V = IR\}$$

$$= \phi\left[R_{l_1} + \frac{R_{l_2}(R_{l_3} + R_g)}{R_{l_2} + R_{l_3} + R_g}\right]$$

Table 2.1 Relation between Magnetic and Electrical Quantities

Quantity and Symbol	C.G.S. Electromagnetic	British Electromagnetic	M.K.S. System Unrationalised	M.K.S. System Rationalised
Magnetic flux (ϕ)	maxwell or line	maxwell or line	weber	weber
Magnetic flux density (B)	guass (lines/cm.^{2})	lines/sq. inch	webers/metre^2	webers/metre^2
Magnetomotive force (F)	gilbert ($0.4\pi\ NI$)	amp-turns	4π amp-turns ($4\pi\ NI$)	amp-turns (NI)
Magnetic field intensity (H)	oersted (gilbert/cm.)	amp-turns/inch	4π NI/metre	NI/metre
Inductance (L)	abhenry	abhenry	henry	henry
Electric flux (Ψ)	4π abcoulomb	4π abcoulomb	4π coulomb	coulomb
Electric field intensity	abvolt/cm	abvolt/cm	volts/metre	volts/metre
EMF or p.d.(V)	abvolt	abvolt	volt	volt
Current (I)	abampere	abampere	ampere	ampere
Charge (Q)	abcoulomb	abcoulomb	coulomb	coulomb
Resistance (R)	abohm	abohm	ohm	ohm
Capacitance (C)	abfarad	abfarad	farad	farad

1 Paragilbert = 10 gilbert = ($4\pi NI$)
1 Paraoersted = 10 oersted = ($4\pi NI$/cm)
1 weber = 10^8 lines
1 Wb/m^2 = 10^4 guass
1 mega (M) = 10^6
1 micro (μ) = 10^{-6}
1 pico ($\mu\mu$) = 10^{-12}

Example 1: A magnetic CKT shown in fig having a cross-section side of square cross-section of 3 cm side. Each air gap is 2 mm wide. Each of the coil is wound with 1000 turns and exciting current is 1.0 A. The relative permeability of part *A* and part *B* may be taken as 1000 and 1200, respectively. Find

(i) Reluctance of part *A* and *B*
(ii) Reluctance of two air gaps
(iii) Total reluctance of the complete circuit
(iv) MMF
(v) Total flux
(vi) Flux density

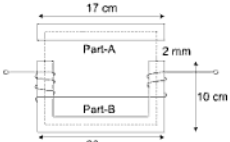


Fig. 2.7

Solution:

$$\text{Reluctance } R_l = \frac{l}{\mu_0 \mu_r A}$$

(i) *For part A*

Fig. 2.8

$$\text{length } l_A = \text{mean length of core}$$
$$l_A = 20 - (1.5 + 1.5) = 17 \text{ cm}$$
$$l_A = 0.17 \text{ m}$$
$$\mu_0 = 4\pi \times 10^{-7}$$
$$\mu_r = 1000, \; A = \text{Area of cross-section} = 3 \times 3 \times 10^{-4} \text{ m}^2$$

$$R_{l_A} = \frac{17 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 9 \times 10^{-4}} = 15.03 \times 10^4 \text{ AT/wb}$$

For part B

Fig. 2.9

length l_B = mean length of core

$= AB + BC + CD$

$= (10 - 1.5) + (20 - 3) + (10 - 1.5)$

$= 8.5 + 17 + 8.5 = 34$ cm

$$R_{l_B} = \frac{l_B}{\mu_0 \mu_{r_B} A} = \frac{34 \times 10^{-2}}{4\pi \times 10^{-7} \times 1200 \times 9 \times 10^{-4}}$$

$$= 25.04 \times 10^4 \text{ AT/wb}$$

(ii) Reluctance for air gap $R_l = \dfrac{l_g}{\mu_0 A}$ {$\mu_r = 1$ for air gap length of mean path

for air gap $= l_g = 2 + 2 = 4$ mm $= 4 \times 10^{-3}$ metre

Area of air gap through which flux passes $= 9 \times 10^{-4}$ m^2

$$R_{l_g} = \frac{4 \times 10^{-3}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 353.5 \times 10^4 \text{ AT/wb}$$

(iii) Electric circuit corresponds to magnetic CKT is

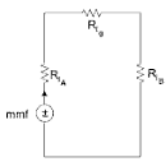


Fig. 2.10

Total reluctance $R_l = R_{lA} + R_{lg} + R_{lB}$

$$R_l = (15.03 + 25.04 + 353.5) \times 10^4$$

$$R_l = 393.57 \times 10^4 \text{ AT/wb}$$

(iv) Exciting current is 1.0 A and number of turns is $N = 1000$.

$$\text{mmf} = NI = 1000 \times 1.0 = 1000 \text{ AT}$$

(v) Total flux $\phi = \dfrac{\text{mmf}}{\text{Reluctance}} = \dfrac{1000}{393.57 \times 10^4}$

$$\phi = 5.08 \times 10^{-4} \text{ weber.}$$

(vi) Flux density $B = \dfrac{\text{Flux}}{\text{Area}} = \dfrac{5.08 \times 10^{-4}}{9 \times 10^{-4}}$

$$= 0.564 \text{ wb/m}^2.$$

Example 2: A steel ring of 25 cm mean diameter and of circular section 3 cm in diameter has an air gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 A. Calculate (i) mmf (ii) flux density of airgap (iii) magnetic flux (iv) reluctance (v) relative permeability of steel ring (vi) Reluctance of steel assume mmf taken by iron path is 35% of total mmf.

Solution:

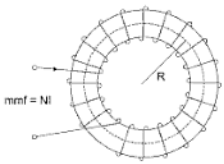


Fig. 2.11

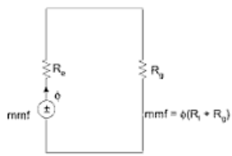


Fig. 2.12

Mean diameter = 25 cm

Number of turns $N = 700$

Current in a coil $I = 2$ A

(i) mmf = NI

$$= 700 \times 2 = 1400 \text{ AT}$$

(ii) reluctance of air gap is $R_g = \dfrac{l_g}{\mu_0 A}$

$$l_g = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ metre,}$$

$$\text{mmf across air gap} = \phi\, R_g \; \{\phi = B_g A_g\}$$

$$(\text{mmf})_g = B_g A_g R_g$$

$$(\text{Total mmf} - 35\% \text{ of total mmf}) = B_g \cdot \frac{l_g}{\mu_0}$$

$$(1400 - 0.35 \times 1400) = B_g l_g / \mu_0$$

$$B_g = \frac{\mu_0 \times 910}{l_g} = \frac{4\pi \times 10^{-7} \times 910}{1.5 \times 10^{-3}} = 0.762 \text{ wb/m}^2$$

(iii) magnetic flux $\phi = B_g A$

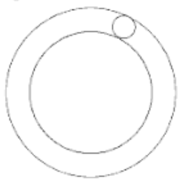


Fig. 2.13

Diameter of circular section ring = 3 cm

Area of cross-section = πr^2

$$= \pi\left(\frac{3}{2}\right)^2 = 7.07 \times 10^{-4} \text{ m}^2$$

$$\text{flux } \phi = 0.762 \times 7.07 \times 10^{-4}$$

$$\phi = 0.538 \text{ m Weber}$$

(iv) Total mmf = 1400 AT

total flux = 0.538×10^{-3} weber

$$\text{total reluctance} = \frac{\text{mmf}}{\text{flux}} = \frac{1400}{0.538 \times 10^{-3}} = 2.6 \times 10^6 \text{ AT/wb}$$

(v) Let relative permeability of steel ring is μ_r

$$R_l = \frac{l_g}{\mu_0 \mu_r A}$$

$$R_g = \frac{l_g}{\mu_0 A} = \frac{1.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 7.07 \times 10^{-4}} = \frac{1.5 \times 10^{8}}{88.79}$$

$$R_g = 0.01689 \times 10^8 = 1.69 \times 10^6$$

Reluctance of steel core $= R_t - R_g$

$$= (2.6 - 1.69) \times 10^6$$

$$= 0.91 \times 10^6 \text{ AT/wb}$$

Example 3: Magnetic CKT shown has cast steel core with dimension.

Mean length from A to B through outer limb = 0.5 m

Mean length from A to B through central limb = 0.2 m

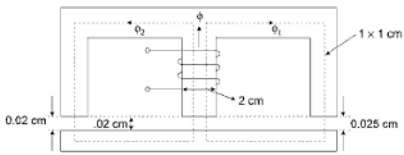


Fig. 2.14

In the magnetic CKT shown it is required to establish a flux of 0.75 mwb in the air gap of the central limb. Determine the mmf of the exciting coil if for the core material (a) $\mu_r = \infty$ (b) $\mu_r = 5000$

(a) $\mu_r = \infty$ there are no mmf drop in magnetic core.

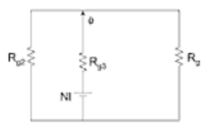


Fig. 2.15

$$\frac{l_c}{\mu_0\mu_r a} = R_{g_1} = \frac{0.025\times10^{-2}}{4\pi\times10^{-7}\times1\times10^{-4}} = 1.99\times10^6$$

$$R_{g_2} = \frac{0.02\times10^{-2}}{4\pi\times10^{-7}\times1\times10^{-4}} = 1.592\times10^6$$

$$R_{g_3} = \frac{0.02\times10^{-2}}{4\pi\times10^{-7}\times(1\times2)\times10^{-4}} = 0.796\times10^6$$

$$Ni = \phi[R_{g_3} + (R_{g_1} \parallel R_{g_2})]$$

$$Ni = 0.75\times10^{-3}(0.796 + .844)\times10^6$$

$$= 1260 \text{ AT}$$

(b) $\mu_r = 5000$

This means that the reluctance of magnetic core must be taken into consideration.

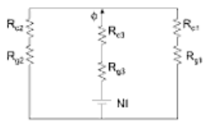


Fig. 2.16

$$R_{c_1} = \frac{l_c}{\mu_0\mu_r a} = \frac{0.5}{4\pi\times10^{-7}\times5000\times(1\times1)\times10^{-4}}$$

$$= R_{c_2} = 0.796\times10^6$$

$$R_{c_3} = \frac{0.2}{4\pi\times10^{-7}\times5000\times(2\times1\times10^{-4})} = 0.1596\times10^6$$

$$R_{c_T} = [(R_{c_1} + R_{g_1}) \parallel (R_{c_2} + R_{g_2})] + (R_{c_3} + R_{g_3})$$

$$= 2.24\times10^6$$

$$\text{mmf} = \phi R_{c_g} \quad [V = I\ R_{c_g}]$$

$$= 0.75 \times 10^{-3} \times 2.24 \times 10^{6}$$

$$\boxed{\text{mmf} = 1680 \text{ AT}}$$

Example 4: An iron ring 10 cm means diameter is made of round iron rod 1.5 cm in diameter of relative permeability 900 and has an air gap 5 mm in length. It has a winding of 400 turns. If the current through the winding is 3.4 amps. Determine (a) MMF (b) total reluctance of the circuit (c) flux in the ring (d) flux density in the ring. Neglect leakage.

Solution: The magnetic circuit is drawn in Fig. 2.17 below.

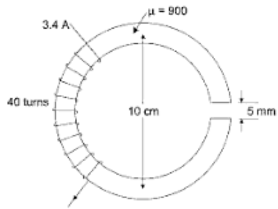


Fig. 2.17

Length of air gap—l_g = 0.5 cm.

Length of iron path—$l_i = (\pi \times 10 - 0.5) = 30.9$ cm

Area of cross-section, $a = \dfrac{\pi \times 15^2}{4} = 1.77 \text{ cm}^2$.

(a) $$\text{MMF} = \frac{4\pi NI}{10} \text{ gilberts}$$

$$= \frac{4\pi \times 400 \times 3.4}{10} = 1710 \text{ gilberts.}$$

(b) Total reluctance = $\dfrac{l_i}{a_i \mu_i}$

$$= \frac{30.9}{1.77 \times 900} + \frac{0.5}{1.77 \times 1}$$

$$= 0.0194 + 0.272 = 0.2914$$

(c) $\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{1710}{0.2914} = 5870$ maxwells

(d) $B = \frac{\phi}{a} = \frac{5870}{1.77} =$ **3320 guass Ans.**

Example 5: In Fig. a ring of composite material is shown. The length of the magnetic path and cross-sectional areas in iron, cast steel and air gap are 100 cm, 200 cm and 1 cm and 20 cm^2, 10 cm^2 and 20 cm^2, respectively. The length of path in iron is divided into two equal halves of 50 cm each. If the relative permeability of iron and cast steel are 300 and 900, respectively, find the current through a coil of 170 turns to produce a useful flux of 9000 lines in the air gap. Take leakage coefficient as 1.2.

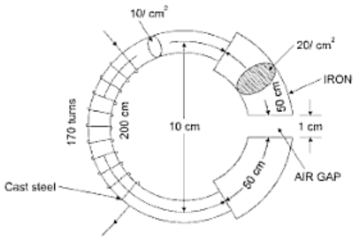


Fig. 2.18

Solution: (In R.M.K.S. Units)

MMF for air gap (*Mg.*)

$$\text{Flux in air gap} = \frac{\text{M.M.F. for air gap (Mg.)}}{\text{Reluctance}}$$

$$\text{Mg.} = \text{Flux in gap} \times \text{Reluctance of air gap}$$

$$\text{Reluctance} = \frac{1g}{\mu . a_g} = \frac{\text{length of the path in gap in m}}{\mu_0 . \mu_r \times \text{area of gap in m}^2}$$

$$= \frac{1\times 10^{-2}}{4\pi\times 10^{-7}\times 1\times 20\times 10^{-4}}$$

$$\therefore \qquad Mg = 9000\times 10^{-8}\times \frac{1\times 10^{-2}}{4\pi\times 10^{-7}\times 20\times 10^{-4}}$$

= 358 Amp-turns.

MMF for Iron path (*Mi*).

Mi = Total flux in iron × Reluctance of iron paths.

Total flux in iron = Leakage coefficient × Flux in gap

$= 1.2\times 9000\times 10^{-8} = 10800\times 10^{-8}$ wb.

$$\therefore \qquad Mi = 10800\times 10^{-8}\times \frac{100\times 10^{-2}}{4\pi\times 10^{-7}\times 300\times 20\times 10^{-4}}$$

= 143 Amp-turns.

MMF for cast steel path (*Mc*).

Mc = Total flux in cast steel × Reluctance of steel paths.

$$= 10800\times 10^{-8}\times \frac{200\times 10^{-2}}{4\pi\times 10^{-7}\times 900\times 10\times 10^{-4}}$$

= 191 Amp-turns.

∴ Total MMF required.

= *Mg* + *Mi* + *Mc*.

= 358 + 143 + 191 = 692 A.T.

$$\text{Hence, current through the coil} = \frac{\text{Total M.M.F. required}}{\text{No. of turns in coil}}$$

$$= \frac{692}{170} = \mathbf{4.06\ Amp} \quad \mathbf{Ans.}$$

Aliter (in c.g.s. system)

Amp-turns for the gap.

From eqn. (5.23)

$ATg = 0.796\ Hg\,.lg.$

$$\text{where, } Hg = \frac{\text{Flux in gap}}{\mu_g\times \text{area of gap}}$$

$$= \frac{9000}{1 \times 20} = 450$$

$$lg = 1 \text{ cm}$$

$$\therefore \quad ATg = 0.796 \times 450 \times 1$$

$$= 358 \text{ Amp-turns.}$$

Amp-turns for the iron.

$$ATi = 0.796\, Hi.li.$$

$$Hi = \frac{\text{Flux in iron}}{\mu \times \text{area of iron}}$$

Flux in iron = Leakage coefficient × Flux in gap.

$$= 1.2 \times 9000 = 10800 \text{ lines.}$$

$$\mu = 300$$

$$\therefore \quad Hi = \frac{10800}{300 \times 20} = 1.8$$

$$\text{So} \quad ATi = 0.796 \times 1.8 \times 100$$

$$= 143 \text{ Amp-turns.}$$

Amp-turns for cast steel.

$$ATc = 0.796\, Hc.lc.$$

$$Hc = \frac{\text{Flux in cast steel}}{\mu_c \times a_c}$$

$$= \frac{10800}{900 \times 10} = 1.2$$

$$\therefore \quad ATc = 0.796 \times 1.2 \times 200$$

$$= 191 \text{ Amp-turns.}$$

So, total ampere-turns required

$$= ATg + ATi + ATc$$

$$= 358 + 143 + 191 = 692$$

Hence, current through the coil,

$$= \frac{\text{Total ampere-turns required}}{\text{No. of turns in the coil}}$$

$$= \frac{692}{170} = \textbf{4.06 Amps} \quad \textbf{Ans.}$$

EXERCISE

1. Explain the terms:

(i) M.M.F.	(ii) Flux	(iii) Flux density
(iv) Reluctance	(v) Permeability	(vi) Permeance
(vii) Fringing	(viii) Coercivity	(ix) Retentivity

2. Find the relation between M.M.F., reluctance and flux for a circular ring having a relative permeability μ_r area of cross-section is A.
3. What are the similarities and disimilarities of magnetic circuits and electrical circuits?
4. Draw a magnetization curve and define the hysterisis and eddy current losses.
5. Explain the effect of AC excitation on magnetic circuits. (Hint: Transformers and ac machines are excited from ac source)
6. A 0.5 m long wire moves at right angles to its length at 40 m/s in uniform magnetic field of 1 Wb/m^2. Calculate the emf induced in the conductor when the direction of motion is (i) Perpendicular to field (ii) inclined at 30° to the direction of field. [**Ans:** 20 V, 10 V]
7. An iron ring has a mean circumferential length of 60 cm with an air gap of 1 mm and a uniform winding of 300 turns, when a current of 1 A flows through the coil, find the flux density. The relative permeability of iron is 300. [**Ans:** 0.1256 T]
8. For the AC excited magnetic circuit given below, calculate the excitation current and induced emf of the coil to produce a core flux of (0.6 sin 314 t) mwb.

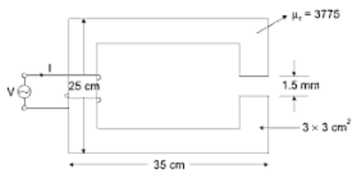


Fig. 2.19

9. A coil of insulated wire of 500 turns and of resistance 4 Ω is closely wound on iron ring. The ring has a mean diameter of 0.25 m and a uniform cross-sectional area of 700 mm^2. Calculate the total flux in the ring when a DC supply of 6 V is applied to the ends of the winding. Assume relative permeability of 550. [U.P.T.U. 2002]

   [Ans: 0.462 mWb]

10. An iron ring of mean length 50 cm and relative permeability 300 has an air gap of 1 mm. If the ring is provided with a winding of 200 turns and a current of 1 A is allowed to flow through. Find the flux density across the air gap. [U.P.T.U. June-2001]

    [**Ans.** 0.0942 T]

11. A coil of 1,000 turns is wound on a laminated core of steel having a cross-section of 5 cm^2. The core has an air gap of 2 mm cut at right angle. What value of current is required to have an air gap flux density of 0.5 T? Permeability of steel may be taken as infinity. Determine the coil inductance $\left(L = \frac{N\phi}{i}\right)$. [U.P.T.U. 2003–2004]

    [**Ans.** 0.314 H]

3

D.C. Network Analysis

3.1 INTRODUCTION

In this chapter we analyse the linear network for Direct-voltage and Direct-current.

For analysing these networks, we introduce two different laws KVL, KCL and different theorems.

3.2 CHARGE

The basic unit of charge is the charge of the electron, when electrons are removed from a substance, that substance becomes positively charged. A substance with an excess of electron is negatively charged.

The electron has a charge of 1.6021×10^{-19} coulomb and represented by Q or q.

3.3 ELECTRIC CURRENT

The phenomenon of transferring charge from one point in a circuit to another is described by the term electric current. It may be defined as the rate of flow electric charge across a cross-sectional boundary. A random motion of electron in a metal does not constitute a current unless there is a net transfer of charge with time.

$$i = \frac{dq}{dt}$$

3.4 AMPERES

If the charge q is given in coulombs and the time t is in sec, then the current is measured in *ampere*.

1 ampere corresponds to the motion of $\dfrac{1}{1.6\times10^{-19}} = 6.24 \times 10^{18}$ electrons past any cross-section of path in one second.

3.5 VOLTAGE

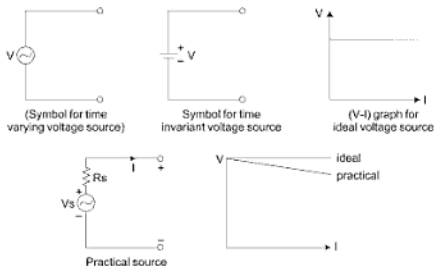
Once the battery circuit is closed by an external connection, the chemical energy (energy in chemical reaction) is expended as work. The energy per unit charge or work per unit charge is given the name *voltage* $\left(v = \dfrac{w}{q}\right)$.

A voltage can exist between a pair of electrical terminals whether a current is flowing or not. (According to principle of conservation of energy, the energy that is expended in forcing charge through the element must appear somewhere) else.

3.6 VOLTAGE SOURCE

Voltage source is assumed to deliver energy with a specified terminal voltage, $v(t)$, and may or may not be dependent of the current from the source.

Voltage source is said to be ideal if the voltage does not depend on current.



(Symbol for time varying voltage source)

Symbol for time invariant voltage source

(V-I) graph for ideal voltage source

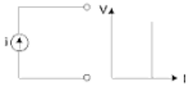
Practical source

3.7 CURRENT SOURCE

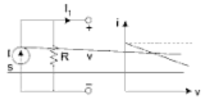
The current source is assumed to deliver energy with with a specified current through the terminals $i(t)$ and may or may not be dependent on terminal voltage.

A current source is said to be ideal if it is independent of voltage.

Transistors, vacuum tubes and photo electric cells make use of current source in their model representation.



Ideal source (I does not depend on v)



Practical current source

3.8 POWER

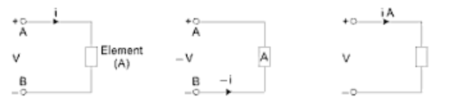
Power is the rate at which energy is expended, and shall represent it by P or P.

If one joule of energy is expended in transferring one coulomb of charge through the device, then the rate of energy expenditure in transferring one coulomb of charge per second through the device is one watt.

This absorbed power must be proportional both to the number of coulombs transferred per second or current and to the energy needed to transfer one coulomb through the element or voltage.

$$P = vi = \frac{\text{Joule}}{\text{Coulomb}} \times \frac{\text{Coulomb}}{\text{Sec}} = \text{Joule/sec (watt)}$$

⇒ If a positive current entering at a +ve terminal, then energy is being supplied to the element. Otherwise, the element is delivering energy to some external device.



(a) Power P = vi is absorbed by the element (Terminal A is +ve the w.r.t. to terminal B)

(b) P = (v) (– i) = vi is absorbed by element

(c) P = (v) (– i) = – vi is absorbed by element or vi is delivered by element

⇒ So, if one terminal of the element is v volts positive with respect to other terminal and if a current i is entering the element through the +ve terminal, then the power $P = vi$ is being delivered to the element or absorbed by the element.

3.9 SOME BASIC DEFINITIONS

Some desirable definitions before going to further discussion.

3.9.1 Passive Network

A network containing circuit elements without energy sources, then the network is called the *passive element* and the elements of the network are passive elements.

Elements, which cannot generate energy but can dissipate or store energy are known as passive elements, for example, resistor, inductor, capacitor.

3.9.2 Active Network

A network is said to be active if it contains energy sources together with other passive elements.

An active element generally supplies electrical energy to the network, for example, battery, generator.

3.9.3 Lumped Network

Physically separate resistors, capacitors and inductors are called lumped parameters. These elements will be connected together by wires (leads) having practically zero resistance, then the network with number of lumped elements and a set of connecting leads, is called a lumped-parameter network. In such a circuit KVL and KCL holds good.

3.9.4 Distributed Network

A network is said to be distributed if the network elements (R, L, C) cannot be electrically separated and individually isolated as separate elements. Examples: Transmission lines, winding, of transformers and generators. If the length of transmission line increases or decreases, value of effective (R, L, C) changes and cannot be separated physically.

3.9.5 Bilateral

The elements to be considered for electric networks are assumed to be bilateral when the voltage and current relations are same irrespective of direction of flow of current Ex- R, L, C, etc. However, for unilateral elements the voltage and current relation are different for two possible direction of flow of currents Example: diodes.

3.9.6 Time Invariant

A system or network in which the parameters do not change with time is called a time-invariant system or time-invariant network.

Thus, in a time-invariant system the relation between its response and excitation always remain the same, regardless at the time of application of the input. Time invariant systems are also called constant-parameter systems.

A network is called *time varying* if it contains one or more time-varying elements.

3.9.7 Network/Circuit

An electrical network is an interconnection of active and passive elements such as resistance, inductance, capacitance and sources.

An electrical circuit is a network that has a closed loop, giving a return path for the current. A network is a connection of two or more components and may not necessarily be a circuit.

3.9.8 Circuit Parameters

Constant of a circuit are parameters and characterizes by the relationship between two variable (voltage, current).

A circuit element is ideal, when its voltage and current are related by

- Constant of proportionality or
- A differential or integral relationship

These relationships can be shown to be linear which means that an ideal circuit element has linear behaviour. Most practical circuits can be modelled by suitable interconnection of these elements like *resistance, capacitance, inductance* and *sources.*

3.9.9 Branch

A line segment representing a circuit element is called a branch or number of elements connected between two nodes constitute a branch.

3.9.10 Node

End point of a branch is called a node or node is a junction when two or more than two branches meet. Fundamental node (junction) is one where more than two branches meet (e.g. (4) and (1, 7, 8).

3.9.11 Path

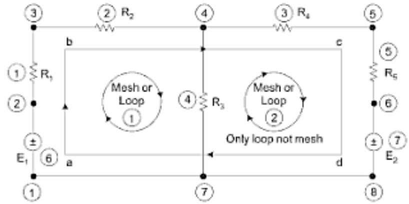
The movement through elements from one node to another node without repeating any node is called a path.

3.9.12 Mesh and Loop

A set of branches forming a closed path is called a loop and a loop that does not contain any loop within it is called a mesh.

Given network having a 7 branches $b = 7$

Given network having a 6 nodes $n = 6$ {1, 7, 8 are same node so treated as a one node},

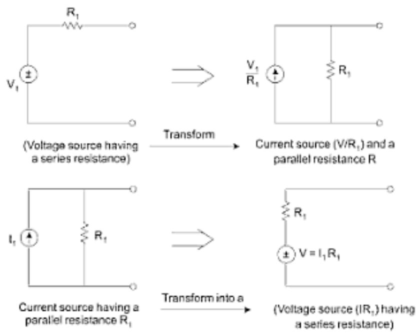


Above network having a 2 meshes or 2 loops + 1 loop (*a b c d a*) (which is not a mesh because this loop contains an another loop (1) and (2)).

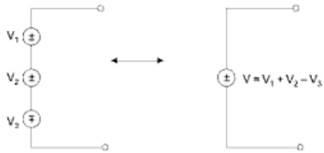
3.10 SOURCE TRANSFORMATIONS

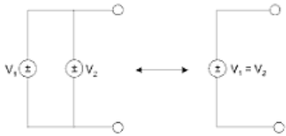
Voltage source having a series resistance may be transformed into a current

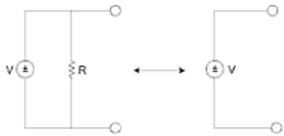
(1) Source having a parallel resistance and vice versa.



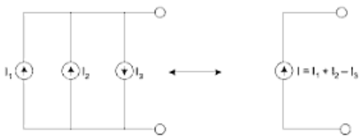
(2) Voltage sources cannot be connected in parallel unless the two sources have identical voltages and similarly the current sources cannot be connected in series unless identical. The paralleling of voltage sources with non-similar voltage, results in heavy currents and damage equipment.

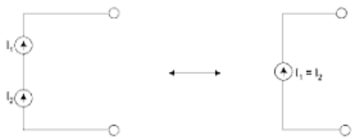




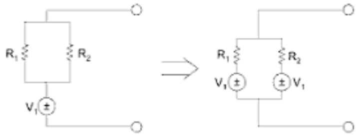


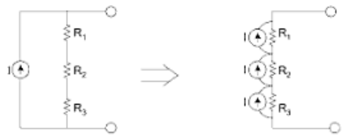
(A resistor in parallel with a voltage source may be ignored or omitted entirely from network)





Resistor in series with current source may be ignored.





3.11 VOLTAGE DIVISION RULE

Consider n resistors that are connected in series. The voltage V_i across any resistor R_i is

$$V_i = I\,R_i = \left[\frac{R_i}{R_1 + R_2 + R_3 + \ldots + R_n}\right] V$$

here, V is total applied voltage.

3.12 CURRENT DIVISION RULE

Consider n resistors that are connected in parallel. Then, current I_i through any resistor R_i ($i = 1, 2 \ldots n$) is

$$R_i = \left[\frac{1/R_i}{(1/R_1 + 1/R_2 + 1/R_3 + \ldots + 1/R_n)}\right] I$$

I is the total current ($I = I_1 + I_2 + I_3 + \ldots + I_n$).

3.13 KIRCHHOFF'S LAWS

There are two Kirchhoff's laws, which have an efficient method for calculating the currents in the branches of a network of conductors. These laws are given below.

3.13.1 Law 1 (Point Law or Current Law)

The algebraic sum of the currents at any node or junction in a network is zero.

Let I_1, I_2 and I_3 be the currents flowing towards the junction and I_4 and I_5 be the currents flowing away from the junction as shown in Fig. 3.1.

Fig. 3.1

If currents flowing towards the junction are assigned positive signs and those flowing away from the junction at negative signs, then

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0 \quad ...(3.1)$$

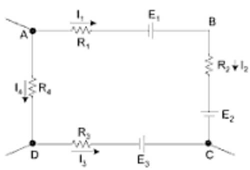
or,

$$I_1 + I_2 + I_3 = I_4 + I_5 \quad ...(3.2)$$

Thus, the currents leaving a junction are equal to the currents entering the junction.

3.13.2 Law 2 (Mesh Law or Voltage Law)

In any closed loop of network having active and passive elements the algebraic sum of the potential drops of passive and active elements around the loop or



mesh is zero. If we consider that the drops in register in the direction of current is positive and take voltage source with sign (+ve or –ve) occur in the way.

Let us travel from A to B to C to D to A in clockwise direction then.

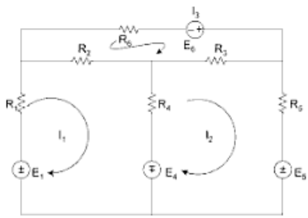
$$I_1R_1 - E_1 + I_2R_2 + E_2 + E_3 - I_3R_3 - I_4R_4 = 0$$

$$\boxed{I_1R_1 + I_1R_2 - I_3R_3 - I_4R_4 = E_1 - E_2 - E_3}$$

3.14 MAXWELL'S MESH OR LOOP METHOD

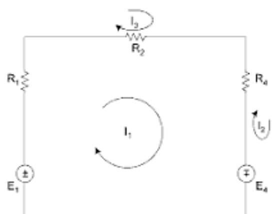
In this method any network is divided into meshes and in each mesh a separate current is assumed to circulate. The direction of all the mesh current can be taken either as clockwise or anti-clockwise, the solution becomes more systematic if the directions of all currents are assumed to be the same.

Method for finding the mesh currents



⇒ Above network can be divided into 3 meshes

⇒ Let each mesh have a currents I_1, I_2 and I_3



Mesh 1

For systematic solution take all the currents in one direction clockwise.

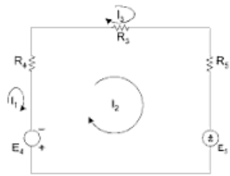
1st taking a Mesh – 1 and apply *KVL*. Branch R_3 is common with mesh 3 and branch R_4 is common with mesh 2

$$-E_1 + I_1R_1 + (I_1 - I_3)R_2 + (I_1 - I_2)R_4 - E_4 = 0$$

$$\boxed{I_1(R_1 + R_2 + R_4) - I_2R_4 - I_3R_2 = E_1 + E_4} \quad ...(1)$$

We are going in the direction of I_1.

Now take mesh 2 and apply *KVL* in the direction of mesh currents.



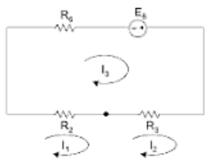
Mesh 2

Apply *KVL* in the direction of I_2

$$E_4 + \underbrace{(I_2 - I_1)R_4}_{\text{drop in the direction of } I_2} + \underbrace{(I_2 - I_3)R_3}_{\text{drop in the direction of } I_2} + I_2R_5 + E_5 = 0$$

$$\boxed{-I_1R_4 + I_2(R_3 + R_4 + R_5) - I_3R_3 = -E_4 - E_5} \quad ...(2)$$

Now take mesh 3 and apply *KVL* in the direction of mesh-current.



Mesh 3

Apply *KVL* in the direction of I_3.

$$(I_3 - I_1)R_2 + (I_3 - I_2)R_3 + I_3R_6 - E_6 = 0$$

$$\boxed{-I_1R_2 - I_2R_3 + I_3(R_2 + R_3 + R_6) = E_6} \quad ...(3)$$

Solving (1) (2) and eq. (3) we can find the mesh currents.

3.15 METHOD FOR WRITING THE MESH EQUATION IN MATRIX FORM

(i) Let *ckt* be divided into a m number of meshes.

(ii) Number of mesh currents ($I_1, I_2, -I_5 I_m$) is same as the number of mesh. Assume all mesh currents are in same direction.

(iii) Let $E_1, E_2 \ldots E_m$ are the algebraic sum voltage sources of each mesh.

⇒ So the generalized matrix with m-number of meshes.

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots & R_{1m} \\ R_{21} & R_{22} & R_{23} & \cdots & R_{2m} \\ R_{31} & R_{32} & R_{33} & \cdots & R_{3m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{n1} & R_{n2} & R_{n3} & \cdots & R_{nm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_m \end{bmatrix}$$

$$\underbrace{[R]_{m\times m}}_{\text{mesh resistance matrix}} \quad \underbrace{[I]_{m\times 1}}_{\text{mesh current vector}} = \underbrace{[E]_{m\times 1}}_{\text{input voltage vector}}$$

⇒ If only voltage sources and resistors are present, then

(iv) All the resistances through which the loop current I_j (current of j[th] loop) flows in the j[th] loop (j = 1, 2, 3 ... m) are summed and the sum is denoted by R_{jj} ($R_{11}, R_{22}, R_{33} \ldots R_{mm}$). Then the coefficient of I_j is R_{jj} is taken with *positive sign*. R_{jj} is called self resistance of j^{th} loop or mesh.

(v) All the resistances through which the loop current I_j (current of j^{th} loop) and loop current I_k (current of k^{th} loop) flows are summed up and denoted by R_{jk} ($R_{12}, R_{13} \ldots R_{1m}, R_{21}, R_{23} \ldots R_2 m$ ...) then the sign of R_{jk} is negative.

R_{jk} is called mutual resistances (common resistance) beween the j^{th} and k^{th} meshes.

(vi) Let Ej be the effective voltage in the j^{th} loop through which the loop current I_j flows. The sign of the term E_j is *positive* if the direction of E_j is same as that of I_j (ot current Ij enters to a –ve terminal of E_j).

(vii) For networks having only passive elements and without any dependent source, the resistance matrix becomes symmetric ($R_{jk} = R_{kj}$).

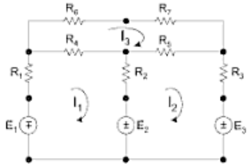
(viii) **Number of mesh equations** are $m = b - n + s$

$b \rightarrow$ number of branches

$n \rightarrow$ number of nodes

$s \rightarrow$ number of separate parts

Example: Write the mesh equation in matrix form for the given Fig.



Total no. of branches $b = 10$

Nodes $n = 8$

$s = 1$

Number of mesh equation $m = 10 - 8 + 1 = 3$

Resistance matrix is form of 3×3.

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1' \\ E_2' \\ E_3' \end{bmatrix}$$

R_{11} = self resistance of loop (1) having current $I_1 = R_1 + R_2 + R_4$

R_{12} = mutual resistance having currents I_1 and I_2

$= -(R_2) = R_{21}$

$R_{13} = R_{31} = -(R_4)$ = common resistance between mesh (1) and mesh (3)

$R_{22} = R_2 + R_5 + R_3$

$R_{33} = R_4 + R_5 + R_6 + R_7$ (total resistance of loop – 3 having current I_3)

$R_{23} = R_{32} = -(R_5)$

E_1^1 = effective voltage of loop – 1

$= -E_1 - E_2$

E_2^1 = effective voltage of loop – 2

$= E_2 - E_3$

E_3^1 = effective voltage of loop – 3

= 0 no source

So matrix is

$$\begin{bmatrix} (R_1+R_2+R_4) & -R_2 & -R_4 \\ -R_2 & (R_2+R_5+R_3) & -R_5 \\ -R_4 & -R_5 & (R_4+R_5+R_6+R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$= \begin{bmatrix} -(E_1+E_2) \\ E_2-E_3 \\ 0 \end{bmatrix} \quad \textbf{Ans.}$$

3.16 NODAL ANALYSIS

Mesh analysis is used to find out unknown currents in the mesh. Nodal analysis is used to find the unknown voltage at different nodes.

Steps for Nodal analysis

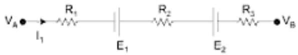
(i) Select a datum node or reference node and assume its potential to be zero.

(ii) Now select different suitable nodes and assign them a voltage with respect to a datum node.

(iii) All branch current is assigned outward from a node at which a nodal analysis have to applied and assume that node should be at higher potential.

(iv) Apply KCL at each node of unknown voltage and write down the equation in terms of node pair voltage and circuit parameter.

⇒ Let us consider a general branch having a current I_1 outwards from node A. V_A and V_B are node voltage

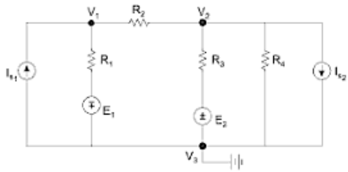


$$V_{AB} = V_A - V_B = I_1R_1 - E_1 + I_1R_2 + E_2 + I_1R_3$$

$$\Rightarrow \qquad I_1 = \frac{V_A - (-E_1) - (E_2) - V_B}{R_1 + R_2 + R_3} = \frac{\text{Voltage between node } A \text{ and } B}{\text{Total resistance between nodes } A, B}$$

⇒ I_1 is outward from node A. Node A will be at higher potential, so we will subtract all the voltages from V_A.

Now consider a circuit



(i) First, select suitable nodes. We will select a node where more than two branches are incident. Select one node as a datum node.

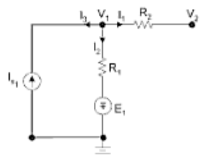
(ii) Assign a node voltage to all the selected nodes. Let the node voltages be V_1, V_2 with respect to datum node $V_3 = 0$ volt.

(iii) Now assign the direction of current at the node at which we have to apply nodal analysis.

Nodal analysis at node 1

(a) We will assume that all the branch currents are *outward* from this particular node.

(b) We will assume that this particular node at which we have to apply nodal analysis should be at higher potential.



(c) Apply KCL at node 1

$$I_1 + I_2 + I_3 = 0$$

$$\boxed{\frac{V_1 - V_2}{R_2} + \frac{V_1 + E_1}{R_1} - I_{s_1} = 0}$$

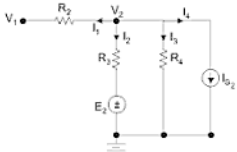
$$\left\{ \begin{aligned} I_1 &= \frac{V_1 - V_2}{R_2} \\ I_2 &= \frac{V_1 - (-E_1) - 0}{R_1} \\ I_3 &= -I_{s_1} \end{aligned} \right.$$

Nodal analysis at node 2

(a) We will assume that all the branch currents are outward at this particular node.

(b) This particular node should be at higher potential.

(c) 4 branches are connected to node 2.



Apply KCL at node 2

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\boxed{\frac{V_2 - V_1}{R_2} + \frac{V_2 - E_2}{R_3} + \frac{V_2}{R_4} + I_{s_2} = 0}$$

$$\begin{cases} I_1 = \dfrac{V_2 - V_1}{R_2} \\ I_2 = \dfrac{V_2 - E_2 - 0}{R_3} \\ I_3 = \dfrac{V_2 - 0}{R_4} \\ I_4 = + I_{s_2} \end{cases}$$

3.17 CHOICE OF METHOD MESH OR NODAL

(1) The number of voltage variables equals the number of fundamental nodes or junction minus one. Nodal equation = $(n - 1)$ and every voltage source connected to reference node reduces the number of unknown by one.

(2) The number of current variables equals the number of meshes. Every current source in a mesh reduces the number of unknowns by one. Number of mesh equation = $b - n + 1$.

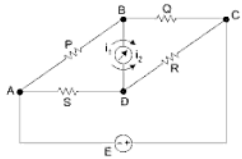
(3) Mesh analysis only applies to planar circuit (circuit that can be drawn using only two dimensions).

(4) Nodal analysis can be applied to both planar and non-planar circuits.

The method with the least unknown to solve is selected. For circuits that are non-planar and cannot be redrawn in a planar form, then nodal analysis is only the choice.

3.18 WHEAT STONE BRIDGE

Wheat stone bridge consists of four ratio arms *AB*, *BC*, *CD* and *DA* and two conjugate arms *AC* and *BD* as shown in figure.



Conjugate arms are those arms for which the condition remains unchanged by interchanging their positions.

When the bridge is balanced, no current flows through the galvanometer and hence *B* and *D* are at the same potential.

$$V_{AB} = V_{AD} \{V_A - V_B = V_A - V_D \text{ and } V_B = V_D\}$$

$$i_1 P = i_2 S \quad \text{...(1)}$$

$$V_{BC} = V_{DC}$$

$$i_1 Q = i_2 R \quad \text{... (2)}$$

From (1) and (2)

$$\boxed{\frac{P}{Q} = \frac{S}{R}}$$

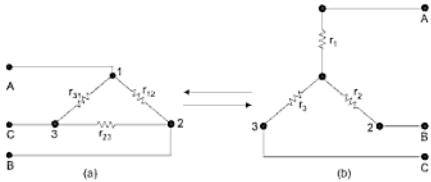
If the value of *R* and ratio *P*/*Q* are known, then unknown resistance *S* can be calculated.

3. 19 DELTA-STAR TRANSFORMATION

A transformation in which delta-connected resistance can be replaced by an equivalent star, so that the resistance measured between any two terminals is unchanged, is known as delta-star transformation.

When *A* is opened, the resistance between the terminals *B* and *C* of circuit (*a*)

$$= \frac{r_{23}[r_{12} + r_{31}]}{r_{23} + [r_{12} + r_{31}]}$$



And the resistance between terminals *B* and *C* of circuit (b).

$$= r_2 + r_3$$

If the circuits (*a*) and (*b*) of figure are identical, then the resistance between any pair of lines will be the same when the third line is opened. Hence,

When *A* is opened, $$r_2 + r_3 = \frac{r_{23}[r_{12} + r_{31}]}{r_{12} + r_{23} + r_{31}} \qquad ...(1)$$

When *B* is opened, $$r_3 + r_1 = \frac{r_{31}[r_{12} + r_{23}]}{r_{12} + r_{23} + r_{31}} \qquad ...(2)$$

When *C* is opened, $$r_1 + r_2 = \frac{r_{12}[r_{23} + r_{31}]}{r_{12} + r_{23} + r_{31}} \qquad ...(3)$$

Adding eqns. (1), (2) and (3) together and dividing both the sides by 2, we obtain,

$$r_1 + r_2 + r_3 = \frac{r_{12}r_{23} + r_{23}r_{31} + r_{31}r_{12}}{r_{12} + r_{23} + r_{31}} \qquad ...(4)$$

Subtracting eqns. (1), (2) and (3) respectively from eqn. (4), we get,

$$r_1 = \frac{r_{12}r_{31}}{r_{12} + r_{23} + r_{31}} \qquad ...(5)$$

$$r_2 = \frac{r_{12}r_{23}}{r_{12} + r_{23} + r_{31}} \qquad ...(6)$$

and, $$r_3 = \frac{r_{23}r_{31}}{r_{12} + r_{23} + r_{31}} \qquad ...(7)$$

3.20 STAR-DELTA TRANSFORMATION

From eqns. (5), (6) and (7), we obtain

$$r_1\,[r_{12} + r_{23} + r_{31}] = r_{31}r_{12} \qquad ...(8)$$

$$r_2\,[r_{12} + r_{23} + r_{31}] = r_{12}r_{23} \qquad ...(9)$$

$$r_3\,[r_{12} + r_{23} + r_{31}] = r_{23}r_{31} \qquad ...(10)$$

Dividing eqn. (10) by eqn. (8)

$$\frac{r_3}{r_1} = \frac{r_{23}}{r_{12}}$$

or,

$$r_{23} = \frac{r_3}{r_1}\,r_{12} \qquad ...(11)$$

Dividing eqn. (9) by eqn. (11)

$$\frac{r_2}{r_1} = \frac{r_{23}}{r_{31}}$$

or,

$$r_{31} = \frac{r_1}{r_2}\,r_{23} = \frac{r_1}{r_2}.\frac{r_3}{r_1}\,r_{12}$$

$$= \frac{r_3}{r_2}.r_{13} \qquad ...(12)$$

Substituting the values r_{23} and r_{31} in eqn. (8)

$$r_1\left[1 + \frac{r_3}{r_1} + \frac{r_3}{r_2}\right]r_{12} = r_{31}r_{12}$$

or,

$$r_{31} = \frac{r_1r_2 + r_2r_3 + r_3r_1}{r_2} \qquad ...(13)$$

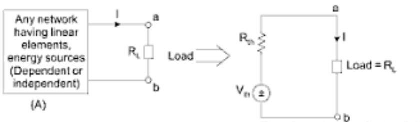
Similarly,

$$r_{12} = \frac{r_1r_2 + r_2r_3 + r_3r_1}{r_3} \qquad ...(14)$$

$$r_{23} = \frac{r_1r_2 + r_2r_3 + r_3r_1}{r_1} \qquad ...(15)$$

3.21 THEVNIN'S THEOREM

Thevnin's theorem states that any two terminal linear network containing energy sources (voltage or current) and impedances or resistances can be replaced with an equivalent circuit consisting of a voltage source (V_{th}) and a series resistance or impedance (R_{th} or Z_{th}) connected to load.



Thevnin equivalent of network A.

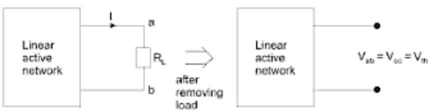
$$\Rightarrow \boxed{I = \frac{V_{th}}{R_{th} + R_L}}$$

Thevnin's theorem is especially useful in analyzing power systems and other circuits where *load* is subject to change and recalculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.

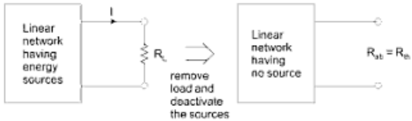
So, Thevnin's theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance and a series load.

3.22 STEPS TO FOLLOW FOR THE THEVNIN'S THEOREM

(1) Find the Thevnin source voltage (V_{th}) by removing, the load resistor from the original circuit and calculate the voltage across the open connection point where the load resistor used to be.



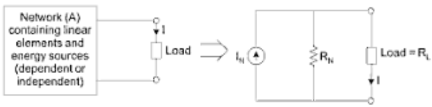
(2) Find the Thevnin resistance (R_{th}) by removing all power sources in the original circuit (voltage sources shorted and current sources open) and then calculate total resistance between the open connection points.



(3) Draw the Thevnin equivalent circuit, with the V_{th} in series with R_{th}. The load resistor reattaches between the two open points of the equivalent circuit.

3.23 NORTON'S THEOREM

Norton's theorem states that any two terminal linear networks containing energy sources and resistances or impedances can be replaced by an equivalent circuit having a current source (I_N) and parallel resistance or impedance connected to a load.

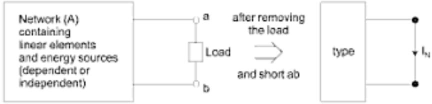


So, Norton's theorem is a way to reduce a network to an equivalent circuit composed of a single current source, parallel resistance (R_N) and parallel load.

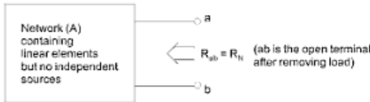
$$I = \left(\frac{R_N}{R_N + R_L}\right) I_N$$

3.24 STEPS TO FOLLOW FOR NORTON'S THEOREM

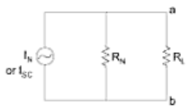
(1) Find the Norton source current (I_N) by removing the load resistor from the original circuit and starting the open terminal. This short circuit current is I_{Norton}.



(2) Find the Norton resistance ($R_N = R_{th}$) by removing all energy sources (independent voltage sources shorted and independent current source open) and calculate total resistance between the open connection point after removing load.



(3) Draw the Norton's equivalent circuit with the Norton's current source in parallel with the Norton's resistance. The load resistor reattaches between the two open points of equivalent circuit.



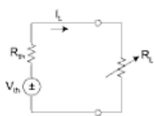
3.25 MAXIMUM POWER TRANSFER THEOREM

This theorem states that the maximum amount of power will be dissipated by a load resistance when that load resistance is equal to Thevnin/Norton resistance of the Network supplying the power. If the load resistance is lower or higher than the R_{th} or R_N, its dissipated power will be less than maximum.

Application

Practical applications of this might include stereo amplifier design (seeking to maximum power delivered to speakers) or electric vehicle design (seeking to maximum power delivered to drive motor). In stereo system design speaker "impedance" is matched to amplifier "impedance" for maximum sound power output.

Proof: For a DC Network



Power delivered to load R_L is P_L

$$P_L = I_L^2 R_L$$

$$= \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$

To find the value of R_L that absorbs a maximum power from the given practical source, we differentiate with respect to R_L.

$$\frac{dP_L}{dR_L} = \frac{(R_{th} + R_L)^2 V_{th}^2 - V_{th}^2 R_L (2)(R_{th} + R_L)}{(R_{th} + R_L)^4}$$

Equate derivative to zero.

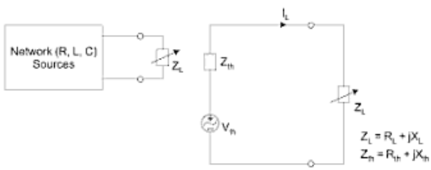
$\Rightarrow$ $$2R_L(R_{th} + R_L) = (R_{th} + R_L)^2$$

$\Rightarrow$ $$\boxed{R_L = R_{th}}$$

$\Rightarrow$ A network delivers the maximum power to a load resistance R_L when R_L is equal to the Thevnin equivalent (or source) resistance of the network.

3.26 MAXIMUM POWER TRANSFER THEOREM FOR AC NETWORK*

The power transferred from an active network to a load depends on load,



If (i) Load is a pure resistance.

(ii) Load resistance and load reactance are independently variable.

(iii) Load has a variable impedance but a constant power factor.

*Not for UPTU (TEE 101/201) student

We have to find the value of Z_L which receives or absorbs maximum power from the network.

$$I_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{V_{th}}{(R_{th} + R_L) + j(X_{th} + X_L)}$$

$$|I_L| = \frac{V_{th}}{\sqrt{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$$

Average power transferred to load is P_L

$$P_L = I_L^2 R_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$$

For maximum power transfer theorem R_L and X_L will be varied independently.

Put $\dfrac{dP_L}{dR_L} = 0$ and $\dfrac{dP_L}{dX_L} = 0$

(i) Let us consider varying X_L and keeping R_L constant

$$\frac{dP_L}{dX_L} = 0$$

$$= \frac{0 - 2V_{th}^2 R_L (X_{th} + X_L)}{[(R_{th} + R_L)^2 + (X_{th} + X_L)^2]^2} = 0$$

$$\boxed{X_L = -X_{th}}$$

(ii) Now, $X_L = -X_{th}$ and $\dfrac{dP_L}{dR_L} = 0$

$$P_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2} \qquad \text{Put } X_L = -X_{th}$$

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}[V_{th}^2 R_L (R_{th} + R_L)^{-2}] = 0$$

$$2R_L = R_{th} + R_L$$
$$R_L = R_{th}$$

So, for absorbing maximum power by load Z_L, Z_L must be equal to $R_{th} - jX_{th} = Z^*_{th}$

$$Z_L = \text{Conjugate of } Z_{th}$$

Hence, maximum power is transferred from an AC network to a load if the load impedance is equal to the complex conjugate of Thevnin equivalent impedance.

$$Z_L = Z^*_{th}$$

Case 2: Consider R_L and X_L are not independently varied but only magnitude (Z_L) of load is varied for constant power factor $\left(\cos\phi = \frac{R_L}{Z_L}\right)$.

$$P_L = \frac{V_{th}^2\, R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$$

$$R_L = Z_L \cos\phi \text{ and } X_L = Z_L \sin\phi$$

$$P_L = \frac{V_{th}^2\, Z_L \cos\phi}{(R_{th} + Z_L \cos\phi)^2 + (X_{th} + Z_L \sin\phi)^2} = \frac{N}{D}$$

$$\frac{dP_L}{dZ_L} = \frac{V_{th}^2 . \cos\phi}{D}\left[D\frac{d}{dZ_L}(Z_L) - Z_L \cdot \frac{d}{dZ_L}(D)\right] = 0$$

$$\frac{d}{dZ_L}(Z_L) = Z_L \cdot \frac{d}{dZ_L}(D)$$

$$D = Z_L\,[2(R_{th} + Z_L \cos\phi)\cos\phi + 2(X_{th} + Z_L \sin\varphi)\sin\varphi]$$
$$D = 2Z_L\,[(R_{th}\cos\phi + X_{th}\sin\varphi) + Z_L]$$

$$R_{th}^2 + Z_L^2\cos^2\phi + \cancel{2R_{th}Z_L\cos\phi} + X_{th}^2 + Z_L^2\sin^2\varphi + \cancel{2X_{th}Z_L\sin\phi}$$
$$= \cancel{2Z_L R_{th}\cos\phi} + \cancel{2Z_L X_{th}\sin\phi} + 2Z_L^2$$
$$(R_{th}^2 + X_{th}^2) = Z_L^2$$
$$|Z_{th}| = |Z_L|$$

So, when magnitude of load impedance is varied but the power factor $(\cos\phi)$ remains constant maximum power is transferred when $|Z_L| = |Z_{th}|$

In transformer to achieve maximum power transfer, where Z_L can be adjusted but ϕ cannot be.

In AC network when $|Z_L| = R_L$ for unity power factor. Load absorbs maximum power when $\boxed{R_L = \sqrt{R_{th}^2 + X_{th}^2}}$.

3.27 SUPERPOSITION THEOREM

This theorem states that in a linear network, containing more than one independent energy source, then the complete response (voltage or current) in any branch of network is equal to the sum of the response due to each independent source acting one at a time with all other ideal independent sources are made inactive (short the voltage source and open the current source).

3.28 STEPS FOR ANALYSING A CIRCUIT

(1) Take any one independent source in the circuit.
(2) Make all other independent sources inactive.
(3) Dependent sources will not be disturbed.
(4) Determine the magnitude and direction of response (voltage or current) in the desired branch by a single source selected.
(5) Now take another independent source and calculate the response in a desired branch using step 1 to 4.
(6) Add all the component of responses in the desired branch. Algebraic addition is to be done for DC networks and phasor addition for AC network.

Limitations

(1) Superposition theorem cannot be applied for calculation of power.
(2) Superposition is the combined properties of additivity and homogeneity of linear network. If any one property is violated then principle of superposition is not valid.
(3) Applicable only for linear networks.
(4) Properties of additivity and homogeneity are fundamental to superposition principle.
(5) There should be more than one source.

Additivity

If X_1 is the excitation (cause) and Y_1 is the response of a given network, related by $Y_1 = CX_1$.

Similarly for an excitation X_2 response $Y_2 = CX_2$

If the network is linear, then for excitation $(X_1 + X_2)$ the response will be $(Y_1 + Y_2)$

$$\boxed{(Y_1 + Y_2) = C\,(X_1 + X_2)}$$

If the excitation and response are related by the linear equation $Y_1 = CX_1 + d_1$. The network is not linear as for double the excitation response will not double.

Homogeneity: If all the sources are multiplied by a constant, the response is multiplied by the same constant.

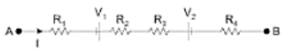
If, $\quad Y_1 = CX_1 \quad$ Let excitation be multiplied by m become mx_1.

Then response $\quad Y = C(mx_1) = mY_1$

SOLVED PROBLEMS

Points to be remembered

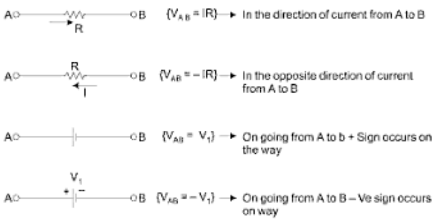
(1) Find $V_{AB} = V_A - V_B$ = Potential difference between A and B.



(1) For finding the potential difference between two points, start from one point and reach to another point by any path.

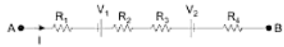
(2) On going from A to B, add all the voltage drop (IR) and source voltages (v_1, v_2, ...).

(3) If we are going in the direction of current take drop *positive* otherwise *negative*.



$$V_{AB} = V_A - V_B = IR_1 + (-V_1) + IR_2 + IR_3 + (V_2) + IR_4$$
$$V_{AB} = IR_1 - V_1 + IR_2 + IR_3 + V_2 + IR_4$$

(4) If we know the node voltage V_A and V_B or V_{AB}, we can find the branch current I.



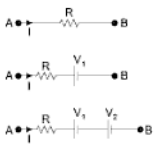
I is the current going out from node (A).

(5)
$$I = \frac{V}{R} = \frac{\text{Total potential difference between } A \text{ and } B}{\text{Total resistance for path } A \text{ to } B}$$

For finding V start from node A and subtract all the voltages from V_A with sign.

$$I = \frac{V_A - (-V_1) - V_2 - V_B}{R_1 + R_2 + R_3 + R_4}$$

(6)

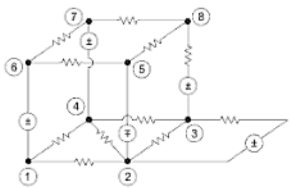


$$\Rightarrow \quad I = \frac{V_A - V_B}{R}$$

$$\Rightarrow \quad I = \frac{V_A - V_1 - V_B}{R}$$

$$\Rightarrow \quad I = \frac{V_A - V_1 - V_2 - V_B}{R}$$

Example 1: How many independent loop equations are there in the given network shown?



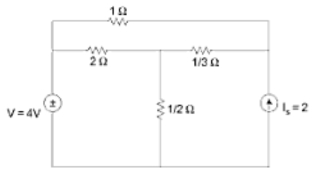
Solution: Total number of independent loop equations are m

$$m = b - n + 1$$

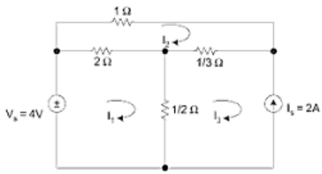
Since, there are 14 branches and 8 nodes, branch (it is a part of between two nodes).

$$m = 14 - 8 + 1 = 7 \text{ equation}$$

Example 2: Solve the given circuit by mesh analysis and determine the current drawn from voltage source V.



Solution:



Total number of nodes = 4

Total branches $b = 6$

Total number of mesh equations = 6 – 4 + 1 = 3

Total number of node equations = $n - 1 = 4 - 1 = 3$

In mesh 1:

$$2(I_1 - I_2) + \frac{1}{2}(I_1 - I_3) - 4 = 0$$

$$\frac{5}{2}I_1 - 2I_2 - \frac{1}{2}I_3 = 4 \qquad ...(1)$$

In mesh 3:

$$I_3 = -2 \text{ Amp} \qquad ...(2)$$

In mesh 2:

$$2(I_2 - I_1) + I_2 + \frac{1}{3}(I_2 - I_3) = 0 \qquad ...(3)$$

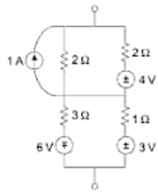
$$-2I_1 + \frac{10}{3}I_2 - \frac{1}{3}I_3 = 0 \qquad ...(4)$$

Solving for (1) (2) and (4) we get

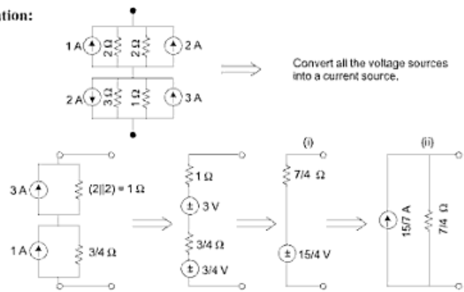
$$I_1 = 2 \text{ Amp}, I_2 = 1 \text{ Amp}, I_3 = -2 \text{ Amp}$$

Current from source V_S is $I_1 = 2$ Amp **Ans.**

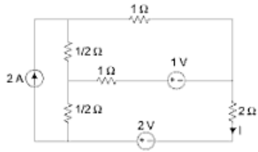
Example 3: Using source transformations, simplify the given network into a (i) Voltage source having a resistance in series (ii) Current source having a parallel resistance.



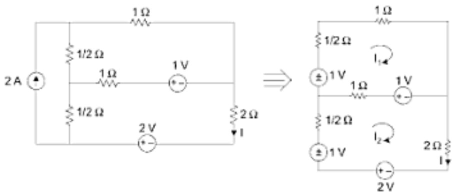
Solution:



Example 4: After converting the current source find I in 2Ω resistor by using mesh analysis.



Solution:



Apply mesh analysis in mesh 1

$$I_1 (1/2 + 1 + 1) - I_2 = 2$$

$$\frac{5}{2} I_1 - I_2 = 2 \qquad ...(1)$$

Mesh 2

$$(I_2 - I_1).1 + 1 + 2I_2 - 2 - 1 + I_2/2 = 0$$

$$-I_1 + I_2 \left(1 + 2 + \frac{1}{2}\right) = 2$$

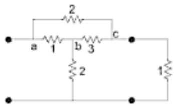
$$-I_1 + \frac{7}{2} I_2 = 2 \qquad ...(2)$$

From equations (1) and (2)

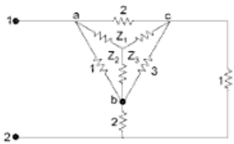
$$I_2 = \frac{7 \times 4}{31}$$

$$I_2 = I = \frac{28}{31} \quad \text{Amp.}$$

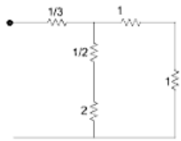
Example 5: Find the equivalent input resistance of the bridge circuit shown in figure.



Solution: Redrawing the figure



Covering the delta to star,



$$Z_1 = \frac{2 \times 1}{6} = \frac{1}{3}$$

$$Z_2 = \frac{3 \times 1}{6} = \frac{1}{2}$$

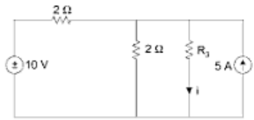
$$Z_3 = \frac{2 \times 3}{6} = 1$$

$$R_{eq} = \frac{1}{3} + \frac{\frac{5}{2} \times 2}{\frac{5}{2} + 2} = \frac{1}{3} + \frac{5}{\frac{9}{2}} = \frac{1}{3} + \frac{10}{9}.$$

$$= \frac{3 + 10}{9} = \frac{13}{9}.$$

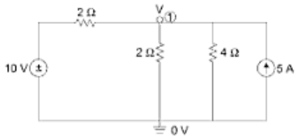
Example 6: Find the current through resistor $R_3 = 4\Omega$, by using

(6) Nodal analysis
(7) Mesh analysis
(8) Superposition theorem
(9) Thevnin theorem
(10) Norton theorem

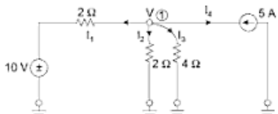


Solution:

(6) Using Nodal analysis:



(a) First select a datum or ground node.
(b) Now select a voltage of node (1) is V.
(c) Number of branches connect from node (1) is **4**.
(d) Assume all the currents from node are outward.



(e) Total sum of current at node is zero

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$I_1 = \frac{V-10}{2},\ I_2 = \frac{V}{2},\ I_3 = \frac{V}{4},\ I_4 = -5 \text{ amp}$$

$$\frac{V-10}{2}+\frac{V}{2}+\frac{V}{4}-5=0$$

$$2V-20+2V+V-20=0$$

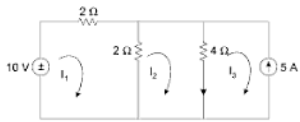
$$V=\frac{40}{5}=8 \text{ Volt}$$

$\boxed{\text{Voltage at node (1) is } V = 8 \text{ Volt}}$

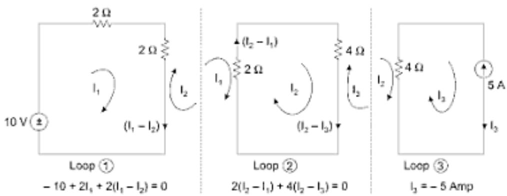
Current through R_3 is $I_3=\dfrac{V}{R_3}=\dfrac{8}{4}=$ **2 Amp** $\boxed{\text{Ans = 2 Amp}}$

(7) Using mesh analysis:

(i) Assume a loop or mesh current in each loop.



(ii) Apply KVL in each loop



Applying mesh analysis loop – 1

$$-10+2I_1+2(I_1-I_2)=0$$

$$\boxed{4I_1-2I_2=10} \qquad ...(1)$$

Applying mesh analysis in loop – 2

$$2(I_2 - I_1) + 4(I_2 - I_3) = 0$$

We can't apply mesh analysis in loop (3) because there is a current source. But $I_3 = -5$ Amp

$$6I_2 - 2I_1 + 20 = 0$$

$$\boxed{-2I_1 + 6I_2 = -20} \qquad ...(2)$$

After solving (1) and (2) we get,

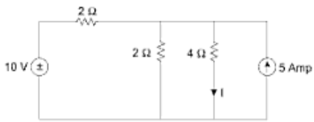
$$I_1 = -1,\ I_2 = -3 \text{ Amp}$$

Current through R_3 is →

$$I = I_2 - I_3 = -3 - (-5)$$
$$= -3 + 5$$

$$\boxed{I = 2 \text{ Amp}} \quad \textbf{Ans.}$$

(8) Using superposition theorem:

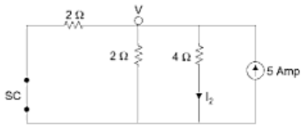


Case – 1: Taking a voltage source of 10 V and deactivate other sources here current source. So open the current source, and find I_1.

Using nodal analysis $\rightarrow \dfrac{V-10}{2} + \dfrac{V}{2} + \dfrac{V}{4} = 0$

$$5V = 20$$
$$V = 4 \text{ Volt}$$
$$I_1 = \frac{V}{4} = \frac{4}{4} = 1 \text{ Amp} \downarrow \text{down}$$

Case – 2: Taking another source (current source) and deactivate the other sources here it is a voltage source. So, short the voltage source or replaced by its internal impedance.



Using nodal analysis $\rightarrow I_2 = \dfrac{V}{4}$ Amp.

$$\frac{V}{2} + \frac{V}{2} + \frac{V}{4} - 5 = 0$$
$$5V = 20$$
$$\boxed{V = 4 \text{ Volt}}$$
$$I_2 = \frac{4}{4} = 1 \text{ Amp} \downarrow \text{downward}$$

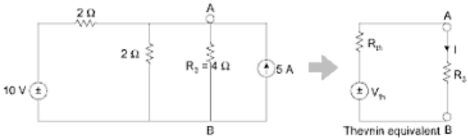
Now according to *superposition principle* net current through R_3 is I.

$$I = I_1 + I_2$$
$$= 1 + 1$$
$$I = 2 \text{ Amp}$$

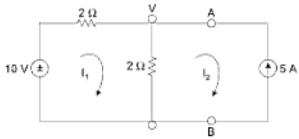
(9) Using Thevnin's theorem:

$$I = \frac{V_{th}}{R_{th} + R_3}$$

So, find I we have to find V_{th} and R_{th}.



(i) For finding V_{th} open the terminal across which we have to find I or have to draw thevnin equivalent. So V_{th} is nothing, it is a open terminal voltage. So open the AB and find V_{AB} which is V_{th}.



Now, V_{AB} can be found by both *mesh* or *nodal*

(i) **by mesh** $I_2 = -5$ Amp

and, $-10 + 2I_1 + 2(I_1 - I_2) = 0$

$$4I_1 - 10 + 10 = 0 \quad \boxed{I_1 = 0 \text{ Amp}}$$

$$V_{AB} = 2(I_1 - I_2)$$

$$= 2(0 + 5) = 10 \text{ Volt}$$

so, $\boxed{V_{th} = 10 \text{ Volt}}$

(ii) **by nodal** $\dfrac{V - 10}{2} + \dfrac{V}{2} - 5 = 0$

$$\boxed{V = V_{AB} = 10 \text{ Volt}} \text{ is } V_{th}$$

For finding R_{th}:

(i) Open the terminal across which we have to find the thevnin equivalent.

(ii) Now deactivate the independent sources.

(a) Current source will be open.

(b) Voltage source will be short or replaced by its internal resistance.

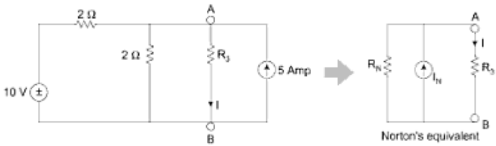
(iii) Now find the equivalent resistance between open terminal AB.

$$R_{AB} = R_{th} = 2 \| 2$$

$$R_{th} = \frac{2 \times 2}{2 + 2} = 1\ \Omega$$

$$\text{So } I = \frac{V_{th}}{R_{th} + R_3} = \frac{10}{1 + 4} = \mathbf{2\ Amp} \quad \textbf{Ans.}$$

(10) Using Norton's theorem:

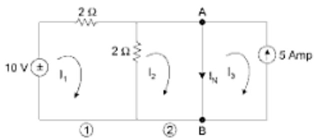


$$\boxed{I = \frac{R_N}{R_N + R_3} I_N}$$

So for finding I we have to find R_N and I_N.

(i) R_N will be same as R_{th}.

(ii) I_N is nothing, it is the short ckt current so short the terminal AB and find the current across short terminal.



$$I_3 = -5 \text{ Amp}$$

in loop 2 $2(I_2 - I_1) + 0\,(I_2 - I_3) = 0$

$$I_1 = I_2$$

in loop 1 $2I_1 + 2(I_1 - I_2) = 10$

$$\boxed{I_1 = 5 \text{ Amp} = I_2}$$

So, $I_N = I_2 - I_3 = I_2 + 5$

$$\boxed{I_N = 10 \text{ Amp}}$$

$$I = \frac{R_N}{R_N + R_3} I_N = \frac{1}{5} \times 10 = \mathbf{2\ Amp} \quad \textbf{Ans.}$$

Example 11: Find V_x across the resistance $R_3 = 1\ \Omega$, using—

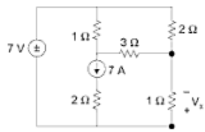
(11) Mesh analysis

(12) Nodal analysis

(13) Superposition

(14) Thevnin theorem

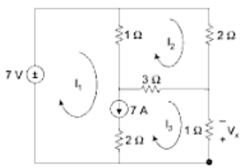
(15) Norton's theorem

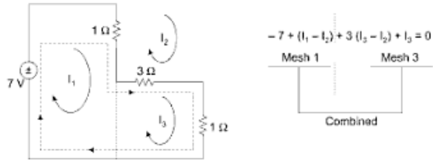


Solution:

(11) Mesh analysis:

We can't apply a mesh analysis in loop 1 and loop 3 because there is a current source. We will take a mesh combining these two *loops* called **super mesh**.





$$-7 + 1(I_1 - I_2) + 3(I_3 - I_2) + I_3 = 0$$

$$\boxed{I_1 - 4I_2 + 4I_3 = 7} \quad ...(1)$$

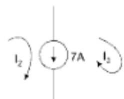
Apply mesh analysis in loop 2

$$(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$$

$$\boxed{-I_1 + 6I_2 - 3I_3 = 0} \quad ...(2)$$

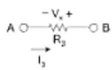
Finally, the independent-source current is related to the assumed mesh currents

$$\boxed{I_1 - I_3 = 7} \quad ...(3)$$



Solving (1), (2) and (3) we get

$$\mathbf{I_1 = 9\ Amp,\ I_2 = 2.5\ Amp,\ I_3 = 2A}$$



$$\boxed{V_{AB} = -V_x = I_3R_3}$$

So ⇒ $-V_x = I_3R_3$

$$\boxed{V_x = 2 \times 1 = -2\text{ Volt}} \quad \textbf{Ans.}$$

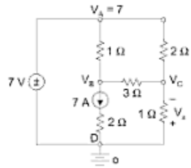
(12) Nodal Analysis:

Let the node voltages are V_A, V_B, V_C and V_D

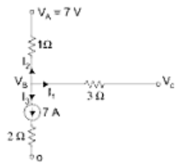
when $V_D = 0$ (ground)

$$V_A - V_D = 7$$

$$\Rightarrow \qquad \mathbf{V_A = 7 \quad Volt}$$



Apply node analysis at node B:



3 branches are at node B. Assume all the currents at node B is outwards.

$$I_1 + I_2 + I_3 = 0$$

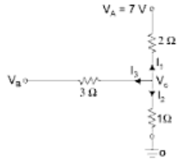
$$\frac{(V_B - V_C)}{3} + \frac{(V_B - 7)}{1} + 7 = 0$$

$$\boxed{4V_B - V_C = 0} \qquad ...(1)$$

Nodal analysis at node C:

3-branches connect at node C

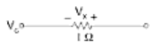
$$I_1 + I_2 + I_3 = 0$$



$$\frac{V_C - 7}{2} + \frac{V_C - V_B}{3} + \frac{V_C}{1} = 0$$

$$\boxed{-2V_B + 11V_C = 21} \qquad ...(2)$$

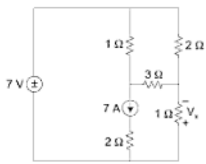
Solving (1) and (2) $\boxed{V_C = 2 \text{ Volt}}$



$$V_x = -V_C = \mathbf{-2\ Volt} \quad \textbf{Ans.}$$

(13) Using superposition theorem:

In superposition theorem we will take one source at a time and rest of the sources will be deactivated.



Case 1: Take a voltage source and deactivate the current source.

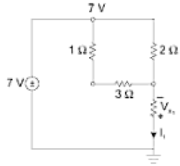
apply nodal analysis at node C

$$\frac{V_C - 7}{4} + \frac{V_C - 7}{2} + \frac{V_C}{1} = 0$$

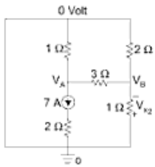
$$V_C - 7 + 2V_C - 14 + 4V_C = 0$$

$$7V_C = 21 \Rightarrow \boxed{V_C = 3 \text{ Volt}}$$

$$V_{x_1} = -V_C = -3 \text{ Volt}$$



Case – 2: Taking ($7A$) current source and deactivate voltage source.



at node – A →

$$\frac{V_A}{1} + \frac{V_A - V_B}{3} + 7 = 0$$

$$\boxed{4V_A - V_B = -21} \qquad ...(1)$$

at node – B

$$\frac{V_B - V_A}{3} + \frac{V_B}{2} + V_B = 0$$

$$2V_B - 2V_A + 3V_B + 6V_B = 0$$

$$\boxed{11V_B = 2V_A} \qquad \boxed{V_A = \frac{11}{2}V_B}$$

Put $V_A = \frac{11}{2} V_B$ *in eq - (1)*

$$4V_A - V_B = -21$$

$$4\left(\frac{11}{2}V_B\right) - V_B = -21$$

$$\boxed{V_B = -1\,\text{Volt}}$$

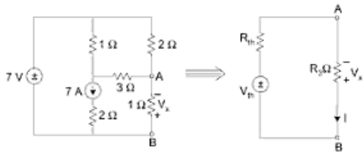
$$\boxed{V_{x_2} = -V_B = 1\,\text{Volt}}$$

Now according to superposition principle net

$$V_x = V_{x_1} + V_{x_2}$$

$$\boxed{V_x = -3 + 1 = -2\,\text{Volt}} \quad \textbf{Ans.}$$

(14) Using Thevnin theorem:



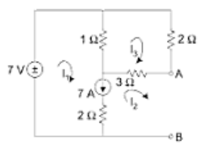
Thevnin equivalent

$$V_x = -IR_3 \quad \text{and} \quad I = \frac{V_{th}}{R_{th} + R_3}$$

For finding V_{th}:

(i) Open the terminal AB, then find V_{AB}, that V_{AB} is V_{th} (open ckt voltage)

Now V_{AB} can be found by Mesh or Nodal analysis.



(a) **First by Mesh**

Mesh 2 is open so $I_2 = 0$

$$I_1 - I_2 = 7\text{ Amp}$$

So $\rightarrow$ $$I_1 = 7\text{ Amp}$$

In mesh 3 → $(I_3 - I_1) + 3I_3 + 2I_3 = 0$

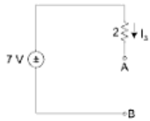
$$6I_3 - I_1 = 0 \Rightarrow \boxed{I_3 = \frac{7}{6}\text{ Amp}}$$

V_{AB} = Start from A reach to B by any path and add all the drops of that path with sign.

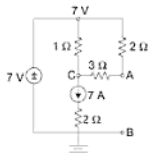
$$V_{AB} = -2I_3 + 7$$

$$= -2 \times \frac{7}{6} + 7$$

$$\boxed{V_{AB} = \frac{14}{3}\text{ Volt}} = V_{th}$$



(b) **By nodal →**



at node – C → $$\frac{V_C - 7}{1} + \frac{V_C - 7}{5} + 7 = 0$$

$$\boxed{6V_C = 7}$$

at node – A → $$\frac{V_A - V_C}{3} + \frac{V_A - 7}{2} = 0$$

$$5V_A - 2V_c = 21$$

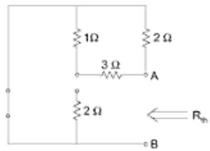
$$5V_A - 2 \times \frac{7}{6} = 21$$

$$V_A = \frac{14}{3}\text{ Volt}$$

$$\boxed{V_{th} = V_{AB} = V_A = \frac{14}{3} \text{ Volt}}$$

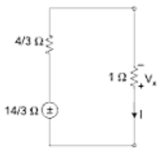
For finding the R_{th}:

Short the voltage source, open the current source, then find the equivalent resistance between the open terminal.



$$R_{AB} = (3 + 1) \| 2$$

$$= \frac{4 \times 2}{6} = \frac{4}{3}\ \Omega$$



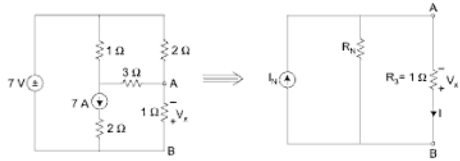
Thevnin equivalent

$$I = \frac{\frac{14}{3}}{1 + \frac{4}{3}} = 2 \text{ Amp}$$

$$V_x = -IR_3 = -2 \times 1$$

$\boxed{V_x = -2 \text{ Volt}}$ **Ans.**

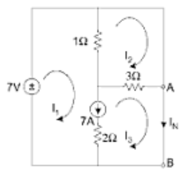
(15) Using Norton's theorem:



$$I = \frac{R_N}{R_3 + R_N} I_N$$

$$\boxed{V_x = -IR_3}$$

For finding I_N, short the terminal AB and find the I_{AB} or I_{SC} short ckt current.



$$I_3 = I_N$$

$$I_1 - I_N = 7 \qquad \text{...(1)}$$

Apply combined mesh analysis in loop – 1 and 3.

$$-7 + (I_1 - I_2) + 3(I_N - I_2) = 0$$

$$\boxed{I_1 - 4I_2 + 3I_N = 7} \qquad \text{...(2)}$$

Apply mesh analysis in loop – 2

$$(I_2 - I_1) + 3(I_2 - I_N) + 2I_2 = 0$$

$$\boxed{6I_2 - I_1 - 3I_N = 0} \qquad \text{...(3)}$$

Put $I_1 = I_N + 7$ in (2) and (3)

$$I_N + 7 - 4I_2 + 3I_N = 7 \Rightarrow \text{in (2)}$$

$$\Rightarrow \boxed{I_2 = I_N}$$

from – 3

$$6I_N - I_N - 7 - 3I_N = 0$$

$$\boxed{I_N = \frac{7}{2}\text{ Amp}}$$

R_N is same as R_{th}

(Norton's equivalent)

$$I = \frac{\frac{4}{3}}{\frac{4}{3}+1}\cdot\frac{7}{2}$$

$$I = \frac{\frac{4}{3}}{\frac{7}{3}} \times \frac{7}{2} = \mathbf{2\ Amp}$$

$$V_x = -IR$$

$$\boxed{V_x = -2\text{ Volt}}\quad \textbf{Ans.}$$

PART – B

Example 16: Find I using—

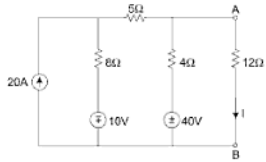
(16) Norton's theorem

(17) Thevnin's theorem

(18) Mesh analysis

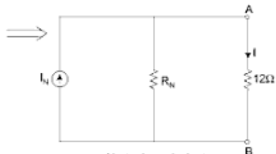
(19) Nodal analysis

(20) Superposition



Solution:

(16) (i) Norton's theorem:



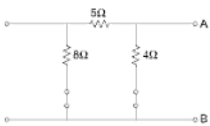
Norton's equivalent

$$I = \frac{R_N}{R_N + 12} . I_N$$

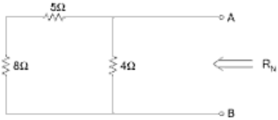
For finding I or for drawing Norton's equivalent ckt. We have to find (i) R_N (ii) I_N

For finding R_N:

(a) Deactivate all the sources, short the voltage source and open the current source.



(b) Now find the equivalent resistance between the open terminals (AB) across which we have to draw the Norton's equivalent.



$$R_N = (8 + 5) \| 4$$

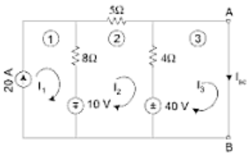
$$R_N = \frac{13 \times 4}{17} = \frac{52}{17}$$

For finding I_{SC} or I_N:

I_N is nothing but a short CKT current.

So, short the terminals (across which the short CKT current have to find) and find current I_{SC} using Mesh or Nodal analysis.

Mesh



$I_1 = 20$ Amp, $I_3 = I_{sc}$

In loop-2

$$17I_2 - 8I_1 - 4I_3 + 40 + 10 = 0$$

$$\boxed{17I_2 - 4I_3 = 110} \quad ...(1)$$

In loop-3

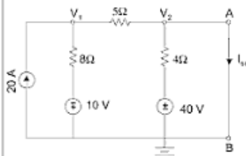
$$4I_3 - 4I_2 = 40$$

$$\boxed{I_2 = I_3 - 10} \quad ...(2)$$

Put-2 into-(1)

$$17I_3 - 170 - 4I_3 = 110$$

Nodal



$\Rightarrow$ *AB* is shorted hence $V_2 = 0$ Volt

so apply nodal at node-1

$$\frac{V_1 + 10}{8} + \frac{V_1 - V_2}{5} = 20$$

but $V_2 = 0$

$$13V_1 = 750$$

$$\boxed{V_1 = \frac{750}{13}}$$

At node-2

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 40}{4} + I_{sc} = 0$$

$13I_3 = 280,\ I_3 = I_{SC}$ $\boxed{I_{sc} = 280/13}$	$\frac{-V_1}{5} - \frac{40}{4} + I_{sc} = 0$ $I_{sc} = 10 + \frac{\overset{150}{\cancel{780}}}{13 \times \cancel{5}}$ $\boxed{I_{sc} = \frac{280}{13}}$ **Ans.**

$$I = \frac{R_N}{R_N + R_L}.I$$

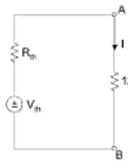
$$I = \frac{52/17}{\frac{52}{17} + 12}.\frac{280}{13}$$

$$I = 4.375 \text{ amp}$$

Now same has to be done by Thevnin theorem.

(17) Thevnin theorem

$$I = \frac{V_{th}}{R_{th} + 12}$$



(Thevnin equivalent)

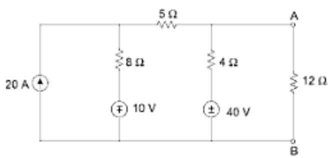
(a) *For finding* R_{th}

R_{th} is found same as R_N.

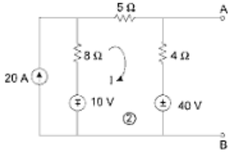
(b) *For finding* V_{th}

V_{th} is nothing it is the open CKT voltage across the terminal across which you have to find Thevnin equivalent CKT.

So open the terminal *AB* and then find $V_{oc} = V_{AB} = V_{th}$ by Mesh or Nodal analysis.



Using Mesh



$V_{th} = V_{AB}$ for finding V_{AB} you have to reach A to B by any path.

$\boxed{V_{AB} = 4I + 40}$ for finding I

apply mesh analysis in mesh 2

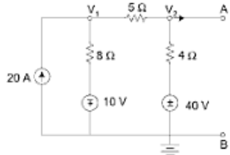
$$17I + 40 + 10 - 160 = 0$$

$$I = \frac{110}{17}$$

$$V_{AB} = V_{th} = 4 \times \frac{110}{17} + 40$$

$$\boxed{V_{th} = \frac{1120}{17} = 65.8 \text{ Volts}}$$

Using Nodal



V_{AB} = open CKT voltage hence

$$V_{th} = V_{AB} = V_2$$

$$\Rightarrow \quad \frac{V_1 + 10}{8} + \frac{V_1 - V_2}{5} = 20$$

$$\boxed{13V_1 - 8V_2 = 750} \qquad \text{...(1)}$$

$$\Rightarrow \quad \frac{V_2 - V_1}{5} + \frac{V_2 - 40}{4} = 0$$

$$\boxed{9V_2 - 4V_1 = 200} \qquad \text{...(2)}$$

$$\text{Put } V_1 = \frac{9V_2 - 200}{4}$$

$$\Rightarrow \quad 13.\frac{(9V_2 - 200)}{4} - 8V_2 = 750$$

$$\boxed{V_2 = \frac{1120}{17}}$$

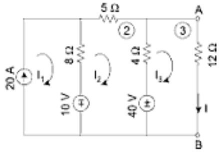
$$V_{th} = \frac{1120}{17} \text{ Volt}$$

$$I = \frac{V_{th}}{R_{th} + 12} = \frac{1120/17}{\frac{52}{17} + 12}$$

$$\boxed{I = \frac{1120}{256} = 4.375 \text{ Amp}} \quad \textbf{Ans.}$$

Same problem is now done by Mesh and Nodal analysis.

(18) Mesh



$I = I_3 = ?,\ I_1 = 20$ Amp

Apply Loop analysis in (2)

$8(I_2 - I_1) + 5I_2 + 4(I_2 - I_3) + 40 + 10 = 0$

$17I_2 - 4I_3 - 8I_1 + 50 = 0$

$17I_2 - 4I_3 - 160 + 50 = 0$

$17I_2 - 4I_3 = 110 \qquad \text{...(A)}$

Mesh analysis in Loop (3)

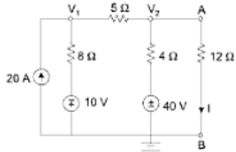
$-4I_2 + 16I_3 = 40 \qquad \text{...(B)}$

$-I_2 + 4I_3 = 10$

from (A) and (B)

$I_3 = 4.375 = I$ **Ans.**

(19) Nodal



At node-1

$$\frac{V_1 - V_2}{5} + \frac{V_1 + 10}{8} = 20$$

$$\boxed{17V_1 - 8V_2 = 750}$$

At node-2

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 40}{4} + \frac{V_2}{12} = 0$$

$$-32V_1 + 92V_2 = 1600$$

$$\boxed{-8V_1 + 21V_2 = 400}$$

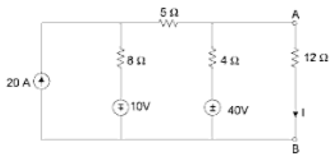
$$V_1 = \frac{21V_2 - 400}{8}$$

Put in (1)

$$17\frac{(21V_2 - 400)}{8} - 8V_2 = 750$$

$\Rightarrow \quad I = \dfrac{V_2}{12} = \mathbf{4.375\ Amp}$ **Ans.**

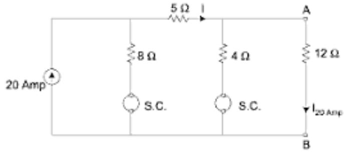
(20) Same problem is by *Superposition Theorem.*



There are three sources hence, current I is the sum of current by individual sources by deactivating other sources (short voltage source and open current source).

$$I = I_{20\text{ Amp}} + I_{10\text{ Volt}} + I_{40\text{ Volt}}$$

Case 1: Taking 20 Amp source and short other voltage source.



by current divider rule $I_{20} = \dfrac{4}{(4+12)} \times I$

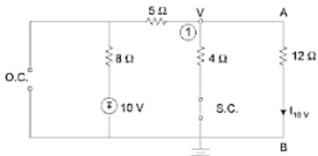
and $I = \dfrac{8}{8+R}$

and $R = 5 + (4 \| 12) = 8\Omega$

$$I = \frac{8}{8+8} \times 20 = 10 \text{ Amp}$$

$\boxed{I_{20} = \dfrac{4}{16} \times 10 = \dfrac{5}{2} \text{ Amp}}$ $\downarrow$ = 2.5 A downward (A to B)

Case 2: Taking 10 V battery source



$$I_{10\text{ V}} = \frac{V}{12}$$

Apply nodal at node 1

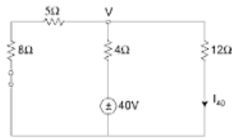
$$\frac{V+10}{13} + \frac{V}{4} + \frac{V}{12} = 0$$

$$48V + 156V + 52V = -480$$

$$V = -\frac{480}{256} \text{ Volt}$$

$$I_{10\text{ V}} = \frac{-\cancel{480}}{\cancel{256} \times \cancel{12}} = \frac{-5}{32} \text{ Amp} \downarrow_{A \text{ to } B} = -.156 \text{ A (downwards)}$$

Case 3: Taking 40 V voltage source and open the current source and short the other voltage source.



$\Rightarrow$ $$\frac{V}{13} + \frac{V-40}{4} + \frac{V}{12} = 0$$

$\Rightarrow$ $$V = \frac{6240}{256} \text{ Volt}$$

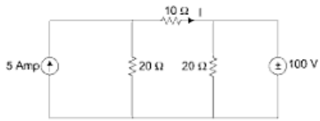
$$I_{40} = \frac{V}{12}$$

$$I_{40} = \frac{\overset{520}{\cancel{6240}}}{256 \times \cancel{12}}$$

$$I_{40} = \frac{\cancel{260}}{\cancel{128}} = \frac{65}{32} = 2.03 \text{ A (downwards)}$$

$I = 2.5 - .156 + 2.03 =$ **4.375 Amp Ans.**

Example 21:

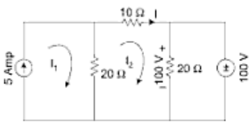


Find *I* using

(21) Mesh analysis
(22) Nodal analysis
(23) Thevnin theorem
(24) Norton's theorem
(25) Superposition analysis **[Ans:** 0 Amp]

Solution:

(21) By Mesh analysis



⇒ In mesh-1 $I_1 = 5$ Amp

⇒ In mesh-2

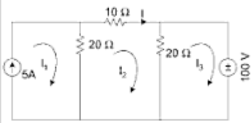
$10I_2 + 100 + 20(I_2 - I_1) = 0$

$30I_2 + 100 - 20I_1 = 0$

Put $I_1 = 5$Amp

⇒ $30I_2 + 100 - 20 \times 5 = 0$

⇒ $I_2 = I = 0$ Amp



In mesh-1, $I_1 = 5$ Amp ...(1)

In mesh-2

$20(I_2 - I_1) + 10I_2 + 20(I_2 - I_3) = 0$

$-20I_1 + 50I_2 - 20I_3 = 0$ ...(2)

In mesh-3

$20(I_3 - I_2) + 100 = 0$

$I_2 - I_3 = 5$ ...(3)

Put $I_1 = 5$ Amp in eqn. (2)

$50I_2 - 20I_3 = 100$ ...(4)

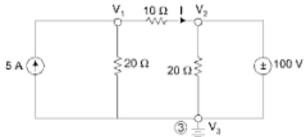
Put $I_3 = I_2 - 5$ from (2) eqn. (4)

$50I_2 - 20(I_2 - 5) = 100$

⇒ $I_2 = 0$ Amp and $I_3 = -5$ Amp

⇒ $I = I_2 = 0$ Amp

(22) By nodal analysis



$$I = \frac{V_1 - V_2}{10} \text{ Amp}$$

Let the node voltages of node –1, 2, 3 be V_1, V_2 and V_3, if node –3 is datum node $V_3 = 0$, then $V_2 = 100$ Volt

Apply nodal analysis at node-1

$$\frac{V_1}{20} - 5 + \frac{V_1 - V_2}{10} = 0$$

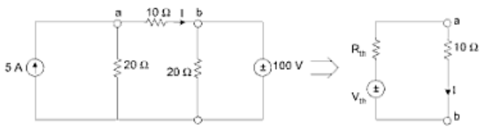
$$V_1 - 100 + 2V_1 - 2V_2 = 0 \quad \text{Put } V_2 = 100 \text{ Volt}$$

$$3V_1 - 200 - 100 = 0$$

$$\boxed{V_1 = 100 \text{ Volt}}$$

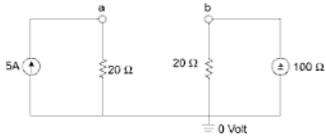
$$\Rightarrow \qquad I = \frac{V_1 - V_2}{10} = \frac{100 - 100}{10} = \mathbf{0\ Amp} \quad \textbf{Ans.}$$

(23) By Thevnin theorem



$$\Rightarrow \qquad I = \frac{V_{th}}{R_{th} + 10} \text{ Amp}$$

For finding the V_{th} open the terminal *ab* then find the open CKT voltage.



$$V_{th} = V_{ab} = V_a - V_b$$

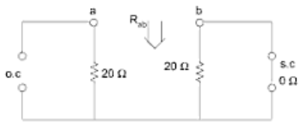
Apply nodal analysis at node *a*

$$\frac{V_a}{20} - 5 = 0 \qquad V_a = 100 \text{ Volt}$$

$$V_b = 100 \text{ Volt}$$

$$V_{th} = V_{ab} = 100 - 100 = 0 \text{ Volt}$$

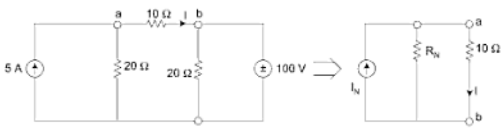
For finding $R_{th} \rightarrow$ open the current source, short the voltage source, then find the equivalent Resistance between the open terminal.



$$R_{th} = R_{ab} = 20 + (20 \| 0) = 20\ \Omega$$

$$I = \frac{V_{th}}{R_{th} + 10} = \frac{0}{20 + 10} = \mathbf{0\ Amp} \quad \textbf{Ans.}$$

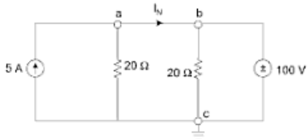
(24) By Norton's theorem



$$I = \frac{R_N}{R_N + 10} \cdot I_N$$

So for finding I by Norton's theorem first we have to draw *Norton's equivalent.*

I_N is finding by short the terminal ab then find the short CKT current $I_{ab} = I_N$.



Apply nodal analysis at node-*a*

$$\frac{V_a}{20} + I_N - 5 = 0$$

$$I_N = 5 - \frac{V_a}{20} \qquad \{\text{when } C \text{ is grounded then } V_b = 100 \text{ Volt}\}$$

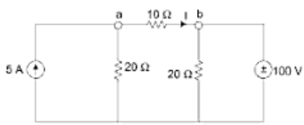
$$I_N = 5 - \frac{100}{20} = 0 \text{ Amp}\{a \text{ and } b \text{ is shorted so } V_a = V_b = 100 \text{ Volt}\}$$

R_N is same as $R_{th} = 20\ \Omega$

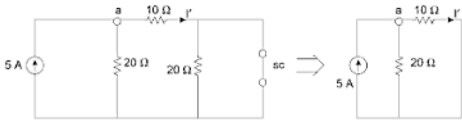
Current through 10 Ω is $I = \frac{R_N}{R_N + 10} \cdot I_N$

$$I = \frac{20}{30} \times (0 \text{ Amp}) = \mathbf{0\ Amp}$$

(25) By superposition theorem



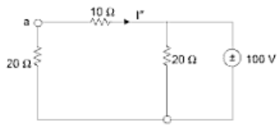
Case 1: Taking 5 Amp current source and deactivate other source (short the voltage source). Let the current through 10 Ω is I'.



By current divider rule 5 A is divided into 20 Ω and 10 Ω.

$$I' = \frac{20}{20+10} \times 5 = \frac{10}{3} \text{ Amp away from node } a$$

Case 2: Taking 100 V voltage source and find I''. Open the current source.



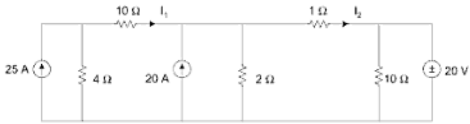
$$I'' = \frac{-100}{20+10} = \frac{-10}{3} \text{ Amp away from } a$$

According to superposition principle net current through ab is

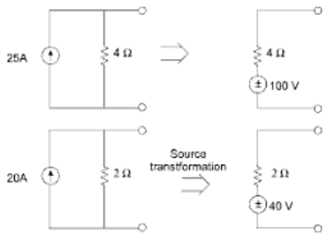
$$I = I' + I''$$

$$= \frac{10}{3} - \frac{10}{3} = 0 \textbf{ Amp} \text{ from } a \text{ to } b$$

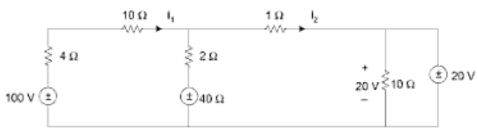
Example 26: First transform all the current sources to a voltage source then by using mesh and nodal analysis, find I_1 and I_2. For the given figure below.



Solution:

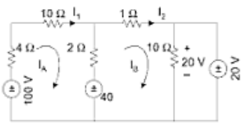


Now the equivalent CKT after source transformation is



Mesh analysis	Nodal Analysis

Mesh analysis



$\Rightarrow$ $I_1 = I_A$ and $I_2 = I_B$

In mesh-1

$$4I_A + 10I_A + 2(I_A - I_B) + 40 - 100 = 0$$

$$16I_A - 2I_B = 60$$

$$8I_A - I_B = 30 \qquad \text{...(1)}$$

In mesh-2

$$-40 + 2(I_B - I_A) + I_B + 20 = 0$$

$$-2I_A + 3I_B = 20 \qquad \text{...(2)}$$

from (1) + eqn. (2) × 4

$$11I_B = 110$$

$$\boxed{I_B = 10 \text{ Amp}}$$

from eqn. (1)

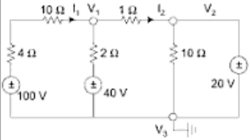
$$8I_A - 10 = 30$$

$$\boxed{I_A = 5 \text{ Amp}}$$

so $I_1 = 5$ Amp

$I_2 = 10$ Amp

Nodal Analysis



Let V_3 is a datum or ground then $V_3 = 0$, $V_2 = 20$ Volt because $V_2 - V_3 = 20$ Volt

At node-1

$$\frac{V_1 - 100}{14} + \frac{V_1 - 40}{2} + \frac{V_1 - 20}{1} = 0$$

$$V_1 - 100 + 7V_1 - 280 + 14V_1 - 280 = 0$$

$$22V_1 = 660$$

$$\boxed{V_1 = 30 \text{ Volt}}$$

$$\Rightarrow I_1 = \frac{100 - V_1}{14} = \frac{100 - 30}{14}$$

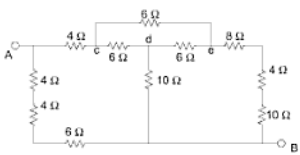
$$\boxed{I_1 = 5 \text{ Amp}}$$

$$\Rightarrow I_2 = \frac{(V_1 - V_2)}{1}$$

$$I_2 = \frac{30 - 20}{1} = 10 \text{ Amp}$$

$$\boxed{I_2 = 10 \text{ Amp}}$$

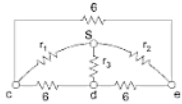
Example 27: Using *delta* to *star* transformation for the given CKT below find equivalent resistance between terminal *AB*.



Solution:

There is a delta *cde*

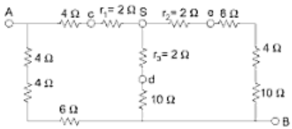
Convert this delta into a star

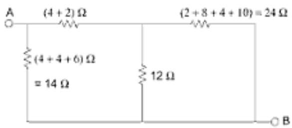


$$r_1 = \frac{6 \times 6}{6+6+6} = 2\ \Omega$$

$$r_2 = 2\ \Omega$$

$$r_3 = 2\ \Omega$$





$$R_{AB} = (14) \parallel [(12//24) + 6]$$

$$= (14) \parallel (14) = \frac{14 \times 14}{2 \times 14} = \mathbf{7\ \Omega} \quad \textbf{Ans.}$$

Example 28: Determine the current *I* through 1 Ω resistor using

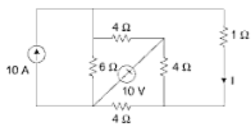
(28) Mesh analysis

(29) Nodal analysis

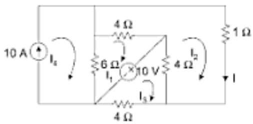
(30) Superposition

(31) Thevnin theorem

(32) Norton's theorem



(28) Mesh Analysis



Without converting the current source into a voltage source.

Applying mesh analysis in **Mesh-1**

Mesh-1

$6(I_1 - I_4) + 4(I_1 - I_2) + 10 = 0$

$10I_1 - 4I_2 - 6I_4 + 10 = 0$

but $I_4 = 10$ Amp

$10I_1 - 4I_2 = 50$ ...(1)

Mesh-2

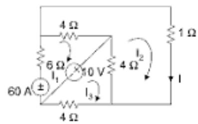
$4(I_2 - I_1) + 4(I_2 - I_3) + I_2 = 0$

$-4I_1 + 9I_2 - 4I_3 = 0$ ...(2)

Mesh-3

$4(I_3 - I_2) + 4I_3 - 10 = 0$

$-4I_2 + 8I_3 = 10$ ...(3)



After converting a voltage source into a current source.

Applying mesh analysis in *mesh-1*

$6I_1 + 4(I_1 - I_2) + 10 - 60 = 0$

$10I_1 - 4I_2 = 50$ ...(1)

Mesh-2

$4(I_2 - I_1) + 4(I_2 - I_1) + I_2 = 0$

$-4I_1 + 9I_2 - 4I_3 = 0$ ...(2)

Mesh-3

$4(I_3 - I_2) + 4I_3 - 10 = 0$

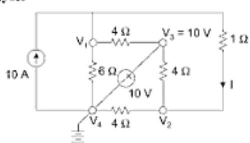
$-4I_2 + 8I_3 = 10$ ...(3)

Solving (1), (2) and (3) we get

$$I_2 = 4.63 \text{ Amp}$$

Hence, current flowing through resistor 1 Ω is $I = 4.63$ Amp

(29) Nodal Analysis



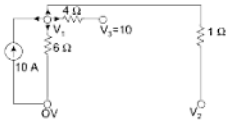
We will consider the node where the more than two branches meets.

If V_4 is grounded or datum node, then, $V_4 = 0$ Volt

$$V_3 - V_4 = 10$$

$$V_3 = 10 \text{ Volt}$$

Applying nodal analysis at Node 1

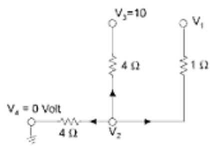


$$\frac{(V_1 - V_3)}{4} + \frac{(V_1 - 0)}{6} + \frac{(V_1 - V_2)}{1} - 10 = 0$$

$$\frac{(V_1 - 10)}{4} + \frac{V_1}{6} + \frac{4(V_1 - V_2)}{4} - \frac{40}{4} = 0$$

$$17V_1 - 12V_2 = 150 \qquad ...(1)$$

Node 2



$$\frac{V_2 - 0}{4} + \frac{V_2 - 10}{4} + \frac{V_2 - V_1}{1} = 0$$

$$-4V_1 + 6V_2 = 10 \qquad ...(2)$$

$$17V_1 - 12V_2 = 150 \qquad ...(1)$$

$$-2V_1 + 3V_2 = 5 \qquad ...(2)$$

$$\text{eq. (2)} \times 4 + (1) = 17V_1 - 8V_1 = 170$$

$$V_1 = \frac{170}{9} = 18.888 \text{ Volt}$$

Now put V_1 in eqn. (2)

$$3V_2 = 42.777$$

$$V_2 = 14.259$$

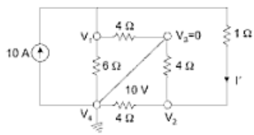
Now current from 1 Ω is $I = \dfrac{V_1 - V_2}{1}$ = 18.888 – 14.259

$$\boxed{I = 4.628 \text{ Amp}}$$

(30) By superposition theorem

According to this theorem, we will take one source at a one time and deactivate other sources.

Case 1: Taking 10 Ampere current source and short the voltage source. Let the current through 1 Ω is I'.



If V_4 is grounded, then V_3 is also zero (shorted)

At Node 1:

$$\frac{V_1}{6} + \frac{V_1}{4} + \frac{V_1 - V_2}{1} - 10 = 0$$

$$17V_1 - 12V_2 = 120 \qquad ...(1)$$

At Node 2:

$$\frac{V_2}{4} + \frac{V_2}{4} + \frac{V_2 - V_1}{1} = 0$$

$$6V_2 - 4V_1 = 0$$

$$\boxed{V_1 = \frac{3}{2}V_2} \qquad ...(2)$$

Put $V_1 = \dfrac{3}{2}V_2$ in eqn. (1)

$$\frac{51}{2}V_2 - 12V_2 = 120$$

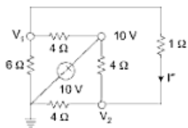
$$V_2 = \frac{120 \times 2}{27} = 8.888$$

and $$V_1 = \frac{3}{2} V_2 = 13.333$$

$$I' = \frac{V_1 - V_2}{1} \downarrow$$

$$I' = \frac{13.33 - 8.88}{1} = 4.444 \downarrow \text{Amp}$$

Case 2: Taking 10 Volt battery source and deactivate (open) current source, and current is I''.



At Node 1:

$$\frac{V_1}{6} + \frac{V_1 - 10}{4} + \frac{V_1 - V_2}{1} = 0$$

$$17V_1 - 12V_2 = 30 \qquad \text{...(1)}$$

At Node 2:

$$\frac{V_2}{4} + \frac{V_2 - 10}{4} + \frac{V_2 - V_1}{1} = 0$$

$$-4V_1 + 6V_2 = 10 \qquad \text{...(2)}$$

From (1) and (2)

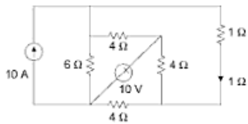
$$V_1 = \frac{50}{9} = 5.55,\ V_2 = \frac{32.22}{6} = 5.370$$

$$I'' = \frac{V_1 - V_2}{1} = \frac{5.55 - 5.370}{1} = 0.19 \text{ Amp} \downarrow \text{down}$$

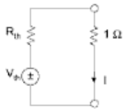
Now according to superposition theorem $I = I' + I''$

$I = 4.444 + 0.19 =$ **4.63 Amp**

(31) By Thevnin theorem

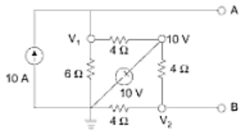


For finding I by using Thevnin theorem first we have to draw thevnin equivalent CKT across 1 Ω resistance in which we have to find current I.



$$\Leftarrow I = \frac{V_{th}}{R_{th}+1}$$

For finding V_{th} we will open the terminal across which we have find to I and find the open CKT voltage.



$V_{AB} = V_{th} = V_1 - V_2$

At node 1

$$\frac{V_1}{6} + \frac{V_1 - 10}{4} - 10 = 0$$

$V_1 = 30$ Volt $= V_A$

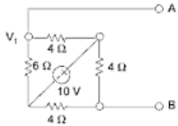
$V_{th} = V_A - V_B = 30 - 5 = 25$ Volt

At node-2

$$\frac{V_2}{4} + \frac{V_2 - 10}{4} = 0$$

$V_2 = V_B = 5$ Volt

For finding R_{th} → Short the voltage source and open the current source, then find the equivalent resistance across the terminals across which we have to draw thevnin equivalent CKT.



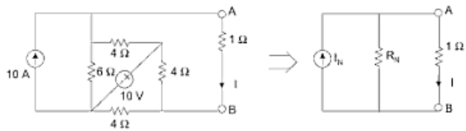
$\Leftarrow R_{AB} = R_{th}$

$$R_{AB} = (6 \parallel 4) + (4 \parallel 4) = 2.4 + 2 = 4.4\ \Omega = R_{th}$$

$$I = \frac{V_{th}}{R_{th} + 1} = \frac{25}{44 + 1} = 4.628 \text{ Amp}$$

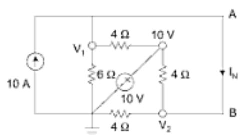
(32) By using Norton's theorem

First we have to draw the Norton's equivalent across the terminal *AB*.



$$I = \frac{R_N}{1 + R_N} . I_N$$

For finding I_N short the terminal *AB*, then find I_{AB} which is I_N.



$$V_1 = V_2 \quad \text{(shorted terminal)}$$

At node-1

$$\frac{V_1}{6} + \frac{V_1 - 10}{4} - 10 + I_N = 0$$

$$5V_1 = 150 - 12I_N$$

$$V_1 = 30 - \frac{12}{5} I_N \quad \text{...(1)}$$

Apply nodal analysis at node-2

$$\frac{V_2}{4} + \frac{V_2 - 10}{4} - I_N = 0 \text{ but } V_2 = V_1$$

$$2V_1 = 4I_N + 10 \quad \text{...(2)}$$

From (1) and (2)

$$30 - \frac{12I_N}{I_5} = (4I_N + 10)\frac{1}{2} \Rightarrow (30 - 5) = \frac{22}{5} I_N$$

$$\boxed{I_N = \frac{125}{22} \text{ Amp}}$$

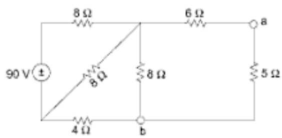
and R_N is same as R_{th} which is 4.4 Ω

$$I = \frac{R_N}{R_N + 1} . I_N$$

$$I = \frac{4.4}{4.4 + 1} \times \frac{125}{22}$$

$$I = \frac{4.4}{5.4} \times \frac{125}{22} = \textbf{4.628 Amp} \quad \textbf{Ans.}$$

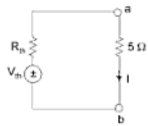
Example 33: Calculate the V_{ab} by using (i) Thevnin's theorem (ii) Norton's theorem for the given Network below:



Solution:

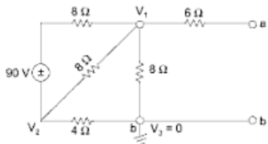
(i) Using Thevnin theorem

First draw the Thevnin equivalent CKT of above given network.



$$V_{ab} = 5I = \frac{5V_{th}}{R_{th} + 5}$$

For finding V_{th} open the terminal *ab*, then find open CKT voltage $V_{oc} = V_{th}$



Let V_3 = Datum node $V_3 = 0$ volt. There is no current from 6 Ω (because it is open CKT) 0 Amp current so

$$V_{1a} = V_1 - V_a = 6 \times 0 = 0 \text{ volt} \qquad \{V_1 = V_a\}$$

$$V_{th} = V_{ab} = V_a - V_b = V_1 - 0 = V_1$$

At node-1

$$\frac{V_1 - 90 - V_2}{8} + \frac{V_1}{8} + \frac{V_1 - V_2}{8} = 0$$

$$3V_1 - 2V_2 = 90 \qquad ...(1)$$

At node-2

$$\frac{V_2 + 90 - V_1}{8} + \frac{V_2 - 0}{4} + \frac{V_2 - V_1}{8} = 0$$

$$-2V_1 + 4V_2 = -90 \qquad ...(2)$$

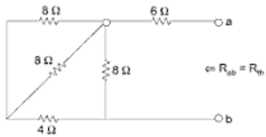
From (1) and (2) ⇒ (1) × 2 + eqn. (2)

$$6V_1 - 2V_1 = 90$$

$$V_1 = \frac{90}{4} \Rightarrow \boxed{V_{th} = \frac{90}{4}\text{ Volt}}$$

For finding R_{th}

Short all the independent voltage sources and open all the independent current sources, then find the equivalent resistance between *ab*.

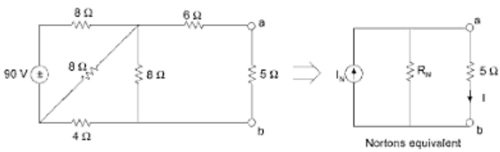


$$R_{th} = \{[(8||8) + 4]//8\} + 6$$

$$= [(4 + 4)//8] + 6 = 10\ \Omega$$

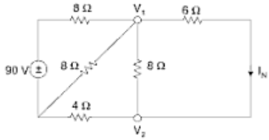
$$V_{ab} = \left(\frac{V_{th}}{R_{th} + R_L}\right)5\ \Omega = \frac{90/4}{10+5}\times 5 = \frac{90\times \not{5}}{4\times \not{15}_{3}} = \mathbf{7.5\ Volt}$$

(ii) *By Norton's theorem*



$$V_{ab} = 5I = 5\left(\frac{R_N}{R_N + 5}\right)I_N$$

For finding I_N or I_{sc} short the terminal *ab*, then find the current through *ab*.



$\Rightarrow \qquad I_N = \dfrac{V_1 - V_2}{6}$

At node 1

$$\frac{V_1 - 90}{8} + \frac{V_1}{8} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_2}{6} = 0$$

$\Rightarrow \qquad 26V_1 - 14V_2 = 540$

$$13V_1 - 7V_2 = 270 \qquad \text{...(1)}$$

At node 2

$$\frac{V_2}{4} + \frac{V_2 - V_1}{8} + \frac{V_2 - V_1}{6} = 0$$

$$6V_2 + 3V_2 - 3V_1 + 4V_2 - 4V_1 = 0$$

$$13V_2 = 7V_1 \qquad \text{...(2)}$$

Put $\qquad V_1 = \dfrac{13}{7} V_2$ in eqn. (1)

$$\frac{169V_2}{7} - 7V_2 = 270$$

$$V_2 = \frac{270 \times 7}{120} = \frac{63}{4} \text{ Volt}$$

From eqn. (2)

$$V_1 = \frac{13}{7} V_2$$

$\Rightarrow \qquad V_1 = \dfrac{13}{7} \times \dfrac{63}{4} = \dfrac{9 \times 13}{4}$ Volt

So, $$I_N = \frac{V_1 - V_2}{6} = \frac{1}{6}\left[\frac{9\times 13}{4} - \frac{63}{4}\right]$$

$$= \frac{1}{6\times 4}\times 54 = \frac{9}{4} \text{ Amp}$$

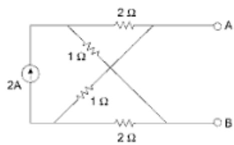
$\Rightarrow R_N$ is same as $R_{th} = 10\ \Omega$

$$\Rightarrow \quad V_{ab} = 5I$$

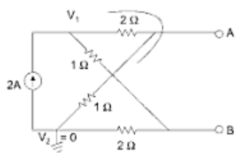
$$V_{ab} = 5\times\left(\frac{10}{10+5}\right)\times\frac{9}{4}$$

$$= 5\ \frac{10}{15}\times\frac{9}{4} = \frac{15}{2} = \textbf{7.5 Volt} \quad \textbf{Ans.}$$

Example 34: Find the Thevnin and Norton equivalent circuit for a given figure.



Solution:



Let V_2 be a datum node.

At Node 1

$$\frac{(V_1 - 0)}{3} + \frac{(V_1 - 0)}{3} - 2 = 0$$

$$\boxed{V_1 = 3 \text{ Volt}}$$

$$V_{th} = \text{Open CKT Voltage} = V_{AB} = V_A - V_B$$

At Node A

$$\frac{V_A - V_1}{2} + \frac{V_A - 0}{1} = 0$$

$$\frac{V_A - 3}{2} + \frac{V_A - 0}{1} = 0 \Rightarrow V_A = 1 \text{ Volt}$$

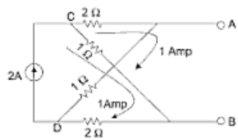
At Node B

$$\frac{V_B - V_1}{1} + \frac{V_B - 0}{2} = 0$$

$$\frac{V_B - 3}{1} + \frac{V_B}{2} = 0 \Rightarrow V_B = 2 \text{ Volt}$$

$\Rightarrow$ $$V_{th} = V_A - V_B = 1 - 2 = -1 \text{ Volt}$$

Alternative method for finding V_{th}

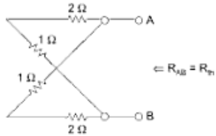


2 Amp current is divide in path *CAD* and *CBD* both paths having equal resistance of (2 + 1) = 3 Ω. So, current in each path is 1 Amp.

$$V_{AB} = \text{Total drop from } A \text{ to } B \text{ (by path } ADB \text{ or } ACB)$$

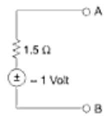
$$= 1 \times 1 + 2(-1) = -1 \text{ Volt.}$$

$R_{th} \rightarrow$ for finding R_{th} open the current source, then find R_{AB}.

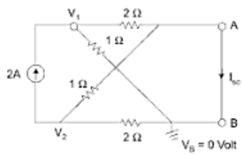


$$R_{AB} = (2+1)//(2+1) = \frac{3}{2} = 1.5\ \Omega$$

Thevnin equivalent across $AB \rightarrow$



For drawing Norton's equivalent we have to find I_N or I_{sc} short CKT current and R_N, R_N is same as R_{th} so $R_N = 1.5\ \Omega$



Let V_B is a datum node $V_B = 0$, then V_A is also 0 Volt.

At node A

$$\frac{0-V_1}{2} + \frac{0-V_2}{1} + I_{sc} = 0$$

$$\boxed{I_{sc} = \frac{V_1}{2} + \frac{V_2}{1}}$$

So for finding I_{sc} we have to find V_1 and V_2.

At Node 1

$$\frac{V_1 - 0}{2} + \frac{(V_1 - 0)}{1} - 2 = 0 \Rightarrow V_1 = \frac{4}{3} \text{ Volt}$$

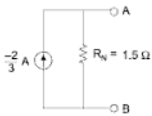
At Node 2

$$\frac{V_2 - 0}{2} + \frac{V_2 - 0}{1} + 2 = 0 \Rightarrow V_2 = -\frac{4}{3} \text{ Volt}$$

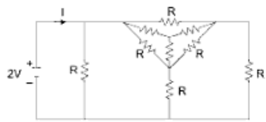
$$I_{sc} = \left(\frac{V_1}{2} + V_2\right)$$

$$I_{sc} = \left(\frac{4}{6} - \frac{4}{3}\right) = \frac{-4}{6} = \frac{-2}{3} \text{ Amp}$$

Norton's equivalent



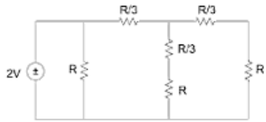
Example 35: Using star-delta transformation calculate the current *I* for the circuit shown in Figure. The value of each resistor is 2 ohms.



Solution: Converting delta to star, as all resistances are equal.

$$Z_y = \frac{1}{3} Z_\Delta = \frac{R}{3}$$

Redrawing,

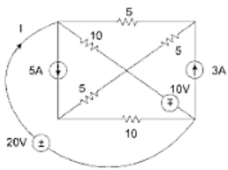


$$\left(\frac{R}{3}+\frac{4}{3}R \,\|\, \frac{4}{3}R\right) \| R = R_{eq}$$

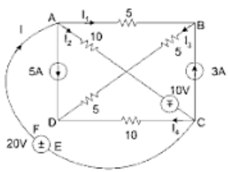
$$= \frac{R}{3}+\frac{2}{3}R = R \,\|\, R = \frac{R}{2} = 1\text{ohm}$$

$$I = \frac{2}{1} = 2\text{A}$$

Example 36: Find the currents flowing in the branches and the total current.



Solution: Let us mark the currents flowing in the various branches as follows:



Node *A*, $I = I_1 + I_2 + 5$ ...(1)

Node *B*, $I_1 + 3 = I_3$ ...(2)

Node *C*, $I_2 = I_4 + 3 + I$ ...(3)

Node *D*, $I_3 + 5 + I_4 = 0$ ...(4)

As there are five unknowns we should have at least 5 equations.

Neglecting the current source loops, we choose voltage source loops for KVL. They are *ACEFA* and *ABDCEFA*.

$$10I_2 - 10 - 20 = 0 \quad ...(5)$$

$$10I_2 = 30$$

$$I_2 = 3 \text{ A} \quad ...(6)$$

$$5I_1 + 5I_3 + 5I_3 - 10I_4 - 20 = 0 \quad ...(7)$$

$$5I + 10I_3 - 10I_4 = 20$$

$$5I_1 + 10I_3 - 10I_4 = 20$$

From (4) $$10I_3 + 10I_4 = -50$$

$$5I_1 + 20I_3 = -30$$

From (2) $$5I_1 - 5I_3 = -15$$

$$25I_3 = -15$$

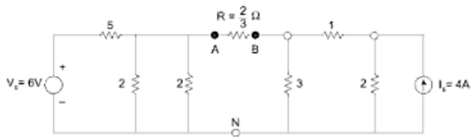
$$I_3 = -\frac{15}{25} = -\frac{3}{5} \text{ A}$$

$$I_1 = -3 - \frac{3}{5} = -\frac{18}{5} = -3\frac{3}{5} \text{ A}$$

$$I_4 = -5 - \left(-\frac{3}{5}\right) = -\frac{22}{5} \text{ A}$$

$$I = -3\frac{3}{5} + 3 + 5 = 4\frac{2}{5} \text{ A}$$

Example 37:



Obtain the Thevnin equivalent for circuits to the left of A and right of B. Hence, when $R = \frac{2}{3}$ ohms is connected across AB, find the current flowing in it.

Solution: Voltage across $AN = V_A$ (open circuit)

Potential division of $V_5 = 6$, gives V_A.

Since, $2 \parallel 2 = 1$, resistance across $V_s = 5 + 1$

$$V_A = 6 \times \frac{1}{6} = 1 \text{ V}$$

To find R_{eq}, the voltage source V_s is shorted

$$R_{eq} = R_{th} = \frac{5 \times 1}{5 + 1}$$

$$= \frac{5}{6} \text{ ohm}$$

V_A and R_{eq} give the Thevnin equivalent to left of AN.

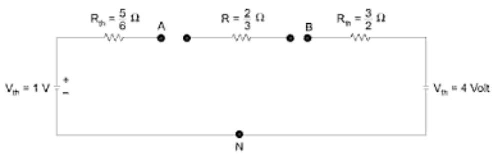
Voltage across $BN = V_B$ (open circuit) is found by current division method.

Current in 3 ohms $= I_s \times \frac{2}{6} = \frac{8}{6}$ A

Voltage $V_B = 3 \times \frac{8}{6} = 4$ Volts

$$R_{eq} = 3 \parallel 3 = \frac{3}{2} \text{ ohms (by setting source } I_s \text{ open)}$$

V_B and R_{eq} give Thevnin equivalent to right of BN



The Thevnin equivalent circuits are shown and $R = \dfrac{2}{3}$ now connected.

Current through R from right to left $= \dfrac{4-1}{\dfrac{5}{6}+\dfrac{2}{3}+\dfrac{3}{2}} = 1$ A

Example 38: For the given figure, find the value of R using (i) Thevnin theorem (ii) nodal analysis.

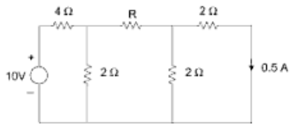
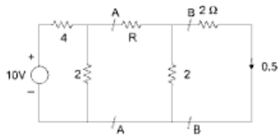


Fig. 1

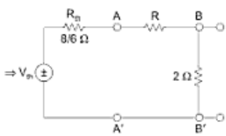
Solution:

Thevnin's theorem.



Open AA'

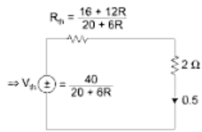
(1) $R_{th} = \dfrac{8}{6}$, $V_{th} = 10 \times \dfrac{2}{6}$



Add and open at BB'

(2) $$R_{\text{th}} = \frac{\left(\frac{8}{6}+R\right)2}{2+R+\frac{8}{6}} = \frac{16+12R}{20+6R}$$

$$V_{\text{th}} = \frac{20}{6} \times \frac{2}{2+R+\frac{8}{6}} = \frac{20\times 2}{20+5R} = \frac{40}{20+6R}$$



(3) $$I = 0.5 = \frac{V_{th}}{R_{th}+2}$$

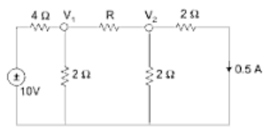
$$I = 0.5 = \frac{40}{20+6R} \div \frac{56+24R}{20+6R}$$

$$= \frac{40}{56+24R}$$

$$12R = 12$$

$$R = 1\ \Omega$$

Alternate method



Given $\frac{V_2}{2} = 0.5 \text{ A}$

$$V_2 = 1 \text{ Volt}$$

At node 1

$$\frac{V_1 - 10}{4} + \frac{V_1}{2} + \frac{V_1 - V_2}{R} = 0$$

$$\frac{V_1 - 10}{4} + \frac{2V_1}{4} + \frac{V_1 - 1}{R} = 0$$

$$3V_1R - 10R + 4V_1 - 4 = 4$$

$$V_1 = \frac{10R + 4}{3R + 4} \quad ...(1)$$

At node 2

$$\frac{1 - V_1}{R} + \frac{1}{2} + 0.5 = 0$$

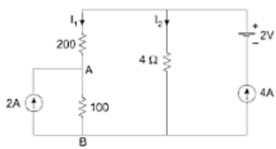
$$1 - V_1 + R = 0$$

$$V_1 = 1 + R \quad ...(2)$$

From (1) and (2)

$$R = 1\ \Omega$$

Example 39: Apply the superposition theorem to the network of Figure to determine the voltage V_{AB}.



Solution: Opening out the two current sources, the circuit is open with no current. Shorting the voltage and opening one current source,

$$I_1 = 4 \times \frac{4}{34} = \frac{16}{34} = 0.471 \text{ A}$$

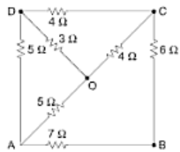
Shorting 2 V and opening out 4 A

$$I_{AB} = 2 \times \frac{24}{34} = 1.41 \text{ A}$$

Total current through $AB = 1.41 + 0.471 = 1.881$ A

$$V_{AB} = 1.881 \times 10 = 18.81 \text{ V}$$

Example 40: Find the resistance between A and B for the given circuit shown.



Solution:

Changing ⅄ ACD into equivalent delta,

$$r_{ac} = \frac{r_a r_c + r_c r_d + r_d r_a}{r_d}$$

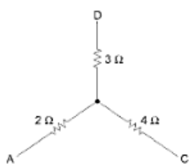


Fig. (1)

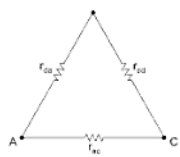


Fig. (2)

where, $r_a = 2\ \Omega$

$r_c = 4\ \Omega$

$r_d = 3\ \Omega$

$$\therefore \qquad r_{ac} = \frac{2\times 4+4\times 3+3\times 2}{3} = \frac{26}{3} = 8.67\ \Omega$$

$$r_{cd} = \frac{r_a r_c \times r_{cd} + r_d r_a}{r_a}$$

$$= \frac{26}{2} = 13\ \Omega$$

$$r_{dc} = \frac{26}{4} = 6.5\ \Omega$$

So the circuit of Fig. (3) reduces to circuit of Fig. (4).

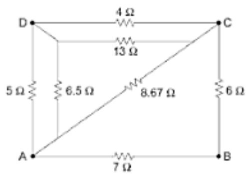


Fig. (3)

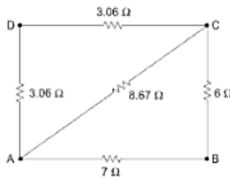


Fig. (4)

Equivalent resistacne between the terminals

$$A\ \&\ D = \frac{5\times 6.5}{5+6.5} = 2.82\ \Omega$$

Equivalent resistance between terminals

$$C\ \&\ D = \frac{13\times 4}{13+4} = 3.06\ \Omega$$

Resistance 2.82 Ω and 3.06 Ω are now in series, so the circuit of Fig. (4) reduces to circuit of Fig. (5)

The resistances 5.88 Ω and 8.76 Ω are now in parallel, so their equivalent resistance

$$= \frac{8.67\times 5.88}{14.55} = 3.5\ \Omega$$

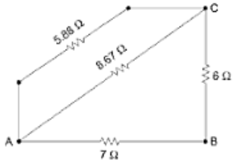


Fig. (5)

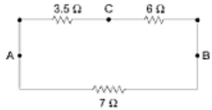


Fig. (6)

The final circuit simplifies the circuit shown in Fig. (7) & Fig. (8)

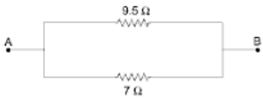


Fig. (7)

Fig. (8)

So, the resistance between the points A & $B = 4.03\ \Omega$ **Ans.**

Example 41: Find the resistance between AB in given circuit below.

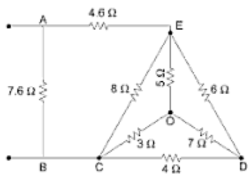


Fig. (1)

Solution:

Transforming ⅄ CDE into equivalent delta,

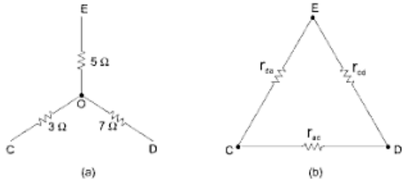


Fig.

$$r_{cd} = \frac{r_c r_d + r_d r_e + r_e r_c}{r_e}$$

where, $r_c = 3\ \Omega$

$r_d = 7\ \Omega$

$r_e = 5\ \Omega$

$$\therefore \qquad r_{cd} = \frac{3 \times 7 + 7 \times 5 + 5 \times 3}{5} = \frac{71}{5} = 14.2\ \Omega$$

$$r_{de} = \frac{71}{3} = 23.67\ \Omega$$

$$r_{ec} = \frac{71}{7} = 10.13\ \Omega$$

Circuit of Fig. (1) simplifies to circuit of Fig. (2)

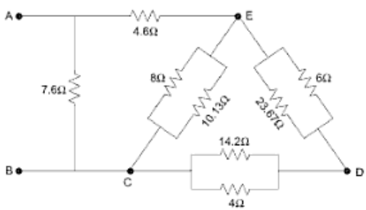


Fig. (2)

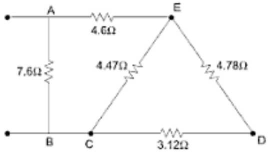


Fig. (3)

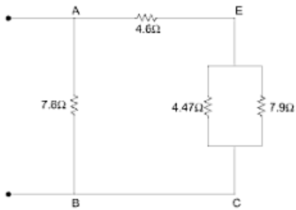


Fig. (4)

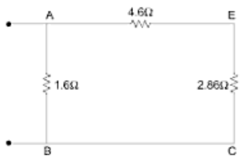


Fig. (5)

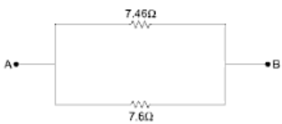


Fig. (6)

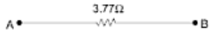
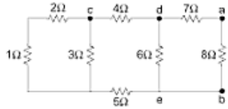


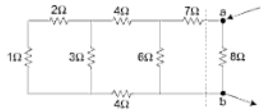
Fig. (7)

Example 42: For the given ckt below, find R_{ab}, R_{cd}, and R_{de}.



Solution:

For finding R_{ab}.



$$R_{ab} = \left\{\left[\underbrace{\{[(1+2)\parallel 3]}_{1.5} + (4+5)\}\parallel 6\right] + 7\right\}\parallel 8$$

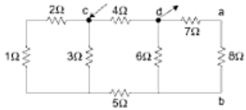
$$= \{[(1.5 + 4 + 5) \parallel 6] + 7\} \parallel 8$$

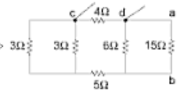
$$= \underbrace{\{[(10.5) \parallel 6]}_{42\parallel 11} + 7\} \parallel 8$$

$$= \left(\frac{42}{22} + 7\right) \parallel 8$$

$$= (10.818) \parallel 8 = \frac{86.55}{18.82} = 4.598\ \Omega$$

(ii) For finding R_{cd}





$$R_{cd} = \{[(1+2) \parallel 3] + 5 + [(6) \parallel (7+8)]\} \parallel 4$$

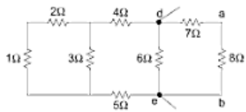
$$= [(3 \parallel 3) + (5) + (6 \parallel 15)] \parallel 4$$

$$= \left(1.5 + 5 + \frac{30}{7}\right) \parallel (4)$$

$$= (10.786) \parallel (4)$$

$$= \frac{43.143}{14.786} = \mathbf{2.92\ ohm} \quad \textbf{Ans.}$$

(iii) For finding R_{de}



$$R_{de} = \underbrace{\{[(1+2) \parallel (3)] + (4) + (5)\}}_{10.5} \parallel (6) \parallel 15$$

$$= (10.5) \parallel (6) \parallel 15$$

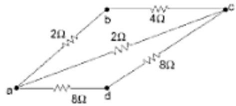
$$= \left(\frac{42}{11}\right) \parallel 15 = (3.82) \parallel 15$$

$$= \frac{57.27}{18.82} = \mathbf{3.043\ ohm} \quad \textbf{Ans.}$$

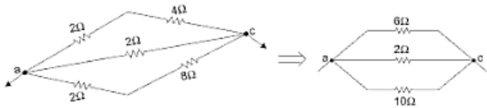
Example 43: Find

(i) R_{ac}

(ii) R_{bd}



(i)



$$R_{ac} = (2 + 4) \parallel 2 \parallel (2 + 8) = 6 \parallel 2 \parallel 10$$

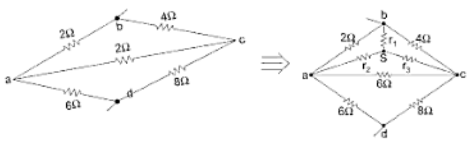
$$= \left(\frac{12}{8}\right) \parallel 10 = \frac{\frac{12}{8} \times 10}{\frac{12}{8} + 10} = \frac{120}{92} \ \textbf{ohm}$$

(ii) R_{bd}

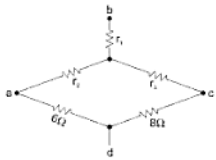
⇒ 2 Ω and 4 Ω are neither parallel nor in series.

⇒ Convert (*abc*) delta into a star.

$$r_1 = \frac{2 \times 4}{12} = \frac{2}{3}\ \Omega,\ r_2 = \frac{12}{12} = 1\ \Omega,\ r_3 = \frac{24}{12} = 2\ \Omega$$



⇒ Now equivalent ckt after conversion



$$R_{bd} = r_1 + [(r_2 + 6) \parallel (r_3 + 8)]$$

$$= \frac{2}{3} + [(1 + 6) \parallel (2 + 8)]$$

$$= \frac{2}{3} + [(7) \parallel 10]$$

$$= \frac{2}{3} + \frac{70}{17}$$

$$= \frac{34 + 210}{51} = \frac{244}{51} = \mathbf{4.78\ ohm}$$

Example 44: Determine the voltage which will be required to pass a current of 40 A through a resistance of 5 ohms.

Solution:

By Ohm's Law,

$$V = IR$$

$$= \mathbf{40 \times 5 = 200\ volts} \quad \textbf{Ans.}$$

Example 45: Calculate the value of resistance of the coil of a heater, which consumes 5 kilowatts across a 200 volts mains.

Solution:

Let R = Resistance of coil in ohms

I = Current flowing in amps.

V = Applied voltage in volts.

W = Power consumed in watts.

Power consumed by coil $W = I^2 R$.

$$\text{Since, } I = \frac{V}{R}$$

$\therefore$ Power consumed, $$W = \frac{V^2}{R}$$

or, $$R = \frac{V^2}{W}$$

Given, $V = 200$ volts, $W = 5 \times 1000$ watts

$$= \frac{200 \times 200}{5 \times 1000}$$

$$= \mathbf{8\ ohms.} \quad \mathbf{Ans.}$$

Example 46: Find the resistance of a metal filament lamp consuming 60 watts on a 240 volts supply. Also state which shall have greater resistance: a 60 W lamp or a 100 W lamp.

Solution:

From eqn.

$$R = \frac{V^2}{W}$$

Resistance of metal filament lamp

$$= \frac{240 \times 240}{60} = \mathbf{960\ ohms} \quad \mathbf{Ans.}$$

If rated voltage is same, resistance is inversely proportional to the wattage. Hence, 60 W lamp will have a greater resistance than 100 W lamp.

Example 47: Which has the greater resistance, a copper wire 20 metre long 0.015 cm in diameter or a platinum silver wire 15 metre long 0.015 cm in diameter at 0°C.

(Sp. resistance for copper and platinum silver alloy are 1.7 and 2.43 microhms-cm, respectively)

Solution:

Copper Wire:

$$\text{Area, } S = \frac{\pi}{4}[0.015]^2$$

$$= 1.77 \times 10^{-4} \text{ sq. cm.}$$

$$R = \rho \frac{l}{S} = 1.7 \times 10^{-6} \times \frac{20 \times 100}{1.77 \times 10^{-4}}$$

$$= 19.2 \text{ ohms}$$

Platinum silver wire:

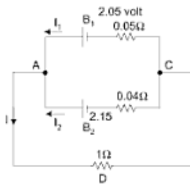
$$R = \rho \frac{l}{S} = 2.43 \times 10^{-6} \times \frac{15 \times 100}{1.77 \times 10^{-4}}$$

$$= 20.6 \text{ ohms.}$$

So, platinum silver wire has greater resistance. **Ans.**

Example 48: Two cells are connected in parallel and supply a current in a circuit of 1 ohm. The emfs of the cells are 2.05 and 2.15 volts and their internal resistances are 0.05 and 0.04 ohms, respectively. Calculate the current of each cell and the potential difference across AC.

Solution:



Let I_1 and I_2 be the currents flowing from the cells B_1 and B_2 towards junction A and I the current flowing away from A.

By Kirchhoff's first law:

$$I_1 + I_2 = I \quad ...(1)$$

Considering the loop AB_1CDA and applying Kirchhoff's second law, we have

$$I_1 \times 0.05 + I \times 1 = 2.05$$

or, $$I_1 \times 0.05 + [I_1 + I_2] \times 1 = 2.05$$

or, $$1.05\, I_1 + I_2 = 2.05$$

Now applying Kirchhoff's second law to loop AB_2CDA we have,

$$I_2 \times 0.04 + [I_1 + I_2] \times 1 = 2.15$$

or,
$$I_1 + 1.04\, I_2 = 2.15$$

Solving equation (3.5) and (3.6), we get

$$I_1 = -0.196 \text{ A}$$

$$I_2 = 2.256 \text{ A}$$

Substituting these values in eqn. (1)

$$I = -0.196 + 2.256$$

$$= 2.06 \text{ A}$$

Terminal P.D. = 2.06 × 1 = 2.06 volts

Result:

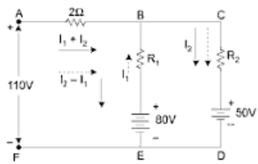
Current of cell B_1 = – 0.196 A.

Current of cell B_2 = 2.256 A.

Terminal P.D. = **2.06 Volts.** **Ans.**

Example 49: What values must, R_1 and R_2 have in circuit of figure given below when,

(a) I_1 = 4 A, and I_2 = 6 A both charging.
(b) I_1 = 2 A discharging and I_2 = 20 A charging and
(c) Under what conditions is I_1 Zero?



Solution:

(a) When currents through both the batteries are charging, the directions of currents are as shown by full arrows.

Applying Kirchhoff's second law to the loops *ABEF* and *ACDF* we have,

$$[I_1 + I_2] \times 2 + I_1 \times R_1 = 110 - 80 \qquad ...(1)$$

and,

$$[I_1 + I_2] \times 2 + I_2 \times R_2 = 110 - 50 \qquad ...(2)$$

Since, $I_1 = 4$ A, $I_2 = 6$ A so from eqn. (1)

$$20 + 4\,R_1 = 30$$

$$= \mathbf{R_1\ 2.5\ \Omega}$$

From eqn. (2)

$$20 + 6\,R_2 = 60$$

$$R_2 = \mathbf{6.67\ \Omega} \quad \textbf{Ans.}$$

(b) When $I_1 = 2$ A discharging and $I_2 = 20$ A charging, the directions of currents are shown by dotted arrows;

Again applying Kirchhoff's second law to the loops *ABEF* and *ACDF* we have

$$[I_2 - I_1] \times 2 - I_1R_1 = 110 - 80 \qquad ...(3)$$

and,

$$[I_2 - I_1] \times 2 + I_2R_2 = 110 - 50 \qquad ...(4)$$

so from eqn. (3)

$$[20 - 2] \times 2 + 2\,R_1 = 30$$

$\therefore$

$$\mathbf{R_1 = 3\ \Omega}$$

From eqn. (4)

$$[20 - 2] \times 2 + 20\,R_2 = 60$$

$\therefore$

$$\mathbf{R_2 = 1.2\ \Omega}$$

(c) When I_1 = Zero, then by Kirchhoff's second law, we have

$$2I_2 + 0.R_1 = 110 - 80 \qquad ...(5)$$

and,

$$[2 + R_2]\,I_2 = 110 - 50 \qquad ...(6)$$

and from eqn. (5) $\quad I_2 = 15$ A

Substituting in eqn. (6) the value of I_2, we have

$$\mathbf{R_2 = 2\ \Omega}$$

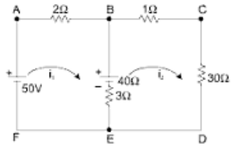
Result:

(a) 2.5 Ω, 6.67 Ω

(b) 3.0 Ω, 1.2 Ω

(c) 2 Ω **Ans.**

Example 50: Find out the currents i_1 and i_2 by loop method.



Solution:

Let i_1 and i_2 be the loop currents in the clockwise direction in the given circuit of above figure.

Applying Kirchhoff's second law to loop 1, we obtain

$$2 \times i_1 + 3[i_1 - i_2] = 50 - 40$$

or, $$5\, i_1 - 3 i_2 = 10$$

Now applying Kirchhoff's law to loop 2, we get

$$1 \times i_2 + 30 \times i_2 + 3(i_2 - i_1) = 40$$

or, $$-3\, i_1 + 34\, i_2 = 40$$

Solving equations (3.26) and (2.27), we obtain

$$i_1 = 2.86 \text{ A}$$

and $$i_2 = 1.43 \text{ A}$$

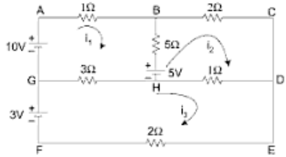
$$\text{Current in } EFAB = i_1 = 2.86 \text{ A}$$

$$\text{Current in } EB = i_2 - i_1 = -1.43 \text{ A}$$

and $$\text{Curent in } BCDE = i_1 = \mathbf{1.43\ A} \quad \textbf{Ans.}$$

This problem can also be worked out by applying Kirchhoff's laws or theorem of superposition.

Example 51: Find out the currents in all the branches of the circuit shown in given figure by loop method.



Solution:

Let i_1, i_2 and i_3 be the loop currents in the clockwise directions as shown in figure.

Applying Kirchhoff's second law to loop – 1

$$1 \times i_1 + 5[i_1 - i_2] + 3[i_1 + i_3] = 10 - 5$$

or, $$9i_1 - 5i_2 - 3i_3 = 5 \quad ...(1)$$

From loop – 2

$$5[i_2 - i_1] + 2i_2 + 1 \times [i_2 - i_3] = 5$$

or, $$-5i_1 + 8i_2 - i_3 = 5 \quad ...(2)$$

From loop – 3

$$3[i_3 - i_1] + 1 \times [i_3 - i_2] + 2i_3 = 3$$

or, $$-3i_1 - i_2 + 6i_3 = 3 \quad ...(3)$$

Multiplying eqn. (2) throughout by 3 and subtracting from eqn. (1)

$$24i_1 - 29i_2 = -10 \quad ...(4)$$

Multiplying eqn. (1) throughout by 2 and adding with eqn. (3)

$$15i_1 - 11i_2 = 13 \quad ...(5)$$

Solving eqn. (4) and (5), we obtain

$$i_1 = 2.85 \text{ A}$$

$$i_2 = 2.705 \text{ A}$$

Substituting these values in eqn. (2) and simplifying, we obtain

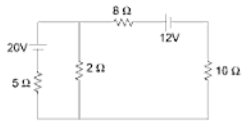
$$i_3 = 2.39 \text{ A}$$

Hence, the currents in different branches are:

$$i_{AB} = i_1 = 2.85 \text{ A}$$

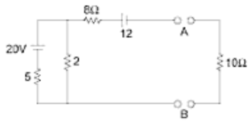
$$i_{BC} = i_2 = 2.705 \text{ A}$$

Example 52: Find the current 10 Ω resistance by thevnin theorem and confirm the result by Norton's theorem.



(CES – 1984)

Solution:



$$R_{th} = 8 + (5 \parallel 2) = 8 + \frac{10}{7} = 9.429\ \Omega$$

$$\text{Voltage across } 2\ \Omega = 20 \times \frac{2}{7} = \frac{40}{7}\ \text{V} \quad \text{(by voltage divide rule)}$$

$$V_{AB} = \left(-12 + 8 \times 0 + \frac{10}{7} \times 0 + \frac{40}{7}\right)$$

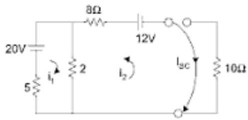
$$V_{AB} = -12 + \frac{40}{7} = -6.29\ V$$

∴ $$V_{Th} = -6.29\ V,\ R_{Th} = 9.429\ \Omega$$

$$\text{Current in } 10\ \Omega = -\frac{6.29}{19.429} = -0.324\ \text{A}$$

$$I_{sc} \text{ from thevnin circuit} = -\frac{6.29}{9.429} = -0.667\ \text{A}.$$

Norton's Theorem



$$7i_1 + 2i_2 = 20$$

$$2i_1 + 10i_2 = 12$$

$$\text{Eliminating } i_1,\ 66i_2 = 44$$

$$i_2 = \frac{44}{66} = 0.667\ \text{A},\quad I_{SC} = -i_2 = -0.667\ \text{A}$$

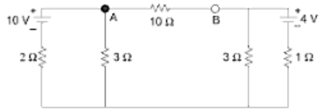
Norton equivalent

$$\text{Current in } 10\ \Omega = -0.667 \times \frac{9.429}{19.429}$$

$$= -0.324\ \text{A}$$

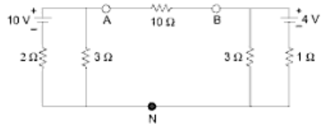
same as in Thevnin circuit.

Example 53: In the circuit given, determine the Thevnin equivalent circuit with respect to terminals A, B and hence current flowing through 10 Ω resistor.



[CES – 1988]

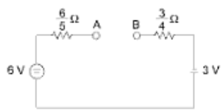
Solution:



$$V_{AN} = 10 \times \frac{3}{5} = 6 \text{ V}, \; V_{BN} = -4 \times \frac{3}{4} = -3 \text{ V}$$

$$R_{AN} = \frac{3 \times 2}{5} = \frac{6}{5} \; \Omega, \; R_{BN} = \frac{3 \times 1}{4} = \frac{3}{4} \; \Omega$$

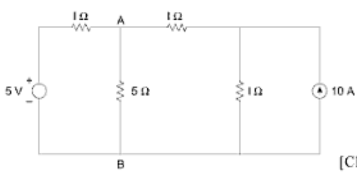
Equivalent circuit



$$V_{AB} = \frac{6}{5} \times 0 + 6 \text{ V} + 3 \text{ V} + \frac{3}{4} \times 0 = 9 \text{ V}$$

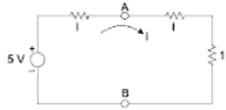
$$V_{AB} = V_{th} = 9 \text{ V}$$

Example 54: Find using Thevnin's theorem the current in the 5 Ω resistor connected across AB in the network shown below:



[CES – 1992]

Solution: By superposition

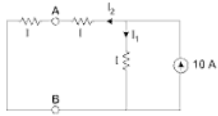


$$I = \frac{5}{3} \text{ A}$$

$$\text{Drop across } 1\,\Omega = \frac{5}{3} \text{ V}$$

$$\therefore \qquad V_{AB} = 5 - \frac{5}{3} = \frac{10}{3} \text{ V}$$

For the current source

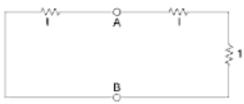


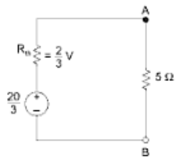
$$I_2 = 10 \times \frac{1}{3} = \frac{10}{3} \text{ A}$$

$$\text{Drop across } 1\,\Omega = \frac{10}{3} \times 1 = \frac{10}{3} \text{ V}$$

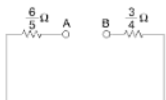
$$\therefore \qquad V_{AB} = \frac{10}{3} + \frac{10}{3} = \frac{20}{3} \text{ V}$$

Thevnin resistance



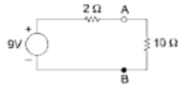


$$R_{th} = 1 \,||\, (1 + 1) = \frac{2}{3}\ \Omega \qquad\qquad I = \frac{20/3}{2/3 + 5} = 1.18 \text{ Amp}$$



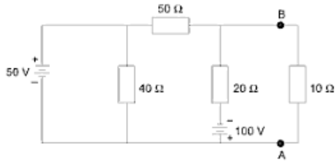
$$\text{Looking into } AB,\ R_{Th} = \frac{6}{5} + \frac{3}{4} = \frac{24 + 15}{20} = \frac{39}{20} = 2\ \Omega$$

Thevnin equivalent circuit

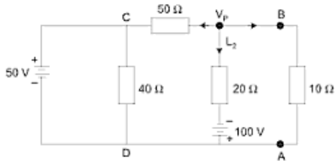


Current through 10 Ω = $\frac{9}{12} = \frac{3}{4}$ A.

Example 55: In the network shown in Figure using nodal analysis find the voltage between *A* and *B*.



Solution: $V_{CD} = 50$ V



Hence, applying Kirchhoff's current law at node *P*, we have

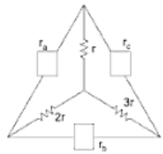
$$\frac{V_P - 50}{50} + \frac{V_P + 100}{20} + \frac{V_P}{10} = 0$$

or, $$V_P = -\frac{400}{17} = -23.53 \text{ V}$$

Hence, $V_{BA} = V_P + 100 = -23.53 + 100 = 76.47$ V

Example 56: Three resistances are connected in star. Find the delta equivalent.

Solution: r, $2r$, $3r$ connected in star:



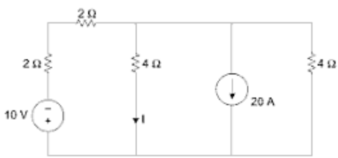
Let the equivalent Δ having a resistance r_a, r_b and r_c.

$$r_a = r + 2r + \frac{r.2r}{3r} = 3r + \frac{2r}{3} = \frac{11}{3}r$$

$$r_b = 2r + 3r + \frac{6r^2}{r} = 11\,r$$

$$r_c = r + 3r + \frac{3r^2}{2r} = 4r + \frac{3r}{2} = \frac{11r}{2}$$

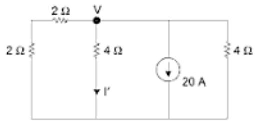
Example 57:



Using superposition theorem, find I

Solution:

Case 1 → taking **20 A** source and short the voltage source.



$$\Rightarrow \qquad I' = \frac{V}{4} \text{ Amp}$$

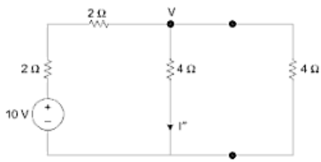
$$\frac{V}{4} + \frac{V}{4} + \frac{V}{4} + 20 = 0$$

$$\boxed{V = -\frac{80}{3} \text{ Volt}}$$

$$I' = \left(-\frac{80}{3}\right) \cdot \frac{1}{4}$$

$$I' = -\frac{20}{3} \downarrow \text{downward.}$$

Case 2 → Taking 10 V voltage source and open the current source. Let current be I''.



$$I'' = \frac{V}{4}$$

$$\frac{V}{4} + \frac{V}{4} + \frac{V - 10}{4} = 0$$

$$\Rightarrow \qquad V = \frac{10}{3} \text{ volt}$$

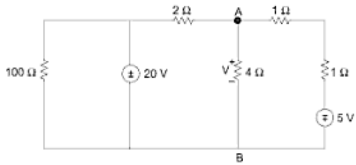
$$I'' = \frac{10}{3 \times 4} = \frac{5}{6} \text{ Amp} \downarrow \text{downward}$$

Now according to superposition principle net current $I = I' + I''$

$$= -\frac{20}{3} + \frac{5}{6} = -\frac{35}{6} \text{ Amp} \downarrow \text{down}$$

$$I = -5.833 \text{ Amp} \downarrow \text{downward}$$

Example 58:

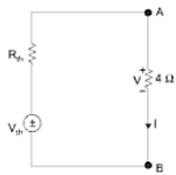


Using Thevnin or Norton find V.

Solution:

(i) *By Thevnin theorem* →

Thevnin equivalent between *AB* is –

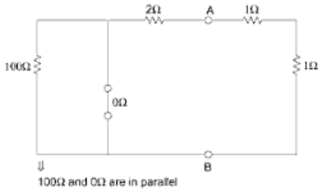


$$I = \frac{V_{th}}{R_{th} + 4}$$

$$V = 4I$$

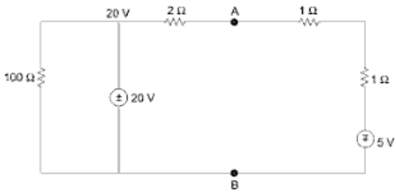
So for finding V, we have to find V_{th} and R_{th}.

(a) For finding R_{th} open the terminal AB and deactivate all the independent sources.



$$R_{AB} = 2 \,||\, 2 = 1\ \Omega$$

(b) For finding V_{th} open the terminal AB, then find the open CKT voltage.



Let B is the dataum node, V_A is voltage at node A.

$$V_{th} = V_{AB} = V_A$$

$$\frac{V_A - 20}{2} + \frac{V_A + 5}{2} = 0$$

$$V_A = +\frac{15}{2} = \mathbf{7.5\ volt}$$

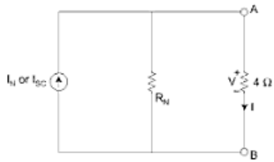
$$\boxed{V_{th} = 7.5\,Volt}$$

$$I = \frac{V_{th}}{4 + R_{th}} = \frac{7.5}{4+1} = \frac{7.5}{5} = 1.5\ \text{Amp}$$

$$V = 4\ I = \mathbf{6\ volt} \quad \textbf{Ans}$$

Using Norton theorem →

Norton's equivalent across AB is

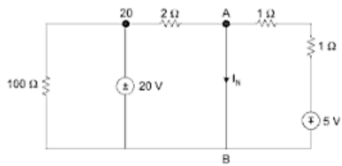


$$I = \frac{R_N}{4 + R_N} \cdot I_N$$

$$\boldsymbol{V = 4I}$$

R_N is same as $R_{th} = 1\ \Omega$

For finding I_N short the terminal AB then find short ckt current I_{sc}.



Let V_B is datum node or at ground, then $V_A = 0$ Volt

$$\frac{V_A - 20}{2} + \frac{V_A + 5}{2} + I_N = 0 \qquad V_A = 0$$

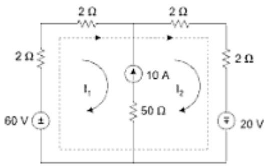
$$-20 + 5 + 2\ I_N = 0$$

$$I_N = \frac{15}{2} = 7.5 \text{ Amp}$$

$$I = \frac{7.5}{4+1} \times 1 = 1.5 \text{ Amp}$$

$$V = 1.5 \times 4 = \mathbf{6\ volt}$$

Example 59: Using mesh analysis find I_1 and I_2.



In loop 1 and loop 2 there is a current source. So we will take a combined loop – (1 + 2) shown by dotted line

$$-60 + 4I_1 + 4I_2 - 20 = 0$$

$$4I_1 + 4I_2 = 80$$

$$I_1 + I_2 = 20 \quad \text{...(1)}$$

Net current through 50 Ω is $(I_2 - I_1) = 50$

$$-I_1 + I_2 = 10 \quad \text{...(2)}$$

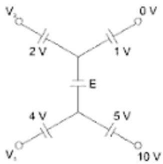
adding [(1) + (2)]

$$2I_2 = 30 \qquad \boxed{I_2 = 15 \text{ Amp}}$$

and, $\boxed{I_1 = 5 \text{ Amp}}$

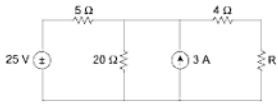
EXERCISE

1. For the shown circuit, find E, V_1 and V_2.



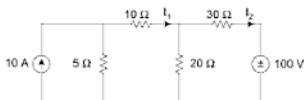
[Ans: $E = -16$ volt]

2. Find the value of R (in ohms) required for maximum power transfer to network shown below.



[Ans: $R = 8\ \Omega$]

3. A resistive network with voltage and current sources shown in fig below. Find currents I_1 and I_2 by using mesh analysis.



[Ans: $I_1 = 0.37$ Amp
$I_2 = -1.85$ Amp]

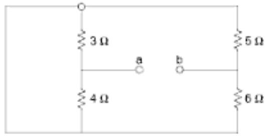
4. Using superposition theorem, find I_2 for the figure of Question (3).

[Ans: $I_2 = -1.85$ Amp]

5. Using Thevnin theorem, find I_1 for the figure of problem – 3.

[Ans: $I_1 = 0.37$ Amp]

6. Determine the resistance between the terminals *a* and *b*.



[**Ans:** 4.44 Ω]

7. A DC circuit comprises two resistors *A* of value 25 Ω and *B* of unknown value, connected in parallel together with a third resistor *C* of value 5 ohms connected in series with the parallel group. The potential difference across *C* is found to 90 V. If the total power in circuit is 4320 Watt, calculate:
   (a) Value of resistor *B*.
   (b) Voltage applied to the end of whole circuit.
   (c) The current in each resistors.

[**Ans:** 12.5 Ω, 240 V, $I_A = 6$ A, $I_B = 12$ A]

8. The resistance in the three arms of the wheat stones bridge are 10,100 and 600 ohms. Find the resistance of the fourth arm. If the voltage of the cell connected to the bridge be 1.5 volt, find branch current in each arm.

[**Ans:** 69 ohm, 0.0136 A, 0.00227 A]

9. With the help of given data in Fig. 1, 2 and 3, find
   (i) The value of load R_L such that a 10 Ampere of current flow from R_L.
   (ii) The value of load R_L for maximum power transfer.

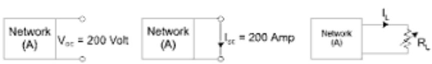
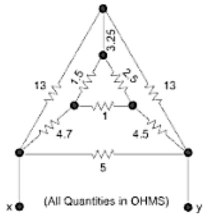


Fig. 1 Fig. 2 Fig. 3

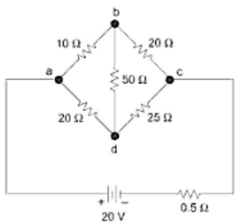
10. (a) Derive relations to show equivalent transformation form delta to star and from star to delta.
    (b) In the circuit shown in figure, determine the resistance between the terminals *x* and *y*. [2.96 ohms.]



[**Ans:** 2.96 Ω]

11. (a) What is Thevnin theorem?

 (b) Find current through a resistance of 50 ohms connected between *b* and *d* by Thevnin's theorem in the network shown in figure.

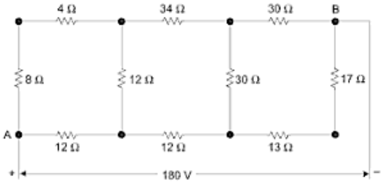


[0.0316 A]

12. Find the current in 10 Ω resistance in the network shown in figure by

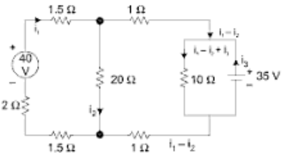
 (a) Star-Delta conversion

 (b) Thevnin's (Helmoltz's) theorem.



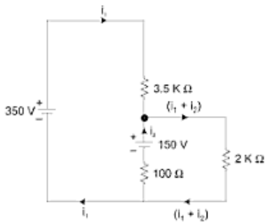
[4 Amp]

13. Find out the current in the various branches of the circuit shown in figure.



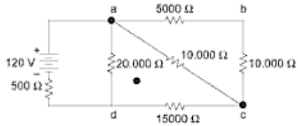
[i_1 = 1.88 A, i_2 = 1.53 A, i_3 = 2.625 A]

14. Estimate the current in each branch of the circuit shown in figure.



[i_1 = 69.5 mA, i_2 = 17.5 mA]

15. Determine the following in the network of figure:
    (a) Current i_1 given by the battery of 240 V;
    (b) P.D. across c and d,
    (c) magnitude and direction of current in ac.

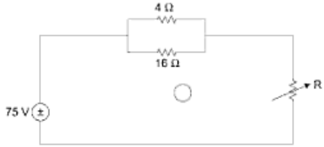


[22.35 mA, 163.5 V, 6.55 mA, from a to c]

16. Find the resistance across terminals RB, BY, RY.

[**Ans:** $RB = 27.5\ \Omega$
$BY = 30.17\ \Omega$
$RY = 33.67\ \Omega$]

17. Power in the 16 ohm resistor is 4 W. Find *R*.

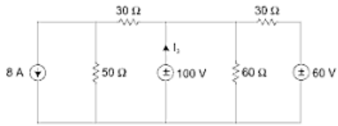


[**Ans:** R = 26.8 Ω]

18. Twelve wires, each of 2 Ω resistance, are joined to a cubical framework. Calculate the resistance between (a) two opposite corners of the cube (b) two adjacent corners (c) two opposite corners of one face.

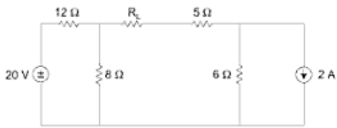
[**Ans:** $\left[\frac{5}{3}\,\Omega, \frac{7}{6}\,\Omega, \frac{3}{2}\,\Omega\right]$]

19. Apply superposition to the given circuit to find i_3

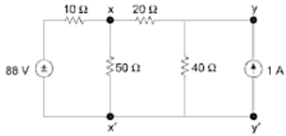


[**Ans:** –0.75 A]

20. Find the value of R_L such that it absorbs maximum power and find the value of that power.

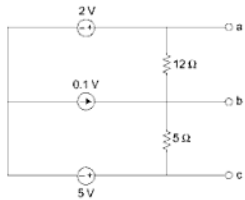


21. Find the Thevnin equivalent of the network in given figure below when viewed from terminals (a) x and x' (b) y and y'

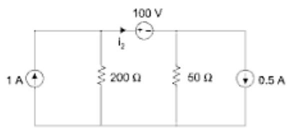


[Ans: (a) 69.3 V, 7.32 Ω
(b) 59.5 V, 16.59 Ω]

22. Find the Norton equivalent across the terminal (i) ab (ii) bc for the given below network.

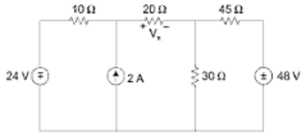


23. Use superposition theorem to find i_2 in the circuit shown below and calculate the power absorbed by each of five circuit elements.

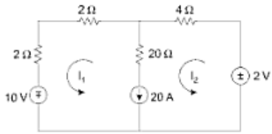


[Ans: 1.3 A, 60 W, 18 W, –130 W, 32 W, 20 W]

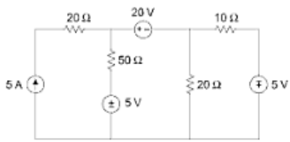
24. Use superposition theorem to find the value of V_x in the circuit.



25. Using mesh analysis find I_1 and I_2



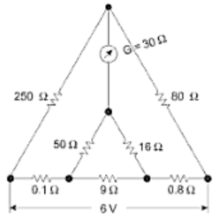
26. Using nodal analysis find the current in each branch.



27. State and prove maximum power transfer theorem for a DC network.
28. Derive the relation between star to delta and delta to star transformation prove if all the three resistances are equal in ⅄ and Δ then

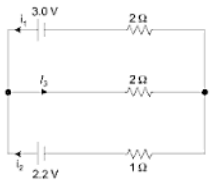
$$R_{\Delta(\text{Delta})} = 3\, R_{\curlywedge(\text{Star})}$$

29. In the network shown in figure determine the current flowing in the galvanometer. [2.38 A]



30. (a) What is the principle of superposition theorem? Write the limitations also.

(b) In the network shown in figure determine the currents i_1, i_2 and i_3. [i_1 = 0.576 A, i_2 = 0.35 A, i_3 = 0.926 A]



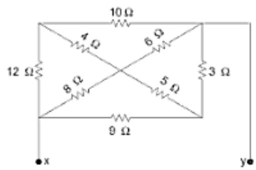
31. Three resistances of 90 Ω, 60 Ω and 36 Ω are connected in parallel. Determine their equivalent resistance graphically and verify the result analytically. [18 Ω]

32. The body of a synchronous motor is connected to three earthing plates having resistances to earth of 5 Ω, 10 Ω and 15 Ω, respectively. Due to fault the body becomes alive. Determine the proportion of total fault energy which is dissipated at each earth connection.

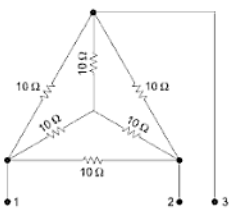
[54.5%, 27.3%, 18.2%]

33. A constant voltage E is applies to N groups of rheostats in series, where each group has M identical rheostats in parallel. One rheostat burns out in one group. Find the percentage increase of current in each rheostat of the faulty group and the percentage decrease of current in each rheostat of the sound groups. $\left[\dfrac{100(N-1)}{MN-N+1}; \dfrac{100}{MN-N+1}\right]$

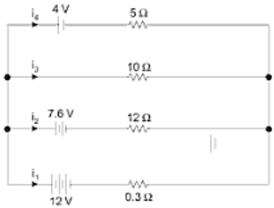
34. In the network shown in figure, determine the equivalent resistance between the points X and Y. If an emf of 20 V is applied across XY, what would be the total current drawn from the supply and what would be the power dissipated in the circuit. [4.91 Ω, 4.06 A, 81.2 W]



35. Determine the resistance between any two terminals of the network shown in figure. [5 Ω]



36. Estimate the current in each branch of the network shown in figure 3.70.

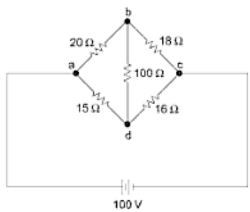


[i_1 = 8.15 A; i_2 = 2.43 A; i_3 = 2.17 A; i_4 = 3.55 A]

37. Two resistors of value 1000 Ω and 4000 Ω are connected in series across a constant voltage supply of 200 volts. Determine (a) Potential difference across 4000 Ω resistor, (b) The change in supply current and the reading on a voltmeter of 12,000 Ω resistance when it is connected across the 4000 Ω resistor.

[160 V, 10 mA, 150 V]

38.
   (a) State Kirchhoff's laws;
   (b) Determine the values of currents flowing in each branch of the circuit shown in figure and the direction of the current in the branch *bd*.

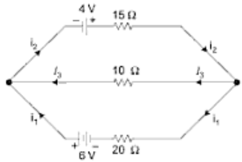


[i_{ab} = 2.614 A, i_{ad} = 3.25 A, i_{bc} = 2.65 A, i_{db}= 0.036 A, i_{da} = 3.214 A]

39. Two batteries of emf 50 V and 40 V are in parallel across an external resistance of 10 ohms. The internal resistances of batteries are 2.5 Ω and 2 Ω, respectively. Find the current in batteries and the external resistor.

[4 A Charging, Zero A discharging; 4 A]

40. Determine the currents and their correct directions in all the branches of the circuit shown in figure by superposition theorem. The internal resistances of the batteries are negligible.



[i_1 = 0.292 A, i_2 = 0.277 A, i_3 = –0.015 A]

41. Determine the value of resistance *R* in the circuit of figure, when no current is flowing through galvanometer *G*. Find also the magnitudes and the directions of the currents in all branches.

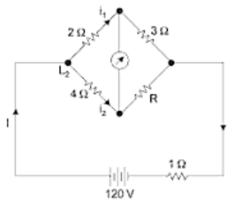


Fig.

[R = 6 Ω, I = 27.7 A, I_1 = 18.47 A, I_2 = 9.23 A]

4

Electrical Measuring Instruments and Measurements

4.1 DIFFERENT TYPES OF MEASURING INSTRUMENTS

For measuring various electrical quantities, different types of measuring instruments are used. More common of them may be summarised below:

(1) *Galvanoscope*–It is used to detect the presence and the direction of an electric current.

(2) *Galvanometer*–It is used not only to indicate the presence of an electric current but to measure its strength also. Galvanometers are also of many types e.g., Tangent galvanometer, Sine galvanometer, Helmholtz galvanometer, Kelvin's galvanometer, Moving coil galvanometer, Moving iron galvanometer, etc.

(3) *Ammeter or Ampere-meter*–It is used to measure the strength of current in an electric circuit and for measurement of week current a milli-ammeter is used.

(4) *Voltmeter*–They are used to measure the potential difference or voltage.

(5) *Wattmeter*–It is used to measure the power or the rate at which energy is utilized in a circuit.

(6) *Energymeter*–It is used to measure the quantity of electricity consumed in a given time in a circuit. Thus, an energymeter does not merely indicate the rate of supply of energy but also takes into account the time for which the power is supplied. It may also be termed as supply meter.

(7) *Ohmmeter*–It is used to measure the resistance of a circuit directly.

(8) *Megger*–It is used to measure very high resistance of the order of megohms i.e., measurement of insulation resistance of cables, etc.

(9) *Avometer (Ampere Volt and Ohmmeter)*–It is a commercial instrument combining an ammeter, voltmeter and Ohmmeter in one instrument, hence, the name avometer. It is classified under the heading "Multimeters".

(10) *Maximum Demand Indicator*–It indicates the maximum current or power supplied in a given duration in a circuit. It is required where the tariff on electrical energy is charged according to maximum demand.

(11) *Power factor meter*–It is used to measure the power factor of load in an AC circuit.

(12) *Frequency meter*–It is used to indicate the frequency in an AC circuit.

(13) *Oscillograph*–It is used to observe and measure the transient phenomena in an electric circuit.

4.2 CLASSIFICATION OF MEASURING INSTRUMENTS

(1) *According to use*–As discussed in § 4.1.

(2) *According to characteristics*–

  (a) Absolute instruments.

  (b) Secondary instruments.

(3) *According to the effects of Electric Current or Voltage* – As will be discussed in § 4.5.

4.3 ABSOLUTE INSTRUMENTS

They give the value of quantity to be measured in terms of the constants of the instruments and their deflection only e.g., tangent galvanometer, etc.

4.4 SECONDARY INSTRUMENTS

They are calibrated instruments and directly give the value of quantity to be measured i.e., current, voltage, power, etc.

The secondary instrument are further classified into:

(1) Indicating Instruments.

(2) Recording Instruments.

(3) Integrating Instruments.

4.4.1 Indicating Instruments

They are fitted with a pointer which moves over a calibrated scale and indicates the electrical quantity to be measured directly on the scale. Ammeters, voltmeters, wattmeters and ohmmeters, etc., refer to this category.

4.4.2 Recording Instruments

They record the electrical quantity to be measured on a graph paper for a desired period. The moving system of the instrument carries an inked pen which rests gently over the graph paper and moves at a low and uniform speed in a direction perpendicular to that of the movement of the pen. The recording instruments are often used in the power stations for recording the variation of power factor and frequency, etc.

4.4.3 Integrating Instruments

They measure the total quantity of electricity consumed in a circuit in a given time. Ampere-hour meters, watt-hour meters and kilo-watt-hour meters refer to this category.

4.5 EFFECTS USED IN MEASURING INSTRUMENTS

The following effects of an electric current or voltage are used in secondary instruments:

(1) *Magnetic Effect*–Used in ammeters, voltmeters, wattmeters, integrating meters, etc.

(2) *Electrodynamic Effect*–Used in ammeters, voltmeters and watt-meters.

(3) *Electromagnetic Induction Effect*–Used in AC ammeters, voltmeters and watt-meters and integrating meters.

(4) *Electrostatic Effect*–Used in voltmeters.

(5) *Chemical Effect*–Used in DC ampere hour meters.

(6) *Heating Effect*–Used in ammeters and voltmeters.

Based upon the above effects, the important types of instruments called by their commercial names are summed up in Table 4.1. It will be seen from the table that moving coil permanent magnet type instruments are suitable on DC only and induction type on AC only. All the rest type of instruments can be used on either AC or DC.

4.6 WORKING OF INDICATING INSTRUMENTS

As explained in § 4.4.1, the indicating instruments are fitted with a pointer which moves over a calibrated scale and indicates the electrical quantity to be measured directly on the scale. The moving system to which the pointer is attached is subjected to the under-noted three torques.

(i) Deflecting (operating) torque.
(ii) Controlling (restoring) torque.
(iii) Damping torque.

4.7 DEFLECTING TORQUE

The deflecting torque is produced on the moving system by one of the effects explained in § 4.5. Under the action of the deflecting torque the moving system of the instrument moves the pointer from its initial position. The movement of the pointer of course depends upon the magnitude of the deflecting torque which in turn depends upon the magnitude of the electrical quantity to be measured i.e., current, voltage, power, etc. the production of the deflecting torque will be discussed in detail in articles to be discussed subsequently.

4.8 CONTROLLING TORQUE

The movement of the moving system under the action of the deflecting torque would be indefinite, if not controlled by means of a controlling force. The controlling torque opposes the deflecting torque and so the pointer is brought to rest at a position where these two torques balance each other. As stated in § 4.7 the deflection of the pointer should depend upon the electrical quantity to be measured. But without a controlling force, the pointer will jump over to the position of the maximum deflection irrespective of the size of the electrical quantity. Further, the pointer will not return to the initial position on disconnecting the supply. Hence, no indicating instrument can function properly in the absence of controlling force.

The controlling or restoring torque is produced by the following two devices:

(i) Spring control
(ii) Gravity control

4.8.1 Spring Control

In Fig. 4.1, a spindle free to turn between two pivots is shown. The moving system is attached to the spindle. A phosphor bronze hair spring is also attached to the spindle. When the spindle turns under the action of the deflecting torque, the phosphor bronze control spring gets twisted and hence develops a restoring torque. The restoring torque (T_r) proportional to the angle of deflection (θ) of the moving system and if the deflecting torque (T_d) proportional to the current passing through the instrument, then

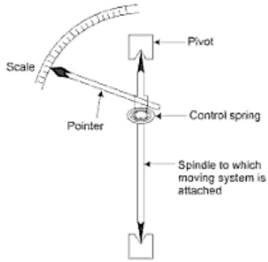


Fig. 4.1

$$\therefore \quad T_c \propto \theta \quad ...(4.1)$$

and, $$T_d \propto I \quad ...(4.2)$$

The pointer comes to a position of rest, when

$$T_c = T_d \quad ...(4.3)$$

$$\therefore \quad \theta \propto I \quad ...(4.4)$$

Since, the deflection is directly proportional to the current hence the spring controlled instruments have a uniform scale over their entire range.

4.8.2 Properties of Material used for Spring

(a) It should be non-magnetic.

(b) It should not develop appreciable fatigue.

If the spring is also used as a lead to the moving system the spring material should also have the following additional properties:

(i) It should have low resistance.

(ii) The temperature coefficient should also be low.

4.8.3 Gravity Control

The instruments having gravity control should be kept vertical for taking the reading. The spindle *S* to which the moving system is attached should be horizontal. The gravity control is obtained by attaching adjustable control and balance weights to the spindle *S* as shown in Fig. 4.2(a).

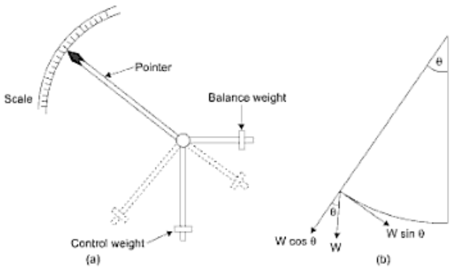


Fig. 4.2

When the spindle *S* turns under the action of the control weight the control and balance weight occupy positions shown dotted. Hence, due to gravity a restoring torque is exerted on the moving system. If *W* is the control weight, the restoring torque (T_r) is proportional to $W \sin \phi$.

i.e., $$T_r \propto W \sin \theta \qquad ...(4.5)$$

Again the deflecting torque,

$$T_d \propto I \qquad ...(4.6)$$

The moving system will occupy a position of rest, when

$$T_d = T_c$$

or, $$I \propto W \sin \theta$$

or, $$I \propto \sin \theta \qquad ...(4.7)$$

In this case we find that I is proportional to sin θ and not θ. Hence, in gravity controlled instruments the scale is not uniform. It is cramped for the lower readings.

Advantages of gravity control are:

(i) It is cheap.
(ii) It is not affected by changes in temperature.
(iii) It does not deteriorate with time.
(iv) It is not subject to fatigue.

Disadvantages of gravity control are:

(i) The surface must be level.

(ii) The instrument has to be kept vertical.

(iii) The scale is cramped.

4.9 DAMPING TORQUE

We have already seen that the moving system of the instrument will tend to move under the action of the deflecting torque. But on account of the control torque it will try to occupy a position of rest where the two torques are equal. Due to inertia of the moving system, the pointer will not come to rest immediately but oscillate about its final deflected position as shown in Fig. 4.3.

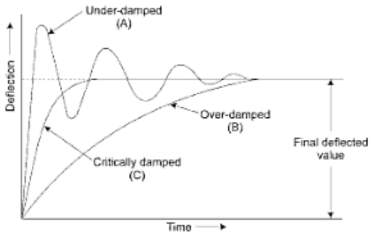


Fig. 4.3

When a damping device is used pointer moves somewhat slowly and reaches the final deflected position without overshooting. There may be two cases:

(i) *Critically damped*–If the pointer moves quickly to its final deflected value, the instrument is called 'critically damped' or 'dead beat' as shown by curve *C* is Fig. 4.3.

(ii) *Over damped*–In this case the pointer moves slowly as shown by curve *B*.

To obtain best results, the instrument is slightly under-damped.

The damping torque is produced by the following methods.

(i) Air friction damping.

(ii) Fluid friction damping.

(iii) Eddy current damping.

4.9.1 Air Friction Damping

A light aluminium piston is attached to the moving system of the instrument which is allowed to travel in a fixed air chamber closed at one end as shown in Fig. 4.4. The clearance between the piston head and the air chamber is very small. The damping is effected by compression and suction of the air.

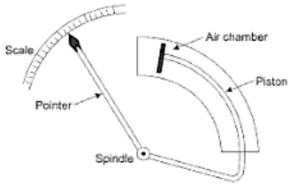


Fig. 4.4

4.9.2 Fluid Friction Damping

In this case the piston moves in a fluid chamber in place of air chamber. Due to greater viscosity of fluid, the damping is more effective.

Disadvantages:

(i) Unsuitable for use in portable instruments.

(ii) Instruments are always required to be placed in upright position.

4.9.3 Eddy Current Damping

It is the most efficient type of damping and is very commonly used in modern instruments. Two ways of achieving this type of damping are discussed below.

(a) Mounting a Thin Disc:

A thin disc of conducting but non-magnetic material like copper or aluminium is mounted on the spindle to which the moving system and the pointer of the instrument is attached as shown in Fig. 4.5.

When the disc rotates it cuts lines of force between the pole of a permanent magnet. Due to motion, eddy currents are induced in the disc which produce a damping torque in a direction opposite to the motion of the disc. Hence, the eddy currents tend to retard the motion of the disc and make the instrument 'dead beat' type.

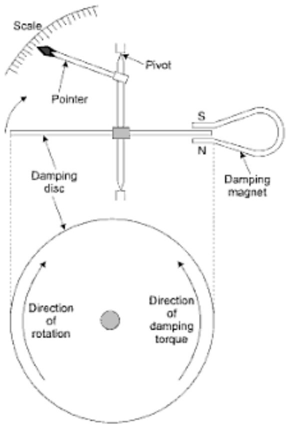


Fig. 4.5

(b) Mounting coil on a Thin Aluminium Former:

In permanent magnet moving coil instruments, the moving coil is wound on a thin light aluminium former in which eddy currents are produced when the coil moves in the field of the permanent magnet. The eddy currents produce a damping torque which opposes motion.

4.10 MOVING IRON INSTRUMENTS

As pointed out in Table 4.1, moving iron instruments are suitable to measure current and voltage in an AC or DC circuit. Such instruments are of two types:

(i) Attraction type

(ii) Repulsion type

4.10.1 Attraction Type

A sectional view of an attraction type of instrument is shown in Fig. 4.6. The working of the instrument depends on attraction of an elliptic disc

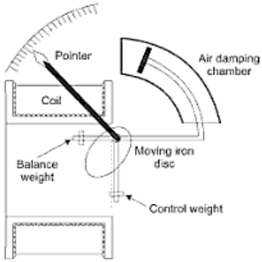


Fig. 4.6

eccentrically pivoted into a magnetic field. The magnetic field is produced by the current following through a coil wound over a solenoid. In case of voltmeter the current through the magnetising coil is proportional to the voltage to the measured.

The instrument has a few turns of thick wire in case of ammeter and many turns of fine wire in that of voltmeter.

When current flows through the coil, a magnetic field is produced along its axis and the moving iron disc gets magnetised in such a way that it will be attracted into the centre of the coil. The movement of the disc will of course depend upon the current strength. If a pointer is fixed to the spindle carrying the iron disc, the pointer will read the current or voltage on a calibrated scale.

It should be noted that the iron disc will always be attracted inwards, whatever may be the direction of the current through the coil so the instrument is called attraction type and is suitable on both DC and AC.

As shown in Fig. 14.6 air friction damping is provided. Now, deflecting torque (T_d) $\propto$ force with which the disc is attracted.

or, $\propto$ field strength (H) of the coil $\times$ pole strength of disc.

But, pole strength of the disc $\propto H$

and, $\propto i$ $\qquad H = \dfrac{Ni}{l}$

$\therefore$ $Td \propto H \times H \qquad H \propto i$

$$\propto i^2 \qquad ...(4.8)$$

Controlling torque $T_c \propto \theta$ ...(4.9)

where, θ = angle of deflection.

When the pointer comes to the position of rest, the two torques should be equal,

$$\therefore \qquad T_c = T_d$$

$$\theta \propto i^2$$

Thus, deflection is proportional to the square of the current at any instant and hence the scale is cramped for the initial readings and these instruments are used for both AC and DC.

4.10.2 Repulsion Type

(i) In the repulsion type there are two rods or pieces of iron inside the coil one fixed and other movable. These are similarly magnetised when the current flows through the coil, and repulsion of the moving iron from the fixed one takes place. The force of repulsion is obviously proportional to the square of current in the coil.

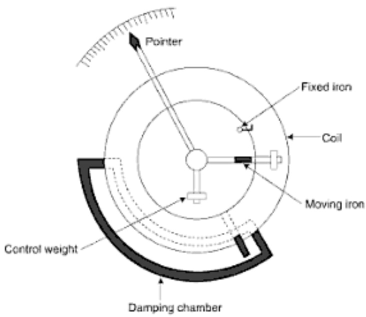


Fig. 4.7

(ii) Whatever be the direction of the current in the coil of the instrument, the magnetization of the moving iron is such that the repulsion always takes place.

(iii) In this type fixed iron consists of a tongue shaped piece of sheet iron bent into a cylindrical form, the moving iron consists of another piece of sheet iron bent and mounted so as to move parallel to the fixed iron and towards its narrower end. It is found that these shapes of the iron give a more uniform scale than is obtained with plain rods.

(iv) The scales of these instruments are uneven being very crowded near the zero. Readings below 1/10 of the maximum are therefore unreliable. According to specification of the Engineering Standards Committee, the useful range commences at 1/4 of the maximum.

4.10.3 Control

In moving iron instruments usually gravity control is used. But in modern instruments spring control is also being used.

4.10.4 Damping

In moving iron instruments air damping is common. However, fluid damping can also be used. But eddy current damping is not possible because the presence of a permanent magnet will affect the deflection of the moving system and hence the reading of the instruments will be incorrect.

4.10.5 Errors in Moving Iron Instruments with both DC & AC

They are liable to errors due to three different causes:

(a) Hysteresis

(b) Stray magnetic field.

(c) Change in resistance of the coil due to temperature changes.

(a) *Hysteresis*–The effect of hysteresis is to make the readings with falling current higher than with rising current, and also to produce a small deflection when the current is again zero because of residual magnetism.

The error is reduced by choosing a low value of flux density in the iron so that the hysteresis effect in the iron is small.

(b) *Stray Magnetic Fields*–Errors that are caused due to the magnetic or electrostatic fields existing around apparatus.

Errors due to this cause may be serious, if not guarded against owing to the weakness of the operating magnetic field.

Such errors are minimised by magnetic screening of the working part of the instrument by an iron case or a thin iron shield.

(c) *Temperature change*–To minimise the errors due to temperature the coil is made of material having low temperature coefficient of resistances.

With AC Only:

The above three kinds of errors take place both on AC and DC. But on AC changes of frequency may produce errors due to change of reactance of the working coil.

4.10.6 Advantages of Moving Iron Instruments

(i) They are cheap and robust.

(ii) They can be used both on AC and DC.

(iii) They can withstand overloads momentarily.

(iv) Since, the deflecting torque is proportional to current square, hence, they have high operating torque.

4.10.7 Disadvantages

(i) Stray magnetic fields affect the readings, so the instrument should be properly shielded.

(ii) The scale is not uniform. It is cramped for the initial readings.

(iii) When used on AC appreciable error is introduced due to changes in frequency.

(iv) Due to hysteresis the readings for increasing values of current and voltage are lower than for decreasing values.

4.11 MOVING COIL PERMANENT MAGNET INSTRUMENT

(i) It consists of a permanent magnet with soft iron pole pieces between which a cylindrical iron core is mounted.

(ii) A rectangular coil of fine wire is pivoted so that it can rotate in the air gap between the pole pieces and the core, and current is led into and out

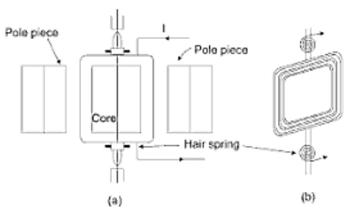


Fig. 4.8

of the coil through two phosphor bronze hair springs, as shown in Fig. 4.8(a) and (b).

(iii) These springs also provide the controlling torque.

(iv) When the coil carries current a deflecting torque is set up, proportional to the product of the current and the strength of the magnetic field in the air gap.

(v) In ordinary instruments the total deflection is only about 60% so that the coil sides may always lie in a field of uniform strength.

(vi) The restoring torque due to the springs is proportional to the deflection, so that the deflection is proportional to the current in the coil. Hence, the scale is uniform.

(vii) Damping is provided by winding the coil on a copper or aluminium frame.

(viii) When used as a voltmeter the instrument is connected in series with a high series resistance, but when used as an ammeter it is connected across a low resistance shunt.

4.11.1 Working of a Moving Coil Permanent Magnet Instrument

(i) The air gap between the magnet poles and iron core is small (about 1 mm) and the flux density is uniform and radial in direction.

(ii) If a current I flows downwards in the left hand side conductor and upwards in the right hand side conductor of the moving coil, forces F and F will act on the two sides of the coil as shown in Fig. 4.9.

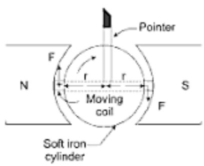


Fig. 4.9

(iii) The torque causing the coil to rotate is thus $F \times 2r$ where, r is the mean distance of the conductor from the axis of rotation.

(vi) If there are N turns in the coil and B is the flux density in wb/m^2 and l is the active length in meter then force F for the given current I is given by

$$F = BIl$$

for N turns of coil

$$F = NBIl \text{ Newton} \quad ...(4.11)$$

(v) Since, the field is radial, the deflecting torque (T_d) is constant for all position of the moving coil, provided its sides are with in the pole area of the magnet.

Deflecting torque (T_d) = Force × Perpendicular distance

$$= (NBIl) \times (2r)$$

$$= NBI(l \times 2r)$$

$$= NBIA \quad \{A = \text{face area of the coil}\}$$

$$T_d = NBIA \quad ...(4.12)$$

is B is constant then

$$T_d \propto I$$

$$T_d = KI \quad ...(4.13)$$

k is constant for a given instrument.

(vi) Such instrument are invariably spring controlled so controlling torque $T_c \propto \theta$

$$T_c = c\theta \quad ...(4.14) \quad \{\theta = \text{angular deflection}\}$$

(vii) For steady state

$$T_c = T_d \quad \theta = \frac{k}{c} I \quad ...(4.14) \quad \{c = \text{spring constant}\}$$

$$\theta \propto I \quad ...(4.15)$$

Note: Moving coil permanent magnet instruments can be used only for direct current measurements, because deflection θ is proportional to current ($\theta \propto I$).

4.11.2 Advantages of Moving Coil Instruments

(i) Low power consumption.

(ii) High torque weight ratio.

(iii) Uniformity of scale and the possibility of a very long scale.

(iv) The possibility of a single instrument being used, with shunt and series resistances, to cover a wide range of both currents and voltages.

(v) Freedom from hysteresis errors due to stray magnetic fields.

(vi) Perfect damping is obtained simply by eddy currents induced in the metal former of the moving coil.

4.11.3 Disadvantages of Moving Coil Instrument

(i) Friction and heating errors are present.

(ii) The weakening of the permanent magnet with the passage of time may introduce a considerable error unless the magnet is carefully aged during manufacture.

(iii) Thermoelectric emfs may introduce errors when these instruments are used shunted for current measurements, but with a well designed shunt such errors should be small.

(iv) Can be used on DC only.

4.12 DYNAMOMETER TYPE AMMETER AND VOLTMETER

The permanent magnet moving coil instruments are only suitable on DC. With an AC supply the deflecting torque produced will be of alternating nature. Owing to the inertia of the moving system, the instrument will not be able to indicate any reading. But if the field is also of alternating nature so that its direction is also reversed in the moving coil. Then the deflecting torque produced would act in one direction only and will cause the moving system to deflect. This principle is employed in dynamometer type of instruments thereby making them suitable on both AC and DC.

The essential feature of a dynamometer type of instrument is that the permanent magnet is replaced by a fixed coil. In Fig. 4.10 two air cored fixed coils F_1 and F_2 and a moving coil M are shown. A pointer is attached to the moving coil amp the controlling torque is obtained by a hair spring.

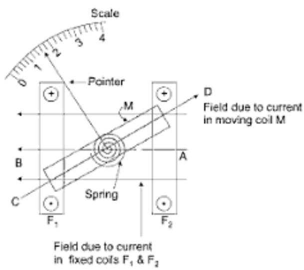
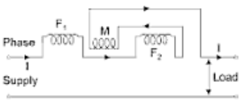


Fig. 4.10

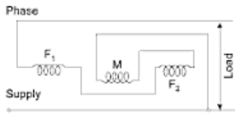
When a current flows through the fixed coils, a magnetic field is produced along AB as shown in Fig. 4.10. Also due to passage of current through the moving coil another field is set up along CD.

Thus, a deflection torque is produced in the clockwise direction which shall deflect the pointer on a calibrated scale.

When the instrument is used as ammeter and voltmeter, the fixed coils and moving coil are connected in series as shown in Fig. 4.11(a) and (b), respectively.



(a) Connection for dynamometer type ammeter



(b) Connection for dynamometer type voltmeter

Fig. 4.11

Deflecting torque (T_d) $\propto H_1 \times H_2$ ...(4.16)

where, H_1 = Field strength due to fixed coils.

H_2 = Field strength due to moving coil.

In case of ammeter same current is passing through the fixed coils and the moving coil.

$\therefore$ $H_1 \propto I$

and, $H_2 \propto I$

where, I = Current through the fixed and moving coils

$\therefore$ Deflecting Torque $\propto I^2$ ...(4.17)

Controlling torque $\propto \theta$ ...(4.18)

$\therefore$ $I^2 \propto \theta$ ...(4.19)

Or, $I \propto \sqrt{\theta}$ ...(4.20)

In case of voltmeter, $I = \dfrac{V}{Z}$

$\because$ Deflecting torque $\propto \left(\dfrac{V}{Z}\right)^2$ ...(4.21) where, Z is the total impedance of the voltmeter.

or, Deflecting torque $\propto V^2$ ...(4.22)

and, Controlling torque $\propto \theta$

$\therefore$ $V^2 \propto \theta$ ...(4.23)

or, $V \propto \sqrt{\theta}$ ...(4.23)

Hence, the readings of current and voltage as given by this type of instrument will be proportional to the square root of the deflection. So, the scale is cramped near the zero.

4.12.1 Control

A phosphor bronze hair spring is used to produce the restoring torque.

4.12.2 Damping

Air or fluid damping is used. Due to week operating fields, eddy current damping is not suitable.

4.12.3 Advantages

(i) They can be used on both DC and AC.
(ii) They are not subject to hysteresis and eddy current errors.

4.12.4 Disadvantages

(i) They have low torque/weight ratio and so a poor sensitivity.
(ii) Costlier than moving iron and moving coil types of instruments.
(iii) The moving system is heavier and so the frictional losses are more.
(iv) On DC they are expensive and inferior to moving coil permanent magnet instruments.

4.13 DYNAMOMETER TYPE WATTMETER

The average power in an AC circuit is given by

$$P = VI \cos \phi$$

where, V = RMS value of voltage

I = RMS value of current.

$\cos \phi$ = power factor.

Since, the term power factor is also involved in the expression of power in an AC circuit, hence power cannot be measured by merely an ammeter and voltmeter. A power factor meter will also be needed. However, if a wattmeter is used, it will take into account the power factor of the circuit and by a single instrument it would be possible to measure the power.

Thus, the more important application of dynamometer type of instruments is in wattmeters. Though power can be measured by this type of wattmeter in DC circuits also, but it is not so important as the product of ammeter and voltmeter readings will directly give the power.

The wattmeter has two coils, current coil and pressure coil as shown in Fig. 4.12. This current coil is the fixed coil and carries load current, while the pressure coil is the moving coil as shown in Fig. 4.10 and carries a current proportional to the voltage. The deflection of the moving coil depends upon the currents in these two coils and upon power factor.

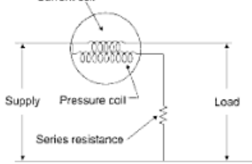


Fig. 4.12

A high non-inductive resistance is connected in series with the pressure coil to make the resistance of the whole pressure coil large in comparison with its reactance and also it helps in reducing the current through the pressure coil.

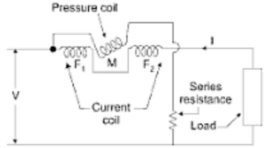


Fig. 4.13

As explained above the two fixed coils F_1 and F_2 will be connected in series with the load as shown in Fig. 4.13. The moving coil with a series resistance is connected across the supply and taken a current proportional to the supply voltage.

Let i_1 = instantaneous value of the current through the current coil.

i_2 = instantaneous value of current through the pressure coil.

v = instantaneous value of supply voltage.

Z = total impedance of the pressure coil circuit.

Then, $$i_2 = \frac{v}{Z}$$

$$\therefore \quad i_2 \propto v \qquad ...(4.25)$$

Since, the coils are air cored, the field strength produced is directly proportional to the currents through them.

$\therefore$ Instantaneous value of deflecting torque

$$\propto i_1 i_2 \qquad ...(4.26)$$

$$\propto i_2 v \qquad ...(4.27)$$

Average value of deflection torque T_d is proportional to the average value of $i_1 v$ over the whole cycle.

$$T_d \propto \frac{1}{2\pi}\int_0^{2\pi} i_1 v\, d\theta \qquad ...(4.28)$$

Let $$v = V_m \sin\theta$$

$$i = I_m \sin(\theta - \phi)$$

where, ϕ = lagging power factor angle of the load.

$$\therefore \quad T_d \propto \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta\, I_m \sin(\theta - \phi)\, d\theta$$

$$\propto \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\cos\phi - \cos(2\theta - \phi)}{2}\, d\theta$$

$$\propto \frac{V_m I_m}{4\pi}\left[\theta\cos\phi - \frac{\sin(2\theta - \phi)}{2}\right]_0^{2\pi}$$

$$\propto \frac{V_m I_m}{4\pi}.2\pi\cos\phi$$

$$\propto \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi \qquad ...(4.29)$$

$$\propto VI \cos\phi \qquad ...(4.30)$$

$$\propto \text{true power} \qquad ...(4.31)$$

where, V and I are RMS values of voltage and current. Since, the deflection is proportional to the average power, hence the scale of a dynamometer wattmeter is uniform.

Control—Spring control.

Damping—Air friction damping.

Advantages:

(i) Can be used on both AC and DC with an equal degree of accuracy.

(iii) Scale is uniform.

Disadvantages:

(i) Error is caused by change in frequency.

(ii) Due to inductance of pressure coil errors are introduced at low p.f. unless suitably designed.

4.14 INDUCTION WATTMETER

Induction wattmeters can only be used on alternating current circuits.

(i) These instruments have two laminated electromagnets as shown in Fig. 4.14, one is excited by the load current and the other by a current proportional to the voltage of the circuit in which the power is to be measured.

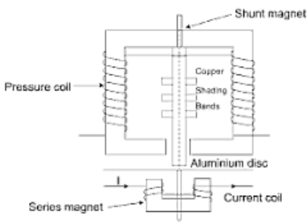


Fig. 4.14

(ii) A thin aluminium disc is mounted so that it is cut by the flux from both of these magnet.

(iii) A deflecting torque is produced by the interaction of these fluxes and the eddy currents which they induce in the disc.

(iv) One or more copper shading bands are fitted on one limb of the shunt magnet in order to cause the resultant flux in the magnet to lag in phase by exactly 90° behind the applied voltage.

(v) The two pressure coils, connected in series, are wound in such a manner that they both send the flux through the centre limb.

(vi) The series magnet in the instrument carries two small current coils in series, these being wound so that they both magnetise the core upon which they are wound, in the same direction.

(vii) The position of the copper shading bands is adjustable so that the correct phase displacement between the shunt and series magnet fluxes may be obtained.

Let V = supply voltage

I = load current

$\cos \phi$ = p.f. of load

ϕ_{sh} = flux through shunt magnet lagging 90° with respect to V. This angle of lag is achieved by the adjustment of shading bands.

ϕ_{se} = flux through the series magnet. This flux will be in phase with I if core losses are negligible.

The vector diagram of the wattmeter is shown in Fig. 4.15.

The flux ϕ_{sh} and ϕ_{se} will induce emfs E_{sh} and E_{se} in aluminium disc lagging 90° behind their respective fluxes. Let I_{sh} and I_{se} be the eddy current produced in the disc by these emfs. If inductance of the eddy current path is assumed negligible , I_{sh} and I_{se} will be in phase with their respective emfs.

There are two torques produced:

(i) by interaction of ϕ_{se} and I_{se} and

(ii) by interaction of ϕ_{se}^{-n}/I_{se}.

These two torques will act in opposite directions. Hence, mean deflecting torque acting on the disc will be given by:

$$T_d = k[\phi_{sh} I_{sh} \cos \phi - \phi_{se} I_{sh} \cos(180 - \phi)]$$

$$= k[\phi_{sh} I_{se} \cos \phi + \phi_{se} I_{sh} \cos \phi] \quad ...(4.32)$$

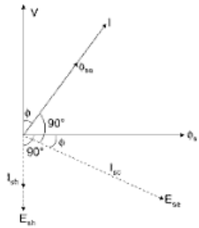


Fig. 4.15

Since,
$$\phi_{sh} \propto V$$
$$I_{se} \propto I$$
$$\phi_{se} \propto I$$
$$I_{sh} \propto V$$

$\therefore$
$$T_d \propto k[VI \cos\phi + IV \cos\phi] \quad ...(4.33)$$
$$\propto 2K\, VI \cos\phi$$
$$\propto VI \cos\phi \quad ...(4.34)$$
$$\propto \text{power in the circuit} \quad ...(4.35)$$

Advantages

(i) Damping is efficient.
(ii) Not much affected by stray fields.
(iii) Long open scale.

Disadvantages

(i) Can be used on AC only.
(ii) Cost is high.
(iii) Power consumption is more.

4.15 METHOD OF CONNECTING THE WATTMETER IN THE CIRCUIT

If the load current is small, the voltage drop in the current coil is small, so the method of connection shown in Fig. 19.16(a) introduces a very small error.

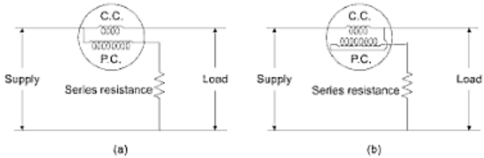


Fig. 4.16

On the other hand, if the load current is large, the watts lost in the pressure coil will be small compared with the watts lost in the current coil and so the method of connection shown in Fig. 4.16(b) is preferable.

4.16 HOT WIRE INSTRUMENTS

The heating effect of current is utilized in hot wire instruments. Hence, they can work both on AC and DC.

Hot wire instruments are usually of two types:

(i) Single sag type.

(ii) Double sag type.

4.16.1 Single Sag Type Hot Wire Instrument

It consists of a fine platinum wire stretched between two points *A* and *B*. Another wire is attached to the mid-point of *AB* and which passes around an axle *C* as shown in Fig. 4.17. This wire is kept stretched by a spring *D* and if there is any sag in *AB* the spring takes it up. The axle carries a pointer which moves over a graduated scale. Now, when current passes through the wire *AB* it is heated up. Due to which it gets elongated. The elongation causes a sag in it which is at once taken up by spring *D*. This causes the rotation of the axle *C*. When the axle rotates the pointer moves over the graduated scale.

4.17 ENERGY METERS

The measurement of energy is essentially the same process as the measurement of power, except that the instrument used must not merely indicate the power or rate of supply of energy, but must take into account also the length of time for which this rate of supply is continued.

Type of energy or supply meters:

There are three general types of energy or supply meters.

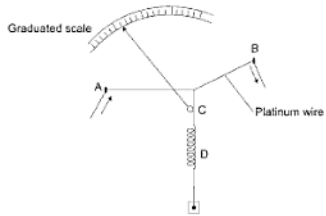


Fig. 4.17

(a) Electrolytic meters—can be used on DC only
(b) Motor meters
(c) Clock meters
(b) and (c) can be used on either DC or AC according to their construction. Only motor meters will be discussed here.

4.18 MOTOR METERS

In this class of meters the moving system is allowed to revolve continuously. The speed of revolution is proportional to the current in the circuit in the case of an ampere hour meter and to the power in the circuit in the case of a watt-hour meter.

The number of revolution made by the meter is recorded by a counting mechanism consisting of a train of wheels, to which the spindle of the rotating system is geared.

The control of speed is brought about by a permanent magnet (brake magnet), so that it induces currents in some part of the rotating system, these currents producing a retarding torque proportional to their magnitude and consequently proportional to the speed of the rotating system.

4.18.1 Ferranti DC Ampere Hour Meter

(i) The armature consists of a thin metal disc rotating in a bath of mercury, this mercury being used to lead the current into and out of the disc as shown in Fig. 4.18.
(ii) In this meter there are two permanent magnets, one for driving purpose and the other for braking. These magnets have mild steel pole pieces.

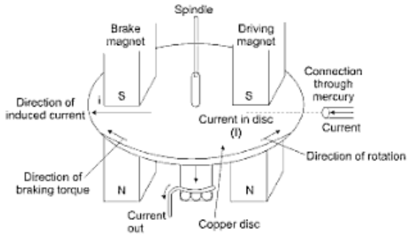


Fig. 4.18

(iii) The space inside the chamber in which the disc is placed is filled with mercury, which exerts a considerable up thrust on the disc and reduces the pressure on the bearings.

(iv) The disc is mounted at the base of spindle, pivoted and working in jewelled cup bearings, upper part of the bearing has a worm cut on it engaged in the gear wheels of the recording train.

(v) The current is led into the disc, through the mercury, at its circumference on the right hand side and flows radially to the centre where it passes out through the spindle.

(vi) A torque is produced owing to the presence of this current in the field of the driving magnet and the disc rotates as shown in Fig. 4.19.

(vii) In its rotation the disc cuts the field of the brake magnet and an eddy current is induced in it which results in a braking torque as shown, this torque controls the speed of rotation of the disc.

Since, the driving torque (T_d) is proportional to the load current I,

$$\therefore \quad T_d \propto I \quad ...(4.41)$$

Let n = speed of the meter.

ϕ = flux of the brake magnet.

and, e = eddy emf induced in the disc.

Then, $$e \propto \phi n \quad ...(4.42)$$

The eddy current induced in the disc,

$$i \propto e$$

$$\propto \phi \cdot n \quad ...(4.43)$$

Braking torque,

$$T_b \propto \phi \cdot i \qquad ...(4.44)$$

$$\propto \phi \cdot \phi n$$

$$\propto n \qquad ...(4.45)$$

(Since, ϕ is constant)

When steady running speed of the disc is attained, the braking torque becomes equal to the driving torque.

$$\therefore \qquad T_b = T_a \qquad ...(4.46)$$

From proportional eqns. (4.41) and (4.45)

$$n \propto I \qquad ...(4.47)$$

So, the speed of the disc is proportional to the load current and hence the number of revolutions on a calibrated counting mechanism give the quantity of electricity passed through the meter in a given time.

Advantages:

(i) Friction is small due to up thrust of mercury.
(ii) Construction is simple.
(iii) Large current carrying capacity without shunting.
(iv) Construction is simple.

Disadvantages:

(i) Suitable on DC only.
(ii) The fluid friction increases as the square of the velocity. To compensate this, two iron bars are placed, one above and the other below the mercury chamber. The lower bar carries a small compensating coil of few turns through which the load current passes.

This coil sets up a local magnetic field, which helps the driving magnet field and weakens the braking magnet field.

This results in compensation of fluid friction.

(iii) The accuracy is affected, if the murcury becomes dirty.

4.18.2 Elihu Thomson Commutator Watthour Meter

Friction Compensation–The two shading bands embrace the flux contained in the two outer limbs of the shunt magnet and thus the eddy currents are induced in them which cause a phase displacement between the enclosed flux and the main gap-flux. As a result a small driving torque is exerted on the disc to compensate for friction torque in the instrument. The friction torque can be adjusted by varying the positions of these bands.

Creep–In some meters a slow but continuous motion is obtained when pressure coils are excited with no load current. This may be due to the incorrect friction compensation or the voltage of supply may be in excess of the normal. To prevent such creeping two holes or cut in the disc on opposite sides of the spindle. The disc tends to remain stationary when one of the holes comes under one of the poles of the shunt magnet.

4.18.3 Polyphase Induction Type Watt Hour Meter

(i) The energy supplied to a three-phase circuit may be measured by a double element meter of the induction type, each element being similar in construction to the single phase meter.

(ii) There are two discs mounted on the same spindle and two separate brake magnets.

(iii) The spindle drives a single counting train.

(iv) In addition to the phase adjustment and friction compensating device in each element, one of the elements has an adjustable magnetic shunt across its shunt magnet, so that the driving torque for the same watts may be made same in the two instruments.

(v) In carrying out this adjustment, the two current coils of the meter are connected in series and the two pressure coils in parallel, the polarities being such that the two driving torques are in opposition.

(vi) Adjustment of the magnetic shunt being continued until the meter ceases to rotate with full rated watts supplied to both elements.

Advantages

(i) It is cheap.

(ii) Robust.

(iii) Damping is efficient.

(iv) No maintenance needed.

Disadvantages

(i) Used only on AC.

4.19 OHMMETER*

It is a direct reading instrument and straightaway gives the resistance of a circuit. In its simplest form an ohmmeter is nothing but a calibrated galvanometer having its scale marked in ohms as shown in Fig. 4.19.

*Not for U.P.T.U. (TEE-101/201) Students

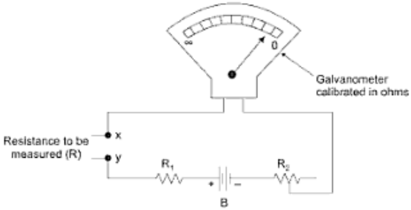


Fig. 4.19

B represents a dry cell, R_1 is a fixed resistance of about 2500 ohms and R_2 is a variable resistance. The terminals *x* and *y* are short circuited initially. The value of resistance *R* is so adjusted that the reading is zero on the calibrated scale. On opening the terminals *x* and *y* the pointer jumps to the position marked '∞'. Thus, the pointer will move from right to left as the resistance to be measured varies from zero to infinity. By putting known standard resistances across *x* and *y*, the instrument is calibrated. Obviously the scale is not uniform, as the deflection is proportional to $\frac{1}{R}$.

Potentiometer: A potentiometer is an instrument for measuring an unknown emf or potential difference by balancing it by a known potential difference produced by the flow of currents in conductor of known constant.

Let v = voltage across the standard resistance as measured by the potentiometer.

r = value of the standard resistance in ohms.

Hence, the reading of the current is,

$$I = \frac{v}{R} \qquad ...(4.48)$$

The given ammeter can now be calibrated.

4.20 EXTENSION OF INSTRUMENT RANGES

4.20.1 Ammeters

Their range can be increased by putting shunts in parallel with the ammeter terminals as shown in Fig. 4.20.

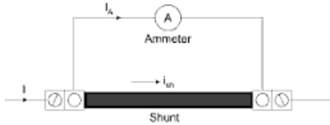


Fig. 4.20

Let i_A = maximum permissible current through ammeter.

r_A = resistance of ammeter including connecting leads.

I = maximum current to be measured.

The current through shunt will be given by,

$$I_{sh} = I - i_A \quad ...(4.49)$$

The voltage drop across the ammeter and the shunt will be same.

$$\therefore \quad i_A \cdot r_A = i_{sh} \cdot R_{sh} \quad ...(4.50)$$

where R_{sh} = Resistance of the shunt.

$$\therefore \quad R_{sh} = \frac{i_A \cdot r_A}{i_{sh}}$$

$$= \frac{i_A \cdot r_A}{I - i_A}$$

$$= \frac{r_A}{\left(\frac{I}{i_A} - 1\right)} \quad ...(4.51)$$

The ratio $\frac{I}{i_A}$ is called the multiplying power of the shunt.

The shunts have low resistance and are made of copper, nickel alloy termed as Eueraka Manganin, etc.

Eueraka or Constantan–Copper 55-60%, Nickel 40-45%.

Manganin–Copper 84%, Manganese 12%, Nickel 4%.

4.20.2 Voltmeter

The range of a voltmeter can be increased by putting a high resistance in series with it as shown in Fig. 4.21.

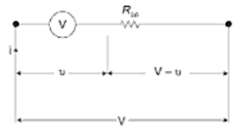


Fig. 4.21

Let v = maximum voltage which can be applied across the voltmeter

r_v = resistance of the voltmeter.

V = voltage to be measured.

R_{se} = resistance to be put in series.

Permissible current through voltmeter,

$$i = \frac{v}{r_v}$$

To measure the voltage V, the series resistance R_{se} should be such that the voltage drop across it = $V - v$.

$$\therefore \qquad R_{se} = \frac{V - v}{i} = \frac{V - v}{v/r_v}$$

$$= \left(\frac{V}{v} - 1\right) r_v$$

Note: (i) Since, an ammeter is placed in series with a load, so its resistance should be very small to have a negligible drop across it.

(ii) Since, a voltmeter is placed across the load, so its resistance should be very high to permit a negligible current to pass through it.

An ideal ammeter should have a zero resistance and an ideal voltmeter an infinite resistance which is of course not possible in practice. The good quality voltmeters have a resistance of about 1000 ohms per volt of their range.

Example 1: A moving coil permanent magnet ammeter consists of 10 turns wound on a rectangular former so that the active length of the conductors is .05 meter. Calculate the deflecting torque exerted on the coil when it carries a current of 5 A and the flux density in the gap is 500 weber/m^2.

Area of the coil is 15×10^{-4} meter^2.

Table 4.1 Common Type of Indicating Instruments

	Type of Instrument	Effect used	Suitable to measure	Used on	Accuracy of Instrument	Control used	Type of damping
1.	Moving Iron	Magnetic effect	Current and voltage	AC/DC	1st Grade (Sub-standard) on AC 2nd Grade on DC	Spring or gravity	Air
2.	Moving Coil (permanent magnet type)	Magnetic effect	Current and voltage	DC	1st Grade	Spring	Eddy current
3.	Dynamometer	Electrodynamic effect	Current, voltage and power	AC/DC	1st Grade	Spring	Eddy current
4.	Induction	Electromagnetic induction effect	Current, voltage, power and energy	AC	2nd Grade	Spring	Eddy Current
5.	Electrostatic	Electrostatic effect	Voltage	AC/DC	1st Grande	Gravity or Spring	Air
6.	Hot wire	Heating effect	Current, voltage	AC/DC	2nd Grade	Spring	Eddy current

Solution: From eq. (4.12), the deflecting torque is given by,

$$T_d = NBIA \qquad \begin{cases} N = 10 \\ B = 500 \text{ web/m}^2 \\ I = 5 \text{ A} \\ A = 15 \times 10^{-4} \text{ met}^2 \end{cases}$$

$$T_d = 10 \times 500 \times 5 \times 15 \times 10^{-4}$$

$$= 37.5 \text{ Newton meter}$$

Example 2: Calculate current through a galvanometer of resistance 10 ohms shunted by a resistance of 1 ohm, when the combination forms the part of the circuit whose total resistance is 12 ohms and a cell of emf 2 volts is sending the current.

Solution:

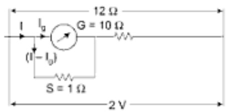


Fig. 4.22

Total resistance of the circuit = 12 ohms.

Current in the circuit, $I = \frac{2}{12} = \frac{1}{6}$ amps.

Current passing through galvanometer is given by $(I - I_g)s = I_g G$

$$I_g = \frac{S}{S+G} \times I$$

$$= \frac{1}{1+10} \times \frac{1}{6} = \frac{1}{66} \text{ amps.} \quad \textbf{Ans.}$$

Example 3: What resistance should a wire have which when connected across the terminals of a galvanometer of 2970 ohms resistance could let $\frac{1}{100}$th of the current to pass through the galvanometer?

Solution: Current through galvanometer is given by eqn. 4.24

$$i_g = \frac{S}{G+S} \times I$$

Here, $$i_g = \frac{I}{100}, \quad G = 2970,$$

So, $$\frac{I}{100} = \frac{S}{2970+S} \cdot I$$

$$100S = 2970 + S$$

or, $$99S = 2970 \text{ or, } S = \mathbf{30\ ohms} \quad \textbf{Ans.}$$

Example 4: A moving coil instrument has a coil resistance of 10 ohms and requires a current of 15 milliamperes for full scale deflection. Calculate the resistances necessary to enable it to be used to read (a) Current upto 100 amps. (b) Voltages upto 500 volts.

Solution:

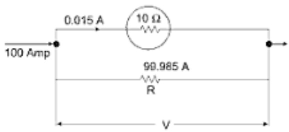


Fig. 4.23

Either the instrument is used as an ammeter or voltmeter it must be arranged so that not more than 15 milliamps passes through the instrument itself.

(a) *To read 0-100 amps.*

The instrument must be shunted with a resistance R which will carry 99.985 amps. at full scale, leaving 0.015 amps. to flow through the meter as shown in Fig. 4.23.

The potential difference across the meter is the same as that across the shunt.

Hence, $99.985 \times R = 0.015 \times 10$

or, $$R = \frac{0.015 \times 10}{99.985} = 0.001501 \text{ ohms}$$

Coil must be shunted with resistance of 0.001501 ohms.

(b) *To read 0-500 volts.*

The meter must be connected in series with a resistance R as shown in Fig. 4.24 which will raise the total resistance to such a value that only 15 milliamps flow when the instrument is connected across 500 volts.

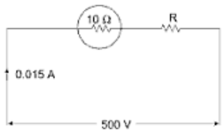


Fig. 4.24

$$\text{Total resistance required} = \frac{500 \text{ Volts}}{0.015 \text{ amp}}$$

$$= 33{,}333.3 \text{ ohms}$$

$$\text{hence the added resistance in series} = 33{,}333.3 - 10$$

$$= 33{,}323.3 \text{ ohms}$$

Example 5: A single phase energy meter has a registration constant of 100 rev/kwh. If the meter is connected to a load carrying 20 A at 230 V and 0.8 power factor for an hour, find the number of revolutions made by it. If it actually makes 360 revolutions, find the percentage error.

Solution: $$\text{Energy consumed in 1 hour} = \frac{V I \cos\theta}{1000} \text{ kWh}$$

$$= \frac{230 \times 20 \times 0.8}{1000}$$

$$= 3.68 \text{ kWh}$$

Now the numbers of revolution for true energy = meter constant × 3.68

= 368

$$\%\text{Error} = \frac{\text{Actual revolutions} - \text{Revolutions for true energy}}{\text{Revolutions for true energy}}$$

$$= \frac{360 - 368}{368}$$

$$= -2.174\%$$

⇒ –ve sign shows that meter will run slow.

Example 6: A moving coil instrument gives a full-scale deflection of 20 mA when a potential difference of 50 mv is applied. Calculate the series resistance to measure 500 V on full-scale. (2004-05)

Solution:

$$I_m = 20 \times 10^{-3} \text{ A}$$

$$V_m = 50 \times 10^{-3} \text{ V}$$

$$R_m = \frac{V_m}{I_m}$$

$$R_m = \frac{500 \times 10^{-3}}{20 \times 10^{-3}} = 2.5\ \Omega$$

Series resistance of the meter to measure 500 V

$$R_s = \frac{V_m}{I_m} - R_m$$

$$= \frac{500}{20 \times 10^{-3}} - 2.5$$

$$= 25000 - 2.5$$

$$R_s = 24997.5\ \Omega$$

Example 7: An energy meter revolves 100 revolutions of disc for one unit of energy. Find the number of revolutions made by it during an hour when connected across load which takes 20 A at 210 V and 0.8 Power factor leading. If energy meter revolves 350 revolutions, find the percentage error. (2004-05)

Solution: Energy consumed by load = $VI \cos \theta$

energy consumed in one hour = $210 \times 20 \times 0.8 \times 1$ Wh

$= 210 \times 20 \times 0.8 \times 1 \times 10^{-3}$ kWh

$= 3.36$ unit

Number of revolution = 3.36×100

$= 336$ (True revolution)

$$\%\text{error} = \frac{\text{Actual} - \text{True revolution}}{\text{True revolution}} \times 100$$

$\therefore$ $$\%\text{error} = \frac{350 - 336}{336} \times 100 = \frac{1400}{336} = 4.17\%$$

Example 8: A moving coil instrument gives full-scale deflection with 15 mA. The resistance of coil is 5 Ω. It is desired to correct this instrument into an ammeter to read upto 2 A. How to achieve it? Further, how to make this instrument to read upto 30 volts. (2005-06)

Solution: Moving coil instrument gives scale deflection when current through it is 15 mA.

Instrument coil resistance has 5 Ω:

(i) **When works as an ammeter**

We can read upto 2 A by connecting (R_{sh}) in shunt as in Fig. 4.25.

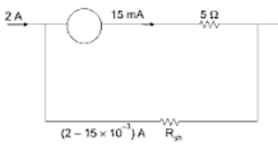


Fig. 4.25

$\Rightarrow$ $$I_m R_m = (I - I_m) R_{sh}$$

$$I_m = I.\frac{R_{sh}}{R_{sh} + R_m}$$

$\Rightarrow$ $$15 \times 10^{-3} = 2 \times \frac{R_{sh}}{R_{sh} + 5}$$

Then, $$15 \times 10^{-3} = 2 \times \frac{R_{sh}}{5} \quad \text{if } R_{sh} << 5$$

So, $$R_{sh} = \frac{15 \times 10^{-3} \times 5}{2}$$

$$R_{sh} = 0.0375\ \Omega$$

(ii) When works as an voltmeter

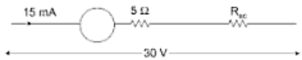


Fig. 4.26

We can read upto 30V by connecting Resistance R_s in series as in Fig. 4.26.

By using formula

$$I_m R_m + I_m R_{se} = V$$

$$I_m = \frac{V}{R_m + R_{se}}$$

So, $$15 \times 10^{-3} = \frac{30}{R+5}$$

$$R_{se} = \frac{30}{15 \times 10^{-3}} - 5$$

$$R_{se} = \frac{30 \times 10^3}{15} - 5$$

$$R_{se} = 20000 - 5$$

$$\boldsymbol{R_{se} = 19995\ \Omega.}$$

Example 9: A 50 A, 230 V meter on full load test makes 61 revolutions in 37 seconds. If the normal disc speed is 500 revolutions per kWh, find the percentage error. (2005-06)

Solution:

Energy consumed in 37 seconds $= \frac{VI \cos\phi}{1000} \times t$ kwh

$$= \frac{230 \times 50 \times \cos\phi}{1000} \times \frac{37}{60 \times 60} \text{ kWh}$$

$$= 0.11819 \times \cos\theta$$

For purely resistive load $\cos\theta = 1$ (assumed)

So energy consumed in 37 sec. $= 0.11819 \times 1$

$= 0.11819$ kWh

Energy consumption registered by the meter

$$= \frac{\text{no. of revolutions made}}{\text{meter const}} = \frac{61}{500} = 0.122 \text{ kWh}$$

Percentage error

$$= 100 \times \frac{(\text{actual registration} - \text{true energy consumption})}{\text{True energy consumption}}$$

$$\%\text{error} = \frac{(0.122 - 0.11819)}{0.11819} \times 100$$

$$= 3.22\%$$

Example 10: Two voltmeters one with a full-scale reading of 100 V and another with a full-scale reaching of 200 V are connected in series across a 100 V supply. The internal resistance of both meters is same. What are the readings? [U.P. Technical Univ. Tutorial Question Bank]

Solution: The internal resistance of each meter be of *R* ohms. Now the two meters are connected in series across 100V supply, the voltage drop across each meter is 50 V, as in Fig. 4.27. So each meter will read 50 V.

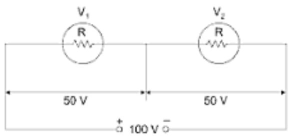


Fig. 4.27

Example 11: Two voltmeters have the same range 0-400 V. The internal impedances are 30,000 Ω and 20,000 Ω. If they are connected in series and 600 V be applied across them, what will be their readings?

[U.P. Technical Univ. Model Question Paper]

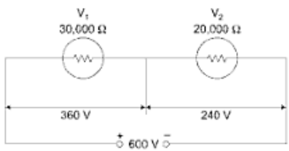


Fig. 4.28

Solution: When two voltmeters of resistances 30,000 Ω and 20,000 Ω are connected in series across 600 V supply, voltage drop across them would be

$$\frac{30{,}000}{30{,}000+20{,}000} \times 600 \text{ and } \frac{20{,}000}{30{,}000+20{,}000} \times 600 \text{ volts i.e., 360 V and 240,}$$

repectively, as shown in Fig 4.28 so the reading of voltmeters V_1 and V_2 would be 360 V, and 240 V, respectively.

Example 12: Two ammeters one with full-scale current of 1 mA and internal resistance of 100 Ω, and the other with a full-scale current of 10 mA and internal resistance of 25 Ω are in parallel. What is the total current these two meters can carry without any meter reading getting out of scale?

[U.P. Technical Univ. Tutorial Question Bank]

Solution: Let the total current be of *I* amperes that these two meters can carry without any meter reading getting out of scale. The current distribution among the two meters will be in inverse proportion to their respective resistances. Thus,

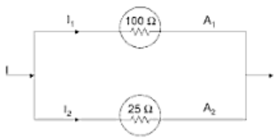


Fig. 4.29

Current flowing through ammeter A_1, $I_1 = I \times \frac{25}{100+25} = 0.2\ I$

Current flowing through ammeter A_2, $I_2 = \frac{I \times 100}{100+25} = 0.8I$

Since, full-scale current of ammeter A_1 is 1 mA so total current *I* comes to be $\frac{1 \text{ mA}}{0.2}$ = **5 mA Ans.**

Current through ammeter $A_2 = 0.8 \times 5 = 4$ mA

EXERCISE

1. Name different types of important measuring instruments. Describe one of them in detail and state briefly the use of each one of them.
2. Distinguish among indicating, recording and integrating instruments. Give at least one example of each type.
3. Differentiate between absolute and secondary instruments and explain, which one of them has a wider practical application.
4. What effects of an electric current are used in secondary instruments? Based on the above effects, give a table of commercial type of instruments indicating their important features.
5. Explain the necessity of deflecting, controlling and damping torques in an indicating instrument. How the instrument would behave if any one of the torques is absent?
6. Compare gravity control with spring control. State their relative merits and demerits.
7. What are three different types of dampings provided in electrical measuring instruments, which one of them is considered most efficient and why? Can it be provided with all types of instruments?
8. What is understood, when it is said that the moving system is over, critical or under damped? What is the state of damping in a dead beat instrument?
9. Explain why a moving iron instrument is called either an attraction type or repulsion type. Describe the principle of working of one of them.
10. Describe the working of moving coil instrument. Explain why it is suitable on DC only.

    A moving coil of a voltmeter consists of 1000 turns wound on a square former of side 3 cm each and the flux devisity in the air gap is 500 lines/cm^2. Calculate the deflecting torque on the moving system, when carrying a current of 10 mA. [4.58 gm-cm]
11. Differentiate between a dynamometer type instrument and a moving coil permanent magnet instrument. Explain why a dynamo meter type of instrument is suitable both on DC and AC.
12. Give the construction and principle of operation of a dynamometer type of wattmeter. What are the two different methods of connecting a wattmeter in the circuit?
13. Describe working of an induction wattmeter. State its advantages and disadvantages.
14. Describe in detail the working of a double sag type hot wire instrument. Can it be used on both AC and DC? If so why?

15. What are different types of energymeters? Describe the working of Ferranti DC Ampere hour meter.
16. How would you measure power of a single phase load by 3 ammeter method?
17. Describe a single phase induction type watthour meter. Why they are so invidely used on AC?

5

Transformer

5.1 INTRODUCTION

A transformer is a static device, which transfers electrical energy from one part of a magnetic circuit to another part of a magnetic CKT by mutual induction. During this transformation, *frequency* is constant. It also steps up and steps down the input voltage.

5.2 CLASSIFICATION OF TRANSFORMER

(i) According to construction
  (a) Core type
  (b) Shell type
  (c) Berry type (circular-shell)

(ii) According to use
  (a) Power transformer: For HV transmission (15,000 KVA and above), for LV transmission (2000 to 15,000 KVA)
  (b) Distribution transformers
  (c) Instrument transformers

(iii) According to winding
  (a) Single phase transformer
  (b) Three-phase transformer
  (c) Auto transformer

(iv) According to type of cooling
  (a) Natural cooled: Natural air cooled and oil immersed natural cooled.
  (b) Forced air cooled: Air blast cooled and oil immersed air blast cooled.

(c) Water cooled: Oil immersed water cooled.

Advantages:

(i) It is economical to transmit the energy at high voltages.
(ii) For distribution purpose.
(iii) Efficiency of transformers is very high.
(iv) It matches the impedance of a source and its load for maximum power transfer in electronic and control circuits.
(v) Isolates the DC from AC while permitting the flow of AC between two circuits.
(vi) Isolates one circuit from another.

Construction:

There are two basic parts of a transformer.

(i) Magnetic core
(ii) Windings

Various types of transformer stampings and laminations are used for making a core of a transformer.

Generally, I shaped and L shaped laminations are used.

Laminations are of high grade silicon steel stampurgs (0.3 to 0.5 mm thick) and are responsible for reduction in hysterer is lasses in the core.

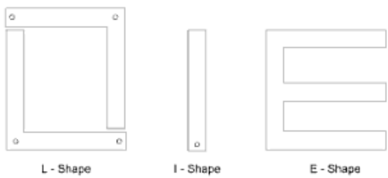


Fig. 5.1

These laminations are isolated from each other through enamelled Varmol for reducing the eddy current losses.

Laminations are overlapped to avoid the air gap at the joints and joints may be staggered otherwise core will have less mechanical strength.

In a transformer there is only HV and LV winding. Winding, which is connected to load is secondary (either HV or LV) and the other connected to input is primary.

5.2.1 Core Type

- It has a single magnetic circuit.
- Core is rectangular in shape of uniform cross-section. It consists of two vertical limbs and the horizontal yokes connecting two limbs.
- Coils used are cylindrical type.
- Coils are wound in helical layers with different layers insulated from each other on two limbs.
- Low voltage coil is placed inside near the core while HV coil surrounds the LV coil.
- Windings are uniformly distributed over the two limbs so natural cooling is more effective.
- In order to minimize leakage flux, half the primary and half the secondary are placed concentrically on each limb.

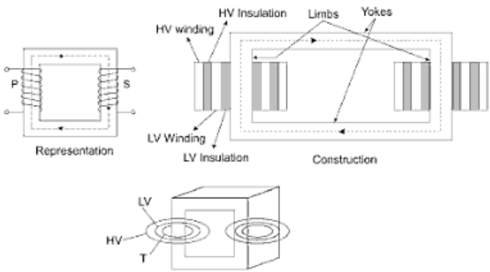


Fig. 5.2

5.2.2 Shell Type

- Magnetic circuit is divided, in two or more parts.
- Core has three limbs.
- Both (HV, LV) windings are placed on central limb.
- Coils used are multilayer disc type or sandwich type.
- HV coils are placed between LV coils.
- LV coils are near to top and bottom of yokes.
- For low capacity shell type transformer is prefered.
- Windings are surrounded by the core so natural cooling does not exist.

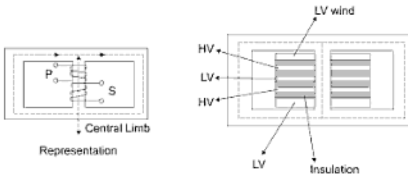


Fig. 5.3

5.2.3 Berry Type

- Distributed magnetic circuit.
- Core construction is like spokes of a wheel.
- This type of transformer gives rugged construction and provides better cooling.

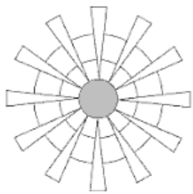


Fig. 5.4

5.3 IDEAL TRANSFORMER

A transformer is said to be ideal if it satisfies the following properties.

(i) It has no core losses (hysteresis and eddy current loss).

(ii) Its winding resistance is zero.

(iii) No leakage flux in Primary and Secondary

(iv) Permeability of core is high so negligible current is required to set up a flux.

5.4 WORKING PRINCIPLE OF TRANSFORMER (1 – θ)

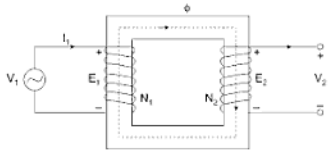


Fig. 5.5

(i) When the primary winding is excited by an alternating voltage V_1.

(ii) This voltage circulates alternating current.

(iii) This alternating current provides an alternating mmf (NI).

(iv) Due to this alternating mmf an alternating flux will be set up in magnetic core given by

$$\phi = \phi_m \sin \omega t$$

(v) Due to this time varying field or flux an emf will be induced in primary and secondary winding, having N_1 and N_2 twins respectrely

$$e_1 = -N_1 \frac{d\phi}{dt} \Rightarrow e_2 = -N_2 \frac{d\phi}{dt}$$

$$\Rightarrow \quad e_1 = -N_1 \omega\, \phi_m \cos \omega t$$

$$\boxed{e_1 = 2\pi f\, N_1 \phi_m \sin(\omega t - \pi/2)} \quad \text{...(A)}$$

$$e_2 = 2\pi f\, N_2 \phi_m \sin(\omega t - \pi/2) \quad \text{...(B)}$$

$$\Rightarrow \quad E_1 = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = \sqrt{2}\, \pi f N_1 \phi_m$$

$$E_2 = \sqrt{2}\, \pi f\, N_2\, \phi_m$$

$$\Rightarrow \quad \boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}} = \text{a (transformer turns ratio)}$$

ϕ = Instantaneous flux

ϕ_m = Maximum flux

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns
f = Frequency of the supply voltage
E_1 = R.M.S. value of primary induced emf
E_2 = R.M.S. value of secondary induced emf

From the equations A and B, it is clear that induced emf lags the flux by $\pi/2$.

Induced emf in primary winding opposes the input voltage V_1 and

$$\bar{V}_1 = -\bar{E}_1$$

Hence, this emf E_1 is also called back emf because it limits the input current.

In case of DC (applied voltage) no self induce emf will generate to oppose the applied voltage, hence very high current will flow into the transformer and shall burn its winding.

5.5 PHASOR DIAGRAM OF 1 – ϕ TRANSFORMER

5.5.1 No Load Condition (Secondary Open)

(i) *Ideal Case* ($R = 0$, $X_l = 0$, no core loss)

R = Winding resistance
X_l = Leakage reactance

Part of the primary flux as well as the secondary flux complete the path through air and links with respective winding only. Such flux is called leakage flux. Due to these leakage fluxes there will be a leakage reactance X in each *wdg*.

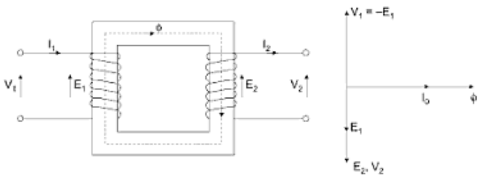


Fig. 5.6

$$\bar{V}_1 = -\bar{E}_1 \quad ...(1)$$

$$\bar{E}_1,\ \bar{E}_2 \text{ lags } \phi \text{ by } \pi/2 \quad ...(2)$$

$$\bar{E}_2 = \bar{V}_2 \quad ...(3)$$

$$\bar{I}_2 = 0 \quad ...(4)$$

$\bar{I}_1 = \bar{I}_0$ (no load current) is in phase with ϕ due to no core loss.

Case 1–Winding resistance included:

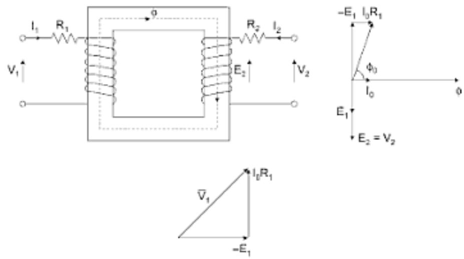


Fig. 5.7

$$\bar{V}_1 = -\bar{E}_1 + \bar{I}_1 R_1 \quad ...(1)$$

$$\bar{E}_1,\ \bar{E}_2 \text{ lags } \phi \text{ by } \pi/2 \quad ...(2)$$

$$\bar{E}_2 = \bar{V}_2 \quad ...(3)$$

$$\bar{I}_2 = 0 \text{ (no load)} \quad ...(4)$$

$\bar{I}_1 = \bar{I}_0$ (no load or exciting current or magnetizing current) is in phase with ϕ because no core loss.

⇒ Core losses are negligible signifing that exciting current is in phase with flux ϕ. There is no core loss component of the exciting current.

⇒ Core losses are governed by the area of *B-H* curve higher the area higher the core loss.

(i) If core losses (hysteresis + eddy current) are negligible means *Area* = 0.
$B \propto H$ or $\phi \propto i$ means ϕ and no load current are in same phase.

(ii) If core losses are present then the *B-H* curve is non-linear and the no load or exciting current leads the flux ϕ, as it may have two components.

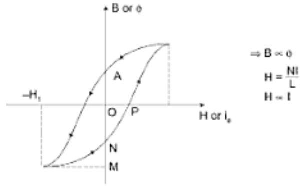


Fig. 5.8

→ one is in Phase with ϕ known as magnetizing component (I_ϕ)

→ other is in Phase with E_1 known as core loss component (I_c)

as $\quad I_c = 0 \quad \phi = -\text{ve}$

$\quad I_c = +\text{ve} \quad \phi = 0$ at (OP)

(iii) So exciting current leads the flux ϕ by some small angle (0 to 10°) known as hystertic angle (δ).

Case 2–Core loss included: (No-load or $I_2 = 0$)

Now I_0 is not in phase with ϕ it leads the flux ϕ.

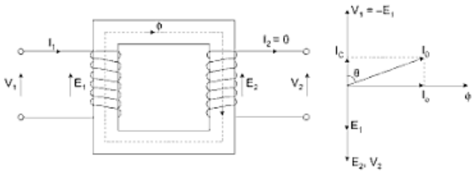


Fig. 5.9

$$\Rightarrow \qquad \bar{V}_1 = -\bar{E}_1$$

$$\bar{I}_1 = \bar{I}_0$$

→ Now I_0 having a two components.

$$\bar{I}_0 = \bar{I}_c + \bar{I}_\phi \qquad \{I_c = I_0 \cos\theta_0, I_\phi = I_0 \sin\theta_0\}$$

→ $\bar{I}_c$ is in phase with V_1 hence responsible for core loss (P_c).

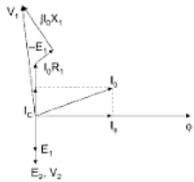
$P_c = V\ I_c \cos\theta_0 = I_c^2 R_c$

V_1 = no load input voltage

R_c = hyphothetical resistance to account for core loss,

$\overline{I}_\phi$ is in phase with flux ϕ. Hence, responsible for magnetizing the core so called magnetization current.

Case 3–At no load ($I_2 = 0$), winding resistance, reactance and core loss included



Example 1: The no load current of a transformer is 10 A at a power factor of 0.25 lagging. When connected to 400 V 50 Hz supply. Calculate

(a) Magnetizing component of no load current

(b) Iron loss

(c) Max. value of flux in core, $N_1 = 500$.

Solution:

At no load $I_2 = 0$, R_1, X_1 = negligible

$$V_1 = E_1 = 400\text{ V},\ \overline{I}_0 = 10\text{ A},\ \cos\theta_0 = 0.25$$

(a) Magnetizing component is I_ϕ

$$I_\phi = I_0 \sin\theta_0$$

$$= 10 \times .968 = 9.68\text{ Amp}$$

(b) Iron loss $\quad P_c = V_1 I_c = V_1 I_0 \cos\theta_0$

$$P_c = 400 \times 10 \times .25 = \mathbf{1000\ W}$$

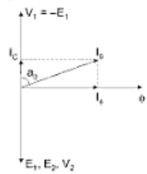


Fig. 5.10

from this we can also calculate R_c and X_ϕ (magnetizing reactance)

$$P_c = I_c^2 R_c \Rightarrow R_c = \frac{1000}{2.5 \times 2.5} = 160\ \Omega$$

$$X_\phi = \frac{400}{968} = 0.413\ \Omega$$

(c) $$V_1 = E_1 = \sqrt{2}\,\pi f \phi_m N_1$$

$$\phi_m = \frac{E_1}{\sqrt{2}\pi f N_1} = \frac{400}{\sqrt{2}\pi \times 50 \times 500}\ \text{Wb} = \mathbf{3.60\ mWb}$$

5.5.2 Transformer on Load

Case 1–(R_1, R_2, X_1, X_2 are negligible, no core losses, secondary connected to lagging load):

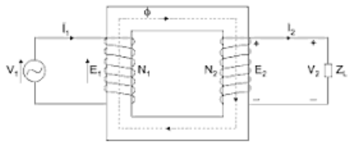


Fig. 5.11

→ When secondary is open then $I_2 = 0$.

→ As we connect a load on secondary side an alternating current $\bar{I}_2$ circulates through N_2 turns.

→ Due to this $\bar{I}_2$, a mmf $N_2 I_2$ will be produced in secondary winding.

→ Due to this mmf $(N_2 I_2)$ a flux ϕ_2 is induced in secondary winding.

→ According to Lenz's law the direction of ϕ_2 is such that it opposes the cause, hence the direction of ϕ_2 is such that it opposes the main flux ϕ.

→ If ϕ reduces then E_1 and V_1 will reduce.

→ Since V_1 (applied voltage) is constant so main flux ϕ has to be remain is constant so mmf $N_2 I_2$ have to be compensate from somewhere else.

→ For compensating mmf $(N_2 I_2)$ we are requiring extra mmf from primary side hence *extra current* I'_2 draws from supply. This current I'_2 is the major part of the primary current to counter the secondary mmf $(N_2 I_1)$

→ Now $$\bar{I}_1 = \bar{I}_0 + \bar{I}'_1$$

$\bar{I}_1$ = Primary supply current

$\bar{I}_0$ = No load or exciting current

$\overline{I}_2'$ = Load component of input current which compensates load current I_2.

$$N_1 \overline{I}_2' = N_2 I_2 \text{ of } I_1$$

→ $\overline{I}_0$ is negligible (2 to 5%) of rated current so $\overline{I}_2' = \overline{I}_1$

→ $$N_1 I_1 = N_2 I_2$$

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}} = \frac{1}{a}$$

5.6 PHASOR DIAGRAM ON LOAD

Case 1–(Ideal transformer on different load):

$$\overline{V}_1 = (-)\overline{E}_1,\ \overline{V}_2 = \overline{E}_2,\ \overline{I}_1 = \overline{I}_0 + \overline{I}_2',\ \overline{V}_2 = \overline{I}_2 Z_L$$

$\overline{I}_0$ is in phase with ϕ because of no core loss. So, ideally $\overline{I}_0 = \overline{I}_\phi$

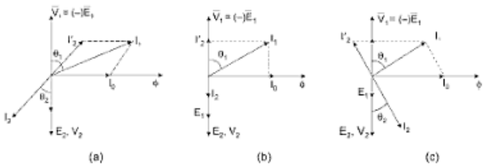


Fig. 5.12

Phasor diagram of ideal transformer

(a) Lagging load (I_2 lags V_2)
(b) Resistive load (I_2 is in phase with V_2)
(c) Leading load (I_2 leads V_2)

Case 2–Conditions:

(a) Winding resistances are negligible.
(b) No leakage flux.
(c) Only core losses (now $\overline{I}_0$ leads flux ϕ by small angle).

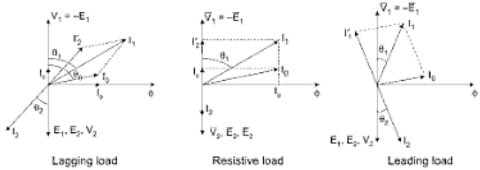


Fig. 5.13

Case 3–Conditions:

(a) Winding resistances are negligible.

(b) There is leakage flux in both primary and secondary.

(c) There is core losses in core.

Due to the leakage flux ϕ_{l1} and ϕ_{l2}, hyphothetical leakage reactances X_1 and X_2 comes into the winding.

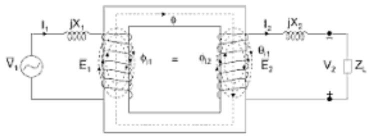


Fig. 5.14

$$\overline{V}_1 = -\overline{E}_1 + jI_1X_1, \quad \overline{E}_2 = \overline{V}_2 + \overline{I}_2 jX_2$$

$$\overline{I}_1 = \overline{I}_0 + \overline{I}_1', \quad \overline{V}_2 = \overline{I}_2 Z_L$$

$$\overline{I}_0 = \overline{I}_c + \overline{I}_\phi$$

$\Rightarrow$ For drawing $\overline{V}_1 = -\overline{E}_1 + j\overline{I}_1 X_1$ we have to add to vectors $-\overline{E}_1$ and (jI_1X_1)

$j\overline{I}_1$ is the vector perpendicular to I_1 in ccw direction.

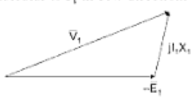


Fig. 5.15

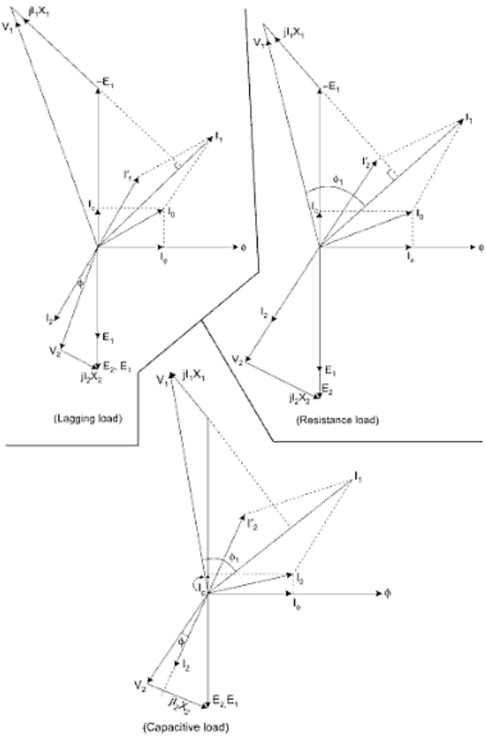


Fig. 5.16

Case 3–Practical Transformer: (R_1, R_2; X_1, X_2; Cove loss included)

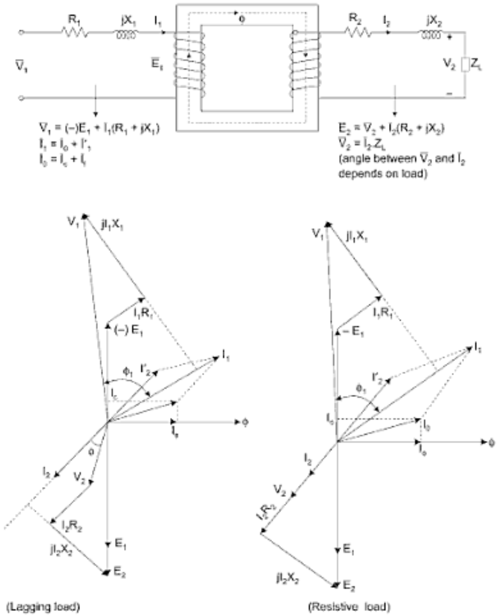
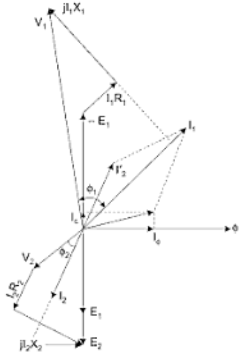


Fig. 5.17(Contd.)



(Capacitive load)

Fig. 5.17

⇒ XKVA, V_1/V_2 Volt, 50 Hz 1-Phase T_X indicates that

- We can apply a X-KVA load at either side of transformer.
- Rated voltage at primary side = V_1
- Rated voltage at secondary side = V_2
- Rated current at primary side is $I_1 = \dfrac{X}{V_1}$ K Amp
- Rated current at secondary side is $I_2 = \dfrac{X}{V_2}$ K Amp

Example 2: A 400/200 V transformer takes 1 Amp at power factor of 0.4 on no load. If the secondary supplies a load current of 50 A at 0.8 lagging power factor, calculate the primary current.

Primary current $\quad \bar{I_1} = \bar{I_0} + \bar{I_2'}$

$\cos\phi_0 = 0.4 \Rightarrow \quad \phi_0 = 66.42°$

$\cos\phi = 0.8 \Rightarrow \quad \phi = 36.87°$

5.7 EQUIVALENT CIRCUIT OF A TRANSFORMER

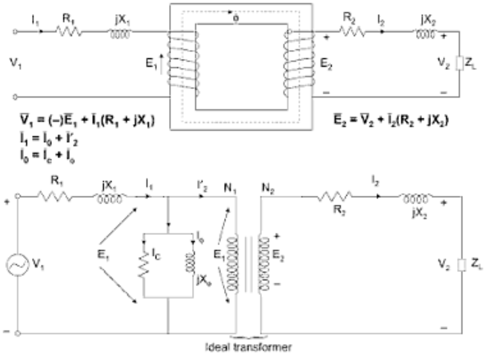


Fig. 5.19 Equivalent CKT

- → Equivalent CKT for an electrical engg. device can be drawn if the equations describing its behaviour are known.
- → If any electrical device is to be analysed and investigated further, then equivalent CKT is necessary.
- → Equivalent CKT can, therefore, be analysed and studied easily by the electric CKT theory.
- → For further simplification, we have to transfer all the elements and sources either primary side or secondary side.

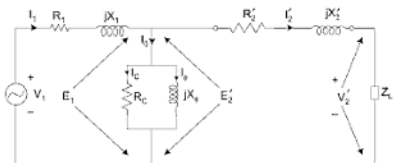


Fig. 5.20 Referred to primary

R_1 = Resistance of the primary winding

X_1 =Leakage reactance of primary side

V_1 = Applied voltage

E_1 = Induced emf at primary side

I_1 = Primary current

R'_2 = Resistance of secondary side refer to primary

X'_2 = Reactance of secondary side refer to primary

V'_2 = Load voltage refer to primary

I'_2 = Load component of the current at primary side

$$\Rightarrow \qquad \frac{I_2'}{I_2} = \frac{N_2}{N_1} \Rightarrow I'_2 = \frac{I_2 N_2}{N_1}$$

$$\Rightarrow \qquad \frac{E_2'}{E_2} = \frac{N_1}{N_2} \Rightarrow E'_2\,(= E_1) = \frac{E_2 N_1}{N_2},\; V'_2 = \frac{V_2 N_1}{N_2}$$

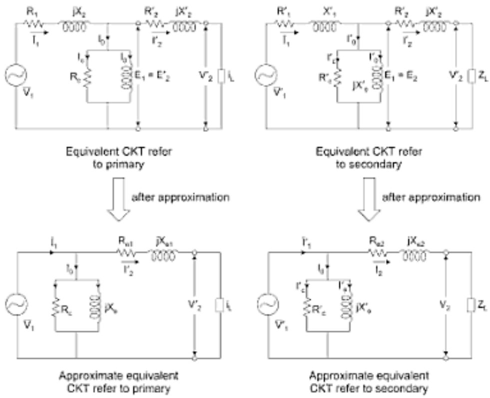


Fig. 5.21

$$R_{e1} = R_1 + R'_2, X_{e1} = X_1 + X'_2, R_{e2} = R'_1 + R_2, Xe_2 = X'_1 + X_2$$

Re_1 = Equivalent resistance refer to primary

R_1 = Resistance of primary side

R'_2 = Resistance of secondary side refer to primary

R'_1 = Resistance of primary side refer to secondary

Xe_1 = Equivalent reactance refer to primary

Xe_2 = Equivalent reactance refer to secondary

X'_2 = Leakage reactance refer to primary.

Let the resistance of secondary side be R_2 and when it refers to primary its value becomes R'_2.

⇒ Then, V_{R_2} = drop in R_2 at secondary side = I_2R_2

V'_{R_2} = drop in R'_2 at primary side = $I'_2R'_2$

⇒ We know, $\dfrac{E_1}{E_2} = \dfrac{V'_{R_2}}{V_{R2}} = \dfrac{N_1}{N_2}$ and $\dfrac{I'_2}{I_2} = \dfrac{N_2}{N_1}$

$$\Rightarrow \quad \frac{V'_{R_2}}{V_{R_2}} = \frac{N_1}{N_2} = \frac{I'_2R'_2}{I_2R_2}$$

$$\Rightarrow \quad R'_2 = R_2\left(\frac{N_1}{N_2}\right).\frac{I_2}{I'_2}$$

$$\boxed{R'_2 = R_2\left(\frac{N_1}{N_2}\right)^2}$$

Similarly, $X'_2 = X_2\left(\dfrac{N_1}{N_2}\right)^2$ = reactance of secondary side refer to Primary

$R'_1 = R_1\left(\dfrac{N_2}{N_1}\right)^2 = \dfrac{R_1}{a^2}$ reactance of Primary side refer to secondary

$X'_1 = X_1\left(\dfrac{N_2}{N_1}\right)^2 = \dfrac{x_1}{a^2}$ reactance of Primary side refer to secondary

$$\text{Power input} = \text{Power output} + \text{Losses}$$
$$= V_2 I_2 \cos\theta_2 + P_c + P_{oh}$$
$$I_2 = \text{load current}$$
$$\cos\theta_2 = \text{Load power factor}$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + P_{oh}}$$

If I_2 is the full load or rated current, then the efficiency is full load efficiency.

Efficiency at full load

$$(\eta_{fl}) = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + P_{oh}} = \frac{(\text{KVA})_{\text{rated}} \cos\theta_2}{(\text{KVA})_{\text{rated}} + P_c + P_{oh}}$$

$$P_{oh} = I_2^2 R_{e_2} = I_1^2 R_{e_1}$$

- If the load becomes half means load current becomes half (1/2). If the load becomes x fraction of rated load means the new current becomes x time of rated current.

$$\boxed{P_{oh} \propto I^2}$$

$$(I_2) \text{ at } x \text{ fraction load} = x\, I_2 \text{ rated}$$

$$\text{(efficiency at fraction } x \text{ load) } \eta_x = \frac{x(V_2 I_2)\cos\theta_2}{x(V_2 I_2)\cos\theta_2 + P_c + x^2 P_{oh}}$$

⇒ For all types of load power factor efficiency formula does not change.

$$\eta_{fl} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + P_{oh}}$$

Efficiency is a function of load or load current assuming $\cos\theta_2$ is constant.

$$\eta_x = \frac{x\theta\cos\theta_2}{x\theta\cos\theta_2 + P_c + x^2 P_{oh(rated)}} \{Q = V_2 I_2$$

For hx to be maximum $\left(1 + \dfrac{P_c}{z\theta\cos\theta_2} + \dfrac{x^2 P_{oh}}{x\theta\cos\theta_2}\right)$

Should be minimum

let $$P = 1 + \frac{P_c}{x\theta\cos\theta_2} + \frac{x P_{oh}}{\theta\cos\theta_2}$$

$\Rightarrow \qquad \dfrac{dP}{dx} = 0 - \dfrac{P_c}{x^2\,\theta\cos\theta_2} + \dfrac{P_{oh}}{\theta\cos\theta_2} = 0$

$\Rightarrow \qquad \dfrac{P_c}{x^2} = P_{oh}$

$\Rightarrow \qquad \dfrac{P_c}{x^2} = P_{oh}$

$$\boxed{P_c = I_2^2 R_{e2} = P_{oh}} \qquad x = \sqrt{\frac{P_c}{P_{oh(\text{rated})}}}$$

$\Rightarrow$ at full load ($x = 1$) efficiency is maximum

when $\boxed{\text{core loss} = \text{ohmic loss}}$

load at maximum efficiency is $Q_{\max \eta}$

$$\boxed{Q_{\text{at}\max\eta} = x.Q_{\text{rated}}}$$

$\Rightarrow \qquad P_c - I_2^2 R_{e2} = 0$

$$\boxed{P_c = I_2^2 R_{e2} = P_{oh}}$$

So at maximum efficiency $P_c = P_{oh}$

Load current at max η is $I_2 = \sqrt{\dfrac{P_c}{R_{e2}}}$

$$(I_2)_{\max\eta} = \sqrt{\frac{P_c}{R_{e2}}}$$

$$(V_2I_2)_{\max\eta} = V_2.(I_2)_{\text{rated}}\sqrt{\frac{P_c}{(I_2)^2_{\text{rated}}R_{e2}}}$$

$$\frac{(V_2I_2)_{\max\eta}}{1000} = \frac{(V_2I_2)_{\text{rated}}}{1000}\sqrt{\frac{P_c}{(P_{oh})_{\text{rated}}}}$$

$$\boxed{(\text{KVA})_{\max\eta} = (\text{KVA})_{\text{rated}}\sqrt{\frac{P_c}{P_{oh\,\text{rated}}}}}$$

$$\boxed{VR\% = \frac{E_2 - V_2}{E_2} \times 100}$$

$\frac{E_2 - V_2}{E_2}$ is per unit regulation.

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 (R_{e2} + jX_{e2}) \quad ...(1)$$

at lagging load I_2 lags by V_2 by angle θ_2.

$$E_2 \approx OC = OA + AB + BC$$

$$E_2 = V_2 + I_2 R_{e2} \cos \theta_2 + I_2 X_{e2} \sin \theta_2$$

$$(E_2 - V_2) = I_2 R_{e2} \cos \theta_2 + I_2 X_{e2} \sin \theta_2$$

$$\frac{(E_2 - V_2)}{E_2} = \frac{I_2 R_{e2}}{E_2} \cos \theta_2 + \frac{I_2 X_{e2}}{E_2} \sin \theta_2$$

$$\boxed{VR = \varepsilon_r \cos \theta_2 + \varepsilon_x \sin \theta_2}$$

$$\varepsilon_r = \frac{I_2 R_{e2}}{E_2} \times 100 = \% \text{ resistance drop}$$

$$\varepsilon_x = \frac{I_2 X_{e2}}{E_2} \times 100 = \% \text{ reactance drop}$$

or simple $\frac{I_2 R_{e2}}{E_2}$ = Per unit resistance drop.

So, $\boxed{V.R.\% = (\varepsilon_r \cos\theta_2 \pm \varepsilon_x \sin\theta_2)100}$

+ sign is for lagging load.

– sign is for leading load.

For lagging PF and unity PF ($E_2 > V_2$)

As load becomes capacitive, (leading) V_2 starts increasing as load increases. At a certain leading power factor $E_2 = V_2$ and regulation becomes zero. If the load is increased further $E_2 < V_2$ and voltage regulation becomes –ve.

$$\varepsilon_r \cos \theta_2 + \varepsilon_x \sin \theta_2 = 0$$

$$\tan \theta_2 = \frac{-\varepsilon_r}{\varepsilon_x} = -\frac{R_{e2}}{X_{e2}}$$

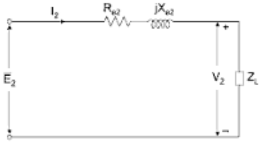


Fig. 5.25

$$\boxed{\theta_2 = \tan^{-1}\left(\frac{-R_{e2}}{X_{e2}}\right)}$$

Voltage regulation is zero at a power factor of $\cos\left[\tan^{-1}\left(\frac{-R_{e2}}{X_{e2}}\right)\right]$.

So, voltage regulation is zero at leading power factor.

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 (R_{e2} + jX_{e2})$$

for lagging load $\quad \bar{I}_2 = I_2 \angle -\theta_2$

$$E_2 = V_2 + I_2 \angle -\theta_2 (R_{e2} + jX_{e2})$$

$$E_2 = V_2 + I_2 (\cos\theta_2 - j\sin\theta_2)(R_{e2} + jX_{e2})$$

$$\boxed{(E_2 - V_2) = I_2(\cos\theta_2 - j\sin\theta_2)(R_{e2} + jX_{e2})}$$

Voltage drop = $(I_2R_{e2}\cos\theta_2 + I_2X_{e2}\sin\theta_2) + j(I_2X_{e2}\cos\theta_2 - I_2R_{e2}\sin\theta_2)$

So, for leading load $\bar{I}_2 = I_2 \angle -\theta_2$

So, we can calculate voltage drop by CKT also.

5.10.1 Power Factor for Zero Regulation

The % regulation will be zero when

$$\varepsilon_r \cos\phi_2 + \varepsilon_x \sin\phi_2 = 0$$

or,
$$\tan\phi_2 = \frac{-\varepsilon_r}{\varepsilon_x} \quad ...(5.1)$$

$$= \frac{-R_{e_2}}{X_{e_2}} \quad ...(5.2)$$

The –ve sign shows that the p.f. will be leading for zero regulation.

5.10.2 Power Factor for Maximum Regulation and Maximum Value of Percentage Regulation

The % regulation is given by,

$$R = \varepsilon_r \cos \phi_2 + v_x \sin \phi_2 \qquad ...(5.3)$$

For maximum value of R, $\dfrac{dR}{d\phi_2} = 0$,

$$\therefore \qquad \frac{dR}{d\phi_2} = -\varepsilon_r \sin \phi_2 + \varepsilon_x \cos \phi_2 = 0$$

or,

$$\tan \phi_2 = \frac{\varepsilon_x}{\varepsilon_r} \qquad ...(5.4)$$

$$= \frac{X_{e2}}{R_{e2}} \qquad ...(5.5)$$

Fig. 5.26

This shows that the maximum regulation occurs at a lagging p.f.
From Fig 5.26.

$$\sin \phi_2 = \frac{\varepsilon_x}{\sqrt{\varepsilon_r^2 + \varepsilon_x^2}}$$

$$\cos \phi_2 = \frac{\varepsilon_r}{\sqrt{\varepsilon_r^2 + \varepsilon_x^2}}$$

Substituting in eqn. (5.3), the maximum % regulation.

$$= \frac{\varepsilon_r}{\sqrt{\varepsilon_r^2 + \varepsilon_x^2}} + \frac{\varepsilon_x}{\sqrt{\varepsilon_r^2 + \varepsilon_x^2}}$$

$$= \sqrt{\varepsilon_r^2 + \varepsilon_x^2} \qquad \text{...(5.6)}$$

$$= I_2 \sqrt{\frac{R_{e2}^2 + X_{e2}^2}{E_2}} \times 100 \qquad \text{...(5.7)}$$

5.11 TESTING OF A 1 – φ TRANSFORMER

For calculating the parameters, efficiency and regulation of a transformer at any load and at any power factor we do some tests. They are

(i) Direct loading test.
(ii) Indirect loading test.

Difficulties, which do not permit the testing of large transformers by *direct load* test are:

(i) Big amount of energy has to be wasted in each loading.
(ii) It is difficult to arrange a large load for very big transformer.
(iii) Appropriate meters (voltmeter, ammeters and wattmeters) are difficult to make available at normal sites.

Performance can be determined by conducting simple tests involving very little power construction, called *non-loading* in direct tests. In these tests the power consumption is simply that which is needed to supply the losses. The two non-loading tests are *open* CKT and *short* CKT test.

5.12 OPEN CIRCUIT TEST

The purpose of this test is to determine the (i) Shunt branch parameters (R_c, X_0), (ii) No load power factor and (iii) Core losses of a transformer.

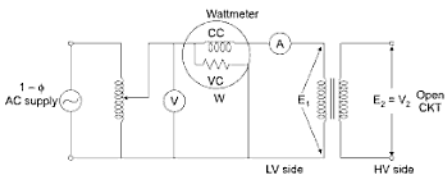


Fig. 5.27

- Secondary side is open CKT, that is why the Ammeter (A) reads a no load current I_0 which is usually 2 to 6% of the rated current.
- Wattmeter reads a total loss of a transformer at no load.

$$W = P_{core} + P_{oh}$$
$$= P_{core} + I_0^2 R$$

I_0 is very low hence $I_0^2 R_e (P_{core})$ is further low and negligible in comparison to core loss $(P_{oh})_{no\ load}$ = (.04 to .36)% of rated copper loss hence the wattmeter indicates practically only *core loss* at no load.

- At no load wattmeter indicates only core loss $(P_h + P_e)$ and the core loss depends on the *Voltage* if the frequency is constant.
- So for getting *rated or complete* core loss we have to apply a *rated voltage*.
- Hence, *open* CKT *test* perform on rated voltage.
- We will perform the O.C. test at LV side because at LV side apparatus of low voltage rating (voltmeter) is easily available and availability of supply easily made available.

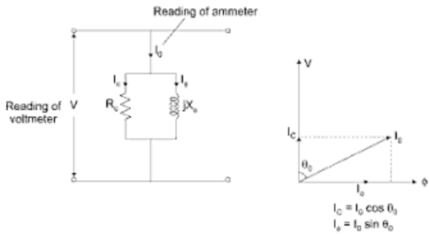


Fig. 5.28

$$\Rightarrow \qquad V = I_c R_c = I_0 X_\phi$$
$$\Rightarrow \qquad W = VI_c = VI_0 \cos \theta_0 \quad \{W = \text{reading of wattmeter}\}$$

$$\cos \theta_0 = \frac{W}{VI_0} \qquad \{V = \text{reading of voltmeter, } I_0 = \text{reading of Ammeter.}$$

From this, we calculate I_c and I_ϕ

$$R_c = \frac{V}{I_c} \text{ and } X_\phi = \frac{V}{I_\phi}$$

5.13 SHORT CIRCUIT (SC) TEST

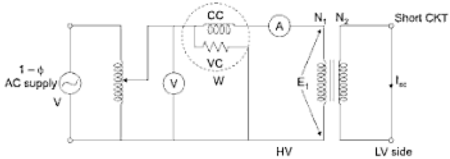


Fig. 5.29

- As secondary is shorted, it may draw a very large current at rated voltage and transformer may burn out. So, this test is *not performed at rated voltage.*
- For getting full load or rated ohmic loss, this test is performed at rated current.
- Rated current may be drawn at only 2 to 10% of the *rated voltage*. So, reading of the voltmeter is 2 to 10% of rated voltage.
- It is convenient to connect high voltage side to supply ratio current at HV side $\left(\frac{\text{kVA}}{\text{kV}}\right)$ is low, so we can solve our purpose with low *rating Ammeter.*

Example: For a 200 kVA, 440/6600 V transform, perform short-cut not at 5% of ratio to be

LV side Rated voltage = 440 Volt

$$\text{Short CKT } V_{sc} = 440 \times \frac{5}{100} = 22 \text{ Volt}$$

Rated current = 200 × 1000/440 = **445 Amp**

HV side $V_{sc} = 6600 \times \frac{5}{100} = 330$ V, rated current = 30 A

So LV side high current is needed for conducting SC test.

While conducting the SC test, the supply voltage is gradually raised from *Zero* to till the transformer draws *full load* or rated current.

- Voltmeter reads short CKT voltage V_{sc} (2 to 10%) of rated voltage.
- Ammeter reads the rated current.
- Wattmeter reads the *core loss* and *ohmic loss* but this test is performed at a short CKT voltage which is 2 to 10% of rated voltage. Hence, the *core loss* is negligible in comparison to copper or ohmic losses.

So wattmeter reads only *ohmic loss*.

$\boxed{W = I_{sc}^2 R_{HV}^2}$ $\{I_{sc}$ = rated current, R_{HV} = equivalent resistance at HV side$\}$

Let the test perform at primary side:

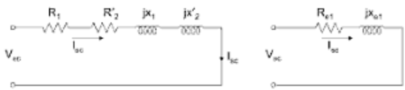


Fig. 5.30

$$\Rightarrow \quad Z_{sc} = \sqrt{R_{e1}^2 + X_{e1}^2} \quad ...(1)$$

$$W = I_{sc}^2 R_{e1} \quad \text{(from this we calculate } R_{e1}\text{)}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} = \sqrt{R_{e1}^2 + X_{e1}^2} \quad \text{(from this we calculate } X_{e1}\text{)}$$

- %η on full load = $\dfrac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + P_{oh}}$

P_c = reading of wattmeter at no load

P_{oh} = reading of wattmeter at rated load current.

I_2 = reading of ammeter (rated current)

V_2 = rated voltage

$\cos\theta$ = load power factor given

- So we can *calculate* η from OC SC test

$$\boxed{V.R. = \frac{I_2 R_{e2}}{E_2}\cos\theta_2 \pm \frac{I_2 X_{e2}}{E_2}\sin\theta_2}$$

- R_{e2} and X_{e2} can be calculated by short CKT test.
- I_2 is rated current.
- We can also draw the equivalent CKT from OC and SC test.

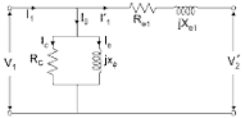


Fig. 5.31

R_c and X_ϕ calculate from OC test.

R_{e1} and X_{e1} calculate from SC test.

Polarity test:

If the primary winding of a transformer is excited with an alternating voltage then emf gets induced in both the winding.

Similar polarity ends of the two windings of a transformer are those ends that acquire simultaneously positive or negative polarity of emf's induced in them.

In determining the relative polarity of the two windings of a transformer, the two windings are connected in series across a voltmeter, while one of the windings is excited from a suitable voltage source.

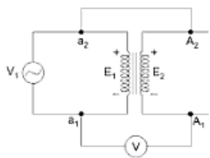


Fig. 5.32

- If the voltmeter indicates $E_1 \sim E_2$, then the ends a_2, A_2 are simultaneously *positive* or *negative* and the polarities are called *subtractive* in nature.

- If the voltmeter indicates $E_1 + E_2$. This confirms that a_2, A_2 are simultaneously opposite polarity and the polarities are called additive. Now the polarity marking one of the winding must be interchanged.

5.14 SUMPNER'S* (BACK TO BACK) TEST

This test is performed on two identical transformers to determine their:

(a) Regulation (b) Efficiency and (c) Temperature rise.

The regulation and efficiency can be determined conveniently with open circuit and short circuit test. For determining the temperature rise of the transformer, it would be essential to fully load it. Smaller transformers may be loaded while testing by means of water loads or lamp loads. But for large rating transformer, it may be very difficult to arrange suitable loads for fully loading it. Further, there is tremendous wastage of the electrical energy. By this test the two transformers may be loaded fully while the power needed from the supply has to feed only iron and copper losses.

The connection diagram to perform Sumpner's (back to back) test is shown in Fig. 5.33.

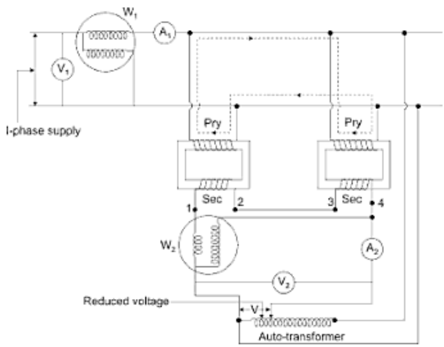


Fig. 5.33

*Not for U.P.T.U. (TEE 101/201) student

The above connections to conduct the test are made in two stages.

Stage 1:

The primary sides of the two transformers are connected in parallel across a single phase supply with a voltmeter V_1, wattmeter W_1 and ammeter A_2 in the circuit keeping the secondary sides open circuited. Now secondaries are required to be connected in opposition. Any two terminals of the secondaries are first connected together and the voltage measured between the other remaining two terminals by a voltmeter V_2. If this voltage is double that of secondary voltage of one transformer, it shows that two transformers are connected in series helping and not in opposition. By interchanging the connections, the two transformers will be in opposition and under that condition the voltmeter V_2 will measure zero voltage. Suppose, when the transformers are in opposition terminals 2 and 3 are connected together and the voltmeter V_2 indicates zero voltage between 1 and 4.

Stage 2:

After achieving the condition of opposition, a reduced voltage which is obtained through an auto-transformer is applied across terminals 1 and 4 after connecting a voltmeter V_2, wattmeter W_2 and ammeter A_2 in the circuit on the secondary side.

When transformers are connected in opposition, no current flows in their secondaries due to primary voltage. Hence, the transformers act as if their secondaries are open circuited. The wattmeter W_1 measures the iron losses of the two transformers.

Now the two transformers are loaded fully by applying a reduced voltage from the auto-transformer. The secondary current will cause a current to circulate in the two primaries having a path as shown dotted in the Fig. 5.34. The arrows show instantaneous direction of current in primaries due to the secondary current. Since, the directions of current are in opposition in the two primaries, hence the reading wattmeter W_1 will not be affected although the primaries and secondaries of the two transformers are loaded fully. The wattmeter W_2 therefore gives the full load copper losses of the two transformers.

Now, $W_1 = 2\ W_i$ = Iron losses of both the transformers

$W_2 = 2\ W_c$ = Full load coppers of both the transformers.

V_2 = Reduced voltage measured by voltage V_2

I_2 = Full load secondary current measured by ammeter A_2.

If Z_s is the total equivalent impedance referred to the secondary of one transformer,

Then, $$2Z_s = \frac{V_2}{I_2}$$

∴ $$Z_s = \frac{V_2}{2I_2}$$

Also, $$2I_2^2 R_s = \text{Copper loss of both transformers} = W_2$$

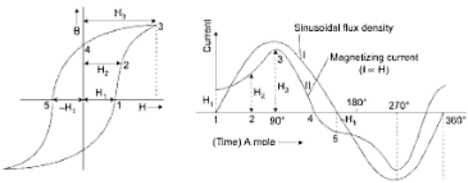
∴ $$R_s = 2\frac{W_2}{2I_2^2}$$

∴ $$X_s = \sqrt{Z_s^2 - R_s^2}$$

The efficiency and regulation can now be calculated easily.

For determining the temperature rise, the two transformers after loading fully in the manner described above are left for few hours till final steady temperature is reached.

*5.15 HARMONIES IN TRANSFORMERS



(a) Hysteresis loop

(b) Magnetizing current wave form

Fig. 5.34

The use of high flux densities in the core of transformers results in reduction of weight and therefore in cost. But this introduces serious troubles due to saturation of the magnetic circuit of the transformer which subsequently results in production of harmonies.

*Not for U.P.T.U. (TEE 101/201) student

*5.16 AUTO-TRANSFORMER

Auto-transformer has one winding only, wound over a closed magnetic circuit of low permeability as shown in Fig. 5.35(a). The electrical representation of a single phase auto-transformer is shown in Fig. 5.35(b). A part of winding is common to both primary and secondary. In theory and operation, an auto-transformer is similar to a two-winding transformer.

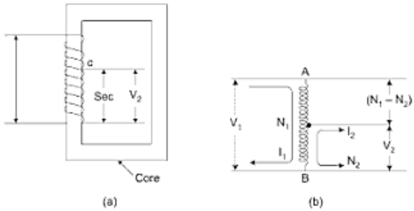


Fig. 5.35

Let the number of turns in primary winding AB be N_1 and in secondary BC be N_2. Let I_1 and I_2 be the primary input and secondary output currents.

Ignoring no load current and iron losses, the transformation ratio a is related as

$$= \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

For the same voltage ratio and rating, an auto-transformer requires less weight of copper than what is required by an ordinary transformer.

Now, weight of copper in a winding

$\propto$ Volume of copper

$\propto$ Length of conductor × cross-section

$\propto$ No. of turns × current carried.

(Since, cross-section of conductor depends upon the current carried by it)

Ordinary-transformer:

Weight of copper in primary

$\propto$ No. of primary turns × current carried

$\propto N_1 I_1$

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Weight of copper in secondary $\propto N_2I_2$

$\therefore$ Total weight of copper $\propto (N_1I_1 + N_2I_2)$

Auto-transformer:

From Fig. 5.35(b), current in section AC having $(N_1 - N_2)$ turns is I_1.

$\therefore$ Weight of copper in section $AC \propto (N_1 - N_2)I_1$

The current in section BC having N_2 turns is $(I_2 - I_1)$, since the currents in primary and secondary are in opposition.

$\therefore$ Weight of copper in section $BC \; \alpha \; N_2(I_2 - I_1)$

$\therefore$ Total weight of copper in auto-transformer

$$\propto \{(N_1 - N_2)I_1 + N_2(I_2 - I_1)\}$$
$$\propto \{(N_1 - 2N_2)I_1 + N_2I_2\}$$

$$\frac{\text{Weight of copper in auto-transformer}}{\text{Weight of copper in ordinary transformer}}$$

$$= \frac{(N_1 - 2N_2)I_1 + N_2I_2}{N_1I_1 + N_2I_2}$$

Dividing numerator and denominator by N_2I_1

$$\therefore \qquad \text{Ratio of weight of copper} = \frac{\left(\frac{N_1}{N_2} - 2\right) + \frac{I_2}{I_1}}{\frac{N_1}{N_2} + \frac{I_2}{I_1}} = \frac{(a-2)+a}{a+a}$$

$$= \frac{\cancel{2}\,(a-1)}{\cancel{2}\,a} = \left(1 - \frac{1}{a}\right)$$

If a is large i.e., one side is at a very high voltage compared with the other then $\left(1 - \frac{1}{a}\right)$ is approximately equal to 1, so the weight of copper in auto-transformer will be nearly equal to the weight in ordinary transformer. Hence, in such a case it is preferable to use an ordinary two winding transformers.

If the transformation ratio a is nearly unity then $\left(1 - \frac{1}{a}\right)$ will be quite small and so there will be sufficient saving of copper in an auto-transformer.

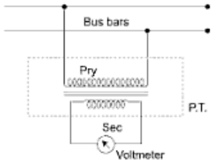


Fig. 5.37

small, so power handled by voltmeter is also quite small. Though a P.T. is similar to a two winding transformers, but it is designed for very low output power. So, the size of instrument transformers is quite small.

*5.18.2 Current Transformer (C.T.)

A current transformer shown in Fig. 5.38 consists of one or few turns in primary of heavy cross-sectional area and a large number of turns in secondary of small cross-sectional area. Thus, a high current in a line can be reduced to such current like 5 to 10 A which may be measured easily by a normal ammeter.

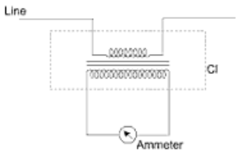


Fig. 5.38

For measurement of very high currents without disturbing the running line, a split core type current transformer as shown in Fig. 5.39 is used. The running line acts as a primary and an ammeter is connected to the secondary. A flux is produced around the line proportional to the line current which induces an emf in the secondary. Hence, the deflection of the ammeter depends upon the current in the line.

Precaution:

Since, the secondary of the C.T. is connected across an ammeter which has a very small resistance, so the voltage across the secondary terminals is

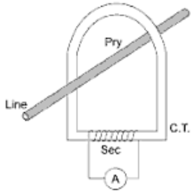


Fig. 5.39

practically equal to zero volt. If at any time the secondary is open circuited, a very high voltage = $\frac{N_2}{N_1} V_1$ (since, $\frac{N_2}{N_1}$ is very large) will be induced in the secondary. This may puncture the insulation of the C.T. core or winding. Hence, to safeguard against such a risk a shunt is used with the ammeter which short circuits the secondary terminals permanently. Further, whenever, the ammeter is removed from the circuit, the secondary terminals are properly short-circuited.

*5.18.3 Application of C.T. and P.T.

The power in a circuit having high voltage and current can be measured easily by using C.T. and P.T. as shown in Fig. 5.39. Let the line voltage be 1100 V and line current by 50 A. By using a P.T. of 1100/110 V and a C.T. of 50/5 A, we can measure power by a wattmeter whose pressure coil is rated for 110 V and current coil for 5 A only.

Note: The secondary of a C.T. cannot be allowed to be open circuited as it would cause the whole of the primary current to act as a magnetizing current resulting in extreme magnetization of the core and totally damaging the same.

SOLVED PROBLEMS

Example 4: 10 kVA transformer having 50 number of turns on primary and 10 number of turns on secondary is connected to 440 V, 50 Hz supply. Calculate:

(a) Secondary voltage on no load
(b) Full load primary and secondary currents
(c) Maximum value of the flux, in the core.

Solution: The given values are,

$N_1 = 50$, $N_2 = 10$, $V_1 = 440$ V, $f = 50$ Hz

(a) According to voltage transformation ratio,

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = K$$

$$\therefore \quad \frac{10}{50} = \frac{V_2}{440} = \frac{1}{5}$$

$$\therefore \quad V_2 = \frac{440}{5} = 88 \text{ V}$$

On no load, $E_2 = V_2 = 88$ V

(b) Now, kVA rating = 10

$$\therefore \quad (I_1)\text{F.L.} = \frac{\text{kVA} \times 1000}{V_1} = \frac{10 \times 1000}{440} = 22.7272 \text{ A}$$

while, $$(I_2)\text{F.L.} = \frac{\text{kVA} \times 1000}{V_2} = \frac{10 \times 1000}{88} = 113.6363 \text{ A}$$

(c) According to the emf equation,

$$E_2 = 4.44 f \phi_m N_2$$

$$\therefore \quad 88 = 4.44 \times 50 \times \phi_m \times 10$$

$$\therefore \quad \phi_m = 39.6396 \text{ mWb}$$

This is the maximum value of the flux in the core.

Example 5: A single phase transformer has 350 primary and 1050 secondary turns. The primary is connected to 400 V, 50 Hz supply. If the net cross-sectional area of the core is 50 cm^2, calculate (i) The maximum value of the flux density in the core (ii) The induced emf in the secondary winding.

Solution: The given values are,

$N_1 = 350$ turns, $N_2 = 1050$ turns

$V_1 = 400$ V, $A = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$

The emf of the transformer is,

$$E_1 = 4.44 f \phi_m N_1$$

$$E_1 = 4.44 B_m A f N_1 \text{ as } \phi_m = B_m A$$

Flux density $B_m = \dfrac{E_1}{4.44\, A\, f\, N_1}$

$$= \frac{440}{4.44 \times 50 \times 10^{-4} \times 50 \times 350} \text{ assume } E_1 = V_1$$

$$= 1.0296 \text{ Wb/m}^2$$

Now, $\dfrac{N_2}{N_1} = \dfrac{1050}{350} = 3$

and, $\dfrac{E_2}{E_1} = 3$

$\therefore$ $E_2 = 3 \times E_1 = 3 \times 400 = 1200$ V

Example 6: (a) A 2200/220 V, 50 Hz single phase transformer has exciting current of 0.6 A and a core loss of 361 watts when its H.V. side is energised at rated voltage. Calculate the two components of the exiting current. (b) If the transformer of part (a), supplies a load current of 60 A at 0.8 p.f. lag on its l.v. side, then calculate the primary current and its power factor. Ignore leakage impedance drops.

Solution: (a) Exciting current $I = .6$ A

Supply current $V'_1 = 2200$ V; core loss $P_c = 361$ W

$\therefore$ Core loss components $I = \dfrac{P_c}{V_1'} = \dfrac{361}{2200} = 0.164$ A from eqn.

Magneting component $I_\phi = \sqrt{I_0^2 - I_c^2}$

$$= \sqrt{(0.60)^2 - (0.164)^2} = 0.577 \text{ A}$$

(b) The primary current component I_1' required to neutralise the effect of secondary current $I_2 = 60$ A is given by

$$I'_1 N_1 = I_2 N_2 \quad I'_1 = \left(\frac{N_2}{N_1}\right) I_2 = \left(\frac{V_2}{V_1}\right) I_2$$

or, $I'_1 = \dfrac{220}{2200}(60) = 60$ A

Expressing I_2 in terms of I_1 gives.

$$P_{oh} = I_1^2 r_1 + \left[I_1\left(\frac{N_1}{N_2}\right)^2\right] r_2 = I_1^2\left[r_1 + r_2\left(\frac{N_1}{N_2}\right)^2\right]$$

$$= I_1^2 r_{e1}$$

if,

$$P_{oh} = \left[I_2\left(\frac{N_2}{N_1}\right)\right]^2 r_1 + I_2^2 r_2 = I_2^2\left[\left(\frac{N_2}{N_1}\right)^2 r_1 + r_2\right]$$

$$= I_2^2 r_{e2}$$

Thus, the total ohmic loss in a transformer = (Equivalent resistance referred to either side). (Square of the current on that side).

Example 8: A 23 kVA, 2300/230 V, 60 Hz, step down transformer has the following resistances and leakage reactance values $R_1 = 4\ \Omega$, $R_2 = 0.04\ \Omega$, $X_1 = 12\ \Omega$, $X_2 = 0.12\ \Omega$. Transformer is operating at 75% of its rated load. If the power factor of the load is 0.866 leading, determine the efficiency of the transformer. Make suitable assumptions.

Solution: Core loss resistance R_c and magnetizing reactance X_ϕ can be negligible (not given).

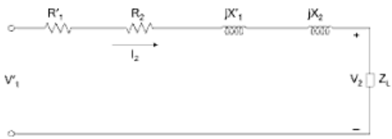


Fig. 5.41

Let the voltage across the load be 230 V (assume)

$$V'_1 = \bar{I}_2 (R_{e2} + jX_{e2}) + V_2$$

$$R_{e2} = R'_1 + R_2 = 4\left(\frac{230}{2300}\right)^2 + 0.04 = 0.08\ \Omega$$

$$X_{e2} = X'_1 + X_2 = 12\left(\frac{230}{2300}\right)^2 + 0.12 = 0.24\ \Omega$$

$$Z_{e2} = (0.08\ \Omega + j0.24)\ \Omega = 0.2529\angle 71.565°$$

(c) Equivalent resistance and reactance of primary referred to secondary.

(d) Total resistance and reactance referred to secondary.

(e) Total copper loss.

Solution: Given,

$$R_1 = 2\ \Omega, \qquad X_1 = 5\ \Omega$$
$$R_2 = 0.02\ \Omega, \quad X_2 = 0.045\ \Omega$$
$$\frac{N_1}{N_2} = \frac{1100}{110} = 10$$
$$\text{Rating} = 22\ \text{kVA}$$

(a)
$$R'_2 = \left(\frac{N_1}{N_2}\right)^2 R_2 = 10^2 \times 0.02 = \mathbf{2\ \Omega} \quad \textbf{Ans.}$$
$$X'_2 = \left(\frac{N_1}{N_2}\right)^2 X_2 = 10^2 \times 0.045 = \mathbf{4.5\ \Omega} \quad \textbf{Ans.}$$

(b)
$$R_{e1} = R_1 + R'_2 = 2 + 2 = \mathbf{4\ \Omega} \quad \textbf{Ans.}$$
$$X_{e1} = X_1 + X'_2 = 5 + 4.5 = 9.5\ \Omega \quad \textbf{Ans.}$$

(c)
$$R'_1 = \left(\frac{N_2}{N_1}\right)^2 R_1 = \frac{1}{10^2} \times 2 = 0.02\ \Omega \quad \textbf{Ans.}$$
$$X'_1 = \left(\frac{N_2}{N_1}\right)^2 X_1 = \frac{1}{10^2} \times 5 = 0.05\ \Omega \quad \textbf{Ans.}$$

(d)
$$R_{e2} = R_2 + R'_1 = 0.02 + 0.02 = 0.04\ \Omega \quad \textbf{Ans.}$$
$$X_{e2} = X_2 + X'_1 = 0.045 + 0.05 = 0.095\ \Omega \quad \textbf{Ans.}$$

(e) Primary current $I_1 = \dfrac{22 \times 10^3}{1100} = 20\ \text{A}$

Secondary current $I_2 = \dfrac{22 \times 10^3}{110} = 200\ \text{A}$

Total copper loss $W_c = I_1^2 R_1 + I_2^2 R_2$

$$= 20^2 \times 2 + 200^2 \times 0.02 = \mathbf{1600\ W} \quad \textbf{Ans.}$$

The total loss can also be found,

$$W_c = I_1^2 R_{e1} = 20^2 \times 4 = 1600 \text{ W}$$

or $$= I_2^2 R_{e2} = 200^2 \times 0.04 = 1600 \text{ W} \quad \textbf{Ans.}$$

Example 15: A 20 kVA single phase transformer designed for 2000/200 V has the following constant: $R_1 = 2.5\ \Omega$, $X_1 = 8\ \Omega$, $R_2 = 0.04\ \Omega$ and $X_2 = 0.07\ \Omega$. Calculate the approximate value of the secondary terminal voltage and % regulation at full load and 0.8 p.f. lagging when primary applied voltage is 2000 V.

Solution: From the given data, we have

$$I_2 = \frac{20 \times 1000}{200} = 100 \text{ A}$$

$$\frac{N_2}{N_1} = \frac{200}{2000} = \frac{1}{10}$$

Total equivalent resistance and reactance referred to the secondary side are given by:

$$R_{e2} = R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1$$

$$= 0.04 + \frac{2.5}{100} = 0.065\ \Omega$$

$$X_{e2} = X_2 + \left(\frac{N_2}{N_1}\right)^2 X_1$$

$$= 0.07 + \frac{8}{100} = 0.15\ \Omega$$

Approximate value of V_2 is given by eqn. (5.14).

$$E_2 = V_2 + I_2 R_{e2} \cos\phi_2 + I_2 X_{e2} \sin\phi_2$$

$\therefore$ $$V_2 = E_2 - (I_2 R_{e2} \cos\phi_2 + I_2 X_{e2} \sin\phi_2)$$

$$= 200 - (100 \times 0.065 \times 0.8 + 100 \times 0.15 \times 0.6)$$

$$= 200 - (5.2 + 9) = \textbf{185.8 V} \quad \textbf{Ans.}$$

$$\% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

$$= \frac{14.2}{200} \times 100$$

$$= \mathbf{7.1\%} \quad \textbf{Ans.}$$

Example 16: In example 15 determine the p.f.s. at which the regulation will be zero and maximum. Find also the maximum value of % regulation.

Solution: Power factor angle at which the regulation is zero, is given by

$$\tan \phi_2 = -\frac{R_{e2}}{X_{e2}}$$

$$= -\frac{0.065}{0.15} = -0.433$$

$\therefore \quad \phi_2 = -23.4°$

$\therefore \quad \cos \phi_2 = \cos 23.4°$

$= \mathbf{0.918}$ **leading** **Ans.**

From eqn. 5.16, the p.f. angle at which the regulation is maximum is given by,

$$\tan \phi_2 = \frac{X_{e2}}{R_{e2}}$$

$$= \frac{0.15}{0.065} = 2.31$$

$\therefore \quad \phi_2 = 66.6°$

$\therefore \quad \cos \phi_2 = \cos 66.6°$

$= \mathbf{0.397}$ **lagging** **Ans.**

$\sin \phi_2 = \sin 66.6° = 0.918$

Maximum value of% regulation

$$= \frac{I_2 R_{e2} \cos\phi_2 + I_2 X_{e2} \sin\phi_2}{E_2} \times 100$$

$$= \frac{100 \times 0.065 \times 0.397 + 100 \times 0.15 \times 0.918}{200} \times 100$$

$$= \frac{2.58 + 13.77}{200} \times 100$$

$= \mathbf{8.18\%}$ **Ans.**

Maximum% regulation can also be formed from eqn. which is

$$= \frac{I_2\sqrt{R_{e2}^2 + X_{e2}^2}}{E_2} \times 100$$

$$= \frac{100 \times \sqrt{0.06^2 + 0.15^2}}{200} \times 100$$

= **8.18% Ans.**

Example 17: A 200 kVA, single phase transformer has 1000 W iron loss 2000 W copper loss at full load, calculate:

(a) Its efficiency at full load and 0.8 p.f. lagging.

(b) Its efficiency at half load and 0.8 p.f. lagging.

(c) Load at which its efficiency is maximum and the maximum efficiency at 0.8 p.f. lagging.

Solution:

(a) Full load output at 0.8 p.f. lagging = 200 × 0.8

= 160 kW

Iron loss at full load w_i = 1000 W

= 1 kW

Copper loss at full load w_c = 2000 W

= 2 kW

∴ Total losses = 1 + 2 = 3 kW

Total input = output + losses

= 160 + 3 = 163

$$\therefore \quad \eta \text{ at full load} = \left(1 - \frac{\text{losses}}{\text{input}}\right) \times 100$$

$$= \left(1 - \frac{3}{163}\right) \times 100$$

= **98.16% Ans.**

(b) Output at half load and 0.8 p.f. lagging

$$= \frac{1}{2} \times 200 \times 0.8$$

$$= 80 \text{ kw.}$$

$$\text{Iron loss} = 1 \text{ kw}$$

(Since, it is independent of output)

$$\text{Copper loss at half load} = \left(\frac{1}{2}\right)^2 \times w_c$$

$$= \left(\frac{1}{2}\right)^2 \times 2 = 0.5 \text{ kW}$$

$\therefore$ Total losses at half load = $1 \times 0.5 = 1.5$ kW.

$$\text{Input at half load} = 80 + 1.5 = 81.5 \text{ kW}$$

$$\therefore \qquad \eta \text{ at half load} = \left(1 - \frac{1.5}{81.5}\right) \times 100$$

= 98.16% Ans.

(c) Let x be the fraction of the full load for which the efficiency is maximum.

$$\text{copper loss} = x^2 . w_c.$$

$$= x^2(2000) \text{ W.}$$

For maximum η this loss will be equal to the iron loss.

$$\therefore \qquad x^2(200) = 1000$$

$$\text{or,} \qquad x = \sqrt{\frac{1}{2}} = 0.707$$

or, **70.7% of full load Ans.**

Output at this load and 0.8 p.f = $0.707 \times 2000 \times 0.8$

$$= 1132 \text{ kW.}$$

$$\text{Iron loss} = 1 \text{ kW}$$

$$\text{Copper loss} = 1 \text{ kW}$$

$$\therefore \qquad \text{Total losses} = 2 \text{ kW}$$

$$\text{Input at this load} = 1132 + 2 = 1134 \text{ kW}$$

$$\therefore \qquad \eta = \left(1 - \frac{2}{1134}\right) \times 100$$

$$= \mathbf{98.24\%} \quad \textbf{Ans.}$$

Example 18: A 400 kVA distribution transformer was loaded as under during a day.

Time	Duration in hours	Load in kVA	Power factor
00-06 hrs	6	0	–
06-12 hrs	6	100	0.8
12-17 hrs	5	400	0.8
17-20 hrs	3	300	0.7
20-24 hrs	4	200	0.85

The full load copper loss and iron loss of the transformer are 4 kW and 1.5 kW, respectively. Calculate its all day efficiency.

Solution:

Energy loss in the core for 24 hours = 1.5 × 24 = 36 kWh

$$\text{Energy loss at any load} = \left(\frac{\text{kVA output}}{\text{kVA rated}}\right)^2 \times \text{Full load copper loss.}$$

Energy losses may be calculated as under:

(i) Energy loss for first 6 hours = 6 × 0 = 0 kWh

(ii) Energy loss for next 6 hours $= 6 \times \left(\frac{100}{400}\right)^2 \times 4 = 1.5$ kWh

(iii) Energy loss for next 5 hours $= 5 \times \left(\frac{400}{400}\right)^2 \times 4 = 20$ kWh

(iv) Energy loss for next 3 hours $= 3 \times \left(\frac{300}{400}\right)^2 \times 4 = 6.75$ kWh

(v) Energy loss for next 4 hours $= 4 \times \left(\frac{200}{400}\right)^2 \times 4 = 4$ kWh

∴ Total energy loss in 24 hours = 32.25 kWh

Total energy output in 24 hours

$= 6 \times 0 + 6 \times 100 \times 0.8 + 5 \times 400 \times 0.8 + 3 \times 300 \times 0.7 + 4 \times 200 \times 0.85$

$= 0 + 480 + 1600 + 630 + 680$ kWh

$= 3390$ kWh

Total energy input in 24 hours $= 3390 + 36 + 32.25$

$= 3458.24$ kWh

$$\therefore \text{All day efficiency} = \frac{\text{kWh output in 24 hours}}{\text{kWh input in 24 hours}}$$

$$= \frac{3390}{3458.25} \times 100$$

= 98.03% Ans.

Example 19: The open circuit and short circuit test on a 10 kVA, 500/250 V, 50 cycle, 1 – phase transformer gave the following results:

Open circuit : 500 V, 2 A, 100 W (H.T. side)

Short circuit : 25 V, 20 A, 90 W (H.T. side)

Compute : (a) Components of no-load current.

(b) Approximate equivalent circuit referred to the primary.

(c) (i) Regulation (ii) Efficiency at full load and 0.8 p.f. lagging.

Solution:

(a) From open circuit test.

$$V = 500\ V,\ I_0 = 2\ \text{A},\ W_i = 100\ \text{W}$$

From eqn.
$$\cos \phi_0 = \frac{W_i}{VI_0}$$

$$= \frac{100}{500 \times 2} = 0.1$$

$$\therefore \quad I_c = I_0 \cos \phi_0$$

$$= 2 \times 0.1 = \mathbf{0.2\ A} \quad \textbf{Ans.}$$

$$I_\phi = \sqrt{I_0^2 - I_c^2} = \sqrt{2^2 - 0.2^2} = \mathbf{1.99\ A} \quad \textbf{Ans.}$$

$$R_0 = \frac{V}{I_c} = \frac{500}{0.2} = 2500\ \Omega$$

$$X = \frac{V}{I} = \frac{500}{1.99} = 251\ \Omega$$

Form short circuit test

$$V_{sc} = 25,\ I_{sc} = 20\ \text{A},\ W_c = 90\ \text{W}$$

From eqns, $\cos \phi_{sc}$

$$= \frac{W_c}{V_{sc} I_{sc}} = \frac{90}{25 \times 20} = 0.18$$

$$R_{e1} = \frac{25}{20} \times 0.18 = 0.225$$

$$Z_{e1} = \frac{25}{20} = 1.25$$

$$\therefore \quad X_{e1} = \sqrt{1.25^2 - 0.22^2}$$

$$= 1.23\ \Omega$$

The approximate equivalent circuit referred to the primary is shown in figure.

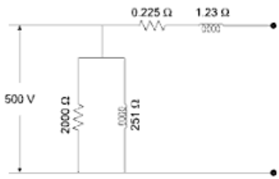


Fig. 5.43

(c) (i) Full load current $I_1 = \frac{10 \times 10^3}{500} = 20\ \text{A},\ \cos \phi_2 = 0.8$

$$\therefore \sin \phi_2 = 0.6$$

% Regulation: $\frac{I_2 R_{e2} \cos\phi_2 + I_2 X_{e2} \sin\phi_2}{E_2} \times 100$

Referred to the primary,

% Regulation

$$= \frac{I_1 R_{e1} \cos\phi_2 + I_1 X_{e1} \sin\phi_2}{E_1} \times 100$$

$$= \frac{20 \times 0.225 \times 0.8 + 20 \times 1.23 \times 0.6}{500} \times 100$$

$$= \mathbf{3.67\%} \quad \textbf{Ans.}$$

(ii) Full load output at 0.8 p.f. = 10 × 0.8 = 8 kW.

Iron loss $W_i = 100$, $W = 0.1$ kW.

Copper loss at full load $W_c = 90$, $W = 0.09$ kW

∴ Total loss = 0.1 + 0.09 = 0.19 kW.

Total input = 8 + 0.19 = 8.19 kW.

$$\therefore \quad \% \eta = \left(1 - \frac{\text{losses}}{\text{input}}\right) \times 100$$

$$= \left(1 - \frac{0.19}{8.19}\right) \times 100$$

$$= \mathbf{97.68\%} \quad \textbf{Ans.}$$

Example 20: A 4 kVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω. Calculate:

(i) The total copper losses on full load.

(ii) The efficiency while supplying full load at 0.9 p.f. lagging.

(iii) The efficiency while supplying half load at 0.8 p.f. leading.

Assume total iron losses equal to 60 W.

Solution: The given values are,

$$V_1 = 200 \text{ V}, \; V_2 = 400 \text{ V}, \; S = 4 \text{ kVA}, \; R_{e1} = 0.15\,\Omega,$$

$$P_i = 60 \text{ W}$$

$$\frac{N_1}{N_2} = \frac{200}{400}$$

$$R_{e2} = \left(\frac{N_1}{N_2}\right)^2 R_{e1}$$

$$= (2)^2 \times 0.15$$

$$= 0.6\,\Omega$$

$$I_1 \text{ rated} = \frac{4\times10^3}{200} = 20 \text{ Amp}$$

$$I_2 \text{ rated} = \frac{4\times10^3}{400} = 10 \text{ Amp}$$

(i) Total copper losses on full load,

$$(P_{cu}) \text{ F.L.} = [(I_2) \text{ F.L.}]^2 R_{e2} = (10)^2 \times 0.6 = 60 \text{ W}$$

(ii) $\cos\phi = 0.9$ lagging and full load

$$\therefore \qquad \% \eta = \frac{VA \text{ rating} \cos\phi}{VA \text{ rating} \cos\phi + P_i + (P_{cu})F.L.} \times 100$$

$$\therefore \qquad \eta = \frac{4\times10^3\times0.9}{4\times10^3\times0.9+60+60} \times 100 = 96.77\%$$

(iii) $\cos\phi = 0.8$ leading half load

As half load, $n = 0.5$

$$(P_{cu}) \text{ H.L.} = n^2 \times (P_{cu}) \text{ F.L.} = (0.5)^2 \times 60 = 15 \text{ W}$$

$$\therefore \qquad \% \eta = \frac{n\times(VA \text{ rating})\cos\phi}{n\times(VA \text{ rating})\cos\phi + P_i + (P_{cu})H.L.} \times 100$$

$$= \frac{0.5\times4\times10^3\times0.8}{0.5\times4\times10^3\times0.8+60+15} \times 100 = \mathbf{95.52\%}$$

Example 21: A 250/125 V, 5 kVA single phase transformer has primary resistance of 0.2 Ω and reactance of 0.75 Ω. The secondary resistance is 0.05 Ω and reactance of 0. 2 Ω.

(i) Determine its regulation while supplying full load on 0.8 leading p.f.

(ii) The secondary terminal voltage on full load and 0.8 leading p.f.

Solution: The given values are,

$$R_1 = 0.2\ \Omega,\ X_1 = 0.75\ \Omega,\ \ R_2 = 0.05\ \Omega,\ X_2 = 0.2\ \Omega,$$
$$\cos\phi = 0.8 \text{ leading}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{125}{250} = \frac{1}{2} = 0.5$$

$$(I_2) \text{ F.L.} = \frac{\text{kVA}}{V_2} = \frac{5\times10^3}{125} = 40 \text{ A} \qquad \text{... full load}$$

$$\therefore \qquad R_2' = R_2\left(\frac{N_1}{N_2}\right)^2 = 0.01\left(\frac{6600}{400}\right)^2 = 2.7225\ \Omega$$

$$\therefore \qquad R_{e1} = R_1 + R_2' = 2.5 + 2.7225 = 5.225\ \Omega$$

It can be observed that primary is high voltage hence high resistance side so while transferring R_2 from low voltage to R_2' on high voltage, its value increases.

To find total equivalent resistance referred to secondary, first calculate R_1'.

$$R_1' = K^2 R_1 = (0.0606)^2 \times 2.5 = 0.00918\ \Omega$$

$$\therefore \qquad R_{e2} = R_2 + R_1' = 0.01 + 0.00918 = 0.01918\ \Omega$$

Example 23: A 33 kVA, 2200/220 V, 50 Hz single phase transformer has the following parameters.

Primary winding (h.v. side): resistance $r_1 = 2.4\ \Omega$, leakage reactance $x_1 = 6.00\ \Omega$.

Secondary winding (l.v. side): resistance $r_2 = 0.03\ \Omega$, leakage reactance $x_2 = 0.07\ \Omega$.

(a) Find the primary resistance and leakage reactance referred to secondary.

(b) Find the secondary resistance and leakage reactance referred to primary.

(c) Find the equivalent resistance and equivalent leakage reactance referred to (i) Primary and (ii) Secondary.

(d) Calculate the total ohmic loss at full load.

(e) Calculate the voltage to be applied to the h.v. side, in order to obtain a short circuit current of 160 A in the l.v. winding. Under these conditions, find the power input also.

Solution:

(a) Primary resistance referred to secondary

$$= r_1' = r_1\left(\frac{N_2}{N_1}\right)^2 = 2.4\left(\frac{220}{2200}\right) = 0.024\ \Omega$$

Primary leakage reactance referred to secondary

$$= x_1' = x_1\left(\frac{N_2}{N_1}\right)^2 = 6.00\left(\frac{220}{2200}\right)^2 = 0.06\ \Omega$$

(b) Secondary resistance referred to primary

$$= r_2' = r_2\left(\frac{N_1}{N_2}\right)^2 = 0.03\left(\frac{2200}{220}\right)^2 = 3.00\ \Omega$$

Secondary leakage reactance referred to primary

$$= x_2' = x_2\left(\frac{N_1}{N_2}\right)^2 = 0.07(10)^2 = 7.00\ \Omega$$

(c) (i) Equivalent resistance referred to primary

$$= r_{e1} = r_1 + r_2' = 2.4 + 3.00 = 5.4\ \Omega$$

Equivalent leakage reactance referred to primary

$$= x_{e1} = x_1 + x_2' = 6.00 + 7.00 = 13\ \Omega$$

(ii) Equivalent resistance referred to secondary

$$= r_{e2} = r_2 + r_1' = 0.03 + 0.024 = 0.054\ \Omega$$

Equivalent leakage reactance referred to secondary

$$= x_{e2} = x_2 + x_1' = 0.07 + 0.06 = 0.13\ \Omega$$

(d) Primary full load current $I_1 = \dfrac{{}^{30}\cancel{33000}}{{}_{2}\cancel{2200}} = 15$ A

Secondary full load current $I_2 = \dfrac{{}^{300}\cancel{33000}}{{}_{2}\cancel{220}} = 150$ A

$\therefore$ ohmic loss at full load $= I_1^2 r_{e1} = (15)^2 \times 5.4 = 1216$ watts.

or, $= I_2^2 r_{e2} = (150)^2 \times 0.054 = 1216$ watts.

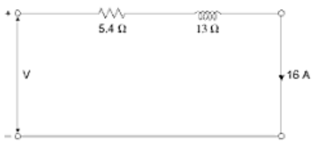


Fig. 5.44

(e) A current of 160 A in the l.v. winding is equivalent to 16 A in the h.v. winding. The equivalent circuit of the transformer, referred to h.v. side is illustrated in Fig. 5.45, from which equivalent leakage impedance referred to h.v. side is $= Z_{e1} = 5.4 + 713$

or, $Z_{e1} = \sqrt{(5.4)^2 + (13)^2} = 14.08\ \Omega$.

∴ The voltage to be applied to the h.v. side. $V = (16)(Z_{e1}) = (16)(14.08)$ = 225.28 volts, power input = $I^2 r_{e1} = (16)^2 \times 5.4 = 1382.4$ watts.

or, power input = $V.I. \cos\theta = (225.28) \times 16 = \dfrac{5.4}{14.08} = 1382.4$ watts.

Example 24: A 10 kVA, 2500/250 V, single phase transformer has resistance and leakage reactance as follows:

$$r_1 = 4.8\ \Omega, \quad r_2 = 0.048\ \Omega.$$
$$x_1 = 11.2\ \Omega, \quad x_2 = 0.112\ \Omega$$

Subscripts 1 and 2 denote high voltage and low voltage windings respectively with primary supply voltage held constant at 2500 V. Calculate the secondary terminal voltage when

(a) The l.v. winding is connected to a load impedance of $5 + j\,3.5\ \Omega$.

(b) The transformer delivers its rated current at 0.8 p.f lagging on the l.v. side.

Solution:

(a) All the quantities may be referred to either the h.v. side or the l.v. side. In this question, i.v. winding is the secondary winding since load is connected across it, with all the quantities referred to the l.v. side, the equivalent circuit of Fig. 5.45(a) is obtained, where

$$r_{e2} = r_2 + r_1\left(\frac{N_2}{N_1}\right)^2 = 0.048 + 4.8\left(\frac{1}{10}\right)^2 = 0.096\ \Omega$$

and, $$x_{e2} = x_2 + x_1\left(\frac{N_2}{N_1}\right)^2 = 0.112 + 11.2\left(\frac{1}{10}\right)^2 = 0.224\ \Omega$$

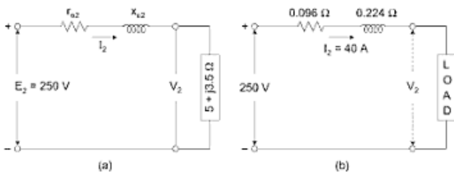


Fig. 5.45

The transformer leakage impedance $r_{e2} + j\,x_{e2}$ and the load impedance are in series. Therefore, the total impedance in the secondary circuit is

$$Z = 5.096 + j\,3.724 = 6.31\ \angle 36.2°$$

Hence, the load current, $I_2 = \dfrac{250}{6.31} = 39.65$ A

The secondary terminal voltage $V_2 = I_2\,Z_L$

$$= (39.65)\left[\sqrt{(5)^2 + (3.5)^2}\right] = (39.65)(6.1) = 242 \text{ V.}$$

(b) The equivalent circuit with all the quantities referred to l.v. side is given in Fig. 5.45(b) The rated current on the l.v. side is

$$I_2 = \frac{10,000}{250} = 40 \text{ A}$$

The phasor diagram for Fig. 5.45(b) is illustrated in Fig. 5.46.

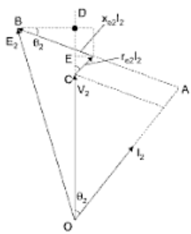


Fig. 5.46

(c) From the geometry of this diagram $OB^2 = OA^2 + AB^2$

or, $$(250) = \sqrt{(V_2\cos\theta_2 + I_2 r_{e2})^2 + (V_2\sin\theta_2 + I_2 x_{e2})^2}$$

$$250 = \sqrt{(0.8V_2 + 3.84)^2 + (0.6V_2 + 8.96)^2}$$

$$(250)^2 = V_2^2 + 16.90\ V_2 + 95.55$$

$$V_2^2 + 16.90\ V_2 - 62,404 = 0$$

$$\therefore \qquad V_2 = \frac{-16.90 + \sqrt{286 + 249{,}616}}{2}$$

$$V_2 = 241.55 \text{ V}$$

Alternatively, the secondary terminal voltage V_2 can be obtained as follows:

From the phasor diagram, it may be seen that OD is approximately equal to $OB = 250$ volts.

$\therefore \qquad V_2 = OD - CD = 250 - CD$

Now, $\qquad CD = CE + ED$

$$= I_2 r_{e2} \cos \theta_2 + I_2 x_{e2} \sin \theta_2$$

$$= (3.84)\,(0.8) + (8.96)\,(0.6) = 8.448 = 8.45 \text{ volts.}$$

$\therefore$ Secondary terminal voltage,

$$V_2 = 250 - 8.45 = 241.55 \text{ V.}$$

The magnitude of the secondary voltage V_2 turns out to be same in both the methods. However, the computational labour in the second method is less than in the first method, therefore, the second method should be preferred.

Example 25: A 15 kVA, 2200/110 V transformer has $R_1 = 1.75\ \Omega$, $R_2 = 0.0045\ \Omega$. The leakage reactances are $X_1 = 2.6\ \Omega$ and $X_2 = 0.0075\ \Omega$. Calculate:

(a) Equivalent resistance referred to primary
(b) Equivalent resistance referred to secondary
(c) Equivalent reactance referred to primary
(d) Equivalent reactance referred to secondary
(e) Equivalent impedance referred to primary
(f) Equivalent impedance referred to secondary
(g) Total copper loss

Solution: The given values are,

$R_1 = 1.75\ \Omega,\ R_2 = 0.0045\ \Omega,\ X_1 = 2.6\ \Omega,\ X_2 = 0.0075\ \Omega$

$$K = \frac{110}{2200} = \frac{1}{20} = 0.05$$

(a) $R_{e1} = R_1 + R'_2 = R_1 + \dfrac{R_2}{K^2} = 1.75 + \dfrac{0.0045}{(0.05)^2} = 3.55\ \Omega$

(b) $R_{e2} = R_2 + R'_1 = R_2 + K^2 R_1 = 0.0045 + (0.05)^2 \times 1.75 = 0.00887\ \Omega$

(c) $X_{e1} = X_1 + X'_2 = X_1 + \dfrac{X_2}{K^2} = 2.6 + \dfrac{0.0075}{(0.05)^2} = 5.6\ \Omega$

(d) $X_{e1} = X_2 + X'_1 = X_2 + K^2X_1 = 0.0075 + (0.05)^2 \times 2.6 = 0.014\ \Omega$

(e) $|Z_{e1}| = R_{e1} + jX_{e1} = 3.55 + j\ 5.6\ \Omega$

$$\therefore \qquad |Z_{e1}| = \sqrt{3.55^2 + 5.6^2} = 6.6304\ \Omega$$

(f) $Z_{e2} = R_{e2} + j\ X_{e2} = 0.00887 + j\ 0.014\ \Omega$

$$\therefore \qquad |Z_{e2}| = \sqrt{(0.00887)^2 + (0.14)^2} = 0.01657\ \Omega$$

(g) To find full load copper loss, calculate full load current.

$$(I_1)\text{ F.L.} = \frac{\text{kVA} \times 1000}{V_1} = \frac{25 \times 1000}{2200} = 11.3636\text{ A}$$

$\therefore$ total copper loss $= [(I_1)\text{ F.L.}]^2\ R_{e1} = (11.3636)^2 \times 3.55 = 458.419$ W

This can be cross-checked as,

$$(I_2)\text{ F.L.} = \frac{\text{kVA} \times 1000}{V_2} = \frac{25 \times 1000}{110} = 227.272\text{ A}$$

$$\begin{aligned} \therefore \text{ total copper loss} &= I_1^2\ R_1 + I_2^2\ R_2 \\ &= (11.3636)^2 \times 1.75 + (227.272)^2 \times 0.0045 \\ &= 225.98 + 232.4365 = 458.419\text{ W} \end{aligned}$$

This is same as calculated above.

Example 26: The open circuit and short circuit test on a 10 kVA, 500/250 V, 50 cycle, 1 – phase transformer gave the following results:

Open circuit : 500 V, 2 A, 100 W (H.T. side)

Short circuit : 25 V, 20 A, 90 W (H.T. side)

Compute : (a) Components or no-load current.

(b) Approximate equivalent circuit referred to the primary.

(c) (i) Regulation (ii) Efficiency at full load and 0.8 p.f. lagging

Solution:

(a) From open circuit test.

$$V = 500\text{ V},\ I_0 = 2\text{ A},\ W_i = 100\text{ W}$$

$$\cos\phi_0 = \frac{W_i}{VI_0}$$

$$= \frac{100}{500 \times 2} = 0.1$$

$$\therefore \qquad I_c = I_0 \cos \phi_0$$

$$= 2 \times 0.1 = \mathbf{0.2\ A} \quad \textbf{Ans.}$$

$$I_\phi = \sqrt{I_0^2 - I_c^2} = \sqrt{2^2 - 0.2^2} = \mathbf{1.99\ A} \quad \textbf{Ans.}$$

(b) From eqns (5.50) and (5.51)

$$R_c = \frac{V}{I_c} = \frac{500}{0.2} = 2500\ \Omega$$

$$X_\phi = \frac{V}{I_\phi} = \frac{500}{1.99} = 251\ \Omega$$

From short circuit test.

$$V_{sc} = 25,\ I_1 = 20\ \text{A},\ W_c = 90\ \text{W}$$

$$\cos \phi_{sc} = \frac{W_c}{V_{sc} I_1} = \frac{90}{25 \times 20} = 0.18$$

$$R_{e1} = \frac{25}{20} \times 0.18 = 0.225$$

$$Z_{e1} = \frac{25}{20} = 1.25$$

$$\therefore \qquad X_{e1} = \sqrt{1.25^2 - 0.22^2}$$

$$= \mathbf{1.23\ \Omega}$$

The approximate equivalent circuit referred to the primary is shown in Fig. 5.47

(i) Full load current $I_1 = \dfrac{10 \times 10^3}{500} = 20$ A, $\cos \phi_2$

$$= 0.8 \quad \therefore \sin \phi_2 = 0.6$$

% Regulation: $\dfrac{I_2 R_{e2} \cos \phi_2 + I_2 X_{e2} \sin \phi_2}{E_2} \times 100$

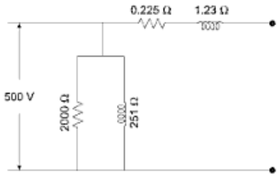


Fig. 5.47

Example 27: A 200 kVA, 1100/400 V, delta-star distribution transformer gave the following test results:

Open circuit test: 400 V, 9 A, 1.50 kW.

Short circuit test: 350 V, rated current, 2.1 kW.

Calculate the equivalent circuit parameters referred to the h.v. side and its efficiency at half full load of unity power factor.

Solution: Problems relating to 3ϕ balanced system are solved by reducing all the quantities to per phase values and so is done here.

Open circuit test: This circuit is performed on the l.v. side, since the applied voltage for this test is equal to the rated voltage on the l.v. side, which is star connected.

∴ Per phase applied voltage $V_1 = \frac{400}{\sqrt{3}} = 231$ V

Per phase exciting current $I_e = 9$ A

Per phase core loss $P_c = \frac{1500}{3} = 500$ W.

Now, $V_1 I_e \cos\theta_0 = P_c$

∴ Core loss current $= I_e \cos\theta_0 = I_c = \frac{P_c}{V_1} = \frac{500}{231} = 2.165$ A

Magnetizing current $I_\phi = \sqrt{I^2 - I_c^2} = \sqrt{9^2 - (2.165)^2} = 8.73$ A

$$R_{cL} = \frac{V_1}{I_c} = \frac{231}{2.165}\ 106.8\ \Omega$$

$$X_{\phi L} = \frac{V_1}{I} = \frac{231}{8.73} = 26.47\ \Omega.$$

But voltage regulation is

$$5 = (r_{e2} \cos \theta_2 + x_{e2} \sin \theta_2) = (1.933 \times 0.8 + x_{e2} \times 0.6)$$

or, $$x_{e2} = 5.756\%$$

$$r_{e2} \text{ in ohms} = \frac{1.933}{100} \times \frac{(1000)^2}{2,00,000} = 0.09665\ \Omega$$

and, $$x_{e2} \text{ in ohms} = 0.2878\ \Omega$$

Core loss resistance, $R_c = \dfrac{(1000)^2}{2319.5} = 431.13\ \Omega.$

Exciting current

$$I_{e2} = \frac{P_c}{V_2 \cos \theta_0} = \frac{2319.5}{1000 \times 0.25} = 9.278 \text{ A}$$

And core loss current, $I_c = \dfrac{2319.5}{1000} = 2.3195$ A

Magnetizing current $I_\phi = \sqrt{9.278^2 - 2.3195^2} = 8.9834$ A

$\therefore$ Magnetizing reactance, $X_\phi = \dfrac{1000}{8.9834} = 111.32\ \Omega$

The approximate equivalent circuit referred to l.v. side is shown in Fig. 5.48.

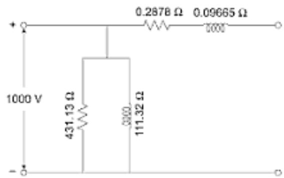


Fig. 5.48

Example 30: A 6600/440 V, single phase transformer has an equivalent resistance of 0.02 p.u. and an equivalent reactance of 0.05 p.u. Find the full-load voltage regulation at 0.8 p.f. log, if the primary voltage is 6600 V. Find also the secondary terminal voltage at full load.

Solution: P.U. voltage regulation

$$= \varepsilon_r \cos\theta_2 + \varepsilon_x \sin\theta_2$$
$$= (0.02)(0.8) + (0.05)(0.6) = 0.046$$

$$\therefore \quad \frac{E_2 - V_2}{E_2} = 0.046$$

For a primary voltage of 6600 V, the secondary no-load voltage E_2 is 440 V.

∴ The change in the secondary terminal voltage

$$E_2 - V_2 = 440(0.046) = 20.25 \text{ V}.$$

and, secondary terminal voltage

$$V_2 = 440 - 20.25 = 419.75 \text{ V}.$$

Example 31: A single phase transformer has 400 primary and 1,000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding is to be connected to a 50 c/s supply at 500 V, calculate the value of maximum flux density in the core and the emf. induced in the secondary winding.

Solution: The number of secondary turns, $N_2 = 1000$

The number of primary turns, $N_1 = 400$

$$V_1 = 500 \text{ V}$$
$$V_1 = E_1 = 4.44\, \phi_m N_1 f \text{ volts}$$
$$500 = 4.44 \times \phi_m \times 400 \times 50$$
$$\therefore \quad \phi_m = 0.563 \times 10^{-3} \text{ Wb}$$

Cross-section of the core = 60 cm^2

$$\therefore \text{ The flux density (max)} = B_m = \frac{\phi_m}{a} = \frac{0.563 \times 10^{-2} \times 10^{-8}}{60}$$
$$= 9{,}383 \text{ lines/cm}^2$$

$$\text{Voltage per turn} = \frac{500}{400} = 1.25$$

$$\therefore \quad \text{Secondary voltage} = 1000 \times 1.25 = 1250 \text{ V}.$$

Example 32: A 50 kVA, 4400/220 V transformer has $r_1 = 3.45\ \Omega$, $r_2 = 0.009\ \Omega$. The values of reactances are $x_1 = 5.2\ \Omega$ and $x_2 = 0.015\ \Omega$. Calculate for the transformer.

(i) Equivalent resistance as referred to primary
(ii) Equivalent resistance as referred to secondary
(iii) Equivalent reactance to both primary and secondary

(iv) Equivalent reactance to both primary and secondary

(v) Total copper loss.

Solution:

Primary full-load current $I_1 = \dfrac{50{,}000}{4{,}400} = 11.36$ A [assuming 100% efficiency]

Secondary full-load current $I_2 = \dfrac{50{,}000}{220} = 227$ A

Turns ratio, $K = \dfrac{220}{4{,}400} = \dfrac{1}{20}$

(i) $R_{e1} = r_1 + \dfrac{r_2}{K^2} = 3.45 + \dfrac{0.009}{\left(\dfrac{1}{20}\right)^2} = 7.05\ \Omega$

(ii) $$R_{e2} = r_2 + k^2 r_1 = 0.009 + \left(\frac{1}{20}\right)^2 \times 3.45 = 0.0176\ \Omega$$

Also, $$R_{e2} = K^2 R_1 = \left(\frac{1}{20}\right)^2 \times 7.05 = 0.0176\ \Omega \text{ (check)}$$

(iii) $$X_{e1} = x_1 + \frac{x_2}{K^2} = 5.2 + \frac{0.015}{\left(\frac{1}{20}\right)^2} = 11.2\ \Omega$$

$$X_{e2} = x_2 + k^2 x_1 = 0.015 + \frac{5.2}{20^2} = 0.028\ \Omega$$

Also $$X_{e2} = K^2 X_1 = \frac{11.2}{400} = 0.028\ \Omega \text{ (check)}$$

(iv) $$Z_{e1} = \sqrt{R_1^2 + X_1^2} = \sqrt{(7.05)^2 + (11.2)^2} = 13.23\ \Omega$$

$$Z_{e2} = \sqrt{R_2^2 + X_2^2} = \sqrt{(0.0176)^2 + (0.028)^2} = 0.0331\ \Omega$$

Also, $Z_{e2} = K^2 Z_1 = 13.23/400 = 0.0331\ \Omega$ (check)

(v) Total copper loss $= I_1^2 r_1 + I_2^2 r_2$

$= (11.36)^2 \times 3.45 + (227)^2 \times 0.009 = 909$ W

Also copper loss = $I_1^2 R_{e1} = (11.36)^2 \times 7.05 = 910$ W

$= I_2^2 R_{e2} = (227)^2 \times 0.0176 = 910$ W

Example 33: A 5 KVA, 400/200 V, 50 c/s 1-phase transformer gave the following results:

No-load : 400 V, 1 A, 50 W (H.V. side)

Short circuit : 12 V, 10 A, 40 W (H.V. side)

Calculate (i) The components of the no-load current (ii) The efficiency and regulations at full-load and factor of 0.8 lagging.

Solution:

(i) $I_o \cos \phi_o = I_c = \frac{W_o}{V_o} = \frac{50}{400} = 0.125$ A

$I_o = 1$ A (given)

I_ϕ the magnetizing component = $\sqrt{1^2 - (0.125)^2} = 0.992$ A

Hence, the core loss component of current, $I_c = 0.125$ A and the magnetizing component of current $I_\phi = 0.992$ A.

(ii) The measurements are made on the primary side again during the short circuit test:

$$Z_{e1} = \frac{12}{10} = 1.2\ \Omega;\ R_{e1} = \frac{40}{10^2} = 0.4\ \Omega$$

$$\therefore \quad X_{e1} = \sqrt{(1.2)^2 - (0.4)^2} = 1.13\ \Omega$$

The full-load current on the primary side,

$$I_1 = \frac{5000}{400} = 12.5 \text{ A}$$

Hence, at full-load and 0.8 lagging power factor,

$$\%\ \text{Regulation} = \frac{I_1(R_1 \cos\theta + X_1 \sin\theta)}{V_1} \times 100$$

$$= \frac{12.5(0.4 \times 0.8 + 1.13 \times 0.6)}{400} \times 100 = \frac{12.5 \times 100}{400} = \mathbf{3.13\%}$$

Example 34: A 50 kVA transformer has 5:1 ratio of turns. The secondary full-load current is 200 A. The primary and secondary resistances are respectively 0.55 Ω and 0.023 Ω. If the transformer is designed for maximum efficiency at 2/3 of full-load, find its efficiency when delivering full-load at 0.8 power factor.

Total energy input = 1880 + 38.4 + 24.7 = 1943.1 kWh

$$\text{The all-day efficiency} = \frac{\text{Energy output per day in kWh}}{\text{Total energy input in kWh}} = \frac{1880}{1943.1} \times 100$$

$= \mathbf{96.8\%}$

Example 36: A 100 kVA, 50 Hz, 440 V/11,000 V, single phase transformer has an efficiency of 98.5% when supplying full-load current at 0.8 p.f., and an efficiency of 99% when supplying half full-load current at unity p.f. Find the iron losses and the copper losses corresponding to full-load current. At what value of load current will the maximum efficiency be attained? (CSE)

Solution:

Let the copper loss at full-load = W_c kW

and the iron loss at full-load = W_i kW

Then,
$$\frac{100 \times 0.8}{100 \times 0.8 + W_c + W_i} = 0.985 \quad \text{...(1)}$$

and,
$$\frac{50 \times 1}{50 \times 1 + \left(\frac{1}{2}\right) W_c + W_i} = 0.99 \quad \text{...(2)}$$

Rearranging equations (1) and (2), we get

$$0.985\ W_c + 0.985\ W_i = 1.2 \quad \text{...(3)}$$

$$0.99\ W_c + 3.96\ W_i = 2 \quad \text{...(4)}$$

Solving equations (3) and (4) we get,

$$W_c = 0.9510 \text{ kW} = 951 \text{ watts and}$$

$$W_i = 0.2673 \text{ kW} = 267.3 \text{ watts}$$

Let the maximum efficiency occurr at a fraction of 'x' times the full-load.

Then, $(x^2)W_c = W_i$

$$x^2 \times 951 = 267.3$$

$$x^2 = \frac{267.3}{951} = 0.2810$$

$$x = 0.53$$

∴ The maximum efficiency occurred at a load of (0.53 × 100) = 53 kVA

$$\text{The full-load current on the primary side} = \frac{100 \times 1000}{440} = 227 \text{ A}$$

Hence, the current at maximum efficiency = 227 × 0.53 = **120 A.**

Example 37: The maximum efficiency of a 500 kVA, 3,300 V/500 V, 50 Hz single phase transformer is 97 per cent and occurs at 3/4 full-load, unity power factor. If the impedance is 10%, calculate the regulation at full-load, power factor 0.8 lagging. (CSE)

Solution:

The full-load current referred to primary $I_1 = \dfrac{500 \times 1000}{3,300} = 151.5$ A

The efficiency $= \dfrac{500 \times 0.75 \times 1}{500 \times 0.75 \times 1 + (P_c + P_i)} = \dfrac{375}{375 + (P_c + P_i)} = 0.97$

Solving the above we get,

$$P_c + P_i = 11.598 \text{ kW}$$

The efficiency is maximum if $P_c = P_i$ and hence $P_c = P_i$

$$= \frac{11.598}{2}\ 5.799 \text{ kW}$$

$$\therefore \quad P_c \text{ for full-load} = \frac{1}{(3/4)^2} \times 5799 = 10309 \text{ W}$$

$$\therefore \quad \text{The value of } R_1 = \frac{10309}{(151.5)^2} = \frac{10309}{22952.25} = 0.449\ \Omega$$

Given that the percentage impedance is 10%.

Hence, $$Z_1 = \frac{3,300 \times 0.1}{151.5} = 2.178\ \Omega$$

$$\therefore \quad X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{(2.178)^2 - (0.449)^2}$$

$$= \sqrt{4.7436 - 0.2015} = \sqrt{4.5421}$$

$$= 2.13\ \Omega$$

The regulation at full-load 0.8 p.f. lagging is,

$$= \frac{I_1 R_1 \cos\theta + I_1 X_1 \sin\theta}{3,300} \times 100$$

$$= \frac{151.5 \times 0.449 \times 0.8 + 151.5 \times 2.13 \times 0.6}{3,300} \times 100$$

$$= \frac{54.4188 + 193.617}{3,300} \times 100 = \frac{248.0358}{3,300} \times 100 = \mathbf{7.516\%}$$

$$\therefore \qquad \phi \propto \frac{E}{f} \quad \therefore B \propto \frac{E}{f}$$

So, here the frequency is halved, also voltage is halved, so B is constant. We know hysteresis losses $W_h \propto B_{max}^{1.6} f$ and eddy current losses $W_e \propto B_{max}^2 f^2$.

Since, B_{max} is constant here, let us take

$$W_{h1} = A \times f$$
$$W_{e1} = B \times f^2$$

Here, A and B constants.

$$\therefore \qquad A = \frac{W_{h1}}{f_1} = \frac{1\%\ \text{output}_1}{50}$$

$$B = \frac{W_{e1}}{f_1^2} = \frac{1\%\ \text{output}_1}{(50)^2}$$

∴ For 25 Hz,

$$W_h = \frac{1\%\ \text{output}_1}{50} \times 25 = \frac{1\%\ \text{output}_1}{2}$$

$$W_e = \frac{1\%\ \text{output}_1}{(50)^2} \times (25)^2 = \frac{1\%\ \text{output}_1}{4}$$

$$\therefore \qquad W_h = \frac{1\%\ \text{output}_2 \times 2}{2} = 1\%\ \text{output}_2$$

$$W_e = \frac{1\%\ \text{output}_2}{4} \times 2 = \frac{1\%\ \text{output}_2}{2}$$

∴ The percentage of *Cu* losses at 110 V, 25 Hz = 2% output

The percentage of hysteresis losses = 1% output

The percentage of eddy current losses = 0.5% output

Example 40: A transformer on no-load has a core loss of 50 W. It draws a current of 2 A (RMS) and has an induced emf of 220 V (RMS). Determine the no-load power factor, core loss and magnetizing components of the no-load current. Neglect winding resistance and leakage flux. Draw the phasor diagram depicting the phasor relationships of induced emf, flux and current. (GATE)

Solution: On no-load, core loss of transformer = 50 watts

No-load current $I_0 = 2$ A

Induced emf = E_0 = 220 V

Neglecting winding resistance then $V_0 = E_0$

We know from no-load test $W_0 = V_0 \times I_0 \times \cos \phi_0$

$\therefore$ $\cos \phi_0$ = No-load power factor

$$= \frac{W_0}{V_0 I_0} = \frac{50}{2 \times 220} = 0.1137$$

$\therefore$ $\sin \phi_0 = \sin \cos^{-1}(0.1137) = 0.9935$

$\therefore$ Core-loss component $I_c = I_0 \cos \phi_0$

$$= 2 \times 0.1137 = 0.2274 \text{ A}$$

Magnetizing component $I_\phi = I_0 \sin \phi_0$

$$= 2 \times 0.9935 = 1.987 \text{ A}$$

The phasor diagram is shown in Fig. 5.49.

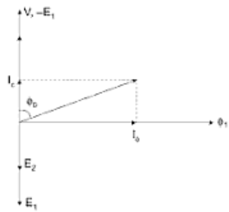


Fig. 5.49

Example 41: A three-phase 550/440 V star-connected auto-transformer supplies a load of 400 kW at 0.8 p.f. lagging. Ignoring magnetizing current, voltage drops and all losses in the transformer, determine:

(a) Current flowing in various sections of the auto-transformer.

(b) Power transferred by transformer action.

(c) Power conducted directly.

Solution:

(a) Let I_1 and I_2 be the currents on primary and secondary sides of transformer, then

$$I_\pi = \frac{\text{kW output}}{\sqrt{3} \times V_2 \times \cos \phi_2} \times 1000$$

Equivalent resistance refer to Primary $R_{e1} = 0.15\ \Omega$, $X_{e1} = 0.37\ \Omega$, core loss resistance $R_c = 600$, magnetizing reactance = 300Ω.

When a transformer is supplying a load of 10 A at a PF of 0.8 lag, calculate primary current and secondary terminal voltage.

[**Ans:** $I_1 = 20.665$ A, 0.75 lag $V_2 = 306.32$ V]

15. A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging PF at which voltage regulation is zero and maximum and the value of this regulation. [**Ans:** 5.585, 0.45 lag]
16. Parameters of a 12300/230 V, 50 Hz transformer are given below:

    $R_1 = 0.286\ \Omega$, $R_2 = 0.319\ \Omega$, $R_0 = 250\ \Omega$

    $X_1 = 0.73\ \Omega$, $X_2 = 0.73\ \Omega$, $X_0 = 1250\ \Omega$

    Secondary load impedance $Z_L = 0.387 + 0.29$. Find:

    1. Primary current
    2. No load current
    3. Efficiency
    4. Voltage regulation

    Normal voltage applied across the primary.
17. A 23 kVA 2300/230 V, 60 Hz step down transformer has the following resistance and leakage reactance values: $R_1 = 4\ \Omega$, $R_2 = 0.04\ \Omega$, $X_1 = 12\ \Omega$, $X_2 = 0.12\ \Omega$, transformer is operating at 75% of its rated road. If the p.f. of the load is 0.866 leading, determine the efficiency of the transformer. [**Ans:** 97.1%]
18. Explain:
    (a) Why the rating of the transformer is in kVA?
    (b) OC and SC test. why these test are generally performed at LV and HV side?
    (c) Losses in a transformer.
    (d) Voltage Regulation at leading PF is negative.
19. Explain with circuit diagram, how open circuit and short circuit tests are performed. Show that the transformer input in open circuit test gives iron losses and the short circuit test copper losses.
20. A 5 kVA, 500/250 V, 50 cycle, 1-phase transformer gave the following reading on high voltage side.

    Open circuit test: 500 V, 1 A , 50 W

    Short circuit test: 15 V, 6 A 21.6 W

    Determine: (a) the efficiency for full-load at 0.8 p.f. lagging (b) the secondary terminal voltage at full-load and 0.8 p.f. lagging and (c) the load for maximum efficiency at unity p.f.

    [**Ans:** (a) 97.3% (b) 240.35 V (c) 9.13 kVA]

21. Describe Sumpner's back to back test. Why this method is preferred most for determining the temperature rise?
23. What are the four common ways of connecting the three-phase transformers. Discuss their relative merits and demerits.
24. What is understood by tertiary winding? State its utility and field of application.

6

Polyphase Circuit

6.1 POLYPHASE CIRCUIT

So far we have discussed single phase circuits. A polyphase circuit is that which has two or more circuits or windings. In a *m*-phase circuit, the electrical displacement between adjacent phases is 360/*m*, except that in case of 2-phase circuits where the phase displacement is 90° electrical.

6.2 GENERATION OF THREE-PHASE VOLTAGES

In AC circuits, 3-phase systems are most common. In Fig. 6.1, a simple 2 pole AC generator is shown. In stator slots 3 coils, *aa*′, *bb*′ and *cc*′ displaced from each other by 120° in clockwise direction, are housed. Let the rotor be revolving in the clockwise direction. When the axis of rotor *N-S* is along *aa*′, the emf induced in this coil is maximum. When the rotor turns by 120°, the *N-S* axis is along *bb*′ and the induced emf is maximum in this coil. Similarly, when the rotor further turns by 120°, the *N-S* axis is along *cc*′ and the induced emf is therefore maximum in this coil.

Hence, three emf's are induced in the three coils *aa*′, *bb*′, *cc*′ which are similar in all respects but displaced in time phase by 120°. If each voltage wave is assumed to be sinusoidal having the maximum value E_m, then the three emf's so generated can be represented in space and time in Fig. 6.2. In practice, in an actual winding each phase consists of a number of coils connected in series.

Let E_a, E_b and E_c be the effective values of voltages induced in the three coils. Since, the emf's are displaced 120°, so they may be represented by the phasor diagram of Fig. 6.3.

The instantaneous values of three emf's may be expressed by:

$$e_a = E_m \sin \omega t = E_m \angle 0 = E_m$$

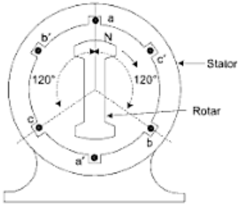


Fig. 6.1

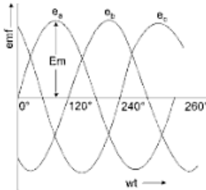


Fig. 6.2

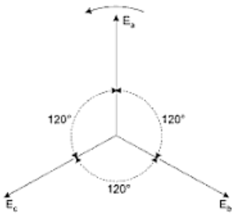


Fig. 6.3

$$e_b = E_m \sin(\omega t\text{-}120°) = E_m \angle -120° = E_m\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$e_c = E_m \sin(\omega t\text{-}240°) = E_m \angle -240° = E_m\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

Resultant of these emf's at an any instant

$$= e_a + e_b + e_c \qquad ...(6.1)$$

$$= E_m \sin \omega t + E_m \sin(\omega t\text{-}120°) + E_m \sin(\omega t\text{-}240°)$$

$$= E_m [\sin \omega t + 2 \sin(\omega t\text{-}180) \cos(60°)]$$

$$= E_m\left[\sin\omega t - 2\sin\omega t \times \left(\frac{1}{2}\right)\right]$$

$$= 0 \qquad ...(6.2)$$

Hence, the sum of the three phase emf at any instant in a balanced 3-phase system are zero. The same is also verified by phasor sum of three emf in Fig. 6.3 as well as by wave ferms of Fig. 6.2.

6.3 PHASE SEQUENCE

The order in which the three phases attain their maximum values is known as phase sequence. In Fig. 6.1-6.3, the direction of rotation of field was assumed clockwise. As such the maximum emf of coil *bb′* was 120° behind that of *aa′* and of coil *cc′* 120° that of *bb′* or 240° that of *aa′*. Hence, the phase sequence of the three-phase voltage is E_a-E_b-E_c as indicated by phasor diagram of Fig. 6.3 and so phase sequence is (*a-b-c*).

If the direction of rotation of the field is reversed, the three-phase would attain their maximum voltages in the order E_a-E_b-E_c as shown by phasor diagram of Fig. 6.4.

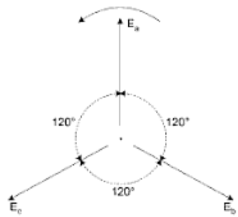


Fig. 6.4

In techanican's language, the three phases are coloured red, yellow and blue and are called by their respective colours.

6.4 ADVANTAGES OF THREE-PHASE SYSTEMS

The factors making three-phase systems most common in practice are summarised below:

(i) Three-phase machines are smaller in size for same output as compared to single phase.

(ii) There is saving in material.

(iii) Efficiency is better.

(iv) P.F. is high.

(v) The power in 3-phase circuit is constant at every instant whereas the same has a constant term superimposed twice frequency pulsating in 1-phase circuits.

(vi) Single phase motors have no starting torque, whereas three-phase motors are self-starting.

(vii) The performance of three-phase machines is much better than those of single phase ones.

(viii) Three-phase system of energy transmission is economical than single phase one.

6.5 STAR AND DELTA CONNECTIONS

The three-phase system may be connected in the following two ways:-

(a) Star (⅄) or Wye.

(b) Delta (Δ) or Mesh.

In Fig. 6.5, three single phase windings are shown. These windings may be connected in either of the two ways shown in Figs. 6.6(a) and 6.7(a). In the former case the connection is ⅄ and in latter Δ. Star and delta connections may also be represented as shown in Figs. 6.6(b) and 6.7(b).

Fig. 6.5

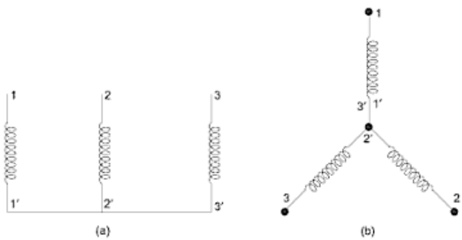
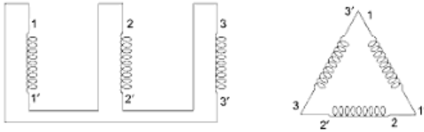


Fig. 6.6 Star Connection



(a) (b)

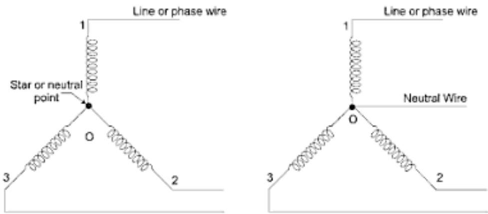
Fig. 6.7 Delta Connection

The common point in star-connections of which either the starting or the finishing ends are joined together is called neutral or star point. There are two ways of connection in star system:

(a) 3-wire system

(b) 4-wire system

The above two systems are shown in Fig. 6.8. In 4-wire system there are 3-phases or line wires and 1 neutral wire.



(a) 3-wire star system (b) 4-wire star system

Fig. 6.8

6.6 VOLTAGE AND CURRENT RELATIONS IN ⅄-(STAR) SYSTEM

In Fig 6.9(a), a 3-phase star circuit is shown. The voltage phasor diagram is shown in Fig. 6.9(b). The phase sequence is abc. The three-phase voltage are

$$E_{a0} = E_{b0} = E_{c0} = E_{ph} \quad \text{(magnitude)}$$

$$\bar{E}_{a0} = E_m \angle 0°$$

$$\bar{E}_{b0} = E_m \angle -120°$$

$$\bar{E}_{c0} = E_m \angle -240°$$

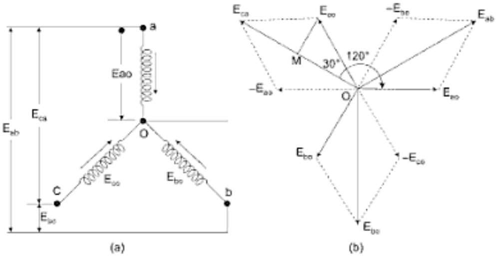


Fig. 6.9

Let E_{ab}, E_{bc} ad E_{ca} be the three line voltages, then

$$\bar{E}_{ab} = \bar{E}_{ao} + \bar{E}_{ob} = \bar{E}_{ao} - \bar{E}_{bo} \qquad ...(6.3)$$

$$\bar{E}_{bc} = \bar{E}_{bo} + \bar{E}_{oc} = \bar{E}_{bo} - \bar{E}_{co} \qquad ...(6.4)$$

$$\bar{E}_{ca} = \bar{E}_{co} + \bar{E}_{oa} = \bar{E}_{co} - \bar{E}_{ao} \qquad ...(6.5)$$

In Fig. 6.9(b) the three phasors E_{ao}, E_{bo} and E_{co} are drawn equal in magnitude and 120° displaced from each other. The line voltage E_{ab} is the vector sum of E_{ao} and $-E_{bo}$. The phasors representing the line voltages E_{ab}, E_{bc} and E_{ca} can therefore be drawn. These phasors are equal in magnitude and are displaced from one another by 120°.

$$\therefore \quad E_{ab} = E_{bc} = E_{ca} = 2OM$$

$$= 2E_{oa} \cos 30°$$

$$= 2E_{ph} . \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}\, E_{ph}$$

Hence, time voltage = $\sqrt{3}$ phase voltage. If line voltage be represented by E, then,

$$E = \sqrt{3}\, E_{ph} \qquad ...(6.6)$$

Mathematically

$$\bar{E}_{ab} = \bar{E}_{ao} + \bar{E}_{ob} = \bar{E}_{ao} - \bar{E}_{bo}$$

$$\bar{E}_{ab} = E_{ph}\angle 0 - E_{ph}\angle -120°$$

$$= E_{ph}\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= E_{ph}\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,E_{ph}\ \angle \tan^{-1}\frac{1}{\sqrt{3}}$$

$$\bar{E}_{ab} = \sqrt{3}\,E_{ph}\angle 30°$$

(Line voltage) = $\sqrt{3}$ time Phase voltage and leads the phase voltage by 30°. The line voltages are also equal in magnitude ($\sqrt{3}\,E_{ph}$) and displaced by 120° w.r.f to each other.

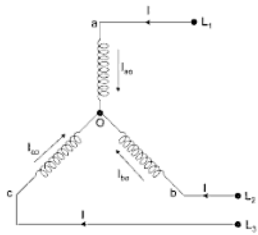


Fig. 6.10

Current Relation:

In a three-phase balanced star connected system, no current flows in the neutral wire. Obviously, the currents in lines and phases are equal. Let I_{ao}, I_{bo} and I_{co} be the three-phase currents, then,

$$I_{ao} = I_{bo} = I_{co} = I_{ph} = I \qquad ...(6.7)$$

where, I is the line current.

Hence, Line current = Phase current

$$I = I_{ph} \qquad ...(6.8)$$

$$\Rightarrow \quad I_{NA} = I_A = I_{ao} \text{ (current in phase A of alternator)}$$
$$I_{NB} = I_B = I_{bo}$$
$$I_{NC} = I_C = I_{co}$$

Thus, the currents flowing through the voltage sources flow through the line conductors and through the load. Hence, the line current is same as phase current both in magnitude and phase.

Line current
$$I_A = \frac{V_{ao}}{Z} = \frac{V_P\angle 0°}{Z\angle\phi} = \frac{V_P}{Z}\angle -\phi$$

$$I_B = \frac{V_{bo}}{Z} = \frac{V_P\angle -120°}{Z\angle\phi} = \frac{V_P}{Z}\angle -120 - \phi$$

$$I_C = \frac{V_{co}}{Z} = \frac{V_P\angle -240°}{Z\angle\phi} = \frac{V_P}{Z}\angle -240 - \phi$$

Magnitude of each line current is V_P/Z and the line currents are displaced by 120° phase angle from each other.

$$I_N = \bar{I}_A + \bar{I}_B + \bar{I}_C$$

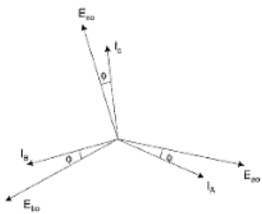


Fig. 6.16 Phasoar diagram

For a balanced load $\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$ so $I_N = 0$

The neutral wire can be omitted without affecting the system.

6.12 BALANCED DELTA-CONNECTED LOAD

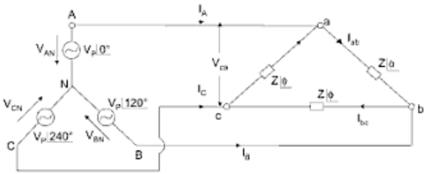


Fig. 6.17

Let the source is star-connected and the load is delta connected.

Source phase voltages are $V_{AN}\angle 0°$, $V_{BN}\ \angle 120°$ and $V_{CN}\angle -240°$, $|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{PH}$.

For delta phase voltage = line voltage = V_{ca}

$$V_{ca} = V_{CN} - V_{AN} = \frac{V_p}{\sqrt{3}}$$

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{V_L\angle 0°}{Z\angle\phi} = \frac{V_L}{Z}\angle -\phi \qquad \{V_L = V_{ab}\}$$

$$I_{bc} = \frac{V_{bc}}{Z} = \frac{V_L\angle -120°}{Z\quad\angle\phi} = \frac{V_L}{Z}\angle -120-\phi$$

$$I_{ca} = \frac{V_{ca}}{Z} = \frac{V_L\angle -240°}{Z\quad\angle\phi} = \frac{V_L}{Z}\angle -240-\phi$$

The line current I_A, I_B and I_C can be found by applying KCL at a, b and c.

$$I_A = I_{ab} - I_{ca}$$

$$I_B = I_{bc} - I_{ab}$$

$$I_C = I_{ca} - I_{bc}$$

In delta connect line current (I_L) is $\sqrt{3}$ times of phase currents and lags the corresponding (reference) phase current by 30°.

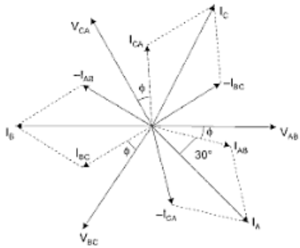


Fig. 6.18

$$\Rightarrow \qquad I_A = \sqrt{3}\, I_{ab} \angle -30° = \sqrt{3}\,.\left(\frac{V_L}{Z}\right)\angle -\phi . \angle -30°$$

$$= \sqrt{3}\,(I_{Ph}) \angle -30° - \phi$$

$$\Rightarrow \qquad I_B = \sqrt{3}\, I_{bc} \angle -30° = \sqrt{3}\, I_{ph} \angle -150° - \phi$$

$$\Rightarrow \qquad I_C = \sqrt{3}\, I_{ca} \angle -30° = \sqrt{3}\, I_{ph} \angle -\phi° - 270°$$

6.13 UNBALANCED DELTA-CONNECTED LOAD

The above delta-load of Fig. 6.19 is unabalanced if $Z_{ab} = Z_1 \angle\phi_1$, $Z_{bc} = Z_2 \angle\phi_2$, $Z_{ca} = Z_3 \angle\phi_3$.

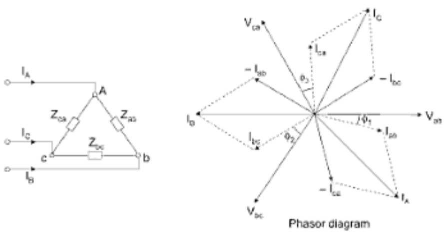


Fig. 6.19

Phase currents

$$\bar{I}_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{V_L\angle 0°}{Z_1\angle\phi_1} = \frac{V_L}{Z_1}\angle-\phi_1 \quad \{V_L = V_{ph} \text{ in delta}\}$$

$$\bar{I}_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{V_L\angle-120°}{Z_2\angle\phi_2} = \frac{V_L}{Z_2}\angle-120-\phi_2$$

$$\bar{I}_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{V_L\angle-240°}{Z_3\angle\phi_3} = \frac{V_L}{Z_3}\angle-240°-\phi_3$$

Line currents

$$\bar{I}_A = \bar{I}_{ab} - \bar{I}_{ca}$$
$$\bar{I}_B = \bar{I}_{bc} - \bar{I}_{ab}$$
$$\bar{I}_C = \bar{I}_{ca} - \bar{I}_{bc}$$

6.14 MEASUREMENT OF POWER IN 3-PHASE CIRCUITS

We can measure the power with the help of wattmeter.

(i) One wattmeter method → For balanced load

(ii) Two wattmeter method → For both balanced and unbalanced load.

(iii) Three wattmeter method → for both balanced and unbalanced load.

6.14.1 One Wattmeter Method

The wattmeter has current coil and pressure coil. Current coil is connected on one phase and the pressure coil is connected between the same phase and star point (neutral). The reading on the wattmeter gives the power per phase and total load power is the 3 times the phase power (Fig. 6.20).

If the neutral point is not available an artificial neutral point has to be created by connecting two resistors or through double throw knife switch.

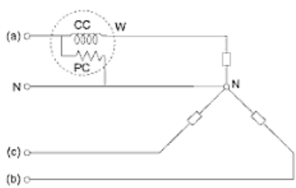


Fig. 6.20 Power measurement when Neutral is available

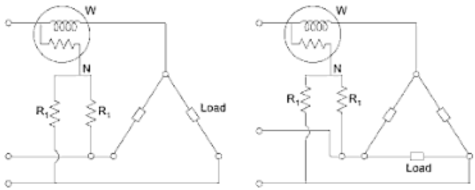


Fig. 6.21 Power measurement when Neutral is not available

6.14.2 Three Wattmeter Method

In this method three wattmeters are connected to each phase, if the neutral is available then the pressure coil terminates to the neutral of the load otherwise they form an artificial neutral as shown in Fig. 6.22.

The arrows in the diagram show the direction of current and voltage and considered positive.

$$\text{Instantaneous power} = e_1 i_1 + e_2 i_2 + e_3 i_3$$

$e_1 i_1 + e_2 i_2 + e_3 i_3$ is the total instantaneous power measured by three wattmeters and the sum of the readings of the wattmeter will give the mean value of the total power.

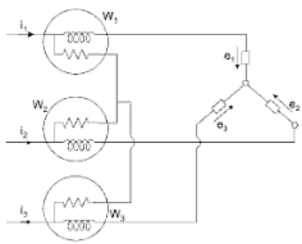


Fig. 6.22

6.14.3 Measurement of Power in 3-phase Load by Two Wattmeter Method

The connections for measurement of power for three-phase load by two wattmeter method are shown in Fig. 6.23.

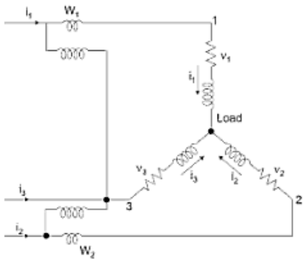


Fig. 6.23

The current coils of the two wattmeters are connected in lines 1 and 2 and their pressure coils across the lines 1 and 3 and 2 & 3, respectively.

Let v_1, v_2 and v_3 be the instantaneous values of voltages and i_1, i_2 and i_3 be the instantaneous values of currents, respectively in the three-phases at any instant. Their directions at that instant are as shown in the figure.

Instantaneous value of power

$$= v_1i_1 + v_2i_2 + v_3i_3 \qquad ...(6.14)$$

By Kirchhoff's first law,

$$i_1 + i_2 + i_3 = 0$$

or,

$$i_3 = -i_1 - i_2 \qquad ...(6.15)$$

$\therefore$

$$\text{Power} = v_1i_1 + v_2i_2 + v_3(-i_1 - i_2)$$

$$= i_1(v_1 - v_3) + i_2(v_2 - v_3)$$

$$= i_1v_{13} + I_2v_{23} \qquad ...(6.16)$$

$$= \text{Reading of wattmeter } W_1 + \text{reading of } W_2 \qquad ...(6.17)$$

So, the total power at any instant is given by the sum of the two wattmeter readings (general statement appreciable for both unbalanced and balanced loads).

The phasor diagram for the load circuit for a balanced load is drawn in Fig. 6.24. Let V_1, V_2 and V_3 be the RMS values of phase voltages and I_1, I_2 and I_3 be line currents. Let ϕ be the lagging power factor angle of the load. Since, we have assumed a balanced load, so I_1, I_2 and I_3 will lag V_1, V_2 and V_3, respectively by an angle ϕ as shown in Fig. 6.24.

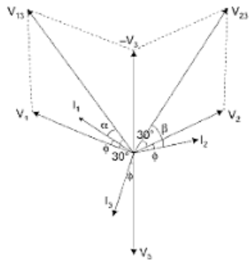


Fig. 6.24

Voltage across $W_1 = V_{13} = V_1 - V_3$

Voltage across $W_2 = V_{23} = V_2 - V_3$

$\therefore$ Reading of $W_1 = V_{13}\, I_1 \cos\alpha$ ...(6.18)

and, Reading of $W_2 = V_{23}\, I_2 \cos\beta$ ...(6.19)

$\angle$ between V_1 and $-V_3$ is equal to 60°

$\therefore$ α = angle between I_1 and V_{13}

$= (30 - \phi)$

β = angle between I_2 and V_{23}

$= 30° + \phi$

If V is the phase voltage and I is the phase current, then,

$$V_1 = V_2 = V_3 = V_{ph}$$

$$I_1 = I_2 = I_3 = I_{ph}$$

and, $$V_{13} = V_{23} = \sqrt{3}\, V_{ph}$$

$$\therefore \quad W_1 = \sqrt{3}\, V_{ph}.I_{ph} \cos(30 - \phi) \quad ...(6.20)$$

$$W_2 = \sqrt{3}\, V_{ph}.I_{ph} \cos(30 + \phi) \quad ...(6.21)$$

Power in the circuit

$$\begin{aligned} P &= W_1 + W_2 \\ &= \sqrt{3}\, V_{ph} I_{ph} [\cos(30 - \phi) + \cos(30 + \phi)] \\ &= \sqrt{3}\, V_{ph} I_{ph} .2 \cos 30° \cos \phi \\ &= 3\, V_{ph} I_{ph} \cos \phi \quad ...(6.22) \end{aligned}$$

This indicates that sum of two wattmeter readings give total 3-phase power.

6.15 POWER FACTOR

From the readings of the two wattmeters, the power factor of the load can also be determined in the following way:

From eqns. (6.20) and (6.21)

$$\begin{aligned} W_1 - W_2 &= \sqrt{3}\, V_{ph} I_{ph} [\cos(30 - \phi) - \cos(30 + \phi)] \\ &= \sqrt{3}\, V_{ph} I_{ph} [2 \sin 30 \sin \phi] \\ &= \sqrt{3}\, V_{ph} I_{ph} \sin \phi \quad ...(6.23) \end{aligned}$$

Dividing values (6.23) by values of (6.22), we obtain

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan \phi$$

$$\therefore \quad \tan \phi = \sqrt{3} . \frac{W_1 - W_2}{W_1 + W_2} \quad ...(6.24)$$

Hence, ϕ and $\cos \phi$ can be determined.

Important Situations:

(i) At $\phi = 0$, $\tan \phi = 0$ or when p.f. = $\cos \phi = 1$ (unity)

From eqn. (6.24)

$$\tan \phi = \sqrt{3} . \frac{W_1 - W_2}{W_1 + W_2} = 0$$

$$\therefore \quad W_1 = W_2$$

Hence, at unity p.f. the two wattmeters will record the same reading.

(ii) At $\phi = 60°$ or $\cos\phi = \frac{1}{2}$

From eqn. (6.20), $W_1 = \sqrt{3}\,VI\cos(30° - 60°)$
$= 1.5\,VI$

From eqn. (6.21), $W_2 = \sqrt{3}\,VI\cos(30° + 60°) = 0$

So, one of the wattmeters reading is zero at 0.5 lagging p.f.

(iii) For $\phi > 60°$

The reading of wattmeter W_2 will be negative, so, the pressure coil or current coil terminals of this wattmeter should be reversed to obtain a forward reading. In this case, the total power in the circuit be equal to $W_1 + (-W_2) = W_1 - W_2$.

(iv) From eqn. (6.17) it is clear that 2 wattmeter method is suitable for measuring power of unbalanced loads also.

Example 1: Three identical impedances, each consisting of a resistance of 15 Ω in series with a capacitive reactance of 10 Ω are connected in star. A 3-phase 400 V, 50 cycle supply is applied to the circuit. Calculate:

(i) Phase voltage.
(ii) Line current.
(iii) Phase current
(iv) P.F. of circuit.
(v) Total power absorbed.
(vi) Total VA and VAR in the circuit.

Solution:

(i) $$\text{Phase voltage} = \frac{\text{Line voltage}}{\sqrt{3}}$$

$$= \frac{400}{\sqrt{3}} = \mathbf{231\ V} \quad \textbf{Ans.}$$

(ii) $$\text{Line current } I = \frac{E_{ph}}{Z_{ph}}$$

$$Z_{ph} = \sqrt{15^2 + 10^2} = 18.02\ \Omega$$

$$\therefore \quad I = \frac{231}{18.02} = \mathbf{12.8\ A} \quad \textbf{Ans.}$$

$$E_{ph} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_{ph} = I = 30 \text{ A}$$

Impedance of each phase,

$$Z_{ph} = \frac{E_{ph}}{I_{ph}} = \frac{231}{30} = 7.7 \ \Omega$$

Power absorbed $P = \sqrt{3}\, V I \cos \phi$

$$\therefore \quad \cos \phi = \frac{P}{\sqrt{3} V I}$$

$$= \frac{12000}{\sqrt{3} \times 400 \times 30} = 0.577$$

From Fig. 6.25

$$\text{Resistance } R_{ph} = Z_{ph} \cos \phi$$

$$= 7.7 \times 0.577 = 4.45 \ \Omega$$

$$\text{Reactance } X_{ph} = \sqrt{Z_{ph}^2 - R_{ph}^2}$$

$$= \sqrt{7.7^2 - 4.45^2} = \mathbf{6.25 \ \Omega} \quad \textbf{Ans.}$$

Example 3: Three similar coils each of resistance 30 Ω and inductance 0.07 H are connected in delta to a 3-phase 400 V, 50 cycle supply. Calculate:

(a) Phase current.

(b) Line current.

(c) Power factor of the circuit.

(d) Power absorbed.

Draw the vector diagram.

(d) Power absorbed = $\sqrt{3}\, EI \cos\phi$

$$= \sqrt{3} \times 400 \times 18.6 \times 0.805$$

$$= \mathbf{10{,}400\ W} \quad \textbf{Ans.}$$

The vector diagram is shown in Fig. 6.27.

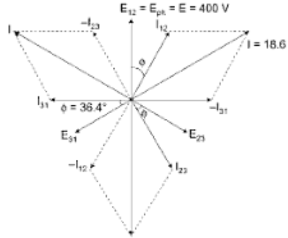


Fig. 6.27

Example 4: A balanced three-phase 866 V, 60 Hz Y connected source feeds a balanced, Δ-connected load having impedance in each phase is $(177 - j246)\ \Omega$ via a 100 km long three wire line. Impedance of each wire of the line is $(1 + j2)\ \Omega$. If the phase sequence is positive, determine the line and the phase currents, power absorbed by the load, and the power dissipated by line.

Solution:

$$V_{\text{line}} = 866 \text{ V}, \quad V_{\text{phase}} = \frac{866}{\sqrt{3}} = 500 \text{ Volt}$$

Assuming phase-a as the reference.

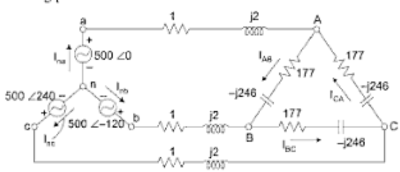


Fig. 6.28

One of the easiest approach is that first convert Δ_{Load} into a star load. $\left(Z_Y = \frac{Z_\Delta}{3}\right)$

$$Z_Y = \frac{177 - j246}{3} = 59 - j82$$

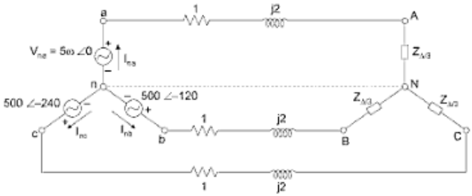


Fig. 6.29

$$\overline{I}_{na} = \frac{500\angle 0°}{1 + j2 + 59 - j82} = 5\angle +53.13° \text{ A}$$

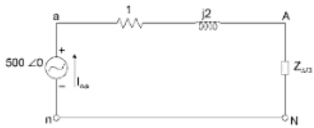


Fig. 6.30 Per phase equivalent CKT

$$\overline{I}_{nb} = 5\angle -120 + 53.13 = 5\angle -66.87° \qquad \{\text{lags a by } 120°\}$$

$$\overline{I}_{nc} = 5\angle -240 + 53.13 = 5\angle -186.87°$$

Now, the $\overline{I}_{na}$, $\overline{I}_{nb}$, $\overline{I}_{nc}$ are the line currents for Δ connection.

$$I_L = \sqrt{3}\, I_{ph} \angle -30° \ \{\text{Line currents lags the phase current by } 30°\}$$

The reading of one wattmeter is 5000 watts and the other reads backwards. On inter-changing the potential terminals the second wattmeter reads 1000 watts. Calculate:

(a) Total power.

(b) P.F. of load.

Solution:

$$W_1 = 5000 \text{ W}$$

Since, the second wattmeter reads backwards, hence its reading is nagative,

$$\therefore \qquad W_2 = -1000 \text{ W}$$

(a) Total power in the circuit

$$= W_1 + W_2$$

$$= 5000 - 1000 = 4000 \text{ W}$$

(b) From equation (6.20) the power factor of load is given by,

$$\tan\phi = \sqrt{3}.\frac{W_1 - W_2}{W_1 + W_2}$$

$$= \sqrt{3}.\frac{5000-(-1000)}{5000-1000}$$

$$= \sqrt{3} \times \frac{6000}{4000} = 2.593$$

$$\therefore \qquad \phi = 68.9^\circ$$

$$\therefore \qquad \cos\phi = \cos 68.9^\circ = \mathbf{0.36} \quad \textbf{Ans.}$$

Example 6: A 3-phase, λ connected 3300 V synchronous alternator supplies power to a 1500 h.p. (metric), *V*-connected induction motor. The efficiency and p.f. of motor are 0.85 and 0.81, respectively. Calculate:

(a) Motor input.

(b) Phase and line current of the alternator.

(c) Phase and line current of the motor.

Solution: The alternator and the induction motor are as connected in Fig. 6.31.

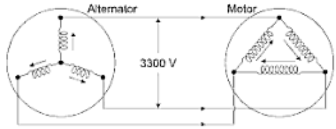


Fig. 6.31

(a) Motor output = 1500 × 735.5 W

$$\therefore \quad \text{Motor input} = \frac{1500 \times 735.5}{0.85} = 1{,}300{,}000 \text{ W}$$

or, Motor input = 1300 **kW** **Ans.**

(b) Now, $\sqrt{3}\, VI \cos\phi = 1{,}300{,}000$

$$\therefore \quad I = \frac{1{,}300{,}000}{\sqrt{3} \times 3300 \times 0.81} = \mathbf{281\ A} \quad \textbf{Ans.}$$

Line, current of alternator = **281 A** **Ans.**

(c) Line current of motor = 281 A

$$\text{Phase current of motor} = \frac{281}{\sqrt{3}}$$

$$= \mathbf{162.4\ A} \quad \textbf{Ans.}$$

Example 7: A 3-phase star-connected system with 200 V between each phase and neutral has non-inductive resistances of 5, 8 and 10 Ω in the 3-phases. Determine the current in each phase and the neutral current. Also calculate the power consumed in each phase and the total power consumed.

Solution:

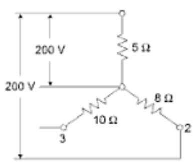


Fig. 6.32

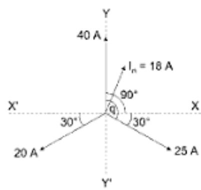


Fig. 6.33

$$E_{ph} = 200 \text{ V}$$

$$\therefore \quad \text{Current in 5 } \Omega \text{ resistor} = \frac{200}{5} = \mathbf{40\ A}$$

$$\text{Current in 8 } \Omega \text{ resistor} = \frac{200}{8} = \mathbf{25\ A}$$

$$\text{Current in 10 } \Omega \text{ resistor} = \frac{200}{10} = \mathbf{20\ A} \quad \textbf{Ans.}$$

Since, the circuit is purely resistive, so currents in respective phases will be in phase with their phase voltages. In a 3-phase circuit phase voltages are displaced 120° from each other and hence the three currents will also be displaced by 120° from each other as shown in phasor diagram of Fig. 6.33. The neutral current I_n is the vector sum of these three currents which could be determined vectorially or by resolving them in two perpendicular coordinate axes.

$$I_n \cos\theta = X\text{-components of the three currents}$$
$$= 40 \cos 90 + 25 \cos 30 - 20 \cos 30°$$
$$= 0 + (25 - 20)\ 0.866 = 4.33 \text{ A}$$
$$I_n \sin\theta = Y\text{-components of the three currents}$$
$$= 40 \sin 90 - 25 \sin 30 - 20 \sin 30°$$
$$= 40 - 22.5 = 17.5 \text{ A}$$

$$I_n = \sqrt{4.33^2 + 17.5^2} = 18 \text{ A}$$

$$\text{Power consumed in first phase} = 200 \times 40 = 8000 \text{ W}$$

$$\text{Power consumed in second phase} = 200 \times 25 = 5000 \text{ W}$$

$$\text{Power consumed in third phase} = 200 \times 20 = 4000 \text{ W}$$

$$\text{Total power consumed} = 8000 \text{ W} + 5000 \text{ W} + 4000 \text{ W} = 17{,}000 \text{ W}$$

Example 8: A balance delta connected load of $(8 + j6)\ \Omega$ per phase is connected to a 3-phase 230 V supply. Find the line current, power factor, power, reactive power, and total volt amperes.

Solution: Phase current in delta

$$I_A = \frac{V_{Pn}}{Z} = \frac{230\angle 0°}{8 + j6} = 30\angle -36.8° \text{ in phase A}$$

$I_A = 23\angle -36.8°$ phase current lags the phase voltage by 36.8°.

Fig. 6.34

So, p.f. = cos 36.8° = 0.8

$$\text{Line current} = \sqrt{3}\, I_{pn} = \sqrt{3}\,.23 = 39.8 \text{ A}$$

$$\text{Power} = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3}\,.230 \times 39.8 \times 0.8 = 12{,}696 \text{ W}$$

$$\text{Reactive Power} = \sqrt{3}\, V_L T_L \sin\phi$$

$$= \sqrt{3} \times 230 \times 39.8 \times 0.6$$

$$= 9.513 \text{ K VAR}$$

$$\text{Total kVA} = \sqrt{3}\, V_L T_L$$

$$= \sqrt{3} \times 230 \times 39.8$$

= 15.855 kVA Ans.

Example 9: A 400 V, 50 Hz 3-phase supply has 100 Ω between AB, j100 Ω between B and C and $-j$100 Ω between CA. Find: (a) Line currents for phase sequence ABC (b) The star-connected balanced resistances for consuming the same power as in delta.

$$P_1 = \text{Power consumed} = \frac{3V_{Ph}^2}{R} = \frac{V_L^2}{R}$$

If one of the resistances is removed

Now power consumed $P_2 = 2.\frac{V_{Pn}^2}{R} = 2.\frac{V_L^2}{4R}$ $\left\{V_{Ph} = \frac{V_L}{2}\right\}$

$$\boxed{P_2 = \frac{V_L^2}{2R}}$$

$$P_I = \frac{V_L^2}{R} \text{ and } P_2 = \frac{V_L^2}{2R} \text{ (after removing a resistor)}$$

Hence, the reduction in load or power is 50%.

(b) For delta-connected load

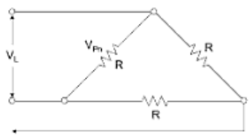


Fig. 6.37

Let the resistance be R

$\{I_L = \sqrt{3}\, I_{ph} \quad V_L = V_{ph}\}$

Power consumed $P_1 = 3V_{ph}\, I_{ph}$ {cos θ = 1 for resistor load}

$$= 3.\frac{V_{Ph}^2}{R} = \frac{3V_L^2}{R}$$

When one resistance is removed

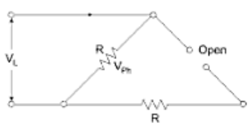


Fig. 6.38

Power consumed $P_2 = V_{ph} I_{ph} = \dfrac{V_{Ph}^2}{R} = \dfrac{V_L^2}{R}$

Net reduction in power $= \dfrac{P_1 - P_2}{P_1} = \dfrac{(3-1)}{2} \times 100 = 66.6\%$

Example 11: A balanced load of 20 kVA is connected to a three-phase three-wire system. Two wattmeters are connected in the usual manner. Determine the readings of the two wattmeters if the power factor of the load is (a) unity (b) 0.866 lagging (c) 0.5 leading (d) Zero lagging. What is the maximum possible reading of either wattmeter?

Solution: The total power consumed by the load = kVA. cos ϕ

$$= 20 \cos \phi \text{ kW}$$

$$P_1 + P_2 = 20 \cos \phi$$

$$\tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$$

$$\tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{20 \cos \phi}$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin \phi$$

(a) When $\cos \phi = 1, \phi = 0, \sin \phi = 0$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin \phi = 0$$

$$P_1 = P_2 \text{ and } P_1 + P_2 = 20 \cos \phi = 20$$

$$P_1 = P_2 = 10 \text{ kW}$$

(b) $\cos \phi = 0.866$ (lagging), $\phi = 30$ lag

$$P_1 + P_2 = 20 \cos \phi = 20 \times 0.866 = 17.32$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin \phi = \frac{20}{\sqrt{3}} \times \sin 30 = 5.773$$

$$P_1 = 11.546 \text{ kW}, \quad P_2 = 5.773 \text{ kW}$$

(c) $\cos\phi = 0.5$ leading ($\phi = 60$ leading)

$$P_1 + P_2 = 20\cos\phi = 20 \times 0.5 = 10 \text{ kW}$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}}\sin\phi = \frac{20}{3}\sin(-60) = -10 \text{ kW} \qquad \text{(for leading)}$$

$$P_1 = 0, \quad P_2 = 10 \text{ kW}$$

(d) For zero lagging ($\phi = 90°$) $\cos\phi = 0$, $\sin\phi = 1$

$$P_1 + P_2 = 20\cos\phi = 0$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}}\sin\phi = 11.547$$

$$P_1 = 5.773 \text{ kW}, \quad P_2 = -5.7736 \text{ kW} \qquad \{P_1 = -P_2\}$$

The maximum reading on either wattmeter occurs when the current through the current coil of the wattmeter is in phase with voltage across the voltage coil. This occurs for W_1 at a phase angle of 30° lagging and for W_2 at a phase angle of 30° leading in either case the maximum reading is 11.546 kW as in (b).

Example 12: Three identical impedances each having a resistance *R* and a capacitance *C* are connected in star across a 3-phase 400 V, 50 Hz supply. The power input to the load is measured by the 2-wattmeter method. The readings of the two wattmeters are 800 W and 2000 W. Determine:

(a) The power factor of the circuit.

(b) The line current and

(c) The resistance and capacitance of each phase.

Solution: Since, each phase has a resistance and capacitance, the power factor will be leading,

$$\text{For leading power factor } \tan\phi = \sqrt{3}\,\frac{\text{(greater – smaller) reading}}{\text{sum of two reading}}$$

$$\tan\phi = \sqrt{3}\,\frac{(2000 - 800)}{2000 + 800}$$

$$= 0.723$$

therefore $\phi = 36.59°$

(a) $-\cos\phi = 0.8029$ (leading)

$$P_1 + P_2 = 800 + 2000 = 2800 \text{ Watt}$$

$$P_{\text{total}} = \sqrt{3}\, V_L I_L \cos\phi$$

(b)
$$I_p = I_L = \frac{P_{total}}{\sqrt{3}\,V_L}\cos\phi$$
$$= \frac{2800}{\sqrt{3}} \times 400 \times 0.8029$$
$$= 5.03 \text{ Amp}$$

(c)
$$Z_p = \frac{V_P}{I_P}$$
$$= \frac{V_L}{\sqrt{3}\,I_p}$$
$$= \frac{400}{3 \times 5.03}$$
$$= 45.886\ \Omega$$
$$R_p = Z_p \cos\phi = 45.886 \times .8029 = 36.84\ \Omega$$
$$1/\omega c = X_C = \sqrt{Z_p^2} - R_p^2$$
$$= \sqrt{(45.886)^2} - (36.84)^2$$
$$= 27.35\ \Omega$$
$$C = \frac{1}{27.35 \times 314}$$
$$= 116.38 \times 10^{-6} \text{ Farad}$$

EXERCISE

1. Derive the relation between voltages (line voltage and phase voltage) and currents (line current and phase current) with the help of suitable vector diagram for a balanced
   (i) Star-connection
   (ii) Delta-connection
2. What are the advantages of star and delta systems?
3. Two wattmeters are used to measure power in 3-phase balance load, prove that the total power measured is $\sqrt{3}\,V_L I_L \cos\phi$ for
   (i) Star-connection (leading load power factor cos ϕ)
   (ii) Delta-connection (lagging load power factor cos ϕ)
   Explain with the help of suitable vector diagram.

4. Three identical coils, each having a resistance of 10 Ω and reactance of 10 Ω are connected in (i) Star (ii) Delta, across 400 V, 3-phase supply. Find in each case the line current and the readings on each of the two wattmeters connected to measure a power.
   (i) $I_L = 16.33$ A, $W_1 = 1690.5$ W, $W_2 = 6309.5$ W
   (ii) $I_L = 48.98$ A, $W_1 = 5070.5$ W, $W_2 = 18924.5$ W
5. Three 40 Ω non-inductive resistances are connected (i) Delta (ii) Star, across 400 V, 3-phase lines. Calculate the power taken from the mains for both the cases. If one of the resistances is disconnected what would be the power taken from the mains? [**Ans:** Delta 1200 W, 8000 W]
6. A balanced three-phase, 866 V, 60 HZ, star-connected source feeds a balanced delta connected load via a long three-wire line. The impedance of each wire of the line is $(i + j2)$ Ω. The per phase impedance of the load is $(177 - j246)$ Ω. If the phase sequence is positive, determine the line and phase currents, power absorbed by the load and power dissipated by line.

   [**Ans:** Line currents as $5\angle 53.13°$ A, $5\angle -66.87°$ A, $5\angle 173.13°$ A]

   Phase currents: $2.887\angle 83.13°$, $2.887\angle -36.87°$

   $P_{\text{load}} = 4425.75$ Watt, $P_{\text{line}} = 75$ Watt
7. Prove that the power in a 3-phase circuit is $\sqrt{3}\,EI\cos\phi$, where E and I are line voltage and line current, respectively irrespective of the fact whether the circuit is star or delta connected.
8. A 3-phase, 400 V supply is given to an induction motor. The motor draws a line current of 20 A. The efficiency and operating power factor of the motor are 0.85 and 0.87, respectively. Estimate the power output of the motor. [**Ans:** 13.9 h.p metric]
9. Three impedances, each of $(5 - j8.66)$ Ω are connected in (a) Star, (b) Delta across a 3-phase $200\sqrt{3}$ V supply. In each case calculate the phase and the line currents and draw the vector diagrams showing these currents in relation to supply voltages. Draw vector diagrams.

   [**Ans:** (a) $I_{ph} = I = 20$ A, (b) $I_{ph} = 20\sqrt{3}$ and $I = 60$ A]
10. A 3-phase star-connected system with 230 V between each phase and the neutral has resistances of 4,5 and 6 Ω, respectively in three phases. Determine the currents flowing in each phase and the neutral current. Find the total power absorbed and draw the complete vector diagram.

    [**Ans:** 57.5 A, 46 A, 38.3 A, 16.7 A, 32.62 kW]

7

Direct-Current Machines

7.1 INTRODUCTION

Electromechanical energy conversion device is that device which converts electrical energy into mechanical energy or mechanical energy into electrical energy. The structure of these devices may vary depending upon the function they perform. These devices may be categorised as under:

(i) Devices used for continuous energy conversion like motors and generators.

(ii) Devices used to produce translational forces or torques with limited mechanical motion like electromagnets, relays, moving iron and moving-coil instruments.

(iii) Devices involving small motion, producing low-energy signals, like telephone receivers, loudspeakers, microphones, gramophones and low signal transducers.

Coupling between the electrical energy and mechanical energy is through magnetic field or electric field. The energy storing capacity of magnetic field is much greater than that of electric field.

7.2 PRINCIPLE OF ENERGY CONVERSION

It is based on law of energy conservation i.e. the energy can neither be destroyed nor be created, it can only be converted from one form to another form. In electromechanical energy conversion device, energy balance equations are:

Total electrical input energy (Pei)

$$= \text{Mechanical energy output } (P_{mo}) + \text{Stored energy } (P_{ms} + P_{es})$$

$$+ \text{ Energy dissipated or losses.} \quad ...(1)$$

Total mechanical input energy = Output electrical energy + Stored energy
+ Energy dissipated ...(2)

Stored energy = Energy stored in magnetic field (P_{es})
+ Energy stored in mechanical system (P_{ms}).

Dissipated energy = Ohmic losses
+ Core losses (hysteresis + eddy current loss)
+ Energy dissipated in mechanical system
(friction and windage loss).

- losses like ohmic (I^2R) losses, coupling field and friction and windage losses are irreversible and these are therefore dissipated as heat.
- magnetic field stored energy in single-phase AC machines does not remain constant because these machines do not have constant air-gap flux.

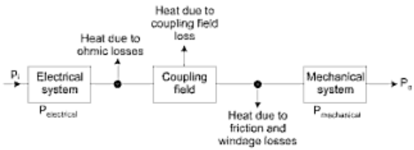


Fig. 7.1 Electromechanical energy conversion device

$$P_i = P_o + P_{\text{field}} + P_{\text{heat}}$$

$$P_i = P_o + P_{\text{field}} + [P_{lf\,(\text{field loss})} + P_{lm\,(\text{mechanical loss})} + P_{le\,(\text{electrical loss})}]$$

$$(P_i - P_{le}) = (P_o + P_{lm}) + (P_{\text{field}} + P_{lf})$$

$$\boxed{\begin{aligned} P_{\text{electrical}} &= P_{\text{mechanical}} + P_{\text{field}} \\ P_e &= P_m + P_f \end{aligned}}$$

If dP_e electrical energy flows in dt time interval to the coupling field, then dP_f be the energy supplied to field and dP_m be the energy converted to mechanical.

$$\boxed{dP_e = dP_m + dP_f}$$

$$dP_{\text{elec}} = dP_{\text{electrical input}} - dP_{\text{electrical losses (ohmic loss)}}$$
$$= V_t\, idt - i^2 r\, dt$$
$$= (V_t - ir)\, idt$$

$$\boxed{dP_{\text{ele}} = e\,idt}$$
$$\boxed{ei\,dt = dP_{\text{mech}} + dP_{\text{field}}}$$

DC MACHINE

7.3 INTRODUCTION

The word machine is commonly used to explain features that are common to both the generator and motor.

In the rotating electrical machines an electromechanical energy conversion takes place. *There are two types of rotating electric energy conversion machines.*

(i) Direct current (DC) machine
(ii) Alternating current (AC) machine

- All DC machines can be operated as either a motor or a generator without making any modifications.
- A rotating machine converting electrical energy into mechanical energy, is called *motor.*
- A rotating machine converting mechanical energy into electrical energy, is called *generator.*

7.4 GENERAL CONSTRUCTION OF ROTATING MACHINE

Whenever there is a relative motion between a conductor and the flux, the emf is induced in the conductor governed by FRH Rule.

Whenever a current carrying conductor is placed in a magnetic field it experiences a mechanical force governed by FLH Rule. Hence, every rotating machine must possess the following parts.

(i) *Stator*: a stationary member
(ii) *Rotor*: a rotating member
(iii) *Shafts*
(iv) Slip ring, brush assembly
(v) Bearing
(vi) *Field winding or exciting winding:* An arrangement of winding which is used as a primary source of flux when current is passed through it. It produces a working flux. Current in this winding is always DC.

(vii) *Armature winding*: An arrangement of conductors in a particular fashion is called armature winding. It works with the working flux produced by field winding and emf is induced in this winding. It always deals with AC.

The following general terms are used in connection with the winding.

(i) *Conductor:* Each individual length of wire lying within the magnetic field is called conductor and it may be made of one, two or more parallel strands.

(ii) *Turn:* A turn consists of two conductors connected in series by end connections.

(iii) *Coil:* A coil consists of one or more turns connected in series.

(iv) *Coil group:* A coil group may have one or more single coils.

(v) *Winding:* It consists of a number of coils arranged in coil groups.

(vi) *Pole pitch:* The number of coil sides per pole is called pole pitch. If Z' is the number of coil sides and p the number of poles, then pole pitch = $\frac{Z'}{P}$.

(vii) *Front pitch:* The distance in terms of number of coil sides spanned by a coil on the commutator end of the armature is called front pitch and is denoted by y_f.

(viii) *Back pitch:* The distance in terms of number of coil sides between the two sides of the coil on the back of the armature is called back pitch and is denoted by y_b. It is also called coil spread or coil span.

(ix) *Resultant pitch:* The distance in terms of number of coil sides between the beginning of one coil and the beginning of the next coil to which it is connected is called resultant pitch and is denoted by y_r.

(x) *Average pitch:* It is denoted by 'y' and is expressed as:

$$y = \frac{y_b + y_f}{2}$$

(xi) *Commutator pitch:* It is the distance measured in terms of commutator segments to which the two ends of a coil are connected and is denoted by y_c.

- Current flowing through the field winding to produce main flux is called magnetizing, exciting or field current.
- Current flowing, through the armature winding varies as the load on the machine (mechanical or electrical) varies. So it is called load current or armature current.

- Generally, it is required to feed in or take out power from the rotor. Thus, a communication between a rotor and the stationary outside device is achieved by using slip rings and brush assembly.
- Slip rings are connected to the rotor winding and rotate along with rotor and behave as a rotating winding terminals.
- Brushes are restings against slip rings and stationary terminal of rotating winding.

7.5 CONSTRUCTION OF A DC MACHINE

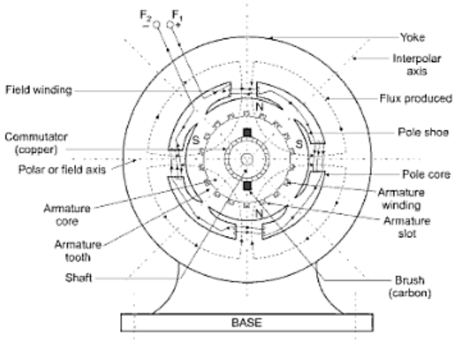


Fig. 7.2 Cross-section of typical DC machine

A DC machine consists of the following parts:

7.5.1 Yoke

(a) Functions:

1. It serves the purpose of outermost cover of the DC machine, so that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes, etc.
2. It provides mechanical support to the poles.

3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

7.5.2 Poles

Each pole is divided into two parts namely, (1) Pole core and (2) Pole shoe. This is shown in Fig. 7.3.

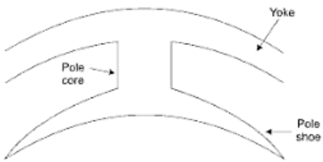


Fig. 7.3 Pole structure

(a) Functions of pole core and pole shoe:

1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core, to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, producing better induced emf shape. To achieve this, pole shoe has been given a particular shape.

(b) Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

7.5.3 Field Winding (F1 – F2)

The field winding is wound on the pole core in a definite direction.

(a) Functions: Field winding carry a D.C. current due to which a stationary poleies and necessary flux are produced.

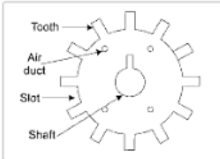


Fig. 7.4 Single circular lamination of armature core

2. Armature winding carry the supplied current by source in case of motor and drawn out current in case of generator.
3. To do the useful work in the external circuit.

(b) Choice of material: As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Armature winding is generally former wound. The conductors are placed in the armature slots which are lined with tough insulating material.

7.5.5 Commutator

We have seen earlier that the basic nature of emf induced in the armature conductors is alternating. This needs rectification in case of DC generator, which is possible by a device called commutator.

(a) Functions:

1. To facilitate the collection of current from the armature conductors.
2. To change internally developed alternating emf to unidirectional (DC) emf.
3. To produce unidirectional torque in case of motors.

(b) Choice of material: As it collects currents from armature, it is also made up of copper segments.

It is cylindrical in shape and is made up of wedge-shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layers of mica. Each commutator segment is connected to the armature conductor by means of copper lug or strip. This connection is shown in Fig. 7.5.

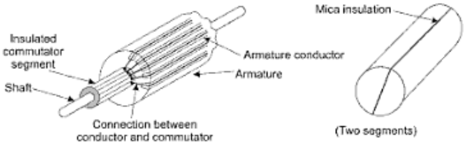


Fig. 7.5 Commutator

7.5.6 Brushes and Brush Gear

Brushes are stationary and resting on the surface of the commutator.

(a) Function: To collect current from commutator and make it available to the stationary external circuit.

(b) Choice of material: Brushes are normally made up of soft materials like carbon.

Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of a lever. A flexible copper conductor called pig tail is used to connect the brush to the external circuit. To avoid wear and tear of the commutator, the brushes are made up of soft materials like carbon.

7.5.7 Bearings

Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

7.6 TYPES OF ARMATURE WINDING

We have seen that there are number of armature conductors, which are connected in specific manner as per the requirement, which is called armature winding. According to the way of connecting the conductors, armature winding has basically two types, namely, a) Lap winding b) Wave winding.

7.6.1 Lap Winding

In this case, if connection is started from conductor in slot 1, then connections overlap each other as winding proceeds, till starting point is reached again.

Developed view of part of the armature winding in lap fashion is shown in Fig. 7.6.

As seen from Fig. 7.6, there is overlapping of coils while proceeding.

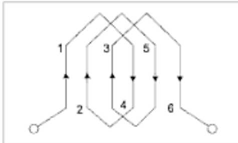


Fig. 7.6 Lap winding

Note: *In lap winding, the total number of conductors divided into 'A' number of parallel paths and therefore A = number of poles in the machine.*

Large number of parallel paths indicate high current capacity of machine hence lap winding is preferred for high current rating machines.

7.6.2 Wave Winding

In this connection, winding always travels ahead avoiding overlapping. It travels like a progressive wave, hence called wave winding.

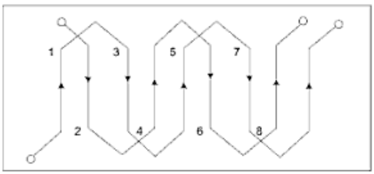


Fig. 7.7 Wave winding

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Note: *In this type of connection, the total number of conductors are always divided in two parallel paths, irrespective of number of poles of the machine.* $A = 2$. *It is preferable for low current, high voltage capacity machines.*

The number of parallel paths in which armature conductors are divided in lap or wave fashion of connection is denoted as A. So $A = P$ for lap connection and $A = 2$ for wave connection.

7.6.3 Comparison of Lap and Wave Type Winding

Lap winding	Wave winding
(1) Number of parallel paths (A) = poles (P)	(1) Number of parallel paths (A) = 2 (always)
(2) Number of brush sets required is equal to the number of poles	(2) Number of brush sets required is always equal to two.
(3) Normally used for generator of capacity more than 500 Amp.	(3) Preferred for generator of capacity less than 500 Amp.

7.7 WORKING PRINCIPLE OF DC MACHINES

(a) Generator

(b) Motor

Magnetic fields is the medium of energy conversion.

7.7.1 Generator

(i) When the rotor or coil is run by a prime-mover (supplying mechanical energy) then conductor cuts the magnetic field and emf is induced in the conductor.

(ii) Direction of an induced emf is governed by FRH rule.

(iii) Induced emf produced in a conductor is alternating.

(iv) This alternating emf becomes unidirectional by a commutator (reverse the connections of a coil in the external ckt in every half cycle).

(v) Now the DC current from commutator is collected by a stationary brush.

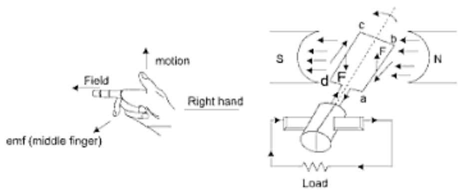


Fig. 7.8

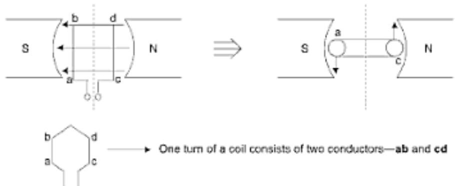


Fig. 7.10

Let the coil abcd move with a speed N r.p.m, then the conductors a and c cuts the flux produced by the poles.

If t sec is the time taken by a conductor a to complete one revolution then the flux cut by conductor is $\boxed{d\phi = P\phi}$

Hence, average emf induced in the conductor is $e_{av} = \dfrac{d\phi}{dt} = \dfrac{P\phi}{t}$

$$\boxed{e_{av} = \frac{P\phi}{t}}$$

t = total time required for one revolution.

$$= \frac{60}{N} \text{ sec}$$

$$\therefore \qquad e_{av} = \frac{P\phi}{t} = \frac{\phi NP}{60}$$

If Z number of conductors are arranged in A number of parallel paths.

Conductors per parallel path = $\dfrac{Z}{A}$

All conductors per parallel path are in series.

Total induced emf in armature is $E_a = e_{av}\cdot\dfrac{Z}{A}$

$$\boxed{E_a = \frac{\phi ZNP}{60A}}$$

$$A = 2 \text{ for wave winding}$$
$$A = P \text{ for } LAP \text{ winding}$$

if speed is in revolution per sec $w = 2\pi \dfrac{N}{60}$

$$\frac{N}{60} = \frac{w}{2\pi}$$

$$E_a = \frac{\phi Z P w}{2\pi A}$$

$$\boxed{E_a = K_a \phi w}$$

$$k_a = \text{constant} = \frac{PZ}{2\pi A}$$

- So in a DC generator when a rotor is rotated by prime mover then the conductor cuts the flux and emf, $E_a = k_a\phi w$, is induced. If there is a load connected at output a current flows through the conductor. This current carrying conductor in generator is produces the counter torque to oppose the prime mover torque.
- In DC motor when a current carrying conductor is placed in a magnetic field it experiences a force and it starts to rotate in a magnetic field.

  Now when this current carrying conductor rotates in a magnetic field it cuts the magnetic field and an emf is induced in a current carrying conductor now according to Lenz's law the direction of the induced emf is such that it opposes the cause (current in conductor).

  Hence, this induced emf have a tendency to reduce the initial current. That is why, it is called a *back emf.*

7.9 TORQUE IN A DC MOTOR

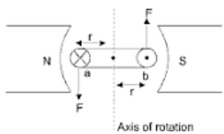


Fig. 7.11

Let the current carrying conductor *ab* be placed in a magnetic field. It experiences a force and begins to rotate about the axis and produces a torque.

$$\boxed{T = F\,r}$$

Let total number of armature conductors $= Z$
total number of poles $= p$
flux/pole $= \phi$ wb
number of parallel path $= A$
current in a armature $= I_a$ Amp

I_c be current in a conductor $= \dfrac{I_a}{A}$

B be magnetic flux density due to pole in which conductor of length l is placed

Force experienced by a conductor is F

$$F = BI_c l \quad \text{...(1)}$$

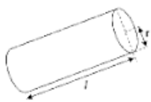
Torque developed by the conductor $= F \times r$ (N-m)

$$T = B\,I_c lr \quad \text{...(2)}$$

Total torque produced by Z conductors, $T = BI_c lr.Z$

$$B = \frac{\text{flux(total)}}{\text{Area}} = \frac{P\phi}{2\pi rl};$$

here Area = Area of cylindrical surface of armature $(2\pi rl)$



Torque $T = BI_c lr.Z$ $\quad \left\{ I_c = \dfrac{I_a}{A} \right.$

$$= B.\frac{I_a}{A} lrZ$$

$$= \frac{P\phi}{2\pi rl} . \frac{I_a}{A} .lrZ$$

$$= \frac{P\phi Z}{2\pi A} . I_a$$

$$\boxed{T = K_a \, \phi \, I_a \text{ (N-m)}} \quad \begin{cases} \text{where } k_a = \dfrac{PZ}{2\pi a} \text{ same as for} \\ \text{generated emf expression} \end{cases}$$

7.10 CKT MODEL OF DC MACHINES

In a DC machine all parallel paths in armature are symmetrical.

Each path has a generated emf E_a and a resistance r_p (net armature resistance $r_a = \frac{r_p}{A}$, A = number of parallel path).

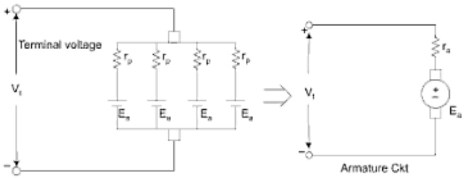


Fig. 7.12 Equivalent Ckt for DC machine

DC machines are classified according to the method of connection of field winding. So the field winding of a DC machine must also be represented in the circuit model.

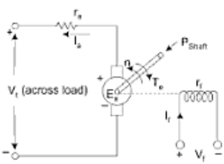


Fig. 7.13 CKT model for generator

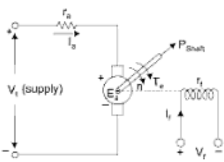


Fig. 7.14 CKT model for motor

7.10.1 Generating Mode

(i) When armature current I_a is in the direction of generated emf E_a, DC machine operates in the generating mode.

(ii) For a generator, electromagnetic torque (T_e) is opposite to the rotor rotation (T_e opposes the prime mover torque) This is essential for the conversion of energy from mechanical to electrical.

(iii) This torque T_e may therefore be called counter torque for a DC generator.

(iv) $V_t = E_a - I_a r_a$ – brush drop

7.10.2 Motoring Mode

(i) When armature current I_a, flows in opposition to generated emf E_a (called counter or back emf), DC machine operates in motoring mode.

(ii) Generated emf that opposes the armature current is called back emf and the electromagnetic torque T_e is in the direction of rotation.

(iii) $V_t = E_a + I_a r_a$ + brush drop

7.11 TYPES OF DC MACHINES

A DC machine can work as an electromechanical energy converter only when its field winding is excited with *direct current*. Hence, DC machines are classified according to methods of excitation.

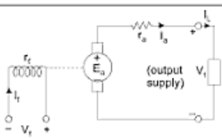
There are, in general, two methods of exciting the field winding, and accordingly DC machines are of two types:

(1) Separately excited DC machine

(2) Self-excited DC machines:

(a) Series excited DC M/C

(b) Shunt excited DC M/C

<table>
<tr><th>Generator</th><th>Motor</th></tr>
<tr><td>


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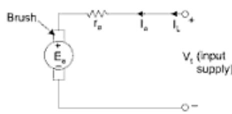


</td></tr>
<tr><td>$E_a = V_t + I_a r_a$ + brush drop</td><td>$E_a = V_t - I_a r_a$ – brush drop</td></tr>
<tr><td>$E_a = K_a \phi w$</td><td>$E_a = K_a \phi w$</td></tr>
<tr><td>$I_a = I_L$</td><td>$I_a = I_L$</td></tr>
</table>

(c) Compound excited DC M/C
   (i) Long shunt
   (ii) Short shunt

7.11.1 Separately Excited DC Machine

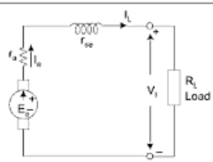
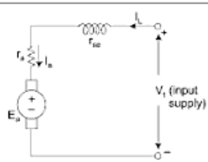
- Separately excited field winding consists of several hundred turns of fine wire.
- Field winding is connected to a separate or external DC source.
- External DC source has no relation with armature voltage.

7.11.2 Self-Excited DC Machine

- When the field winding is excited by its own armature, the machine is said to be self-excited DC machine.
- Field poles must have a residual magnetism.
- A self-excited DC machine can be subdivided as—
  (a) Series excitation
  (b) Shunt excitation
  (c) Compound excitation
     (i) Long shunt
     (ii) Short shunt

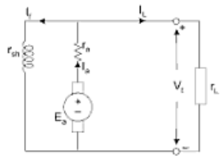
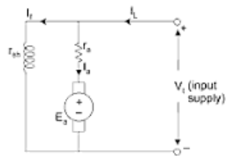
7.11.2.1 Series (excited) DC machine

- Field winding consists of *few turns* of *thick* wire.
- Field winding is connected in series with armature so a series field may be called a current operated field.
- Series field winding are characterized by low resistance.

Generator	Motor
	
$I_a = I_L = I_{rse} = I_f$ I_f = field current $\propto I_a$ $E_a = V_t + I_a (r_a + r_{se})$ + BD field varies with armature current $E_a = K_a \phi I_a = K_a I_a^2$ (unsaturated field $\phi \propto I_a$)	$I_a = I_L = I_{rse} = I_f$ $I_f \propto I_a$, $\varnothing \propto I_a$ $E_a = V_t - I_a (r_a + r_{se})$ – BD Torque $= K_a \phi I_a$ $= K I_a^2$ (unsaturated field $\phi \propto I_a$)

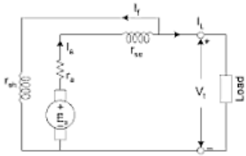
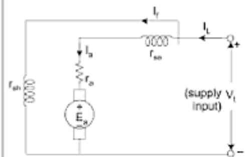
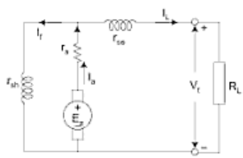
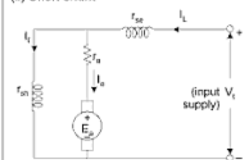
7.11.2.2 *Shunt excited DC machine*

- Field winding consists of a large number of turns of fine wire.
- Field winding is connected in parallel with the armature.
- Voltage across the armature terminal (V_t) and shunt field is same so it is called *voltage-operated* field.
- Shunt field winding are characterized by high resistance.

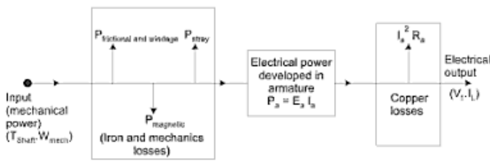
Generator	Motor
	
$I_a = I_L + I_f$ $V_t = I_f r_{sh}$ (as V_t changes I_f changes and ϕ will change) $E_g = V_t + I_a r_a + \text{BD}$	$I_a = I_L - I_f$ $V_t = I_f r_{sh}$ (as V_t changes I_f changes) $E_a = V_t - I_a r_a - \text{BD}$

7.11.2.3 *Compound (excited) DC machine*

- This machine has both series and shunt winding
- If the series field flux aids the shunt-field flux, M/C is called *cumulatively compound* DC machine.
- If the series field flux opposes the shunt flux, the machine is called a *differentially compound*.
  (i) long shunt
  (ii) short shunt

Generator	Motor
(i) Long shunt  $V_t = I_f r_{sh}$ $I_a = I_L + I_f$ $E_g = V_t + I_a (r_a + r_{se})$ + BD	(i) Long shunt  $I_a = I_L - I_f$ $V_t = I_f r_{sh}$ $E_b = V_t - I_a (r_a + r_{se})$ – BD (Brush Drop)
(ii) Short shunt  $I_a = I_L + I_f$ $V_t = -I_L r_{se} + I_f r_{sh}$ $E_g = I_a r_a + I_L r_{se} + V_t$ + Brush drop	(ii) Short shunt  $I_a = I_L - I_f$ $V_t = I_L r_{se} + I_f r_{sh}$ $E_b = V_t - I_a r_a - I_L r_{se}$ – Brush droop

7.12 POWER STAGES IN DC GENERATORS

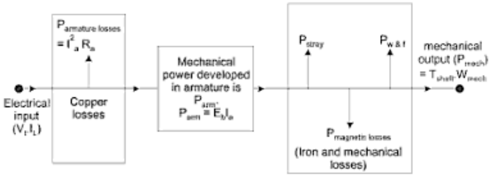


$$E_a = V_t + I_a r_a$$

$\Rightarrow$ $E_a I_a = V_t I_L + I_a^2 r_a$ (power equation of generator)

$\Rightarrow$ (Gross mechanical input to armature equivalent to $E_a I_a$) = Electrical output + Copper losses

7.13 POWER STAGES IN DC MOTORS



$$V_t = E_b + I_a r_a \text{ (in motor)}$$

$$V_t I_a = E_b I_a + I_a^2 r_a \text{ (power equation of motor)}$$

(net electrical power input to armature) = (Electrical equivalent of gross mechanical power developed $P_m = E_b I_a$) + armature copper loss.

7.14 LOSSES AND EFFICIENCY OF DC MACHINES

Losses in a DC machine can be classified as under.

(i) ***No-load Rotational loss:***

(a) Iron loss at working flux and speed.

(b) Mechanical losses (friction and windage losses at the operating speed) is 10 to 20% of full load losses.

(ii) ***Copper or I^2R loss:***

(a) Armature ckt loss $I_a^2 r_a$.

where, r_a = Armature resistance + brush contact resistance + interpole and compensating winding resistance.

(b) Field ohmic loss $V_f I_f$. This loss includes the field-rheostat loss also. (In series machine, field ohmic loss forms a part of armature circuit loss).

(iii) ***Stray load losses*****:** These are produced by

(a) The distortion of the air gap due to armature reaction.

(b) Current in the commutated coil.

$$\eta_{motor} = 1 - \frac{\text{Losses}}{\text{Input}}$$

$$\eta_m = 1 - \frac{I_a^2 r_a + V_f\ I_f + w_o}{V_t I_L}$$

Efficiency is max $\left(\frac{d\eta_g}{dI_L} = 0 \ or\ \frac{d\eta_m}{dI_L} = 0\right)$ when variable armature *ckt* loss = constant loss.

$$I_a^2 r_a = (V_f I_f + \text{No load rotational loss})$$

$$I_a^2 r_a = P_o \text{ (constant losses)} = P_o$$

Armature current at max efficiency = $\sqrt{\frac{P_o}{r_a}}$

7.16 CHARACTERISTICS OF DC GENERATORS

The behaviours of various types of DC generators can be studied by their characteristics:

(i) *Magnetization Characteristic*: $E_a = f\ (I_f)$ speed for a fixed. This characteristic is also called *no load, saturation,* or *open circuit* characteristic.

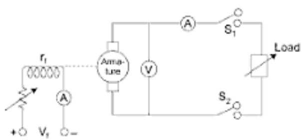
(ii) *Load or External Characteristic*: $V_t = f\ (I_L)$ with both I_f and speed are kept constant.

7.16.1 Separately-excited generator

These generators are used for wide range of output voltage.

7.16.1.1 No load or magnetization characteristic

This characteristic gives the variation of armature generated emf (E_a) with field current I_f for zero armature current (no load) for a fixed speed.



$E_a = k_a \phi w$, $\phi \propto I_f$ at no load s_1, s_2 is open $I_a = 0$, w is constant

$E_a \propto I_f$

OA = residual voltage

- Armature is driven at rated speed by prime mover and switch is kept always open.
- Though the field winding is not energised, the voltmeter indicates a small voltage (2 to 6 volts), due to the presence of residual flux in the main poles.
- Residual flux voltage is shown by OA.
- The field winding is now energised and I_f increases in steps.
- Field current I_f is increased till E_a is about 1.1 to 1.25 times the rated voltage.
- After certain time, I_f core gets saturated and flux (ϕ) also remains constant, though I_f increases.

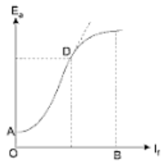


Fig. 7.16 Magnetization characteristic

- Hence, after saturation, voltage E_a does not change linearly with I_f.
- If the field current is now decreased from OB, E_a versus I_f plot will not original curve. It will lie above it due to hysteresis.

7.16.1.2 External characteristic

This characteristic gives the variation of armature terminal voltage (V_t) with load current (I_L) for constant speed and fixed field current.

- generator is run at rated speed
- its field winding is excited to give rated terminal voltage at no load.
- now close the switches and vary load resistance in steps.
- $V_t = E_a - I_a r_a$

  when I_a or I_L is zero, $V_t = E_a$

  as we increase I_a, then drop in $V_t = (AB) + (BC)$ where

  AB = drop in E_a due to armature reaction or armature reaction drop.

$BC = I_a r_a$ drop

(As I_a increases then at each point E_a decreases due to armature reaction)

- $V_t = E_a - I_a r_a$ (so V_t also drops due to $I_a r_a$ drop)

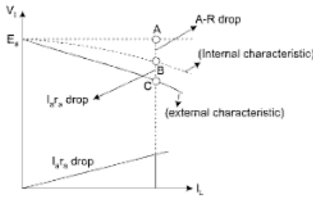


Fig. 7.17

7.16.2 Shunt Generator

These generators are frequently employed because no separate source for excitation is required. However, the load current must be well below the maximum current for avoiding large dips in terminal voltage.

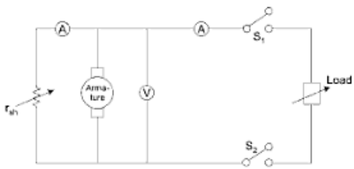


Fig. 7.18

7.16.2.1 No load or magnetization characteristic

If the field winding ckt of the shunt generator is disconnected from the armature ckt and separately excited, then the no load characteristic with separate excitation will not differ from that obtained with shunt excitation.

This is due to the fact that small amount of current (1 to 4% of rated) flowing in armature has negligible effect on the main flux.

- For drawing *OCC* for shunt generator, it is convenient to run it as a separately excited generator.
- Run the armature at constant speed.
- Now increase the field current I_f in steps

$$E_a = ka\phi w \quad (w \text{ is constant})$$

$$\text{So,} \quad E_a \propto I_f$$

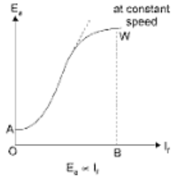


Fig. 7.19

7.16.2.2 External characteristic

- Run the armature at rated speed.
- Adjust the field current to give rated voltage at no load.
- Now close the switch and increase load gradually (from Fig. 7.18).

  for shunt generator equations are
- $V_t = E_a - I_a r_a$ and $V_t = I_f r_f$
  - (i) at $I_L = 0$ or $I_a = 0$, $V_t = E_a$
  - (ii) as I_L or I_a increases then due to armature reaction E_a will decrease. Hence terminal voltage (V_t). will decrease. ($BC = AR$ drop)
  - (iii) As I_a increases then due to $I_a r_a$ drop V_t further decreases ($BA = I_a r_a$ drop)
- Due to two drops (AR drop and $I_a r_a$ drop) the terminal voltage V_t will drop and $V_t = I_f R_f$ so I_f will decrease due to drop in V_t hence E_a (generated emf) will further decrease.
- Drop in generated emf in shunt generator is more than separately excited generator, in shunt generator drop in $E_a = BD = BC + CD$

  DC = drop due to $I_f R_f$

  BC = drop due to armature reaction
- Hence, drop in terminal voltage is more in shunt generator ($I_f R_f + A.R + I_a r_a$) than in separately excited ($A.R + I_a r_a$) generator.

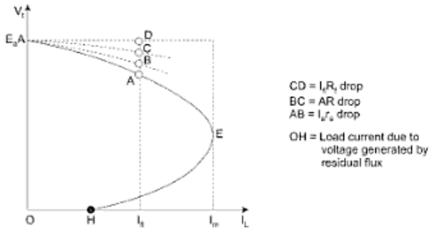


Fig. 7.20

- As the load resistance decreases, load current increases and terminal voltage drops until point E is reached. If the load resistance further decreases then the current increases momentarily. This momentarily increase in load current, produces more armature reaction thus causing a reduction in terminal voltage and the field current. The net reduction in terminal voltage is so large, then the load current decreases and *external characteristic turns back.*

  In case, machine is shortcircuited, the curve terminates at point H. Here OH is the load current due to voltage generated by residual flux.

7.16.2.3 Voltage build up in shunt generator

In a separately excited generator when armature is driven it cuts the field flux (by separately excited winding) and generate a voltage.

In a shunt generator when the armature is driven at a speed for which magnetization curve is given.

(1) The residual pole flux generates a small voltage *oc*, with zero field current.

(2) Due to this small residual voltage *oc* produces a small field current *od*.

(3) Which in turn raises the generated voltage to *de*.

(4) This generated voltage *de* raises the field current to *of*.

(5) Further raises the generated voltage to *fg*.

(6) This happens until point *a* is reached. The point *a* is known as stable point.

(7) Stable point *a* is determined by the intersection of field resistance line *op* and the magnetization or saturation curve.

(8) Beyond the point *a*, the generated emf given by magnetization curve is less than that required to maintain the corresponding field current (*OE*).

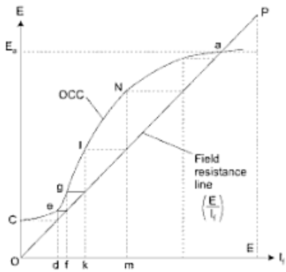


Fig. 7.21

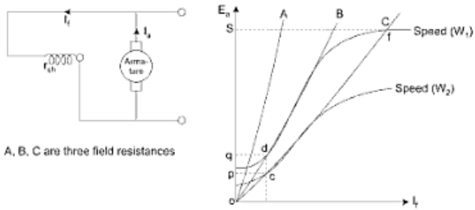


Fig. 7.22

(i) For magnetization curve at speed (w_1) for this if the field resistance is *C* then voltage build upto *os*, *f* is the stable point.

- Now if field resistance increases to *B*, then the stable point is at *d*. voltage is upto *oq*. Beyond this field resistance no voltage will be built up so *OB* is known as the critical field resistance at speed *w*.
- So **critical field resistance** is the maximum resistance beyond which no voltage will build up.

(ii) If we decrease the speed of a DC shunt generator than that minimum speed after which no voltage will be built up, is known as **critical speed.**

7.16.2.4 Failure to voltage build up

The causes of failure, the method of detection and the corresponding remedy for the self excited generators are given in Table 7.1.

7.16.3 Series Generator

These generators are used mainly as series boosters connected in the line to neutralise the effect of line ohmic drop.

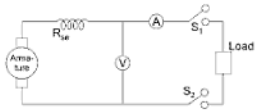
7.16.3.1 No-load characteristic

- In a series generator armature winding, field winding and load resistance are connected in series, therefore the $I_f = I_L = I_a = 0$ at no-load, so no load magnetization curve can be obtained by separately exciting its field from its low voltage source, so that armature current is always zero. So *same ckt and characteristic as separately* excited generator.

Table 7.1

Cause	Method of detection	Remedy
1. Absence of residual magnetism due to ageing.	Zero reading on voltmeter after rotating armature.	Operate the generator as separately excited first and then as a self-excited.
2. Wrong field winding connections. Due to this, flux gets produced in opposite direction to residual flux. So residual flux cancels the main flux.	Voltmeter reading decreases rather than increasing as generator is started.	Interchange the field connections.
3. Field resistance is more than the critical resistance.	Voltmeter shows zero reading.	Reduce the resistance of field circuit using proper field diverter.
4. Generator is driven in opposite direction.	This wipes out the residual flux and fails to excite.	Drive the generator in proper direction.
5. Speed less than the critical speed.		Drive the generator at more than critical speed.

7.16.3.2 External characteristic or load character



$V_t = E_a - I_a r_{se}$

In series motor $I_a = I_L = I_f$

At no load $I_f = 0, I_L = 0, I_a = 0$

$I_f = 0$ so E_a should be zero but due to residual magnetism there is some residual voltage (OC).

(i) $I_f = I_a = I_L = 0, V_t = E_{\text{residual}}$

(ii) Now if $I_a = I_L = I_f$ increases then E_a increases.

$V_t = E_a - I_a r_a$ (increases from residual voltage to V_t)

- If total resistance drop $I_a r_a$ is added to the ordinates of curve C, the internal characteristic shown by curve B is obtained.

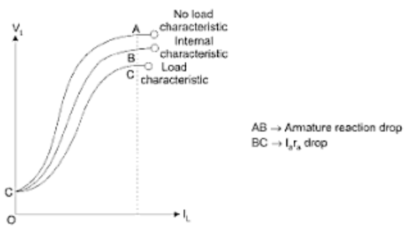


Fig. 7.23

- It is obvious from the shape of the external characteristic that a series generator is a variable voltage generator. *It is therefore never used as a voltage source.*

7.16.4 Compound Generator

Cumulatively compounded generators are more common because these can furnish almost constant voltage from no load to full load.

- Degree of compounding can be controlled by connecting a suitable low resistance called diverter in parallel with the series field winding.

Applications of various types of generators

(i) *Separately excited generators:*

As a separate supply is required to excite field, the use is restricted to some special applications like electroplating, electrorefining of materials, etc.

(ii) *Shunt generators:*

Commonly used in battery charging and ordinary lighting purposes.

(iii) *Series generators:*

Commonly used as boosters on DC feeders, as a constant current generators for welding generators and arc lamps.

(iv) *Cumulatively compound generators:*

These are used for domestic lighting purposes and to transmit energy over long distance.

(v) *Differential compound generators:*

The use of this type of generators is very rare and it is used for special applications like electric arc welding.

7.17 OPERATING CHARACTERISTICS OF A DC MOTOR

- In a DC motor emf (E_a) generated in the armature is called counter or back emf.
- For DC motors, the supply voltage is usually constant. The following are the important characteristics of DC motors:
  - (i) Speed-armature current characteristic.
  - (ii) Torque-armature current characteristic.
  - (iii) Speed-torque characteristic.

7.17.1 DC Shunt Motor

- V_t terminal voltage is constant so flux ϕ is constant.

$$E_a = k_a\phi w_m$$

$$w_m = \frac{E_a}{k_a\phi}$$

$$E_a = V_t - I_a r_a$$

$$\boxed{w_m \propto \frac{1}{\sqrt{T_e}}}\ \text{hyperbolic equation}$$

7.17.3 Compound Motor

- A DC compound motor has both the series and shunt windings.
- So compound motor characteristic depends on the fact whether the motor is cumulatively compound or differential compound.
- Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor. However, it is not having disadvantage of series motor even at light or no load.
- Differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load, the motor may try to *run with dangerously high speed. So differential compound motor* is generally not used.

$$V_t = E_a + I_a\,(r_a + r_s)$$

$$E_a = k_a\,(\phi_{sh} + \phi_{se})\,w_m$$

$$w_m = \frac{1}{k_a\,(\phi_{sh} + \phi_{se})}\,[V_t - I_a\,(r_a + r_{sh})]$$

$$T_e = k_a\,(\phi_{sh} + \phi_{se})\,I_a$$

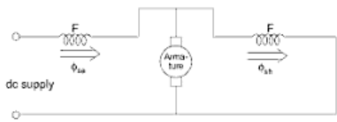


Fig. 7.31 Cumulatives

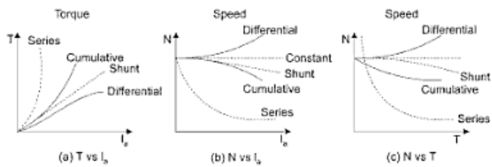


Fig. 7.32 Characteristics of compound motor

7.18 APPLICATIONS OF DC MOTORS

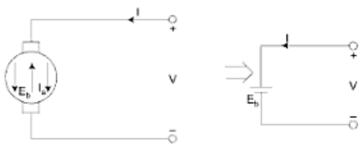
Instead of just stating the applications, the behaviour of the various characteristics like speed, starting torque, etc., which makes the motor more suitable for the applications, is also stated in the Table 7.2.

Table 7.2

Type of motor	Characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1) Blowers and fans 2) Centrifugal and reciprocating pumps 3) Lathe machines 4) Machine tools 5) Milling machines 6) Drilling machines
Series	High starting torque. No load condition is dangerous. Variable speed.	1) Cranes 2) Hoists, elevators 3) Trolleys 4) Conveyors 5) Electric locomolives
Cumulative compound	High stating torque. No load condition is allowed.	1) Rolling mills 2) Punches 3) Shears 4) Heavy planers 5) Elevators
Differential compound	Speed increases as load increases.	Not suitable for any practical applications

7.19 SIGNIFICANCE OF BACK EMF

According to Lenz law induced emf opposes the main terminal voltage, so it is referred to as counter or back emf E_b.



The rotating armature generating the back emf is like a battery of emf E_b put across a supply mains of V volts. Power required to overcome this is $E_b I_a$.

$$I_a = \frac{V - E_b}{R_a}, \quad E_b = \frac{\psi Z N \text{ P}}{60 \text{ A}}$$

If speed is high, E_b is high then I_a is small.

So E_b acts like a governor (it makes a motor self-regulating, so that it draws as much current as is necessary).

7.20 SPEED CONTROL OF DC MOTORS

The speed of a DC motor is given by,

$$W = k_a \frac{E_b}{\phi}$$

where, κ_a = a constant

E_b = back emf

$= V_t - I_a R_a$

ϕ = flux per pole

V_t = applied voltage or terminal voltage

I_a = armature current

R_a = armature circuit resistance

$$N = \kappa_a \frac{V_t - I_a R_a}{\phi}$$

Thus the speed of a DC motor can be varied by varying the:

(i) Armature circuit resistance R_a (known as rheostatic control).

(ii) Flux ϕ (known as field control)

(iii) Applied voltage V_t.

The speed control of shunt and series motor will be discussed separately by the above methods.

7.20.1 Speed Control of Shunt Motors

(a) Rheostatic control.

(b) Field control.

(c) Variation of applied voltage.

  (i) Ward-Leonard control.

  (ii) Modified Ward-Leonard control.

7.20.1.1 Rheostatic control of shunt motors

Let R be the adjustable resistance placed in series with the armature as shown in Fig. 7.33(a), then back emf is given by

$$E_b = V_t - I_a (R_a + R)$$

where, I_a = Armature current.

$$W = K_a \frac{V_t - I_a(R_a + R)}{\phi} \qquad \text{...(1)}$$

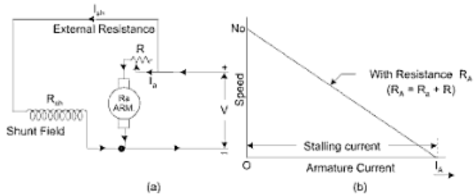


Fig. 7.33

Let W_o be the motor speed at no load. At no load armature current I_a would be quite small and if R is assumed to be zero, then back emf would be approximately equal to the applied voltage V_t.

$$\therefore \qquad W_o = k_a \frac{V_t}{\phi} \qquad \text{...(2)}$$

From equation 1 and 2

$$\frac{W}{W_o} = \frac{V_t - I_a(R_a + R)}{V_t} \qquad \text{...(3)}$$

or,

$$W = W_o \left\{ 1 - \frac{R_a + R}{V_t} \cdot I_a \right\}$$

Putting, $R_a + R = R_A$

$$\therefore \qquad W = W_o \left\{ 1 - \frac{R_A}{V_t} \cdot I_a \right\}$$

This shows that the speed of the motor drops linearly with the armature current I_a for a given resistance R_A as shown in Fig. 7.33(b) The drop in speed depends upon the value of R_A and hence upon the external resistance R.

There will be a load current I_A for which the motor speed will be zero. This current is known as stalling current and is given by

$$0 = W_o\left\{1 - \frac{R_A}{V_t}\cdot I_A\right\}$$

or, $$I_A = \frac{V_t}{R_A}$$

From Fig. 7.33(b) it is obvious that as the load on the motor increases the speed drops from N_0 to zero in a linear way for a given external resistance R.

Power wasted in controller for load of constant torque: Since, the external resistance is in series with the armature, so full armature current flows through it. This will result in heavy power loss in the controlling resistance.

$$\text{Power input to the armature circuit} = V_t.I_a$$

$$\text{Power converted into the mechanical power} = E_b.I_a$$

$$= (V_t - I_aR_A)I_a$$

where, $R_A = R_a + R$

$$\frac{\text{Power converted into mechanical power}}{\text{Power input to the armature circuit}} = \frac{(V_t - I_aR_A)I_a}{V_tI_a}$$

$$= \frac{V_t - I_aR_A}{V_t}$$

$$= \frac{W}{W_o} \text{ [By equation (3)]}$$

Assuming power loss in the armature is negligible,

Power wasted in controller = power input – power converted

$$= \text{power input}\left(\frac{1 - \text{Power converted}}{\text{Power unit}}\right)$$

$$= \text{power input}\left(1 - \frac{W}{W_o}\right)$$

$$= \text{power input} \; \frac{W_o - W}{W_o}$$

$$= \text{power input} \times \frac{\text{Change of speed}}{W_a}$$

Hence, the power wasted in controller is directly proportional to the change in speed. If the speed N is zero, power wasted in controller would be equal to power input as shown in Fig. 7.34.

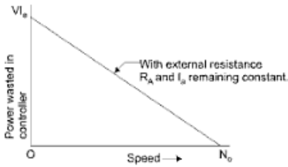


Fig. 7.34

***Power wasted in controller when load connected to motor is such that the torque varies as the square of the speed:** In case of fans and centrifugal pumps, etc., the torque varies as the square of the speed, i.e.,

$$T \propto W^2$$

In a shunt motor

$$T \propto I_a$$

$$\therefore \qquad I_a \propto W^2$$

Let W_0 be the no load speed and T_m be the maximum torque at this speed and the motor current at this instant be I_m, then

$$I_m \propto W_0^2$$

$$\frac{I_a}{I_m} = \left(\frac{W}{W_0}\right)^2$$

or,
$$I_a = I_m \cdot \left(\frac{W}{W_0}\right)^2$$

*Not for U.P.T.U. (Tee-101/201) Student

Also, speed at no load $W_0 \propto V_t$.
and, speed at any load $N \propto E_b$.

$$\therefore \qquad \frac{E_b}{V_t} = \frac{W}{W_0}$$

$$\text{or,} \qquad E_b = V_t \cdot \frac{W}{W_0}$$

Voltage drop in the controller

$$= V_t - E_b$$

$$= V_t - V_t \cdot \frac{W}{W_0}$$

$$= V_t\left(1 - \frac{W}{W_0}\right)$$

Power wasted in the controller

P_c = Voltage drop × Current in controller.

$$= V_t\left(1 - \frac{W}{W_0}\right) . I_a$$

$$= V_t\left(1 - \frac{W}{W_0}\right) . I_m\left(\frac{W}{W_0}\right)^2$$

$$= V_t\, I_m\left(1 - \frac{W}{W_0}\right)\left(\frac{W}{W_0}\right)^2$$

When, $\quad W = 0$

$$P_c = 0$$

Hence, the power wasted in the controller is zero, both when speed is either zero or maximum.

$$P_c = V.I_m\left[\left(\frac{W}{W_0}\right)^2 - \left(\frac{W}{W_0}\right)^2\right]$$

Putting, $\quad \dfrac{W}{W_0} = \beta$

$$P_c = V.I_m\,(-\beta^2 - \beta^3)$$

For P_c to be maximum

$$\frac{dP_c}{d\beta} = O$$

or, $$\frac{d}{d\beta}(\beta^2 - \beta^3) = O$$

or, $$2\beta - 3\beta^2 = O$$

or, $$\beta = \frac{2}{3}$$

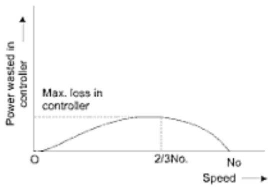


Fig. 7.35

or, $$\frac{W}{W_0} = \frac{2}{3}$$

or, $$W = \frac{2}{3} W_0$$

$\therefore$ Maximum loss in controller $= V.I_m\left[\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3\right]$

$$= 0.148.V\,I_m$$

Hence, the maximum power loss in controller is 14.8 per cent of the motor intake. The curve of power wasted is shown in Fig. 7.35. So when the load connected to the motor is such that the torque varies as the square of the speed, this method of controlling the speed is not very expensive.

Demerits of rheostatic method of speed control: The rheostatic method of speed control suffers from the following demerits:

(i) The speed of the motor changes with the change of load current. If the speed of the motor is to be maintained constant when the load is variable, the value of the extra resistance R has to be adjusted accordingly.

(ii) Speeds above normal cannot be obtained, only speed below normal are possible.

(iii) Considerable amount of power is wasted in the controller, which lowers the efficiency of the motor.

(iv) Costly controller with proper arrangement for heat dissipation is required.

7.20.2 Field Control of Shunt Motors

The speed is given by = $\kappa_a \dfrac{E_b}{\phi}$.

The speed is inversely proportional to the flux. If an external resistance is inserted in the field circuit as shown in Fig. 7.36, the field current will decrease and so the flux will also decrease. This will result in an increase in speed of the motor and decrease in developed torque.

By this method, speeds above normal are obtained. This method of speed control is very commonly used and the external resistance R is called shunt regulator.

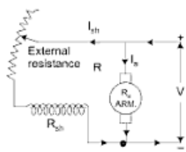


Fig. 7.36

Advantages:

(i) This is an easy and convenient method.

(ii) It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small field current.

(iii) The speed control by this method is independent of load on the machine.

Disadvantages:

(i) Only speeds higher than the normal speed can be obtained since the total field circuit resistance cannot be decreased beyond shunt field winding resistance.

(ii) There is a limit to the maximum speed obtainable by this method.

(iii) Shunt field cannot be opened because its speed will increase to an extremely high value.

7.20.3 Speed Control by Variation of Applied Voltages

The speed is given by,

$$W = k_a \frac{V_t - I_a R_a}{\phi}$$

If an external resistance R be inserted in the supply line of the motor as shown in Fig. 7.37 and I be the motor input current, then

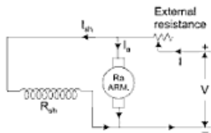


Fig. 7.37

Applied voltage across motor = $V - IR$.

Hence, by increasing R, the applied voltage across the motor would decrease, which would result in decrease of speed. The **disadvantages** of this method are:

(i) Heavy power loss in the external resistance.

(ii) Speeds above normal cannot be obtained.

The variable voltage which is fed to the shunt motor can also be obtained from a separate DC generator as it would be discussed in the next chapter. This method of speed control is very popular and is known as Ward-Leonard system of speed control.

7.20.3.1 Ward-Leonard system of speed control of shunt motor

In this system the variable voltage for the motor M whose speed is to be controlled, is obtained by means of a separate generator G driven by a motor M'. The connection diagram is shown is Fig. 7.38.

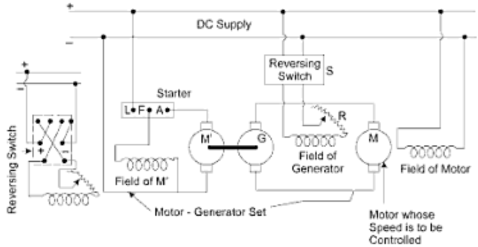


Fig. 7.38

The desired voltage across the terminals of the motor *M* can be obtained by varying the field of the generator *G*. The field of the motor *M* is excited by the supply mains which remains constant. By this system the speed of the motor can also be controlled in reverse direction. Since, for reversing the speed of the motor either the field or the supply voltage across armature of the motor should be reversed. In this system, field remains unchanged. The supply voltage across the motor armature terminals can be reversed by means of the reversing switch *S*. The motor will now run in the reverse direction and by adjusting the generator field desired motor speed can be obtained. The driving motor *M* can as well be an induction or a synchronous motor. In that case only a small DC power is required for the field excitation of generator *G* and motor *M*.

Advantages:

(i) Very fine speed control over the whole range from zero to full speed in both directions is obtained.
(ii) Uniform acceleration can be obtained.
(iii) Speed regulation is good.

Disadvantages:

(i) Low overall efficiency
(ii) Costly arrangement, since two extra machines are required.

Applications: This system is commonly used in:

(i) Colliery winders.
(ii) Elevators.

shortcircuited according to the requirement. The speed of the motor is minimum when full field is in the circuit. The speed can be increased in steps by shortcircuiting some of the series turns. This method is usually employed in electric traction.

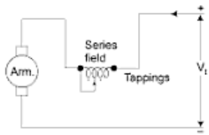


Fig. 7.42 Tapping of field

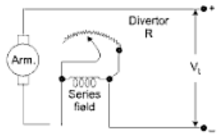


Fig. 7.43 Field divertor

By Field Divertor: This method of speed control is most common. A variable resistance R known as divertor is placed in parallel across the series field. By adjusting the divertor resistance any desired value of current can be passed through it and thus the current through it and thus the current through the series field may be reduced. Hence, the field flux would decrease which will result in an increase of motor speed. By reducing the divertor resistance the speed will increase and vice versa. This method is used to give speeds above the normal.

By Armature Divertor: A variable resistance is connected across the armature which is known as armature divertor as shown in Fig. 7.44. If the motor is driving a constant load torque device, any reduction in armature current due to the divertor will tend to decrease the torque. Since $T \propto \phi . I_a$, so ϕ must increase to maintain the torque constant. This would result to an increase in current form the supply which in turn would increase ϕ. Since speed $\propto \frac{1}{\phi}$, so the speed will decrease. The speed can be controlled by varying the divertor resistance.

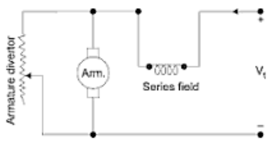


Fig. 7.44 Armature divertor

This method is used for obtaining speeds lower than the normal speeds.

Series and parallel control of series motors: This method requires two series motors mechanically coupled together. Consider two series motors connected in series as shown in Fig. 7.45.

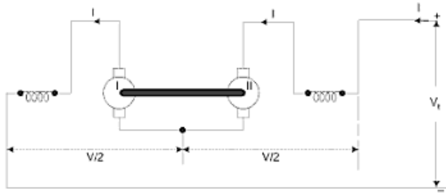


Fig. 7.45 Series control

Let V_t = Supply voltage

I = Input current

Then, $$\text{speed} \propto \frac{\text{Back emf}}{\text{Flux}}$$

Since, Back emf ∝ applied emf

and, Flux ∝ I

$$\therefore \quad \text{Speed } (W_s) \propto \frac{V_t/2}{I}$$

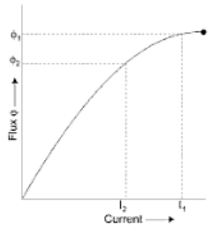
$$\propto \frac{V_t}{2I}$$

and, $$\text{Torque } (T_s) \propto \phi . I$$

$$\propto I.I$$

$$\propto I^2$$

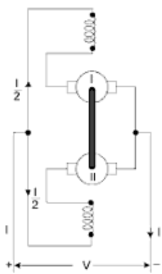
Series motor starter resistance: In case of a series motor the flux does not remain constant but depends upon the motor current. In Fig. below the magnetization characteristic of a series motor be shown. Let I_1 and I_2 be the maximum and minimum currents. Let ϕ_1 be the flux corresponding to the current I_1 and ϕ_2 be the flux corresponding to the current I_2 as shown in Figure below.



Consider two series motors connected in parallel as shown in Figure below assuming that the supply voltage and input current remains the same.

Let V_t = Supply voltage

I = Input current



Parallel control

Now, speed $W_p \propto \dfrac{V_t}{I/2}$

$$\propto \frac{2V_t}{1}$$

and, Torque $(T_p) \propto$ flux $\times$ current

$$\propto \left(\frac{I}{2}\right)\cdot\left(\frac{I}{2}\right)$$

$$\propto \frac{I^2}{4}$$

$$\frac{\text{Speed }(W_s)}{\text{Speed }(W_p)} = \frac{V/(2I)}{2V/I} = \frac{I}{4}$$

Hence, the speed of the set, when the two series motors are connected in series is one-fourth of the speed when the motors are in parallel.

$$\frac{\text{Torque}(T_s)}{\text{Torque}(T_p)} = \frac{I^2}{I^2/4} = 4.$$

When the two series motors are in series then the torque produced is 4 times the torque produced by the motors in parallel.

*7.21 STARTING OF DC MOTORS

Need of Starter

At the time of starting, the motor armature is at rest and so there is no back emf. When a full supply voltage is applied a very heavy current flows through the armature at the instant of starting and the armature windings may burn out.

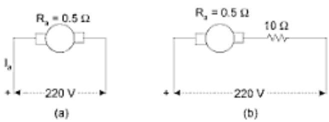


Fig. 7.46

Consider a 5 hp 220 V DC motor having an armature circuit resistance of 0.5 ohm.

Full load current of the motor $I_a = \dfrac{5 \times 735.5}{220}$

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(Assuming 100% efficiency)

$$= 16.75 \text{ A}$$

Since, there is no back emf at the time of starting, so starting current is given by Fig. 7.46(a),

$$I_s = \frac{220}{0.5} = 440 \text{ A}$$

$$\therefore \qquad \frac{I_s}{I_f} = \frac{440}{16.75} = 26.2$$

Thus, the starting current is very heavy as compared with the full load current and hence if the protective devices fail to operate the armature is likely to burn and there would be pitting on the commutator sectors. Hence, the DC motors must not be switched on to supply directly.

If a resistance of say 10 ohms is connected in series with the armature at the time of starting as shown in Fig. 7.46(b) the new starting current would be,

$$I_s = \frac{220}{10.5} = 21 \text{ A}$$

Thus, by putting a resistance in series with the armature, the starting current can be limited to a desired value. The value of the series resistance is so adjusted that starting current is 1.5 to 2 times the full load current to develop sufficient starting torque.

For fractional horsepower motors such as DC fans, additional series resistance may not be necessary, when they are switched on. Because they have quite large armature resistance which limits the starting current. Also their moment of inertia is small so they readily pick up the speed.

The starters may be classified in two categories:

(a) Manual starters.

(b) Automatic starters.

7.21.1 Manual Starters

In manual starters there is an external resistance which is inserted in series with the armature at the time of starting which is cut-off step by step by hand as the motor gains speed. The use of manual starters is limited to motors upto 50 hp.

7.21.2 Automatic Starters

In automatic starters, the successive sections of the starting resistance are cut-off by means of contactors which are actuated by electromagnets. The number of sections of the starting resistances is less in automatic starters as compared with the manual starters. The present trend is towards increased use of automatic starters.

Simple shunt motor starter

In Fig. 7.47, the simplest type of DC shunt motor starter is shown. It consists of a series resistance which is divided into several sections and connected to the brass studs. There is a brass arc which is connected to the shunt field as shown. To start the motor the DC supply is switched on. The handle is moved, as soon as it comes in contact with stud 1, the entire resistance is inserted in the

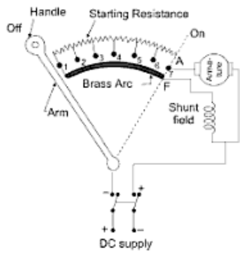

Fig. 7.47

armature circuit. The motor speeds up and this resistance is completely cut-off in steps. Such a starter is quite alright from the viewpoint of starting, but the following are the chief objections for its use:

(a) If the supply breaks when the motor is running, the motor will come to rest and the handle will remain in its on position. If the supply is restored again, the full voltage will be applied across the armature terminals which will result in burning of the armature winding.

(b) There is no protective device fitted on the starter to break the circuit of the motor at the time of overload. This may also result in the burning of armature.

Three-point shunt motor starter with no-volt and overload protections

In Fig. 7.48, a three-point shunt motor starter with no volt and overload coils is shown.

motor is running. The electromagnet of the overload release is designed in such a way that it cannot attract the soft iron piece *M* upto its full load current. When the motor is overloaded, the input current becomes greater than the full load current and *M* gets attracted and shortcircuits the no-volt coil. This is achieved by connecting the ends of the coil of the no-volt release to two brass terminals 1 and 2 as shown by dotted line in the Fig. 7.48. As soon as *M* is attracted, the terminals 1 and 2 get electrically connected and the field current does not pass through the no-volt coil, but completes its path through the shorted studs. The electromagnet of the no-volt release gets demagnetized and hence the arm returns to the off-position. Thus, the motor is automatically safe-guarded against the overload current. In some starters, the setting of the soft iron piece *M* is adjustable for two or three values of overload currents. In that case, the soft iron piece *M* may be set according to the requirement.

Precaution: The handle should be moved slowly so as to enable the motor to gain speed. If the handle is suddenly moved from off to on position, the armature of the motor is likely to be burnt.

Four-point shunt motor starter

For a variable speed motor, a three-point starter is not suitable because in that case the field current will vary sufficiently. The no-volt coil may be strongly magnetized at higher values of the field current and weakly magnetized at the lower values. In some cases the handle may jump back to 'off' position during normal running due to weak magnetization of the no-volt coil. To overcome this difficulty, a four-point starter is used as shown in Fig. 7.49.

This starter is similar to a 3-point starter in construction, except that the no-volt coil has high resistance and is connected directly across the line. When the supply fails, the no-volt coil demagnetizes and permits the arm to return back to 'off' position.

Disadvantage:

The advantage of 'No-field release' is sacrificed in a four-point starter.

Starter 5 and 6 is shortcircuited and finally when the controller is at stud 5, the entire starting resistance is shortcircuited. Thus, for controlling the speed of the motor the controller may be kept on any of the studs. Similarly, when the contact is with stud 1 on the left, the direction of current through the armature will reverse, whereas the direction of current remains same in the field. As a result of this, the motor will run in the reverse direction. The speed of the motor will be maximum when the controller is at stud 5 i.e., when entire starting resistance is shortcircuited.

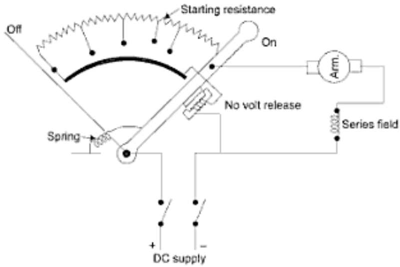

Fig. 7.50

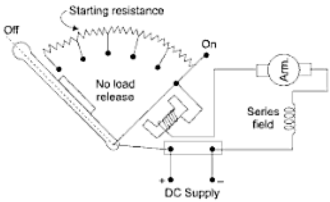

Fig. 7.51

*7.22 BRAKING OF DC MOTORS

The following three methods are employed for quickly stopping of DC motors.

(a) Dynamic braking.

(b) Regenerative braking.

(c) Plugging braking.

7.22.1 Dynamic Braking

In case of electric trams, rolling mills, hoists, etc., the inertia of the moving load may be utilized in driving the motor as a generator during the retarding

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period. A resistor is connected across the armature which dissipates the energy so produced and automatically puts braking action. The controlling switches should be so arranged that when the braking resistor is connected across the armature the field winding gets sufficient current to maintain the field at its full or even somewhat greater strength. In case of a shunt motor there is no difficulty and it would act as a generator without any change in the connections of the field winding with the supply. But in case of a series motor with current limiting resistor across the supply during the braking period as shown in Fig. 7.54. This method of quickly stopping the DC motors is called dynamic braking.

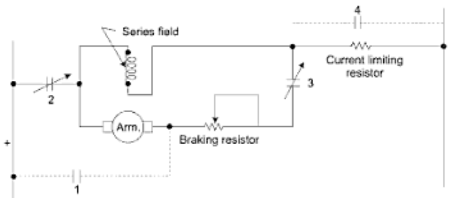


Fig. 7.54

Contacts 1 and 4 are closed and 2 and 3 are opened during the normal running of the motors. When the motor has to be quickly stopped contacts 1 and 4 are opened and 2 and 3 are closed as shown in Fig. 7.54. The series field of the motor, thus, gets power for its excitation from the supply mains and also from the generated emf of the machine as it is being driven in the same direction by its inertia during braking. The generated energy is dissipated in the braking resistor.

7.22.2 Regenerative Braking

This is a modified form of dynamic braking in which the energy stored in the moving system is delivered back to the supply circuit instead of being consumed in a resistor. Thus, it would relieve a part of power from the system load. This may be used when electric trains moving down the slope.

7.22.3 Plugging

When the power is applied in the reverse direction before a motor has fully stopped is called plugging. This is used when a motor has to be stopped quickly and then immediately reversed as in case of rolling mill service.

Field current $I_f = 1.64$ Amp

Rotational losses $= 540$ W

Armature resistance $r_a = 0.15\ \Omega$

(i) Armature induced emf

$$E_a = V_t + I_a r_a$$

$$E_a = 250 + (I_L + I_f) r_a$$

$$\Rightarrow \quad I_L = \frac{10,000}{250} = 40 \text{ Amp}, \quad I_f = 1.64 \text{ Amp}$$

$$E_a = 250 + (40 + 1.64)0.15$$

$$E_a = 256.25 \text{ volt}$$

(ii) Torque is developed from the electromagnetic power ($E_a I_a$).

$$E_a I_a = T_e \omega$$

$$T_e = \frac{E_a I_a}{\omega} = \frac{256.25 \times 41.64}{2\pi \frac{n}{60}}$$

$$T_e = \frac{10670}{2\pi\left(\frac{1000}{60}\right)} = 101.9 \text{ Nm}$$

(iii) Efficiency $\eta = \frac{\text{Output}}{\text{Input}}$

Output = 10 kW

Input = Output + Losses

Losses = $P_{\text{rotational}}$ + Armature losses + Field losses

$= P_{\text{rotational}} + I_a^2 r_a + V_t I_f = 540 + (41.64)^2 \times 0.15 + 250 \times 1.64$

$= 540 + 260 + 410 = 1210$ W

Input = 10,000 + 1210 = 11,210 Watt

$$\eta = \frac{10,000}{11,210} = 0.892$$

$$\boxed{\eta = 89.2\%}$$

Example 4: The full load current of a 240 V shunt generator is 200 A. The resistance of the shunt field is 60 ohms and its full load efficiency is 90%. The stray losses are 800 W. Find (a) armature resistance, (b) load current corresponding to maximum efficiency.

Solution:

(a) $$\text{Output} = 240 \times 200 = 48{,}000 \text{ W}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$\therefore \quad \text{Input} = \frac{48{,}000}{0.9} = 53{,}330 \text{ W}$$

$$\therefore \quad \text{Total loss} = 53{,}330 - 48{,}000 = 5{,}330 \text{ W}$$

$$I_f = \text{Shunt field current} = \frac{240}{60} = 4 \text{ A}$$

$$\therefore \quad I_a = 200 + 4 = 204 \text{ A}$$

$$\text{Shunt copper loss} = I_f^2 r_f = 4^2 \times 60 = 960 \text{ W}$$

$$\text{Stray losses} = 800 \text{ W} \quad (\text{Iron + friction losses})$$

$$\text{Constant losses} = 960 + 800 = 1760 \text{ W}$$

$$\text{Armature copper loss} = 5{,}330 - 1760 = 3{,}570 \text{ W}$$

$$\therefore \quad I_a^2.R_a = 3570 \text{ W}$$

$$\therefore \quad R_a = \frac{3{,}570}{204^2} = 0.0858 \text{ ohm}$$

(b) For maximum efficiency

$$I_a^2.R_a = \text{Constant losses} = 1760 \text{ W}$$

$$\therefore \quad I = \sqrt{\frac{1760}{0.0858}} = 143.2 \text{ A}$$

Example 5: A 200 V shunt generator delivers a current of 50A. The armature and shunt field resistances are 0.1 ohm and 100 ohms, respectively. The core and friction losses are 500 W. Find:

(a) Induced emf

(b) B.H.P. of the prime mover

(c) Commercial, electrical and mechanical efficiencies.

Solution:

(a) Shunt field current $I_{sh} = \dfrac{200}{100} = 2$ A

Armature current $I_a = I_{sh} + I$

$= 2 + 50 = 52$ A

Induced emf $= V_t + I_a.R_a$

$= 200 + 52 \times 0.1$

= 205.2 volts Ans.

(b) Armature copper loss $= I_a^2 R_a = 52^2 \times 0.1 = 270$ W

Shunt field copper loss $= I_{sh}^2 R_{sh} = 2^2 \times 100 = 400$ W

Stray losses = 500 W

Total losses = 270 + 400 + 500 = 1,170 W

Output = 200 × 50 = 10,000 W

Input = 10,000 + 1,170 = 11,170 W

B.H.P. of the prime mover $= \dfrac{11{,}170}{735.5}$ = **15.2 H.P. (metric)**

(c) Commercial efficiency $= \dfrac{\text{Output}}{\text{Input}}$

$= \dfrac{10{,}000}{11{,}170} \times 100$ = **89.5%**

Electrical power developed $= E_a I_a = 205.2 \times 52 = 10670.4$ watt

Mechanical efficiency $= \dfrac{10{,}670}{11{,}170} \times 100$ = **95.5%**

Electrical efficiency $= \dfrac{10{,}000}{10{,}670}$ = **93.7%**

$$P_{\text{losses}} = 2240 \text{ watt}$$

$$P_{\text{inp}} = 14920 + 2240 = 17,160 \text{ watt}$$

$$\eta = \frac{14,920}{17,160} = 0.869 = 86.9\%$$

(v) Shaft torque $= \dfrac{\text{Shaft power}}{w_m} = \dfrac{14920}{120} = 124$ N-m

So the difference between shaft torque and electromagnetic torque is the amount needed to overcome the rotational losses.

Example 7: For the above problem, if the shaft load or load torque remains fixed and field flux is reduced to 80% of its value by means of the field rheostat. Find the new operating speed.

Solution: New flux $\phi_2 = 0.8\phi_1 = 0.8\phi$ (if $\phi_1 = \phi$)

Load torque is constant $\Rightarrow \phi I_a =$ const

$$\phi_2 I_{a2} = \phi_1 I_{a1}$$

$$I_{a2} = \frac{\phi I_a}{0.8\phi} = \frac{10}{8} I_{a1}$$

$$I_{a2} = \frac{10}{8} \times 73 = 91.3 \text{ Amp.}$$

$$E_b = \kappa_a \phi w$$

$$\boxed{\frac{E_{b2}}{E_{b1}} = \frac{\phi_2}{\phi_1} \cdot \frac{W_2}{W_1} = \frac{\phi_2}{\phi_1} \frac{N_2}{N_1}}$$

$$E_{b1} = 216.3 \text{ volt}, \quad N_1 = 1150 \text{ rpm}$$

$$E_{b2} = V_t - I_{a2} r_a$$

$$= 230 - 91.3(0.180) = 212.8 \text{ volt}$$

$$\Rightarrow \quad N_2 = \frac{E_{b2}}{E_{b1}} \cdot \frac{\phi_1 N_1}{\phi_2}$$

$$N_2 = \frac{212.8}{216.3}\left(\frac{1}{0.8}\right)1150 = 1414 \text{ rpm}$$

Example 8: A 250 V, DC shunt machine has armature and shunt field resistances of 0.1 ohm and 125 ohms, respectively. As a generator, it delivers 20 Kw at 250 V when running at 1000 rpm.

Calculate:

(a) The speed of the same machine when running as a shunt motor and taking 20 Kw at 250 V.

(b) The internal power developed when working as a:

(i) generator delivering 20 kW.

(ii) motor taking 20 kW.

Solution: As Generator

Output current

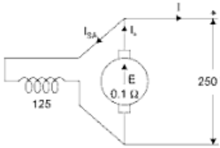
$$I_{lg} = \frac{20 \times 10^3}{250} = 80 \text{ A}$$

$$I_{sh} = \frac{250}{125} = 2 \text{ A}$$

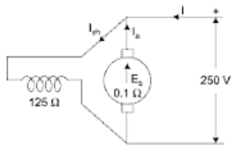
$$I_{ag} = I_{lg} + I_{sh}$$
$$= 80 + 2 = 82 \text{ A}$$

Induced emf

$$E_g = V_t + I_{ag} R_a$$
$$= 250 + 82 \times 0.1$$
$$= 258.2 \text{ V}$$



As generator



As motor

As motor

$$I = \frac{20 \times 10^3}{250} = 80 \text{ A}$$

$= E_m I_{am}$ watts.

$$= \frac{242.2 \times 78}{1000} \text{ kW}$$

$= 18.9$ kW

Example 9: A 4 pole 230 V lap wound shunt motor takes a current of 52 A from the supply mains. The total number of conductors in armature is 600 and shunt field resistance is 115 ohms. The airgap diameter is 30 cm and effective length of pole is 20 cm, and the average flux density in the airgap is 4100 guass. Determine the torque developed in the motor.

Solution: Given

$p = 4$

$V_t = 230$ V

$I_L = 52$ A

$Z = 600$

$R_{sh} = 115\ \Omega$

Airgap dia. $d = 30$ cm $= 0.3$ m (1 guss $= 10^{-4}$ wb/m^2)

Effective length of pole $l_p = 20$ cm $= 0.3$ m

$B = 4100$ guass $= 4100 \times 10^{-4}$ wb/m^2 $= 0/41$ wb/m^2

Shunt field current $I_{sh} = \dfrac{230}{115} = 2$ A

∴ Armature current,

$I_a = I - I_{sh}$

$= 52 - 2$

$= 50$ A

$$\text{Pole area} = \frac{\pi d}{p} \times l_p$$

$$= \frac{\pi \times 30}{4} \times 20 = \frac{0.06\,\pi}{4} \text{ m}^2$$

Flux per pole = Pole area × average flux density

$$= \frac{0.06\,\pi}{4} \times .41$$

$= 0.0193$ wb

Form eqn. torque developed, $T = \dfrac{\phi\,\phi Z\,I_a}{2\pi A}$ N-m ($P = A$ lap wdg)

$$= \frac{0.0193 \times 600 \times 50}{2\pi}$$

$$= \frac{3 \times 193}{2\pi} = \frac{579}{6.28} \text{ N-m}$$

= 92 Nw-m Ans.

Example 10: A 6 pole DC machine has 400 conductors and each conductor is capable of carrying 80A. Flux/pole is 0.020 Wb and the machine is driven at 1800 rpm. Calculate:

(i) Total current

(ii) emf

(iii) Power developed in armature and electromagnetic torque if conductors are:

  (a) Wave connected

  (b) Lap connected

Solution: Given data

Pole $P = 6$

Total conductors $Z = 400$

Flux per pole $\phi = 0.020$ Wb

Speed $N = 1800$ rpm

(a) For wave connected

A = parallel paths = 2

Total current I_a = Current per conductor × A

= 80 × 2 = 160 Amp

emf $E_a = \dfrac{\phi Z N P}{60\,A}$

= 720 V

Power developed in armature = $E_a I_a$ = 720 × 160 = 115.2 Kw

$$T_e = \frac{115.200}{2\pi \times \dfrac{1800}{60}} = 611.46 \text{ Nm}$$

(b) For lap connected

$$A = P$$

Total current $\quad I_a = 80 \times 6 = 480$ Amp

Emf $\quad E_a = \dfrac{\phi Z N P}{60\,A}$

$$= \frac{0.020 \times 400 \times 1800 \times 6}{60 \times 6} = 240 \text{ Volts}$$

Power developed in armature $= E_a I_a$

$$= 480 \times 240$$

$$= 115.2 \text{ Kw} \quad \text{(same as before)}$$

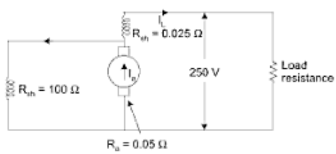
Electromagnetic torque $T_e = \dfrac{E_a I_a}{W_m}$

$$T_e = \frac{115200}{2\pi \times \dfrac{1800}{60}} = 611.46 \text{ N-m}$$

⇒ DC machine rating in terms of electromagnetic power and internal torque remains unaltered whether the armature winding is lap or wave connected.

Example 11: A 20 kW compound generator works on full load with a terminal voltage of 250 volts. The armature, series and shunt windings have resistances of 0.05, 0.025 and 100 ohms, respectively. Calculate the total emf generated in the armature when the machine is connected as short shunt.

Solution:



$$I_l = \frac{20 \times 1000}{250} = 80 \text{ A}$$

Voltage drop in series winding = $80 \times 0.025 = 2$ V

Voltage across the shunt field winding = $250 + 2 = 252$ V

$\therefore$ Shunt field current $I_{sh} = \dfrac{252}{100} = 2.52$ A

$$\therefore \quad I_a = I_{sh} + I_l = 2.52 + 80 = 82.52 \text{ A}$$

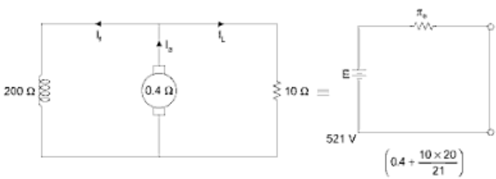
Armature resistance drop

$$I_a r_a = 82.52 \times 0.05$$
$$= 4.126 \text{ V}$$
$$E_a = V_t + I_a r_a + I_L r_{se}$$

$\therefore$ The generated emf $E_a = 250 + 4.126 + 2$

$$= 256.126 \text{ Volts}$$

Example 12: A 4-pole, wave wound, 750 rpm, shunt generator has armature and field resistances of 0.4 Ω and 200 Ω, respectively. The armature has 720 conductors and the flux per pole is 2.895 mega lines. If load resistance is 10 Ω, determine the terminal voltage of the machine.



Solution:

Given,
$$P = 4$$
$$A = 2$$
$$Z = 720$$
$$N = 750 \text{ rpm}$$
$$\phi = 2.895 \times 10^6 \text{ lines}$$
$$= 2.895 \times 10^6 \times 10^{-8} \text{ Wb}$$
$$= 2.895 \times 10^{-2} \text{ Wb}$$

The armature voltage drop, $I_a R_a = 44 \times 0.1 = 44$ V

Brush drop $= 2 \times 1 = 2$ V

$\therefore$ The total emf. generated $= 22 + 4.4 + 2 = 226.4$ volts

Example 14: A shunt generator has a full load current of 196 A at 220 V. The stray losses are 720 watts and the shunt field coil resistance is 55 Ω. It has a full load efficiency of 88%, find the armature resistance. Also find the load current corresponding to maximum efficiency.

Solution: Output = 220 × 196 = 43,120 watts ($V_t I_L$)

$$\text{Electrical input} = \frac{\text{output}}{\text{efficiency}} = \frac{43{,}120}{0.88} = 49{,}000 \text{ W}$$

$\therefore$ The total losses = 49,000 – 43,120 = 5880 W

$$I_f = \text{Shunt field current} = \frac{220}{55} = 4 \text{ A}$$

Armature current $I_a = 196 + 4 = 220$ A

Shunt field copper loss = 220 × 4 = 880 W

Stray losses = 720 W

$\therefore$ The constant losses = 880 + 720 = 1600 W

Armature copper loss = 5880 – 1600 = 4280 W

$$I_a^2 R_a = 4280$$

$$\therefore \quad R_a = \frac{4280}{(200)^2} = 0.107 \ \Omega$$

For maximum efficiency $I_a^2 R_a$ = Constant losses = 1600 watts

$$\therefore \quad I_a = \sqrt{\frac{1600}{0.107}} = 122.3 \text{ A}$$

Example 15: A 230 V, DC shunt motor takes an armature current of 3.33 A at rated voltage and at a no-load speed of 1000 rpm. The resistance of the armature circuit and field circuit are respectively 0.3 Ω and 160 Ω. The line current at full load and rated voltage is 40 A. Calculate the full load speed if the armature reaction weakens the no-load flux by 4%.

Solution: At no-load, the counter emf is,

$$E_{a1} = V_t - I_{a1} r_a$$
$$= 230 - 3.33 \times 0.3 = 229 \text{ V}$$

Field current, $I_f = \frac{230}{160} = 1.44$ A

At full load, $I_{a2} = I_L - I_f = 40 - 1.44 = 38.56$ A

∴ Counter emf at full load = $E_{a2} = 230 - 38.56 \times 0.3 = 218.43$ V

At full load the field flux, $\phi_2 = 0.96\ \phi_1$ (given)

$$\frac{E_{a1}}{E_{a2}} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

$$N_2 = \frac{E_{a2}}{E_{a1}} \times \frac{\phi_1}{\phi_2} \times N_1$$

$$= \frac{218.43}{229} \times \frac{\phi_1}{0.96\phi_i} \times 1000$$

$$N_2 = 994 \text{ rpm}$$

Example 16: A 250 V, 4 pole, shunt motor has two circuit armature windings with 500 conductors. The armature circuit resistance is 0.25 Ω, field resistance is 125 Ω and the flux per pole is 0.02 Wb. Armature reaction is neglected, if the motor draws 14 A from the mains, then compute,

(i) Speed and the internal (total or gross) torque developed

(ii) The shaft power, shaft torque and efficiency with rotational losses equal to 300 watts.

Solution: $P = 4$, $Z = 500$, $r_a = 0.25\ \Omega$, $r_f = 125\ \Omega$ and $f = 0.02$ Wb; $A = 2$

Constant shunt field current = $\frac{250}{125} = 2$A

(i) The armature current $I_a = 14 - 2 = 12$A

∴ Counter emf, $E_a = V_t - I_a r_a$

$= 250 - 12 \times 0.25 = 247$ V

Using the relation, $E = \frac{\phi ZNP}{A \times 60}$ volts

$$N = \frac{247 \times 2 \times 60}{500 \times 4 \times 0.02} = 741 \text{ rpm}$$

Electromagnetic power $P_e = E_a I_a = 247 \times 12$

$= 2964$ watts

$\therefore$ Internal torque developed $= \dfrac{P_e}{\omega} = \dfrac{2964}{2\pi N/60}$

$$= \frac{2964 \times 60}{2\pi \times 741} = 38.197 \text{ Nm}$$

(ii) Shaft power, $P = P_e$ – Rotational losses

$= 2964 - 300 = 2664$ watts

Shaft torque $= \dfrac{2664 \times 60}{2\pi \times 741}$ 34.33 Nm

Efficiency $= \dfrac{\text{Output at the shaft}}{\text{Power input}}$

$$= \frac{2664}{250 \times 14} \times 100 = 76.1\%$$

Example 17: A 200 V DC shunt motor takes 22 A at rated voltage and runs at 1000 rpm. Its field resistance is 100 Ω and armature circuit resistance (including brushes) is 0.1 Ω. Compute the value of additional resistance required in the armature circuit to reduce the speed to 800 rpm when

(i) the load torque is independent speed (as in a reciprocating pump)

(ii) the load torque is proportional to speed

(iii) the load torque varies as square of the speed (as in a fan motor)

(iv) the load torque increases as the cube of speed

Solution: Constant field current,

$$I_f = \frac{200}{100} = 2 \text{ A}$$

$\therefore$ Armature current, $I_a = 22 - 2 = 20$ A

The speed is to be controlled from 1000 to 800 rpm by armature resistance control method.

(i) Since, load torque is independent of speed, the torque is constant at both speeds,

$$T \propto \phi_1 I_{a1} \propto \phi_2 I_{a2}$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

or, $I_{a1} = I_{a2} = 20$ A (since, $\phi_1 = \phi_2$)

At 1000 rpm, the counter emf $= E_{a1} = V_t - I_{a1} r_a$

$$= 200 - 20\,(0.1) = 198 \text{ V}$$

At 800 rpm, the counter emf $E_{a2} = V_t - 20\,(0.1 + R_s)$ where, R_s is the additional resistance inserted.

Now, $$\frac{E_{a2}}{E_{a1}} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{N_2}{N_1}$$

or, $$\frac{800}{1000} = \frac{200 - 20(0.1 + R_s)}{198}$$

Solving the above equation

$$R_s = 1.98\ \Omega$$

(Loss in $R_g = (20)^2 \times 1.98 = 792$ watts)

(ii) Here, load torque $T_L \propto N$

But the electromagnetic torque $T_e = k\,\phi I_a$

$\therefore$ $$k\,\phi_1 I_{a1} \propto N_1$$

$$k\,\phi_2 I_{a2} \propto N_2$$

i.e., $$\frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1}$$

or, $$I_{a2} = 20\left(\frac{800}{1000}\right) = 16 \text{ A}$$

Now, $E_{a2} = 200 - 16(0.1 + R_s)$ and $E_{a1} = 198$ V

$$\frac{E_{a2}}{E_{a1}} = \frac{200 - 16(0.1 + R_s)}{198} = \frac{800}{1000}$$

Solving the above we get,

$$R_s = 2.5\ \Omega$$

(Loss in $R_s = (16)^2\,(2.5) = 640$ watts)

(iii) The load torque in this case, $T_L \propto N^2 \propto \phi I_a$

Using similar approach, the value R_s is found to be 3.15 Ω

∴ The equation (4) will be

$$\frac{238}{248-0.5\,I_{a_2}} = \frac{I_{a_2}}{30}$$

$$\therefore \qquad 7140 = 248\,I_{a_2} - 0.5\,I_{a_2}^2$$

or, $I_{a_2}^2 - 496\,I_{a_2} + 14280 = 0$

$$\therefore \qquad I_{a_2} = \frac{496 \pm \sqrt{(496)^2 - (4\times 14280)}}{2}$$

$$I_{a_2} = 30.7 \text{ or } 465.3$$

Since, 465.3 A is too high as this makes E_2 very very small, not acceptable of increase is special

$$I_{a_2} = 30.7 \text{ A}$$

∴ From equation (1)

$$\frac{\phi_2}{\phi_1} = \frac{I_{a_1}}{I_{a_2}}$$

$$\phi_2 = \frac{I_{a_1}}{I_{a_2}} \times \phi_1 = \frac{20}{30.7}\,\phi_1 = 0.6517\,\phi_1.$$

∴ The flux to be reduced $= \dfrac{(1-0.6517)}{\phi}\,\phi_1 \times 100$

$$= 34.8\% \text{ of main flux.}$$

Example 21: A 10 kW, 6 pole DC generator develops an emf of 200 V at 1500 rpm. The armature has a lap connected winding. The average flux density over a pole pitch is 0.9 tesla. The length and diameter of the armature are 0.25 m and 0.2 m, respectively. Calculate:

(a) the flux per pole,

(b) the total number of active conductors in the armature and

(c) the torque developed by the machine when armature supplies a current of 50A. [GATE]

Solution: Rating of DC generator = 10 kW

No of poles = 6

Induced emf $E_g = 200$ V

RPM = 1500

Generator is lap connected.

Average flux density over a pole pitch = 0.9 tesla = 0.9 Wb/m^2

Length of armature = 0.25 m

Diameter of armature = 0.2 m

Pole pitch is defined as the periphery of the armature divided by the number of poles or the area of armature between two adjacent poles.

$$\text{Area of armature} = 2\pi r \times l$$

$$= 2 \times \pi \times \frac{0.2}{2} \times 0.25$$

$$= 0.157 \text{ m}^2$$

$$\text{Flux density over one pole pitch} = \frac{\text{Flux per pole}}{\text{Area of armature between two poles}}$$

(a) ∴ Flux per pole = Flux density × Area of armature

$$= 0.9 \times 0.157$$

$$= 0.1413 \text{ Wb}$$

(b) Induced $\quad \text{emf} = \dfrac{\phi Z N}{60} \times \dfrac{P}{A}$

Since, it is lap connected $E_g = \dfrac{\phi Z N}{60}$

$(A = P)$

$$\therefore \qquad Z = \frac{60 \times E_g}{\phi \times N} = \frac{60 \times 200}{0.1413 \times 1500} = 56$$

Total number of active conductors = 56

(c) Armature current = 50 A

$E_g = 200$ V

Power developed $= E_g \times I_a$

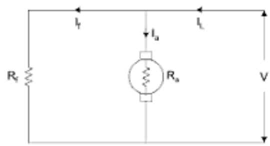
$200 \times 50 = 10$ kW

$$\therefore \text{ Torque developed } = \frac{\text{Power}}{\frac{2\pi N}{60}} \text{ where, } N \text{ is in rpm.}$$

$$= \frac{10 \times 10^3}{2 \times \pi \times \frac{1500}{60}}$$

$$= 63.66 \text{ Nm}$$

Example 22: A shunt wound motor runs at 600 rpm from a 230 V supply when taking a line current of 50 A. Its armature and field resistances are 0.4 Ω 104.5 Ω, respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (i) The no-load speed if no-load line current is 5 A (ii) The resistance to be placed in the armature circuit in order to reduce the speed to 500 rpm when the motor is taking a line current of 50 A, (iii) Percentage reduction of flux/pole in order that the speed may be 750 rpm when the armature current is 30 A with no added resistance in the armature circuit. [CSE]



Solution: Here, $N_1 = 600$ rpm V = 230 V, $I_{L_1} = 50$ A

$R_a = 0.4\ \Omega$, $R_F = 104.5\ \Omega$, Brush drop = 2 V.

(i) No-load line current $I_{L_2} = 5$ A

From the diagram $\quad I_{a_1} = I_{L_1} - \dfrac{V}{R_F}$

$$= 50 - \frac{230}{104.5} = 47.8 \text{ A}$$

$\therefore \qquad E_{b_1} = V - I_{a_1} R_a$ – brush drop

$$= 230 - (47.8 \times 0.4) - 2$$

$$= 208.88 \text{ V}$$

$E_{b_2} = V - I_{a_2} R_a$ – brush drop

$$= 230 - \left[5 - \frac{230}{104.5}\right] \times 0.4 - 2$$

$$= 226.88 \text{ V}$$

$$\therefore \qquad \frac{E_{b_2}}{E_{b_1}} = \frac{N_2}{N_1}$$

$$\therefore \qquad N_2 = \frac{E_{b_2}}{E_{b_1}} \times N_1$$

$$\therefore \text{ Speed at no-load} = \frac{226.88}{208.88} \times 600$$

$$= 651.7 \text{ rpm}$$

As may be expected from a shunt motor's speed current curve no-load speed is slightly higher than loaded speed.

(ii) $N_2 = 500$ rpm, $I_{L_2} = 50$ A

We know $E_{b_1} = 208.88$ V.

$$E_{b_2} = V - \left[I_{L_2} - \frac{V}{R_F}\right] \times (R_a + R_{se}) - \text{brush drop}$$

$$E_{b_2} = 230 - \left[50 - \frac{230}{104.5}\right] (0.4 + R_{se}) - 2$$

$$E_{b_2} = 228 - 47.8(0.4 + R_{se})$$

Also, $\qquad E_{b_2} = \dfrac{N_2}{N_1} \times E_{b_1}$

$$= \frac{500}{600} \times 208.88 = 174.07 \text{ V}$$

$$\therefore \qquad 0.4 + R_{se} = \frac{228 - 174.07}{47.8}$$

$$= 1.128$$

$$\text{Additional resistance} = R_{se} = 1.128 - 0.4$$

$$= 0.728\ \Omega$$

(iii) $N_2 = 750$ rpm, $I_{a_2} = 30$ A

$$\frac{E_{b_2}}{E_{b_1}} = \frac{\phi_2 N_2}{\phi_1 N_1}$$

$$\therefore \qquad \phi_2 = \frac{E_{b_2}}{E_{b_1}} \times \frac{N_1}{N_2} \times \phi_1$$

$$E_{b_1} = 208.88 \text{ V}$$

$$E_{b_2} = 230 - (30 \times 0.4) - 2$$

$$= 216 \text{ V}$$

$$\therefore \qquad \phi_2 = \frac{216}{208.88} \times \frac{600}{750} \times \phi_1$$

$$\therefore \qquad \phi_2 = 0.827\ \phi_1$$

$$\therefore \qquad \%\ \text{reduction} = \frac{(1 - 0.827)}{\phi_1} \phi_1 \times 100$$

$$= 17.3\%$$

Example 23: A 230 V DC shunt motor has an armature circuit resistance (r_a) of 0.4 Ω and field resistance (r_f) of 115 Ω. The motor drives a constant load torque and takes an armature current of 20 A at 800 rpm. If the motor speed is to be raised from 800 to 1000 rpm, find the resistance that must be inserted in field circuit for increasing the speed.

Solution: Let the back emf at 800 rpm be E_{b1} and at 1000 rpm be E_{b2},

$$\frac{E_{b_1}}{E_{b_2}} = \frac{k_a \phi_1 N_1}{k_a \phi_2 N_2} \qquad \text{...(1)}$$

$$E_b = V_t - I_a r_a$$

$$\Rightarrow \qquad \frac{V_t - I_{a_1} r_a}{V_t - I_{a_2} r_a} = \frac{\phi_1 N_1}{\phi_2 N_2} \qquad \text{...(2)}$$

Example 24: A 250 V DC shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω when driving a constant load torque at 600 rpm the motor draws 21 A. What will be the new speed of the motor if an additional 250 Ω resistance is inserted in the field circuit? (GATE-1988)

Solution:

$$\frac{E_{b_1}}{E_{b_2}} = \frac{V_t - I_{a_1} r_a}{V_t - I_{a_2} r_a} \qquad ...(1)$$

Given, $V_t = 250$ V, $r_a = 0.5\ \Omega$, $r_f = 250\ \Omega$

$$I_{f_1} = \frac{250}{250} = 1 \text{ Amp at 600 rpm}$$

$$I_{a_1} = I_L - I_{f1} = 21 - 1 = 20 \text{ amp at 600 rpm}$$

Let the armature current at speed N_2 is I_{a_2} for constant load torque

$$\phi_1 I_{a_1} = \phi_2 I_{a_2}$$

$$I_{a_2} = \frac{\phi_1 I_{a_1}}{\phi_2} \quad \phi_1 \propto \frac{1}{r_{f_1}} \propto \frac{1}{250},\ \phi_2 \propto \frac{1}{r_{f_2}} \propto \frac{1}{500}$$

$$\Rightarrow \quad \frac{\phi_1}{\phi_2} = \frac{r_{f_2}}{r_{f_1}} = \frac{500}{250} = 2$$

$$\Rightarrow \quad I_{a_2} = 2 I_{a_1} = 40 \text{ Amp at } N_2 \text{ rpm.}$$

from equation (1)

$$\frac{k_a \phi_1 N_1}{k_a \phi_2 N_2} = \frac{V_t - I_{a_1} r_a}{V_t - I_{a_2} r_a} \qquad \left\{\frac{\phi_1}{\phi_2} = 2\right.$$

$$\frac{2 \times 600}{N_2} = \frac{250 - 20 \times 0.5}{250 - 40 \times 0.5}$$

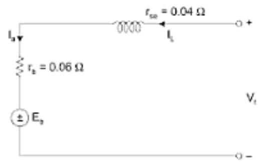
$$N_2 = \frac{\cancel{2} \times {}^{50}\cancel{600} \times 23\cancel{0}}{{}_{\cancel{12}}\cancel{240}} = \textbf{1150 rpm} \quad \textbf{Ans.}$$

SOLVED PROBLEMS ON DC SERIES MACHINE

Example 26: What is a booster?

Solution: It is a series generator and is used to inject its voltage into a circuit to compensate for a drop of voltage in the circuit. There is a drop of feeder due to its resistance. Since this voltage drop in a feeder is proportional to the current so the voltage which is injected by the booster should also be proportional to the current booster is driven at a constant speed by shunt motor.

Example 27: A 200 V DC motor has an armature resistance of 0.06 Ω and series field resistance of 0.04 Ω. If the motor input is 20 kW, find (a) counter emf of motor (b) Power developed in armature.



Solution:

$$I_L = I_a = I_{se} = \frac{20,000}{200} = 100 \text{ Amp}$$

(a) $$E_b = V_t - I_a(R_a + R_{se})$$
$$= 200 - 100\,(0.06 + 0.04)$$
$$= 190 \text{ Volt}$$

(b) Power developed in armature $P_a = E_b\, I_a$

$$P_a = 190 \times 100 \text{ watts}$$

$$\boxed{P_a = 19 \text{ kw}}$$

Example 28: The induced emf in the linear region of a series generator is given E_a = 120 Volt. Generator has R_{se} = 0.03 Ω and R_a = 0.02 Ω. It is used as a booster between a 240 V station bus-bar and a feeder of 0.25 Ω resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of 300 A.

Solution: Voltage drop across R_{se}, R_a and feeder is

$$V = 300\,(0.02 + 0.03 + 0.25) = 90 \text{ V}$$

Hence, the voltage between far end and the bus bar is

$$V_t = 240 + 120 - 90 = 270 \text{ Volts}$$

The net increase of 30 V may be beyond the desired limit the placement of field diverter resistance may be necessary to regulate the far-end terminal voltage.

Example 29: A DC series motor having a resistance of 1 Ω between terminals runs at a speed of 800 rpm at 200 V with a current of 15 A. Find the speed at which it will run when connected in series with a 5 Ω resistance taking the same current at the same supply voltage.

Solution: Given $R_a = 1\ \Omega,\ N = 800$ rpm, $I_a = 15$ Amp

$$E_{b_1} = \text{back emf} = V_t - I_a R_a$$

$$E_{b_1} = 200 - 15 \times 1 = 185 \text{ volt}$$

When 5 Ω resistance (R_s) is inserted, then E_{b_2} be the back emf.

$$E_{b_2} = V_t - I_a (R_a + R_s)$$

$$= 200 - 15\,(5 + 1) = 110 \text{ Volt}$$

Since, the field current is same in both the cases, so we have $\phi_1 = \phi_2$

$$\Rightarrow \qquad \frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2}$$

$$\Rightarrow \qquad N_2 = \frac{E_{b_2}}{E_{b_1}} . N_1$$

$$= \frac{110}{185} \times 800 = \mathbf{476\ rpm}$$

Example 30: A series motor develops 5 hp (metric) at 100 rpm. when taking a current of 30 A. Find the starting torque in nw-metres, if the starting current is limited to 45 A. Assume that the flux is proportional to the current and neglect the armature reaction.

Solution: HP developed in watts $= \dfrac{2\pi NT}{60}$

Where, N = speed in rpm.

T = torque in nw-metres.

$\therefore \quad T = \dfrac{\text{hp (Watts)} \times 60}{2\pi \times N}$

or, $\quad T = \dfrac{5 \times 735.5 \times 60}{2\pi \times 1000}$

$= 35.2$ nw-metres.

Now, torque $\propto \phi \times I_a$

Since, $\phi \propto I_a$,

$\therefore$ torque $\propto I_a^2$

$$\frac{\text{Starting torque } T_s}{T} = \frac{(\text{Starting current})^2}{I^2}$$

or, $\quad T_s = 35.2 \times \left(\dfrac{45}{30}\right)^2$

= **79.2 nw-m Ans.**

Example 31: A series motor whose combined resistance of armature and field circuit is 0.1 ohm is connected across 230 V supply mains. The armature takes 100 A and its speed is 1000 rpm. Find the speed when the armature takes 200 A and flux being increased by 20%.

Solution:

$$E_{b_1} \propto \phi_1 N_1$$

$\therefore \quad (V_t - I_{a_1} R_a) \propto \phi_1 N_1$

and, $\quad (V_t - I_{a_2} R_a) \propto \phi_2 N_2$

or, $\quad \dfrac{V_t - I_{a_1} R_a}{V_t - I_{a_2} R_a} = \dfrac{\phi_1 N_1}{\phi_2 N_2}$

Here, $V_t = 230$ V, $I_{a_1} = 100$ A, $I_{a_2} = 200$ A, $N_1 = 1000$,

$R_a = 0.1\ \Omega$ and $\phi_2 = 1.2\ \phi_1$

$\therefore \quad \dfrac{230 - 100 \times 0.1}{230 - 200 \times 0.1} = \dfrac{\phi_1 \times 1000}{1.2\phi_1 \times N_2}$

$\therefore \quad N_2 = \dfrac{1000}{1.2} \times \dfrac{210}{220}$

= **795 rpm Ans.**

Example 32: A 250 V series motor takes 20 A and runs at 1000 rpm. The motor has 4 poles and the resistance of the field coil on each pole is 0.05 ohm and the armature has a resistance of 0.2 ohm. Find the speed at which the motor runs while developing the same torque with:

(a) 10 ohms resistance parallel with the armature

(b) 0.5 ohm divertor resistance in parallel with the series field

Assume unsaturated magnetic circuit.

Solution:

Armature resistance $R_a = 0.2\ \Omega$

Series field resistance $R_{se} = 4 \times 0.05$ (as all field coils placed on poles are connected in series)

$= 0.2\ \Omega$

Resistance of the motor $= R_a + R_{se}$

$= 0.2 + 0.2$

$= 0.4\ \Omega$

Back emf of the motor while taking 20 A current,

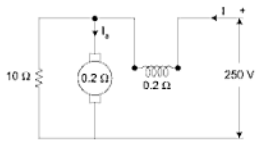
$$E_{b_1} = 250 - 20 \times 0.4$$

$$= 242 \text{ V}$$

Torque, $T_1 \propto \phi_1 \times I_a$.

Since, the magnetic circuit of the motor is unsaturated, flux would be directly proportional to the field current.

$\therefore$ $T_1 \propto 20 \times 20.$



(a) 10 ohms resistance in parallel with the armature

Let I be the input current. Drop in series field = $0.2\ I$

Voltage across the armature terminals = $250 - 0.2\ I$

$\therefore$ Current in 10 Ω resistance = $\dfrac{250 - 0.2\,I}{10}$

Armature current

$$I_a = I - \frac{250 - 0.2\,I}{10}$$

$$= \frac{10.2\,I - 250}{10}$$

Torque developed,

$$T_2 \propto \phi_2\, I_a$$

$$\propto I \times \frac{10.2\,I - 250}{10}$$

Since, the torque developed in two cases is equal,

$$\therefore \quad T_2 = T_1$$

or $$I \times \frac{10.2\,I - 250}{10} = 400$$

or $$1.02\,I^2 - 25\,I - 400 = 0$$

or $$I = \frac{25 - \sqrt{625 + 1632}}{2.04}$$

$$= \frac{25 - 47.6}{2.04}$$

$$= 35.3 \text{ A}$$

The other value of I being negative has been ignored.

$$\therefore \quad I_a = \frac{10.2 \times 35.3 - 250}{10}$$

$$= 11 \text{ A}$$

Back emf developed

$$E_{b_2} = 200 - 11 \times 0.2$$

$$= 247.8 \text{ V}$$

Again, $$E_{b_2} \propto \phi_2 . N_2$$

$$\propto 35.3\, N_2$$

and, $$E_{b_1} \propto \phi_1\, N_1$$

$$\propto 20 \times 1000$$

$$\therefore \qquad \frac{E_{b_2}}{E_{b_1}} = \frac{35.3 N_2}{20 \times 1000}$$

$$\text{or,} \qquad \frac{247.8}{242} = \frac{35.3 N_2}{20 \times 1000}$$

$$\therefore \qquad N_2 = \frac{247.8}{242} \times \frac{20 \times 1000}{35.3}$$

$$= \mathbf{580 \text{ rpm}} \quad \textbf{Ans.}$$

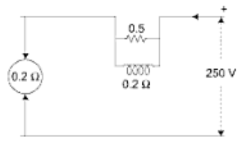
(b) 0.5 ohm diverter resistance:

Let I be the input current in this case

Current in field winding

$$= \frac{0.5}{0.5 + 0.2} I$$

$$= \frac{5}{7} I$$



$$\therefore \text{ Torque} \qquad T_3 \propto \phi_3 I_a$$

$$\propto \frac{5}{7} I.I$$

$$\propto \frac{5}{7} I^2$$

$$\text{Since,} \qquad T_3 = T_1$$

$$\text{or,} \qquad \frac{5}{7} I^2 = 400$$

or, $$I = \sqrt{\frac{7 \times 400}{5}}$$
$$= 23.6 \text{ A}$$
$$E_{b_3} = 250 - 23.6 \times 0.2$$
$$= 245.28$$
also, $$E_{b_3} \propto \phi_3 N_3$$
$$\propto \frac{5}{7} . I N_3$$
$$\propto 16.9 \, N_3$$
and, $$E_{b_1} \propto 20 \times 1000$$
$$\frac{E_{b_3}}{E_{b_1}} = \frac{16.9 N_3}{20 \times 1000}$$
$$\frac{245.28}{242} = \frac{16.9 N_3}{20 \times 1000}$$
$$N_3 = 1200 \text{ rpm}$$

Example 33: A 230 V DC series motor develops its rated output at 1500 rpm while taking 20 A. Armature and series field resistances are 0.3 Ω and 0.2 Ω respectively. Determine the resistance that must be added to obtain rated torque:

(a) at starting
(b) at 1000 rpm

Solution: For series motor $\phi \propto I_a$

so $$T_e \propto I_a^2$$

rated armature current $I_{a1} = 20$ A

(a) at starting $E_b = 0$ and $V_t = I_{a1}(r_a + r_{se} + R_{ext})$

$$R_{ext} = \frac{V_t - I_{a1}(r_a + r_{se})}{I_{a1}} = \frac{230 - 20 \times 0.5}{20} = \frac{220}{20}$$
$$R_{ext} = 11 \ \Omega$$

(b) for developing rated torque at 1000 rpm

$$I_{a2} = I_{a1} = 20 \text{ amp}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2\phi_2}{N_1\phi_1} = \frac{N_2 I_{a2}}{N_1 I_{a1}} = \frac{N_2}{N_1} = \frac{1000}{1500}$$

$$\frac{1000}{1500} = \frac{V - I_{a2}(r_a + r_{se} + R_{ex})}{V - I_{a1}(r_a + r_{se})}$$

$$I_{a2} = I_{a1} \quad \text{(rated torque)}$$

$$= \frac{230 - 20\times(0.5)}{230 - 20\times(0.5)} - \frac{20\times R_{se}}{220}$$

$$\frac{1}{11}R_{se} = \left(1 - \frac{10}{15}\right)$$

$$R_{se} = \frac{\not{5}\times 11}{_3\not{15}} = \frac{5}{15}$$

$$= \mathbf{3.667\ \Omega}$$

COMPOUND MACHINE

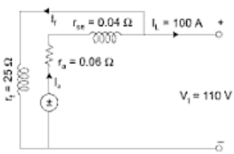
Example 34: The armature series field and shunt field resistances are 0.06, 0.04 and 25 Ω, respectively of a 110 V compound generator. The generator supplies a load current of 100 A. Find the total emf and armature current when the machine is connected as:

(a) long shunt compound generator

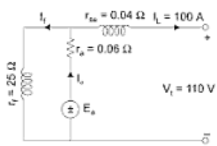
(b) short shunt compound generator

Solution:

Long shunt



Short shunt



Example 36: The full load output of a 250 V long shunt compound generator is 150 A. The core loss at full load is 1200 W, and mechanical losses are 800 W. The resistance of various windings of the machine are armature including brush contact 0.08 Ω, shunt field 62.5 Ω; series field 0.03 Ω and interpole field 0.02 Ω. Calculate:

(a) Constant losses

(b) Full load efficiency of the machine

Solution:

(a) Output = 250 × 150 = 37,500 W

Shunt field current $I_{sh} = \dfrac{250}{62.5} = 4$ A

Armature current $I_a = 150 + 4 = 154$ A.

Total armature circuit resistance

= 0.08 + 0.03 + 0.02 = 0.13 Ω

Armature circuit copper loss = $(154)^2 \times 0.13$

= 3,080 W

Shunt field copper loss = 250 × 4 = 1,000 W

Core loss = 1,200 W

Mechanical losses = 800 W

Constant losses = 1000 + 1200 + 800 = **3000 W** **Ans.**

(b) Total losses = 3080 + 3000 = 6080 W

Input = 37,500 + 6080 = 43,580 W

$$\therefore \quad \eta = \frac{37{,}500}{43{,}580} \times 100 = \mathbf{86\%} \quad \textbf{Ans.}$$

Example 37: The total core loss in a 50 kW, 250 V DC machine at its rated speed and excitation is 5000 W. If the excitation remains the same but the speed is reduced to half rated speed, the total core loss is 2000 W. Calculate the hysteresis and eddy current losses at (a) full speed (b) 125% of the full speed.

Solution: Since, the excitation remains unchanged so the flux density remains the same, so

hysteresis loss $W_h \propto N$

and eddy current loss $We \propto N^2$

or, $W_h = AN$

and, $W_e = BN^2$

where A and B are constant.

Total core loss at rated speed and excitation would be,

$$W = W_h + W_e$$

or $$5000 = AN + BN^2 \quad \text{...(i)}$$

When the speed is reduced to half rated speed, then

$$2000 = A(0.5\,N) + B(0.5\,N)^2$$

or $$2000 = 0.5\,AN + 0.25\,BN^2 \quad \text{...(ii)}$$

Multiplying (i) by 0.5

$$2500 = 0.5\,AN + 0.5\,BN^2 \quad \text{...(iii)}$$

Subtracting (ii) from (iii)

$$500 = 0.25\,BN^2$$

$$\therefore \quad BN^2 = \frac{500}{0.25} = 2000 \text{ W.}$$

From (i)

$$AN = 5000 - BN^2$$

$$= 5000 - 2000 = 3000 \text{ W}$$

(a) Hence, at full speed

$$W_h = 3000 \text{ W}$$

and, $$W_e = 2000 \text{ W}$$

(b) At 125% of full speed

$$W_h = A(1.25\,N)$$

$$= 1.25 \times 3000 = 3750 \text{ W}$$

and, $$W_e = B(1.25\,N)^2$$

$$= 1.25^2 \times 2000 = 3125 \text{ W}$$

Example 38: A 215 V DC machine has an armature resistance of 0.4 Ω. It is supplying 5 kW as a generator when run at 1000 rpm and is excited to give a terminal voltage of 215 volts. At what speed would it run as a motor if it is fed the same terminal voltage draws the same armature current but the flux/pole is increased by 10%?

Solution: *As generator*

$$I_{ag} = \frac{5 \times 1000}{215} = 23.26 \text{ A}$$

$$E_{ag} = 215 + 0.4 \times 23.26 = 224.3 \text{ V}$$

As a motor

$$I_{am} = \frac{5 \times 1000}{215} = 23.26 \text{ A}$$

$$E_m = V_t - I_{am} r_a$$

$$= 215 - 0.4 \times 23.26 = 205.7 \text{ V}$$

$$\frac{E_g}{E_m} = \frac{\phi_{ag} N_{am}}{\phi_{bm} N_{bm}} \qquad \{(\phi_b = 1.1\phi_a \text{ given})$$

$$\frac{1}{1.1} \times \frac{1000}{N_m} = \frac{224.3}{205.7}$$

N_m = **834 rpm Ans.**

EXERCISE

1. State the principle of conversion of energy and apply it to an electrical motor as an electromechanical energy conversion device.
2. What is magnetic characteristic of a DC machine? How it can be expressed in terms of field current and induced emf? Why then magnetic characteristic for decreasing field current lies above than that for increasing field? Explain fully.
3. How the magnetic characteristic of a separately excited generator are determined experimentally? What would be the effect of speed on the magnetic characteristic?
4. Explain in detail how a DC shunt generator builds up a voltage. What limits the voltage to which the machine can build up?
5. A shunt generator, when driven at its normal speed fails to self excite. Discuss the reason for this and state how will you rectify the fault.
6. Explain why does the terminal voltage rise in a series generator with the increasing load. How this characteristic is utilized to boost the feeder voltage?
7. Give three reasons why the voltage of a shunt generator drop when load is applied. Why the three effects are cumulative? If the load goes on increasing, why does the load characteristic turns over? Why there are two values of critical resistance for a shunt generator?
8. What is the effect of saturation on the load characteristic of a shunt generator? Explain why saturation is essential for the stable operation of a shunt generator.

23. Explain that for a DC motor the armature circuit method of speed control is called a constant torque drive while field-flux method is called a constant power drive.
24. Enumerate the losses occurring in a DC machine. Which of these losses are (i) Constant (ii) Proportional to current (iii) Proportional to current squared.
25. Discuss how power input and motor torque get adjusted automatically as load on the shaft of the shunt series and cummulative compound motor verges.
26. A DC shunt motor is running at a certain speed. Discuss the effect on the speed of this motor if its:
    (i) line voltage is reduced to half,
    (ii) armature terminal voltage is reduced to half but its field current is kept constant; and
    (iii) Field current is reduced to half but its armature terminal voltage is fixed.
27. Describe with circuit diagram the working of an automatic starter. How is it advantageous over a hand operated starter?
28. What are the factors which control the speed of a DC motor? Compare and contrast them.
29. Explain how the speed of a shunt motor may be varied both below and above the normal speed.
30. What is meant by 'Rheostatic control' and 'Flux control' Methods? Compare and contrast their merits and demerits.
31. Derive expressions of the power wasted in controller when the load is of (a) Constant torque (b) Torque varying as the square of the speed. State the inference derived from the above expressions.
32. Describe Ward-Leonard system of speed control of shunt motor. What are its chief advantages and disadvantage? Where is this system commonly employed?
33. What special advantage is obtained in modified Ward-Leonard control over the normal Ward-Leonard control? Compare the two.
34. Discuss field control method for varying the speed of a series motor.
35. What will happen, if a DC motor is switched on to supply directly at the time of starting?
36. What is a starter and why a starter is necessary for a motor?
37. In case of a shunt motor in what circuit is the starting rheostat connected? What will happen, if it is connected in line?

38. Give a sketch of a 3-point shunt motor starter with no-volt and overload coils and explain the function of each component.
39. Sketch a 4-point DC shunt motor starter and state in what way it is superior to a 3-point starter. With what chief disadvantage does a 4-point starter suffer?
40. How would you calculate the resistance between the successive studs of a shunt motor starter?

    A 10 hp 230 V shunt motor with an armature of 0.2 ohm resistance has an efficiency of 85% at full load. If the maximum permissible current at starting is the full load current and if the current is allowed to fall to 75% of its full value before altering the resistance, find the resistance necessary in the first three steps of the starter. Verify the results graphically. [5.83 Ω, 4.32 Ω, 3.32 Ω]
41. What type of starter is used with a series motor? Sketch the connections of a drum controller. What additional function it can perform?
42. What is a face-plate type of starter and in what way does it differ from a drum controller.

    A 20 hp 500 V series motor has 1 ohm as confined resistance of armature and field circuit. The maximum and minimum values of current during starting are, respectively 2 and 1.5 times the full load current. The full load efficiency of the motor is 80% The flux increases by 10% as the current changes from 1.5 times to 2 times full load current. Find the number of sections and the resistance of each section of the starter used with the above motor.

    [4; 1.85, 1.52 1.27 and 1.05 ohms]

NUMERICAL PROBLEMS

DC Shunt Machine

1. Calculate the emf generated by a 4 pole wave wound armature with 45 slots with 18 conductors per slot when driven at 1000 rpm the flux/pole is 0.02 Wb. [**Ans:** 540 V]
2. Determine the power output of a DC armature having 1,152 lap-connected conductors carrying 150 A and rotating at 300 rpm in a 12 pole field the flux/pole is 60 m Wb. [**Ans:** 51.84 kW]
3. A 20 kw, 220 V DC shunt generator has an armature resistance of 0.07 Ω and a shunt field resistance of 200 Ω. Determine the power developed in armature when it delivers rated output. [**Ans:** 20.835 kW]
4. A 2 pole lap wound DC shunt motor with 360 conductors operates at a constant flux level of 50 mWb. The motor armature has a resistance of

0.12 Ω and is designed to operate at 240 V taking a current of 60 A at full load.

(i) Determine the value of external resistance to be inserted in the armature circuit so that armature current does not exceed twice its full-load value at starting. [**Ans:** 1.88 Ω]

5. A belt driven 60 kW shunt wound generator running at 500 rpm is supplying full load to a bus bar at 200 V. At what speed will it run if the belt breaks and the machine continues to run taking 5 kW from the bus, bar and the armature and field resistances are 0.1 Ω and 100 Ω respectively. Brush contact drop may be taken as 2 V.

[**Ans:** 421.404 rpm]

6. A DC shunt motor runs at 750 rpm from 250 V supply and is taking a full load line current of 60 amp its armature and field resistances are 0.4 Ω and 125 Ω, respectively.

Assuming 2 V brush drop and negligible armature reaction effect, calculate.

(i) no load speed for a no-load line current of 6 amp

(ii) resistance to be added in series with the armature circuit to reduce the full load speed to 600 rpm. [**Ans:** 822 rpm. 0.7552 Ω]

7. A DC shunt motor takes an armature current of 50 A at its rated voltage of 240 V. Its armature-circuit resistance is 0.2 Ω. If an external resistance of 1 Ω is inserted in series with the armature and the field-flux remains unchanged. Calculate

(a) Percentage decrease or increase in speed of the same load torque.

(b) Percentage decrease or increase in speed for half to the load torque

[**Ans:** 21.739% decrease 8.7% decrease]

8. The emf developed in the armature of a shunt generator at 1155 rpm is 240 volts for a field current of 4.5 amperes and 255 volts for a field current of 5 amperes. The generator is now used as a motor on a 260 V supply and takes an armature current of 75 amperes. Find the motor speed when the field current is adjusted to 4.8 A. Armature resistance is 0.12 Ω. [**Ans:** 1164.3 rpm]

9. A 200 V DC shunt motor takes 27 Amp at rated voltage and runs at 800 rpm. Its field resistance is 100 Ω. If an additional resistance 20 Ω inserted in the armature circuit, compute the motor speed and the line current in case load torque varies as the square of the speed.

[**Ans:** 370.66 rpm, 7.367 Amp]

10. A 240 V shunt motor takes a current of 3.5 A on no load. The armature circuit resistance is 0.5 Ω and the shunt field winding resistance is

8

Synchronous Machine

8.1 INTRODUCTION

(i) Like a DC machine, Syn machine is also a doubly excited M/C.

(ii) There are two windings (a) Armature winding is placed on stationary member, called stator. (b) Field winding is placed on rotating member called rotor.

(iii) In case of *Motor Stator winding* is fed from 3-phase AC Supply while rotor winding is supplied with DC through two slip rings by a separate DC source.

(iv) In case of *Alternator* 3-Phase AC supply is taken from stator while rotor winding is supplied with DC and a mechanical power is given to rotor by a prime move.

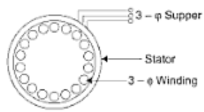
8.2 CONSTRUCTION—ALTERNATOR AND MOTOR

The basic construction of a synchronous generator and a synchronous motor is the same. Main parts of a machine are:

8.2.1 Stator

(i) It is a stationary part of a machine and is made up of sheet-sheet laminations having slots on its inner periphery.

(ii) A three-phase winding is placed in slots on stator and serves as armature winding.



8.2.2 Rotor

(i) Rotor carries a field winding which is supplied with direct current through two slip rings by a DC source.

(ii) Type of rotor construction depends upon the type of prime movers used to drive the synchronous generator.

(a) Salient (or projecting) pole type (for low speed).

(b) Non-salient (or cylindrical) pole type (for high speed).

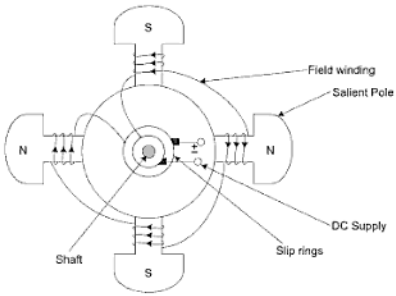
(a) Salient Pole Type

(i) In this type, salient or projecting poles are mounted on a large circular steel frame which is fixed to the shaft.

(ii) The individual field pole windings are connected in series in such a way that when the field winding is energized by the DC exciter adjacent poles have opposite polarities.

(iii) Low and medium speed alternators (120-600) have salient pole type rotor. They are driven by hydrolic (water) turnibes and I.C. engines in case of disel generators.

(iv) Salient-pole type have large dia and short axial length.



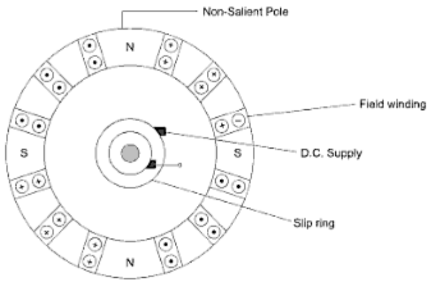
(b) Cylindrical Pole Type

(i) In this type, the rotor is made of smooth solid forged steel radial cylinder having a number of slots along the outer periphery.

(ii) Field windings are embedded in these slots and are connected in series to slip rings through which they are energized by the DC exciter.

(iii) For forming the poles some portion left unslotted.

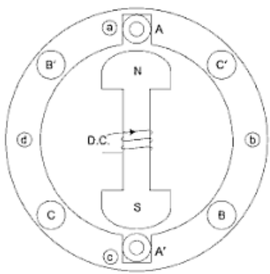
(iv) High speed alternators (1500 or 3000 rpm) are driven by steam turbines are used cylindrical rotor.



(Non-Salient Rotor)

8.3 SYNCHRONOUS GENERATOR OR ALTERNATOR

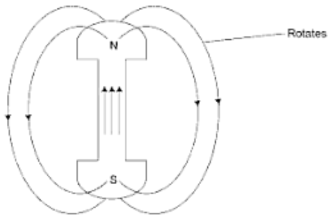
(a) Working Principle



Elementary-2-Pole Syn M/C Stator have a three-phase winding A, B, C

(i) Let a rotor is excited by a DC supply.

(ii) North and south (stationary) poles produced in a rotor.



(iii) Rotor is rotated by some prime mover then rotor *poles* will move means field due to a North and South Pole will rotate.

(iv) As the rotor rotates, the flux wave form sweeps the coil sides (AA′, BB′, CC′).

(v) By Faraday's law, a voltage is induced in the stator coils. Direction of the induced emf can by found by *FRH* rule and frequncy of induced emf is given by $f = \frac{NP}{120}$.

(vi) If the stator coil is short circuited the induced emf would cause a current to flow in the direction that would oppose any change in the flux linkage in stator coil.

(vii) This stator current will also produce a rotating field which reacts on the rotor field.

(viii) The common flux is responsible for generating the induced emf.

(ix) The stator and rotor field must be stationary w.r.t. to each other so the stator should have a same number of poles as rotor.

So, the frequency of induced emf (cycle/sec) is same as *rotor speed* (*rev/sec*).

So, the *electric frequency* is synchronized with the mechanical (rotor) speed. That is why, it is called a *synchronous generator*.

8.4 EMF EQUATION

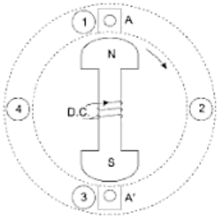
Let T = be the number of turns in coils connected in series in each phase

ϕ = flux per pole in webers

P = number of poles

N = the *r p m* of rotor

- Magnetic flux cut by a conductor of stator in one revolution of the rotor poles = $P\phi$ weber.

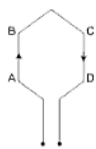


- Time taken by the rotor poles to make one revolution is $t = \dfrac{60}{N}$ sec.
- Flux out per second by a conductor of stator = $\dfrac{PN\phi}{60}$ wb/sec.
- Average emf in conductor = flux cut per sec

$$= \frac{P\phi N}{60} \text{ Volt}$$

- T is total number of turn in each phase so total number of armature conductor is

$$\boxed{Z = 2T}$$



One coil turn having two conductors AB and CD

- Average emf induced/phase = E_{av}

$$E_{av} = \frac{PN\phi Z}{60} \text{ volts}$$

- For a sinusoidal distributed flux the wave shape of the induced emf will be sinusoidal. So, we will take RMS value instead of average value.

- For a sinusoidal wave form factor = 1.11 = $\frac{E_{rms}}{E_{av}}$
- $E_{rms} = 1.11.E_{av} = 1.11\ \frac{PN\phi Z}{60}$
- $E_{rms} = 1.11.\ \not{P}.\left(\frac{{}^{2}\not{120} f}{P}\right).\phi.\frac{2T}{\not{60}}$
- $E_{rms} = 4.44\phi f T$

$N = \frac{120f}{P}$
$\rightarrow f$ = frequency of induced emf
$\rightarrow P$ = number of poles
$\rightarrow Z = 2T$

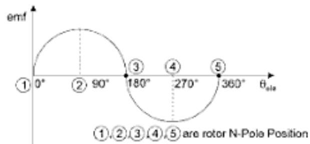
- Above equation is same as transformer, emf equation.
- Number of coils per pole per phase is usually refered to as a phase group or phase belt.
- When the stator of a three-phase, 4 pole synchronous generator has 24 slots, the number of coils in each phase group is $\left(\frac{24}{3\times 4}\right)$ **two.** There are 12 phase groups (poles × phase). All coils in a phase group are connected in series.
- Each coil in a phase group can be wound as a full pitch coil (coil span is 180° electrical)
- If the coil span differs from 180°, then it is a fractional pitch coil and in a fractional pitch coil the induced emf in it is smaller than in a full pitch coil. The reason is that flux linking the fractional pitch coil is smaller than that of full pitch coil.
- Ratio of flux linking the fractional pitch coil to the flux that would link in a full pitch coil is called the **pitch factor** (k_p)
- In order to make the induced emf approach a *sinusoidal function*, there are always more than one coil in a phase group, and the coils are not concentrated in a single slot, but distributed uniformly along the air gap periphery.
- Since, the coils are distributed spatially w.r.t. each other, so the induced emfs in these coils are not in phase. If all the coils are placed in a same slot (not distributed) then induced emfs are in phase so due to this *distribution comes* into picture

$$\text{distribution factor } k_d = \frac{\text{Phasor sum of coils emfs}}{\text{Arithmetic sum of coil emfs}}$$

- The distribution factor is unity (phasor sum = arithmetic sum of emfs) when all the coils are placed in same slot.

- For a given alternator or all rotating AC machines the product $K_p K_d$ is constant and is refereed to as the *winding factor* $kw = K_p K_d$. This value is always less than unity.
- So $$\boxed{E_{rms} = 4.44\phi f T\, k_p\, k_d}$$
- If k_p and k_d are not given, then take it unity.

8.5 FREQUENCY OF INDUCED EMF



For a 2-Pole generator

(i) Consider a conductor AA' when rotor is at position (1) induced emf in AA' is zero, rotor rotates with a speed w_m rad/sec.

(ii) When rotor rotates and comes to position (2) emf in $(AA)''$ is max.

(iii) At position (3) again induced emf is zero.

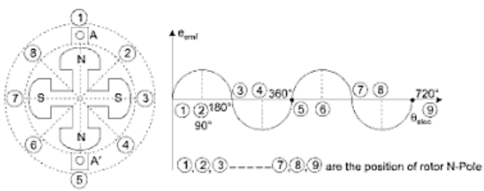
(iv) At position (4) induced emf in coil is minimum (–ve).

For a two pole machine when rotor rotates 360° (θ_m), then one cycle (θ_{elec} = 360°) of induced emf is generated.

$$\theta_m = \theta_{ele} \text{ for a two pole.}$$

For a 4-pole generator

(i) Consider a 4-pole rotor. When rotor is at position (1) induced emf in coil is zero and at position (2) it is max.



(ii) When rotor rotates one revolution ($\theta_m = 360°$), then two cycles of emf are generated ($\theta_{elec} = 720°$)

$$\boxed{\theta_{elec} = 2\theta_m}$$

So for a P Mole machine—

$$\boxed{\theta_{elec} = \frac{P}{2}\theta_{mec}}$$

$$\frac{d\theta_{elec}}{dt} = \frac{P}{2}\frac{d\theta_{mec}}{dt}$$

$$w_{ele} = \frac{P}{2} w_{mech}$$

w_{mec} = speed of rotor or speed of rotating field in mech rad/sec.
w_{ele} = electrical rad/sec = $2\pi f$

$$2\pi f = \frac{P}{2} w_{mech}$$

$$w_{mech} = \frac{4\pi f}{P} \text{ rad/sec}$$

$$2\pi \frac{N}{60} = \frac{4\pi f}{P}$$

$$\boxed{N = \frac{120 f}{P}} \textbf{ rev/min} \quad \text{or} \quad f = \frac{PN}{120} \text{ (Hz)}$$

So, the speed of rotor or rotating field which generates a frequency f of induced emf at stator is N rpm. and in order to fixe frequency the speed $N = N_s$ (fixed) and is called synchronous speed.

Rating of Alternators

(1) Rating of AC machinery, such as alternators, transformers is given by their heating losses in them.

(2) Losses in these machines are ohmic ($I^2 R$) losses, core losses and some small amount of friction and windage losses.

(3) $I^2 R$ losses depends on current and core losses on voltage.

(4) So, these losses are almost unaffected by the load power factor.

(5) Hence rating of AC machinery to supply a given load is found by *volt-ampere* of that load and not by the load power alone.

8.6 ADVANTAGES OF FIELD WINDING ON ROTOR AND STATIONARY ARMATURE WINDING

In DC M/C the field system is stationary and the armature winding is rotating. The same arrangement can be done in a syn m/c, but due to a number of reasons the field system is made rotating and armature winding is placed on stator slots. Particularly for high capacity (high voltage and higher current) synchronous machines.

(i) Ease of construction:

(a) For large three phase syn m/c. armature winding is more complex than the field winding so it is easy to place on a stationary part.

(b) Coil and phase connection including bracing of the winding can be done more easily on a stationary structure.

(ii) Number of slip ring required:

(a) When armature winding is made rotating at least three slip rings are needed to receive the generated power for the output circuit.

(b) For large syn M/C (MVA's) transferring power through brush and slip ring may cause some problems only *two slip ring* required for DC.

(c) It is also difficult to insulate the slip ring from the rotating shaft for high voltage.

(iii) Improved Ventilation arrangement:

(a) Arrangement for forced-air cooling or hyd cooling for large m/c can easily made on a stationary armature.

8.7 SYNCHRONOUS MOTOR

(i) A synchronous motor runs at synchronous speed only. Its speed is constant at all load and is given by $N_s = \dfrac{120\,f}{p}$

(ii) By adjustment of field excitation, a syn-motor can be made to operate over a wide range of power factors (lagging, unity or leading)

(iii) Syn-motors are generally of the salient pole type.

(iv) A syn-motor is not self-starting and an auxiliary means has to be used for starting it.

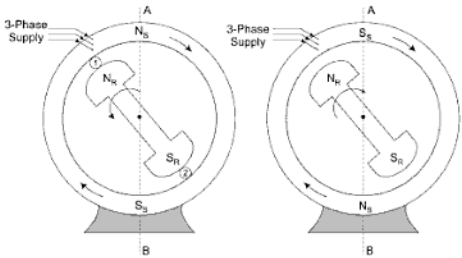
8.8 OPERATING PRINCIPLE (SYNCHRONOUS MOTOR IS NOT A SELF-STARTING MOTOR)

The fact that a synchronous motor has no starting torque can be easily explained.

(i) Let a 3-phase voltage is fed to stator winding, then stator winding produces a rotating field (stator poles rotates) which revolves round the stator at a synchronous speed $\left(N_s = \dfrac{120\,f}{p}\right)$. If the rotor is having two poles than stator is also having a two poles.

(ii) The direct current is given to rotor a field winding which set up a two stationary poles ($N_R - S_R$) on the rotor.

(iii) Now we are having a two rotating poles ($N_s - S_s$) on stator and two stationary poles on rotor.



(iv) Let at any instant, stator Ns poles is at position A rotor N_R pole is at position (1) and stator S_S and rotor S_R pole is at position B and (2), as shown in Fig. (i). It is clear stator N_S poles repel rotor S_R pole and rotor have a tendency to move **anticlockwise** direction.

(v) Due to inertia rotor can't move instantly, as rotor began to move, stator south pole (S_S) comes at position A and attract the rotor N_R pole, so rotor tends to move in **clockwise direction**.

(vi) Since, the stator poles change their polarities rapidly, they tend to pull the rotor first in one direction and then after a half cycle $\left(\dfrac{1}{2f}\right)$ in other direction.

(vii) Due to high inertia of rotor, the rotor practically remains at standstill.

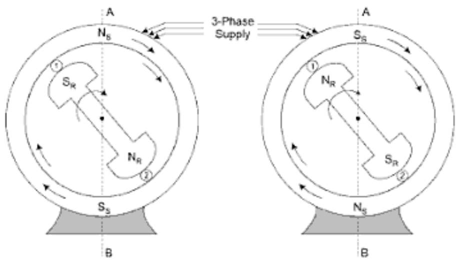
Hence, a synchronous motor has no self-starting torque i.e., a synchronous motor cannot start by itself.

8.9 HOW TO GET CONTINUOUS UNIDIRECTIONAL TORQUE?

For getting continuous unidirectional torque we have to rotate rotor poles by some external means at a speed such that they interchange their positions alongwith the stator poles,

(i) It means that when stator N_S pole comes to position A then at the same instant rotor S_S pole comes to position (1), shown in Fig. (i) below.

(ii) Now when stator pole changes its position (stator S_S poles comes to position A) then rotor pole also have to change its position [rotor N_R pole should come at position (1)] as shown in Fig. (ii) given below.



(i) (ii)

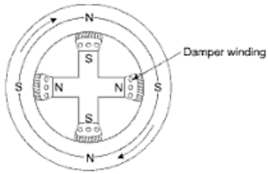
(iii) if now the prime mover driving the rotor is removed, rotor will continue to rotate at synchronous speed in the direction of rotating magnetic.

8.10 MAKING SYNCHRONOUS MOTOR SELF-STARTING

A synchronous motor cannot start by itself. In order to make the motor self-starting, a squirrel cage winding (also called damper winding) is provided on

*Since, the rotor poles are being rotated by an external means at such a speed that they interchange their positions along with the stator poles.

the rotor (same as 3-phase induction motor). The damper winding consists of copper bars embedded in the pole faces of the salient poles of the rotor as shown in Fig. below. The bars are short-circuited at the ends to form in effect a partial squirrel cage winding.



Uses to start the motor

(i) To start with, 3-phase supply is given to the stator winding while the rotor field winding is energized. The rotating stator field induces currents in the damper or squirrel cage winding and rotor starts as an induction motor.

(ii) As the motor approaches the synchronous speed, the rotor is excited with direct current. Now the resulting poles on the rotor face poles of opposite polarity on the stator and a strong magnetic interaction is set up between them. The rotor poles lock in with the poles of rotating flux. Consequently, the rotor revolves at the same speed as the stator field i.e., at synchronous speed.

(iii) Because the bars of squirrel cage portion of the rotor now rotate at the same speed as the rotating stator field, these bars do not cut any flux and, therefore, have no induced currents in them. Hence, squirrel cage portion of the rotor is, in effect, removed from the operation of the motor.

(iv) It may be emphasized here that due to magnetic interlocking between the stator and rotor poles, a synchronous motor can only run at synchronous speed. At any other speed, this magnetic interlocking of rotor poles facing opposite polarity stator poles) ceases and the average torque becomes zero. Consequently, motor comes to a halt with a severe disturbance on the line.

(v) It is important to excite the rotor with direct current at the right moment. For example, if the DC excitation is applied when N-pole of the stator faces N-pole of the rotor, the resulting magnetic repulsion will cause a

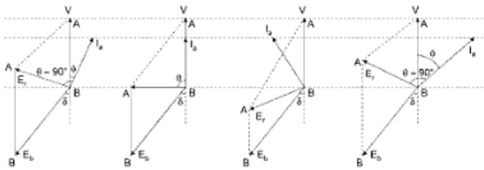
8.13 V-CURVE

V-curves show the relation between the armature current (I_a) and field current with constant shaft load and for constant terminal voltage.

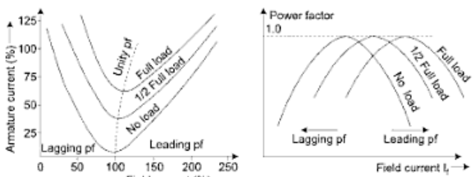
Mechanical load as well as speed is constant so power input to motor is constant. V is constant

$$P = VI_a \cos\theta = \text{const} \quad \Rightarrow \quad I_a \cos\theta = \text{constant}$$

If I_a increases $\cos\theta$ or power factor decreases.



- Armature current is minimum at unity power factor
  (1) When excitation is such that $E_b = V$ (normal excitation) motor works at lagging load armature current lags the V.
  (2) As we increase the excitation ($AB = V$) E_b increases hence E_r rotates clockwise hence I_a rotates anticlockwise so value of I_a decreases and comes in phase with V (Power factor becomes lagging to unity).
  (3) Further increase in the excitation E_r again rotates anticlockwise and I_a increases and power factor becomes leading from unity.



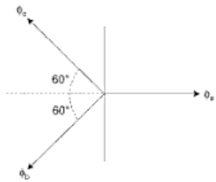
(i) Maximum value of resultant flux is equal to 1.5 times the maximum flux produced by one phase of stator winding.

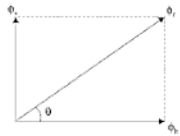
(ii) Resultant flux is rotating at the constant angular velocity, $w\,(2\pi f)$.

(iii) Field produced by the resultant mmf rotates at synchronous speed,

$$\left\{ n_s = \frac{120\,f}{P} \right\}.$$

2nd Method





Let the resultant flux be ϕ_r.

$$\phi_h = \phi_a - (\phi_b + \phi_c)\cos 60^\circ$$

$$\phi_h = \phi_m \sin wt - \frac{\phi_m}{2}[\sin(wt - 120^\circ) + \sin(wt + 120^\circ)]$$

$$\boxed{\phi_h = \frac{3}{2}\phi_m \sin wt}$$

$$\phi_v = \phi_c \cos 30^\circ - \phi_b \cos 30^\circ$$

$$= \phi_m [\sin(wt + 120^\circ) - \sin(wt - 120^\circ)] \cos 30^\circ$$

$$\phi_v = \phi_m \frac{\sqrt{3}}{2}[2 \cos \mathrm{wt} \sin 120^\circ]$$

$$\boxed{\phi_v = \frac{3}{2}\phi_m \cos wt}$$

$$\phi_r = \sqrt{\phi_h^2 + \phi_v^2} = \sqrt{\left(\frac{3}{2}\phi_m \sin wt\right)^2 + \left(\frac{3}{2}\phi_m \cos wt\right)^2}$$

$$\boxed{\phi_r = \frac{3}{2}\phi_m} \quad \text{and} \quad \theta = \tan^{-1}\frac{\phi_v}{\phi_n}$$

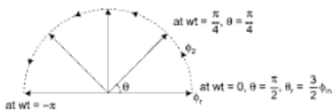
$$\theta = \tan^{-1}\left(\frac{\frac{3}{2}\phi_m \cos wt}{\frac{3}{2}\phi_m \sin wt}\right)$$

$$\theta = \tan^{-1}\cot wt$$

$$\tan\theta = \tan\left(\frac{\pi}{2} - wt\right)$$

$$\boxed{\theta = \frac{\pi}{2} - wt}$$

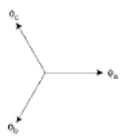
- Resultant ϕ_r is independent of time, so it is a constant flux of magnitude equal to $\frac{3}{2}\phi_m$.
- As wt (time) varies θ (position of resultant flux) changes.



So, resultant flux is rotating in nature with a constant angular velocity $w = 2\pi f$ and $f = \frac{PN_s}{120}$

- Direction of rotating flux depends upon the phase sequence.

Graphical method



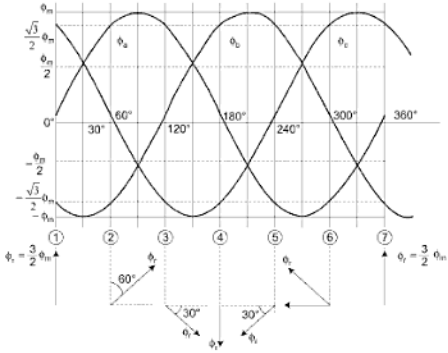
$\phi_a = \phi_m \sin wt$

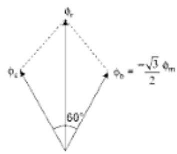
$\phi_b = \phi_m \sin (wt - 120°)$

$\phi_c = \phi_m \sin (wt - 240°)$

- *at instant (1) wt = 0*

$\phi_a = 0$ and the resultant flux is due to only





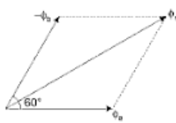
$$\phi_b = \frac{\sqrt{3}}{2}\phi_m \text{ and } \phi_c = \frac{\sqrt{3}}{2}\phi_m$$

$$\phi_r = \sqrt{\phi_c^2 + \phi_b^2 + 2\,\phi_c\,\phi_b \cos 60^\circ}$$

$$\phi_r = \sqrt{\frac{3\phi_m^2}{4} + \frac{3\phi_m^2}{4} + 2\phi_m^2\,\frac{3}{2}} = \frac{\sqrt{3}}{2}\phi_m$$

$$\boxed{\phi_r = \frac{\sqrt{3}}{2}\phi_m}$$

• *at instant (2) – when wt = 60°*



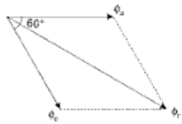
$$\phi_a = \frac{\sqrt{3}}{2}\phi_m,\ \phi_b = \frac{-\sqrt{3}}{2}\phi_m,\ \phi_c = 0$$

so resultant flux is due to ϕ_a and ϕ_b.

$$\phi_r = \sqrt{\phi_a^2 + \phi_b^2 + 2\,\phi_a\,\phi_b \cos 60^\circ}$$

$$\phi_r = \sqrt{\frac{3}{4}\phi_m^2 + \frac{3}{4}\phi_m^2 + \frac{3}{4}.\cancel{2}\,\phi_m^2.\frac{1}{\cancel{2}}}$$

$$\boxed{\phi_r = \frac{3}{2}\phi_m}$$



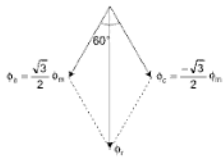
• *at instant (3) – when wt = 120°*

$$\phi_a = \frac{\sqrt{3}}{2}\phi_m,\ \phi_b = 0,\ \phi_c = \frac{-\sqrt{3}}{2}\phi_m$$

so resultant flux is due to ϕ_a and ϕ_c.

$$\phi_r = \sqrt{\phi_a^2 + \phi_c^2 + 2\phi_a\phi_c\cos 60°}$$

$$\boxed{\phi_r = \frac{\sqrt{3}}{2}\phi_m}$$



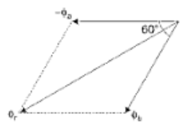
• *at instant (4) – when wt = 180°*

$$\phi_a = 0,\ \phi_b = \frac{\sqrt{3}}{2}\phi_m,\ \phi_c = \frac{-\sqrt{3}}{2}$$

so resultant flux is due to ϕ_b and ϕ_c.

$$\phi_r = \sqrt{\phi_b^2 + \phi_c^2 + 2\phi_b\phi_c\cos 60°}$$

$$\boxed{\phi_r = \frac{\sqrt{3}}{2}\phi_m}$$



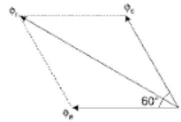
• *at instant (5) – when wt = 240°*

$$\phi_a = 0,\ \phi_b = \frac{\sqrt{3}}{2}\phi_m,\ \phi_c = \frac{-\sqrt{3}}{2}\phi_m$$

so resultant flux is due to ϕ_b and ϕ_a.

$$\phi_r = \sqrt{\phi_a^2 + \phi_b^2 + 2\phi_a\phi_b\cos 60°} = \frac{\sqrt{3}}{2}\phi_m$$

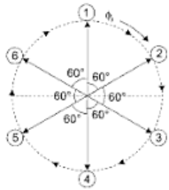
• *at instant (6) – when wt = 300°*



$$\phi_a = \frac{-\sqrt{3}}{2}\phi_m,\ \phi_b = 0,\ \phi_c = \frac{\sqrt{3}}{2}\phi_m$$

$$\phi_r = \sqrt{\phi_c^2 + \phi_a^2 + 2\,\phi_a\,\phi_b \cos 60°}$$

$$\boxed{\phi_r = \frac{\sqrt{3}}{2}\phi_m}$$



So, due to three fluxes having a phase difference of 120°, the resulting flux is rotating in nature and having a constant magnitude of magnitude $\frac{3}{2}\phi_m$.

8.15 CHARACTERISTIC FEATURES, ADVANTAGES AND DISADVANTAGES

The following characteristic features of synchronous motor are worth noting:

1. *It runs either at synchronous speed or not at all:* The speed can be changed by changing the frequency only $\left(\text{since, } N_s = \frac{120\,f}{P}\right)$.
2. *It is not inherently self-starting:* It has to be run up to synchronous or near synchronous speed by some means before it can be synchronized to the supply.
3. *It can operate under a wide range of power factors both lagging and leading.*
4. *On no-load the motor draws very little current from the supply to meet the internal losses:* With fixed excitation the input current increases

with the increase in load. After the input current reaches maximum no further increase in load is possible. If the motor is further loaded, the motor will stop.

Advantages: Synchronous motors entail the following advantages:

1. These motors can be used for *power factor correction* in addition to supplying torque to drive loads.
2. They are more efficient (when operated at unity power factor) than induction motors of corresponding output (kW) and voltages rating.
3. The field pole rotors of synchronous motors can permit the use of wider air-gaps than the squirrel cage designs used on induction motors, requiring less bearing tolerance and permitting greater bearing wear.
4. They give constant speed from no-load to full-load.
5. Electromagnetic power varies linearly with the voltage.

Disadvantages: The disadvantages of synchronous motors are:

1. They require DC excitation which must be supplied from external source.
2. They have a tendency to hunt.
3. They cannot be used for variable speed jobs as speed adjustment cannot be done.
4. They require collector rings and brushes.
5. They cannot be started under load. Their starting torque is zero.
6. They may fall out of synchronism and stop when overloaded.
7. They are costly as compared to equivalent size of 3-phase induction motor.

8.16 APPLICATIONS

The synchronous motors have the following fields of application:

1. *Power houses and sub-stations:* Used in power houses and sub-stations in parallel to the bus-bars to improve the power factor.
2. *Factories:* Used in factories having large number of induction motors or other power apparatus, operating at lagging power factor, to improve the power factor.
3. *Mills-industries, etc:* Used in textile mills, rubber mills, mining and other big industries, cement factories for power applications.
4. *Constant speed equipment:* Used to drive continuously operating and constant speed equipment such as:
    - Fans
    - Blowers.

9

Polyphase Induction Motor

9.1 INTRODUCTION

(i) Polyphase induction M/C is a single-excited (only stator or rotor winding will excite).

(ii) Normally stator wound of 3-phase winding is directly connected to a 3-phase AC source.

(iii) Its rotor winding receives energy from stator by means of induction (*transformer action*)

(iv) Induction motor can't run at *synchronous speed* $\left(n_s = \dfrac{120\,f}{p}\right)$

(v) Induction motors are of two types on the basis of rotor construction:

- (a) Squirrel cage rotor
- (b) Slip ring or wound rotor (used high starting torque and wide speed control)

9.2 CONSTRUCTION

(i) An induction motor consists of mainly two parts:

- (a) Stator
- (b) Rotor

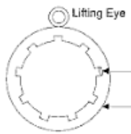
9.2.1 Stator

(i) Stator core is made of laminated steel stamplings and has slots and teeth on its inner periphery to house stator windings.

(ii) Stator carries a 3-phase winding having space displacement of 120° electrical.

(iii) The 3-phase winding is either star or delta connected and is fed from 3-phase supply.

(iv) The radial ventilating ducts are provided along the length of the stator core.



9.2.2 Rotor

(i) Rotor comprises a cylindrical laminated iron core, with slots on outer periphery.

(ii) Like stator, rotor laminations are punched in one piece for small M/C.

(iii) In larger M/C the laminations are segmented.

(iv) If there are ventilating ducts on the stator core, an equal number of such ducts is provided on rotor core.

According to windings rotor are of two types:

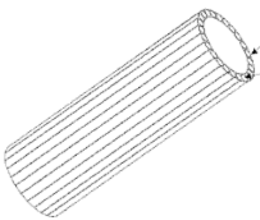
(a) Squirrel cage rotor

(b) Slip ring or wound rotor

(1) Squirrel Cage Rotor:

(i) This rotor consists of a cylindrical laminated core with parallel slots.

(ii) Rotor slots are usually not quite parallel to the shaft but *for reducing the magnetic hum and locking tendency rotor slots are slight skew.*



(iii) in rotor slots heavy copper, aluminium or alloy bars are housed.

(iv) Rotor bars are permanently short circuited at the ends. This limts that no external resistance insertion is possible.

(2) Slip Ring or Wound Rotor:

(i) The rotor is wound for the same number of poles and number of phase as that of stator.

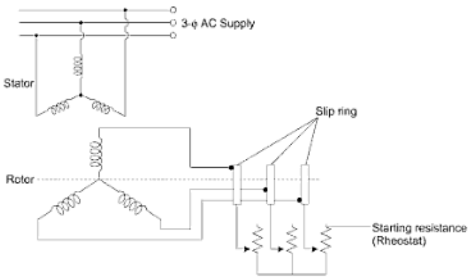
(ii) Rotor winding is either star or delta but star connection is preferred.

(iii) The three star terminals are connected to three brass slip rings mounted on rotor shaft.

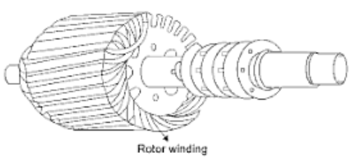
(iv) These slip rings are insulated from rotor shaft.

(v) Slip rings connected with brushes and three brushes can further be connected externally to 3-variable rehostats.

(vi) This makes possible introduction to additional resistance in the rotor circuit during starting period.



(Slip ring Induction Motor)



(Wand Rotor)

9.3 COMPARISON OF SQUIRREL CAGE AND SLIP RING INDUCTION MOTOR

Table clearly indicates the advantage and disadvantages of two types of induction motor.

Comparison of Squirrel Cage and Slip Ring Induction Motor

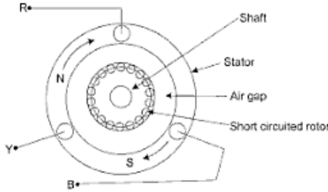
Squirrel cage Induction motor	*Slip ring Induction motor*
Advantages	**Advantages**
(i) Rugged in construction	(i) Much higher starting torque (by inserting resistances in rotor circuit)
(ii) No slip rings, brush gears, etc.	(ii) Comparatively lesser starting current (2 to 3 times the full load current)
(iii) Minimum maintenance	(iii) Capable of starting with load demanding high starting torque
(iv) Trouble-free performance	(iv) Speed control (by varying resistance in the rotor circuit)
(v) Cheaper	(v) Can be started directly on lines, (resistance in the rotor circuit acts like a starter and reduces the starting current)
(vi) Comparatively higher efficiency	
(vii) Possible to obtain medium starting torque by using double cage rotor or deep bar rotor	
(viii) Relatively better cooling conditions	
(ix) Comparatively better pull out torque and overload capacity.	
Disadvantages	**Disadvantages**
(i) Low starting torque	(i) Higher Cost
(ii) Higher starting current (5 to 6 times the full load current)	(ii) Comparatively lower efficiency
(iii) No speed control	(iii) Higher degree of maintenance
(iv) Needs a starter	(iv) Extra losses in external resistances, specially when operated at reduced speed
(v) Cannot be used for loads demanding high starting torque	(v) Extra slip ring, brush gears, etc.

9.4 PRINCIPLE OF OPERATION

(1) Why a 3-ϕ I/M is self-starting?

(2) How does the rotor rotates?

(3) Why 3-ϕ I/M do not run at syn-speed?



3-ϕ Induction motor (shorted rotor) (two pole)

(i) When a 3-ϕ stator winding having a space displacement of 120° electrical is energized from a 3-ϕ supply having 120° time displacement a rotating magnetic field is set up in stator.

(ii) This rotating magnetic field rotates with synchronous speed $\left(n_s = \dfrac{120\,f}{p}\right)$ with respect to stationary stator in the air gap.

(iii) This rotating field passes through the air gap and *cuts the stationary rotor conductors.*

(iv) Due to the *relative speed* between the rotating flux and the stationary rotor EMFs are induced in the rotor conductors.

(v) If the rotor conductors are short circuited, *currents start flowing in the rotor conductor*.

(vi) According to *Lenz's Law* the direction of the *induced current* is such that it opposes the *cause.*

(vii) Cause is the relative speed between rotating field and stationary rotor.

(viii) Hence, a rotor has a tendency to *reduce the relative speed.*

(ix) So rotor begins to move in the direction of rotating field and continues towards synchronous speed and the machine runs at a speed near but below synchronous speed depending upon load on shaft.

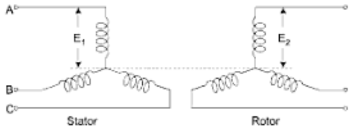
(x) As the speed of rotor reaches to synchronous speed (speed of field) relative speed is zero. Hence no emf, no current and therefore no torque at synchronous speed. Hence rotor never reaches to synchronous speed.

(xi) At syn speed *current is zero* in rotor conductor hence *no force acting on rotor* conductor and *rotor slip back, somewhat less* speed than syn, speed.

9.5 INDUCTION MOTOR AS A TRANSFORMER

(i) Stator winding and rotor winding are analogous to primary and secondary winding of a transformer.

(ii)



(iii) Rotor winding are assumed open, so current in rotor is zero and no electromagnetic torque.

(iv) Three phase-balance voltage at line frequency f_1 given to stator winding causes the production of rotating field.

(v) This rotating flux cuts both the stationary stator and rotor winding.

(vi) EMF at line frequency f_1 is induced in both of them when rotor is stationary.

$E_1 = \sqrt{2}\,\pi f_1\, k_{w1}\, N_1\, \phi$ $\qquad$ $E_2 = \sqrt{2}\,\pi f_1\, k_{w2}\, N_2\, \phi$

N_1 and N_2 are stator and rotor series turns per phase.

Comparison:

	Induction Motor	**Transformer**
(i)	One (rotor) winding is rotating	Both the winding are stationary
(ii)	Lower efficiency as no load of losses are more due to large number load or magnetizing current	Higher efficiency as no load losses are very small due to small no load current (no air gap) and no mechanical loss
(iii)	$\frac{E_1}{E_2} \neq \frac{N_1}{N_2}$ (for moving rotor)	$\frac{E_1}{E_2} = \frac{N_1}{N_2}$
(iv)	On loading, the rotor mmf affects stator winding current.	On loading, mmf of secondary affects primary current.
(v)	Very high magnetizing current.	Very low magnetizing current.

9.6 SLIP SPEED

The relative speed between the rotating magnetic field (n_s) and rotor (n_r) is called slip speed.

$$\boxed{\text{Slip speed} = n_s - n_r} \quad rps$$

Slip: Percentage change in slip speed is slip.

$$\boxed{\underset{(s)}{\text{Slip}} = \frac{n_s - n_r}{n_s} \times 100} \quad \Rightarrow \quad n_r = n_s\,(1 - s)$$

(i) When rotor is stationary ($n_r = 0$), $s = 1$ or 100%

9.7 FREQUENCY OF ROTOR EMF/CURRENT

(i) When the rotor is stationary rotor emf having same frequency as stator emf.

$$E_s = \sqrt{2}\,\pi f N_1 \phi \qquad E_r = \sqrt{2}\,\pi f N_2 \phi$$

(ii) $\text{Frequency} = \dfrac{\text{Poles} \times \text{relative speed or slip speed}}{120}$

Frequency of rotor induced emf $f_r = \dfrac{P(n_s - n_r)}{120}$ $\quad\Big|\quad s = \dfrac{(n_s - n_r)}{n_s}$

$$f_r = \left(\frac{Pn_s}{120}\right) \qquad\Big|\qquad f_{\text{stator}} = \frac{Pn_s}{120}$$

$$\boxed{f_r = sf_{\text{stator}}}$$

(iii) As rotor picks up speed hence rotor current frequency decreases.

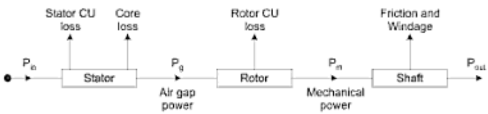
(iv) When rotor is rotating

$$E_{\text{stator}} = \sqrt{2}\,\pi f_s N_1 \phi\, k_w \quad \text{and} \quad f_r = s\, f_{\text{stat}}$$

$$E_{\text{rotor}} = \sqrt{2}\,\pi f_r N_2 \phi\, k_w$$

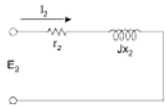
$$\Rightarrow \qquad \boxed{E_r = SE_s}$$

9.8 POWER FLOW DIAGRAM



Power Flow Diagram

9.9 ROTOR E.M.F CURRENT AND POWER



(i) Let the rotor resistance/phase be r_2 and stand still reactance/phase be x_2.

(ii) When rotor rotates

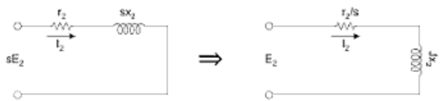
(a) Resistance is constant

(b) Reactance $(2\pi fL)$ becomes sx_2 $[2\pi\,(sf)\,L]$

(c) EMF will become sE_2

(d) Frequency $f_r = sf$

$$J_2 = \frac{sE_2}{r_2 + Jsx_2} = \frac{E_2}{\frac{r_2}{s} + Jx_2}$$



(iii) Per phase rotor current is I_2

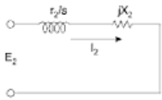
$$\boxed{I_2 = \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 = x_2^2}}} = \frac{E_2}{Z}\angle\theta_2 = \frac{E_2}{Z}\angle\tan^{-1}\left(\frac{x_2}{r_2/s}\right)$$

(iv) Per phase power input to rotor is P_g

$$P_g = E_2\, I_2 \cos\theta_2 \qquad \left\{\cos\theta_2 = \frac{r_2/s}{Z}\right.$$

$$I_2\, Z . I_2 . \frac{r_2/s}{Z}$$

9.10 TORQUE-SLIP CHARACTERISTICS



(Equivalent Ckt/phase)

$$T_e = \frac{3P_g}{w_s} = KI_2^2 . r^2/s$$

$$\boxed{T_e = \frac{k E_2^2 . \frac{r_2}{s}}{\left[\left(\frac{r_2}{s}\right)^2 + x_2^2\right]}} \quad ...(1)$$

$$\left\{ \begin{aligned} k &= \frac{3}{w_s} \\ T_e &= \frac{3P_m}{w_r} = \frac{3P_n(1-s)}{w_s(1-s)} \\ T_e &= \frac{3P_g}{w_s} \end{aligned} \right.$$

- If the rotor resistance is high than starting torque ($s = 1$) is high.
- E_2 = induced emf at rotor
  V = supply voltage
  $E_2 \propto V$
  So, Torque $\propto$ (Supply Voltage)^2
- So, torque is very sensitive to supply voltage

The variation of torque with slip, or speed, of an induction motor can be plotted from equation (1) for different values of slip s and with the motor connected to constant frequency voltage source.

Shape of the *torque-speed* or *torque-slip* curve depends upon the value of slip and induction motor can have the following operating regions.

(a) Motoring mode:

(1) Slip on motoring mode is $0 < s \leq 1$ and rotor revolves in the direction of rotating magnetic field.

(2) $$T_e = \frac{k E_2^2 . \frac{r_2}{s}}{\left(\frac{r_2}{s}\right)^2 + X_2^2}$$

$$\frac{T_{est}}{T_{efl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl}$$

$$\boxed{\frac{T_{est}}{T_{efl}} = x_1^2\left(\frac{I_{sc}}{I_{fl}}\right)^2 S_{fl}} \qquad ...(1)$$

Now if the per phase starting current from line is I_L start. Then, $I_{L\,\text{start}} = x^2 I_{sc}$

From 1

$$\frac{T_{est}}{T_{efl}} = x^2\left(\frac{1}{x^2}\frac{I_{L\,\text{start}}}{I_{fl}}\right)^2 s_{fl}$$

$$\boxed{\frac{T_{est}}{T_{efl}} = \frac{1}{x^2}\left(\frac{I_{L\,\text{start}}}{I_{fl}}\right)^2 S_{fl}}$$

So with auto-transformer, starting current (I_2 start) from mains and starting torque are reduced to x^2 times their corresponding values with DOL.

Advantages

(i) It requires only three terminals for connection.

(ii) It can be used for the motor designed on the basis of star-delta mode (starters) operation.

(iii) It provides variable torque.

Disadvantages

(i) Costlier

(ii) It reduces the power factor.

9.12.4 Star-Delta Starting

An induction motor (designed with delta run) with six terminals of the stator winding e.g., START terminals *A*, *B* and *C* connected to 3-phase power supply through switch and other FINISH terminals *A′*, *B′* and *C′* connected to the two-way switch, are shown in Figure as a star-delta starter.

The stator windings are first connected in star mode by turning the two-way switch to the left side. This position is known as 'START' position. The moment steady state is reached, two-way switch is turned to the right hand side we have, $I_Y = \frac{1}{\sqrt{3}} I_\Delta$

The current I_Y is also the starting current for star-delta starter.

$$\frac{\text{Starting current (line) for star-delta starter}}{\text{Starting current (line) with direct switching (Delta mode)}}$$

$$= \left(\frac{V_L}{\sqrt{3}\,Z}\right)\Bigg/\left(\sqrt{3}\,\frac{V_L}{Z}\right) = \frac{1}{3}$$

i.e., with star-delta starter, the starting current scales down to one-third of the current when connected in delta mode (direct switching). Similarly, the torques concerned are proportional to the square of the voltages i.e.,

$$\frac{\text{Starting torque with star-delta starter}}{\text{Starting torque with direct switching to supply } (\Delta \text{ mode})}$$

$$= \frac{(V_L/\sqrt{3})^2}{V_L^2} = \frac{1}{3}$$

i.e., with star-delta starter, the starting torque also scales down to one-third of the torque when connected in delta mode (direct switching).

In case of auto-transformer starter, with value of $k = 1/\sqrt{3}$ the starting current $I' = \frac{I}{3}$ and the starting torque $\tau_s = \frac{\tau_L}{3}$. Both starting current and torque scale down to one-third of their respective values at direct switching in delta mode.

We can say logically, that auto-transformer starter is analogous to star-delta starter as far as working is concerned, with $k = \frac{1}{\sqrt{3}}$.

Advantages

Star-delta starter has the following advantages:

1. It is cheaper in making.
2. It requires least maintenance.
3. It can improve the efficiency of the motor under low load conditions.

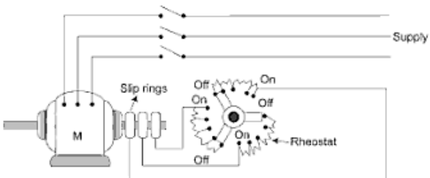
Disadvantages

(i) For applications requiring starting torque not exceeding about 50% of rated torque, it is a suitable method.

(ii) For line voltage 3.3 kV star-delta starter is not used because then stator winding in delta requires large turns making the motor becomes expensive.

9.12.5 Starting of Slip Ring Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Figure.



(i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.

(ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

9.13 APPLICATION OF 3-ϕ INDUCTION MOTOR

(i) For constant speeds and low starting torques, squirrel cage induction motor is a good choice. So squirrel cage induction motor with low rotor resistance are used for fans, centrifugal pumps, most machine tools, wood working tools, etc.

Squirrel cage induction motor with high rotor resistance (full-load slip 3 to 7%) are used for compressors, crushers, reciprocating pumps.

(ix) Equivalent load resistance, $r_l = r'_2\left(\frac{1-s}{s}\right)$

(x) Power input, $P_{in} = 3V_1 I_1 \cos\theta_1$ = stator copper loss + stator core loss + air-gap power (P_g)

(xi) Air-gap power, P_g = Mechanical power developed (P_m) + Rotor copper losses ($I_2^2 r_2$)

(xii) Mechanical power develops, P_m = Useful mechanical output (P_0) + Rotational losses

(xiii) $P_g : P_m : P_{rc} : 1 : (1-s) : s$

(xiv) Mechanical torque developed, $T_{em} = \frac{P_m}{\omega_r} = \frac{P_g}{\omega_s} = \frac{3I_2^2.r_2/s}{\omega_s}$

(xv) Starting torque, $T_{st} = K'.\frac{R_2}{R_2^2 + X_2^2}$ where, $K' = \frac{3E_2^2}{\omega_s}$

(xvi) Maximum torque, $T_{max} = \frac{3}{2}.\frac{E_2^2}{\omega_s.X_2}$, occurs at $s = \frac{r_2}{x_2}$

SOLVED EXAMPLES

Example 1: A 3-phase induction motor is sometimes called a generalized transformer. Discuss how a 3-ϕ induction motor operates under the following conditions if

(a) Rotor frequency (f_r) = stator frequency (f_s)

(b) $f_r < f_s$

(c) $f_r > f_s$

(d) Rotor generated voltage and rotor current maximum

(e) Rotor emf E_2 and rotor current are zero.

(f) Both E_r and I_r are minimum

(g) Both E_r and I_r are negative.

Solution: Frequency of stator induced emf = $\frac{P}{120}$ (speed of rotating field w.r.t. stator)

$$\boxed{f_s = \frac{Pn_s}{120}} \quad ...(1)$$

(vii) If the applied voltage per phase is 230 V, find the rotor induced emf at stand still and at 10% slip, with stator to rotor turn ratio of 1:0.5.

Solution: Given speed of induction motor at full load is 1140 rpm so speed of rotating field or synchronous speed with respect to stator is more than (round about 1140 rpm) 1140 rpm.

(i) $$n_s = \frac{120f}{p} = \frac{120 \times 60}{p} \qquad \boxed{P = \frac{120f}{n_s}}$$

$$P = \frac{120 \times \cancel{60}^{\,3}}{{}_{57}\cancel{1140}} \text{ (approx equal to syn speed)}$$

$$\boxed{P = 6.32}$$

So, number of poles is whole number either **6 or 7** but poles or *even* so $\boxed{P = 6}$

$$n_s = \frac{120f}{p} = \frac{120 \times 60}{6} = \mathbf{1200\ rpm.}$$

(ii) Full-load speed $N_r = 1140$ rpm

% slip at full-load $$S_{fl} = \frac{N_s - N_r}{N_s} \times 100 = \frac{1200 - 1140}{1200} \times 100$$

$$\boxed{S_{fl} = 5\%}$$

(iii) Freq. of rotor voltage $f_r = Sf_{sator}$

$$\boxed{f_r = 0.05 \times 60 = 3 \text{ Hz}}$$

(iv) Rotating magnetic field produced by rotor currents is rotor field with respect to rotor.

⇒ Speed of rotor field with respect to rotor $= \dfrac{120f}{p}$

$$= \frac{120 \times 3}{6}$$

or $= \mathbf{60\ rpm}$

Same rotating field is cut by the stator and rotor. The speed of rotating field on rotor is n_s and speed of rotor is n_r, then

Solution:

$$\frac{T}{T_{\max}} = \frac{2}{\dfrac{s}{s_m} + \dfrac{s_m}{s}}$$

$$s_m = \text{slip at max torque} = \frac{0.2}{2} = 0.1$$

(a) Direct on line starting

At starting $s = 1$

$$\frac{T_{\text{starting}}}{T_{\max}} = \frac{2}{\dfrac{1}{0.1} + \dfrac{0.1}{1}} = 0.198$$

$$\boxed{T_s = 0.198\, T_{\max}}$$

Given $\boxed{T_m = 2\,T_{fl}}$

$$T_s = (0.198 \times 2)T_{fl}$$
$$= 0.396 T_{fl}$$

(b) When star-delta starter is used, the starting torque is (1/3) of starting torque with DOL.

$$T_s = \frac{1}{3}(0.396)T_{fl}$$

$$T_s = 0.132\, T_{fl}$$

(c) When auto-transformer with 70% tapping.

$$Ts = (0.7)2\ Ts \text{ (on DOL)}$$
$$= (0.7)2 \times 3.96 T_{fl}$$
$$= 0.194\, T_{fl}$$

Example 5: A 12-pole, 3 phase alternator driven at a speed of 500 rpm supplies a power to an 8 pole, 3-phase induction motor. If the slip of the motor is 0.03 Pu, calculate its speed.

Solution: Alternator rotates at syn speed n_s = 500 rpm

P = pole of alternator = 12

f = freq. of supply generated by alternator and fed to induction motor.

$$f = \frac{n_s \times P}{120} = \frac{500 \times 12}{120} = \mathbf{50\ Hz}$$

Slip of I/M = 0.03 PU

Speed of rotating magnetic field on *8 pole* Induction motor due to a 3-ϕ supply of **50 Hz**.

$$n_s = \frac{120f}{P} = \frac{^{15}\cancel{120} \times 50}{\cancel{8}} = \mathbf{750\ rpm}$$

speed of motor $n_r = n_s(1 - s)$

$n_r = 750\ (1 - 0.03)$

$n_r = 750 \times 0.97$

$$\boxed{n_r = 727.5 \text{ rpm}}$$

Example 6: In a 3-phase, slip ring 4-pole induction motor, the rotor frequency is found to be 2.0 Hz while connected to a 400 V, 3 phase star-connected supply, determine motor speed, if the stator to rotor turn ratio is 1:0.5 find the rotor induced emf at above frequency 2.0 Hz. Assume rotor is also star-connected.

Solution:

Rotor freq. $f_r = 2.0$ Hz

Supply freq. $f_{stator} = 50$ Hz

Slip $S = \frac{f_r}{f_{stator}} = \frac{2}{50} = 0.08 = 8\%$

Speed of rotor $n_r = n_s(1 - s)$

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{4} = \mathbf{1500\ rpm}$$

$n_r = 1500\ (1 - 0.08)$

$= 1500\ (0.92)$

$$\boxed{n_r = 1380 \text{ rpm}}$$

Stator emf per phase $= \frac{E}{\sqrt{3}} = \frac{400}{\sqrt{3}}$ volt $= E_{stat}$

E_{rotor} at a freq of 2.0 Hz and slip at this freq = **0.08**

E_{rotor} at standstill $\Rightarrow \dfrac{E_{rotor}}{E_{stator}} = \dfrac{\text{rotor turn}}{\text{stator turn}}$

E_{rotor} at stand still $= \dfrac{400}{\sqrt{3}} \times \dfrac{0.5}{1} = \dfrac{200}{\sqrt{3}}$

Rotor induced emf at 2.0 Hz is E'_r.

$$E'_r = S\,E_{rotor} \text{ at standstill}$$

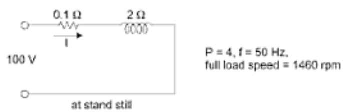
$$= 0.08 \times \frac{200}{\sqrt{3}} = \mathbf{9.237\ volt}$$

$$\boxed{E'_r = 9.237 \text{ volts}} \text{ per phase}$$

Example 7: A 4-pole, 3-phase, 50 Hz induction motor has a star-connected rotor, the rotor has a resistance of 0.1 Ω/phase and standstill reactance of 2 Ω/phase. Induced emf between the slip ring is 100 V. If the full-load speed is 1460 r.p.m., calculate

(i) Slip also the slip at which max torque occurs
(ii) EMF induced in rotor/phase
(iii) Rotor reactance/phase
(iv) Rotor current
(v) Rotor power factor.

Solution:



(i) $$S = \frac{n_s - n_r}{n_s}$$

$$n_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S_{fl} = \frac{1500 - 1460}{1500} = \frac{40}{1500} = \mathbf{2.66\%}$$

(a) Rotor standstill voltage is E_2

Stator voltage, $E_1 = 3000/\sqrt{3}$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\Rightarrow \qquad E_2 = \frac{3000}{\sqrt{3}} \times \frac{1}{3.6} = 481.125 \text{ volt}$$

$$\text{Rotor current} \quad I_2 = \frac{E_2}{\sqrt{r_2^2 + X_2^2}} \quad \text{(at starting)}$$

$$I_2 = \frac{481.125}{\sqrt{(0.1)^2 + (1.13)^2}} = \frac{481.125}{1.13 = Z}$$

$$\boxed{I_2 = 424.1 \text{ Amp}}$$

$$\text{Starting current in } I_{\text{stator}} = \frac{I_2}{3.6} = \frac{424.1}{3.6}$$

$$\boxed{I_{\text{stator}} = 117.4 \text{ Amp}}$$

$$T_{\text{starting}} = \frac{3}{w_s}.\frac{E_2^2 . r_2}{Z^2} \qquad \left\{ w_s = 2\pi \times \frac{1000}{60} = 104.7 \text{ rad/sec} \right.$$

$$= \frac{3}{104.7} \times \frac{(481.125)^2 \times 0.1}{(1.13)^2} = \mathbf{513\ Nm}$$

(b) Starting current $I_{\text{stator}} = 30$ A.

$$I_{\text{rotor}} = 30 \times 3.6 = \mathbf{108\ Amp}$$

Let the rotor resistance for limiting the current $I_2 = 108$ Amp is r_2.

$$I_{\text{rotor}} = \frac{E_2}{\sqrt{r_2^2 + X_2^2}}$$

$$108 = \frac{481.125}{\sqrt{r_2^2 + X_2^2}}$$

Motor efficiency $\eta = \dfrac{\text{output power}}{\text{input power}}$

$$\eta = \frac{41.75}{48} \times 100 = 87\%$$

Example 11: A four pole induction motor, running with 5% slip, is supplied by a 60 Hz synchronous generator. (a) Calculate the speed of the motor. (b) What is the generator speed if it has 6 poles?

Solution:

(a) Speed of motor $n_r = n_s (1 - s)$

$$n_r = \frac{120 f}{P} (1 - s)$$

$$= \frac{120 \times 60}{4} (1 - 0.05)$$

$$= 1710 \text{ rpm}$$

(b) generator rotates at synchronous speed ns.

$$n_s = \frac{120 f}{6} = \frac{120 \times 60}{6} = 1200 \text{ rpm.}$$

Example 12: A three-phase 440 V distribution circuit is designed to supply not more than 1200 amperes. Assuming that a three-phase squirrel cage induction motor has a full-load efficiency of 0.85 and a full load power factor of 0.8 and that starting current at rated voltage is 5 times the rated full-load current. What is the maximum permissible kW rating of the motor.

(a) If it is to be started at full voltage?

(b) If it is to be started using an auto transformer stepping down the voltage to 80%?

(c) If it is designed for use with a star-delta starter? (I.E.S, 1978)

Solution:

(a) Maximum permissible line current that the three phase induction motor can take from the distribution circuit is 1200 A at the time of starting. It is given that the starting current at the rated voltage is 5 times the rated current of the induction motor. Therefore the rated line current of three-phase induction motor with full voltage is 1/5 of steady current is 1200/5 = 240 A. Thus, the max. permissible induction motor rating when started at full voltage.

$$= \sqrt{3}\, V_1 I_1 \cos \varphi_1 \times \text{Efficiency}$$

$$= \sqrt{3}\,(440)\,(240)\,(0.8)\,(0.85) \text{ watts}$$

$$= 124.371 \text{ kW.}$$

(b) Maximum permissible starting current from supply mains

$$I_{supply} = x^2 I_{sc}$$

$$I_{supply} = 1200 = x^2 I_{sc} = x^2 (5 I_{fl})$$

$$1200 = (0.8)^2 (5 I_{fl})$$

$$I_{fl} = \frac{1200}{(0.8)^2 \times 5} = 375 \text{ A.}$$

∴ Maximum permissible induction motor rating

$$= \sqrt{3}\,(440)\,(375)\,(0.8)\,(0.85) \text{ watts}$$

$$= 194.33 \text{ kW}$$

(c) A star-delta starter is equivalent to auto transformer starter with 57.8% tapping.

∴ $$1200 = (0.578)^2 (5 I_{fl}) = \left(\frac{1}{\sqrt{3}}\right)^2 (5 I_{fl})$$

∴ $$I_{fl} = 720 \text{ A.}$$

Maximum permissible induction motor rating

$$= \sqrt{3}\,(440)\,(720)\,(0.8)\,(0.85) = 373.113 \text{ kW.}$$

Example 13: A 3-phase, 50 Hz induction motor has a full-load speed of 1440 r.p.m. For this motor, calculate the following:

(a) Number of poles.

(b) Full-load slip and rotor frequency.

(c) Speed of stator field with respect to

  (i) stator structure and

  (ii) rotor structure.

(d) Speed of rotor field with respect to

  (i) rotor structure

  (ii) stator structure and

  (iii) stator field.

for part 'c' and 'd' answer should be given in the rpm and rad/sec.

Solution:

(a) $$N_s = \frac{120 f}{P} = \frac{120 \times 50}{P}$$

for $$P = 4, \quad n_s = 1500 \text{ rpm}$$
$$= 6, \quad n_s = 1000 \text{ rpm}$$
$$= 2, \quad n_s = 3000 \text{ rpm}$$

Speed of rotating magnetic field in induction motor should be somewhat more than the full load motor speed (1140 rpm). So, permissible speed of rotating magnetic field is 1500 rpm. It is possible when the number of poles is 4. So

$$\boxed{P = 4 \text{ Poles}}$$

(b) Synchronous speed,

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Slip $$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$$

Rotor frequency $f_2 = sf = 0.04 \times 50 = 2$ Hz

(c) (i) Speed of stator field with respect to stator structure = N_s = 1500 r.p.m.

$$\therefore \quad w_s = \frac{2\pi N_s}{60} = \frac{2\pi \times 1500}{60} = 150.08 \text{ rad/sec.}$$

(ii) Speed of stator field w.r.t revolving rotor structure

$$= 1500 - 1440 = 60 \text{ r.p.m} = \frac{2\pi \times 60}{60} = 6.283 \text{ rad/sec.}$$

(d) (i) Speed of rotor field w.r.t rotor structure

$$= \frac{120 \text{ (rotor frequency)}}{\text{Poles}} = \frac{120 \times 2}{4} = 60 \text{ r.p.m} = 6.283 \text{ rad/sec.}$$

(ii) Speed of rotor field w.r.t stator structure

= (Mechanical speed of rotor) + (speed of rotor field w.r.t rotor structure) = 1440 + 60 = 1500 r.p.m = 150.0.8 rad/sec.

(iii) Since, both the stator and rotor fields are rotating at synchronous speed of 1500 r.p.m. with respect to stator structure, speed rotor field with respect to stator field is zero. Thus, the stator and rotor fields are stationary with respect to each other.

Example 14: A 10 KW. 400 V, 3ϕ, 4 pole, 50 Hz delta connected induction motor is running at no load with a line current of 8 A and an input power of 660 watts. At full load, the line current is 18 A and the input power is 11.20 kW. stator effective resistance per phase is 1.2 Ω and friction, winding loss is 420 watts for negligible rotor ohmic losses at no load, calculate,

(a) Stator core loss.
(b) Total rotor losses at full-load.
(c) Total rotor ohmic losses at full load.
(d) Full load speed.
(e) Internal torque, shaft torque and motor efficiency.

Solution:

(a) At no load, total power input is equal to the sum of stator core loss, friction and windage loss, stator no-road I^2R loss and negligible rotor core loss.

∴ Stator core loss

= Power input at no load – friction and windage loss – stator I^2R loss at no load

$$= 660 - 420 - 3\left(\frac{8}{\sqrt{3}}\right)^2 (1.2) = 163.2 \text{ W}.$$

(b) Air gap power at full-load,

P_g = stator input at full load – stator core loss – stator full load ohmic loss.

$$= 11200 - 163.2 - 3\left(\frac{18}{\sqrt{3}}\right)^2 (1.2) = 10{,}648 \text{ watts}.$$

∴ Total rotor loss = rotor input power, P_g – shaft power

= 10,648 – 10,000 = 648 watts.

(c) Total rotor loss consists of rotor ohmic loss and friction, windage loss.

∴ Rotor ohmic loss = $3I_2^2 r_2$ = 648 – 420 = 228 watts

(d) Now, $$\frac{3I_2^2 r_2}{S} = P_g$$

11. A 3-ϕ, 6 pole, 400 V, 50 Hz induction motor develops maximum torque at a speed of 792 rpm. The rotor resistance is 0.2 Ω/phase. Find rotor reactance per phase at standstill. [**Ans:** 0.9615 Ω]
12. A 3-phase, 50 Hz star connected, 208 V induction motor has an effective stator to rotor turn ratio of 3:1. It requires 52 V to produce rated stator line current at standstill. The rotor resistance and standstill reactance per phase are 0.1 Ω and 0.5 Ω. Find rotor current with rotor blocked and at 5% slip.
    [**Ans:** 19.62 A, 19.43 A]
13. A 3-phase, 6 pole, 60 Hz induction motor has a no-load speed of 1196 rpm and full-load speed of 1150 rpm. At standstill-rotor induced emf/phase is 55 Volt. Find (a) slip at no load and full-load (b) rotor induced emf at no load and full load. [**Ans:** 0.33%, 4.16% (b) 183 V, 2.29 V]
14. A 3-phase, 4 pole, 50 Hz, 208 V induction motor has a starting line current of 700 A and a starting torque of 225 N-m. If a reduced voltage of 120 V is applied to stator at the time of starting, find starting torque and starting line current. [**Ans:** 75 N-m, 403 A]
15. A 3-phase, 60 Hz, 10 pole induction motor has a full-load speed of 690 rpm, the rotor resistance and standstill reactance per phase are 0.3 Ω and 1.1 Ω, respectively. Find (a) Speed at full-load torque (b) additional rotor resistance per phase to obtain maximum torque at starting (c) Full-load speed with added rotor resistance.
    [**Ans:** (a) 523.64 rpm (b) 0.8 Ω (c) 610 rpm]
16. Explain the terms air gap power P_g, internal mechanical power developed and shaft power P_{sh}. Show that
    $$P_g : \text{rotor ohmic loss} : P_m = 1 : s : (1-s)$$
17. A 4-pole, 3 phase, 50 Hz, synchronous machine has its rotor directly coupled to that of a 3-ϕ slip ring induction motor. Stators of both machines are connected to the same 3-ϕ, 50 Hz supply. It is desired to use such as arrangement to generate 150 Hz across the rotor terminals of the induction motor. Determine the number of poles for which the induction machine should be wound.
    [**Ans:** 8 or 16 poles]
18. A 3-phase, 50 Hz induction motor has a starting torque which is 1.25 times full-load torque and a maximum torque which is 2.5 times full-load torque. Neglecting stator resistance and rotational losses and assuming constant rotor resistance, find
    (i) The slip at full-load
    (ii) Slip at max torque
    (iii) The rotor current at starting in per unit of full-load rotor current.
    [**Ans:** (a) 25 Hz or 125 Hz (b) 0.056, 0.268, 4.722]

3rd Proof—13-3-07

10

Single-Phase I/Motor

10.1 CONSTRUCTION

(i) 1 – ∅ induction motor physically looks similar to that of a 3 – ∅ induction motor.

(ii) Its stator is provided with a 1 – ∅ winding.

(iii) Rotor construction is identical to that of 3 – ∅ induction motor (*squirrel cage type*)

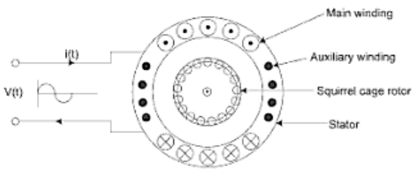
(iv) Rotor of any 1 – ∅ *I/M* is interchangeable with that of a 3 – ∅ *I/M* (induction motor).

(v) A simple 1 – ∅ winding would produce *no rotating magnetic field* and no *starting torque*. Hence, 1 – ∅ *I/M* is not self-starting.

(vi) For making it self-start the stator winding is split into two windings (a) starting (auxiliary) winding (b) main (running) winding.

(vii) 1 – ∅ motors are built with two-phase windings having space displacement of 90° electrical.

(viii) Time phase shift is achieved by connecting resistance, inductance or capacitance in series with starting winding and such arrangement is connected in parallel to main winding and then to 1-phase AC supply.



10.2 TYPES

Single-Phase (1 – ∅) induction motors are named according to the *starting methods*.

(i) Split phase method
- Split phase resistance start
- Split phase capacitor start
- Split phase capacitor start capacitor run motor

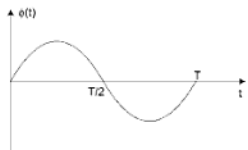
(ii) Shaded pole type

(iii) Reluctance start

10.3 WORKING PRINCIPLE

(i) When a distributed stator winding is fed from a single-phase supply.

(ii) An alternating (pulsating) field is produced along one space axis only.



(iii) Due to this alternating, pulsating or time varying flux. An alternating EMF hence alternating current is induced in the rotor conductor.

(iv) Now these current carrying conductors experience a force (*when a current carrying conductor is placed in magnetic field it experiences a force*).

(v) But after each half = (after T/2 sec) cycle the direction of the induced current is changed. Hence, the direction of force or torque is changed after each cycle.

(vi) So pulsating flux acting on a stationary squirrel-cage rotor *cannot produce rotation* and therefore 1 – ∅ induction motor is not self-start.

(vii) However, if the rotor of such a M/C is given initial start by hand or otherwise in either direction, then immediately a torque arises and motor accelerates in that direction.

The above peculiar behaviour can be explained by

(i) Double-field revolving theory

(ii) Cross-field theory.

10.4 DOUBLE-FIELD REVOLVING THEORY

According to this theory, an alternating sinusoidal flux can be represented by two revolving flux.

- Each flux equal to half the value of alternating flux.
- Each flux rotating synchronously $\left(N_S = \dfrac{120f}{P}\right)$ in opposite direction.

(i) Let the alternating flux be a maximum value of $\varnothing_m$.

(ii) Now, let us have two fluxes $\left(\varnothing_A = \varnothing_B = \dfrac{\varnothing_m}{2}\right)$ rotating in opposite direction synchronously.

(iii) At instant (a) resultant flux $\varnothing_r = \varnothing_m$

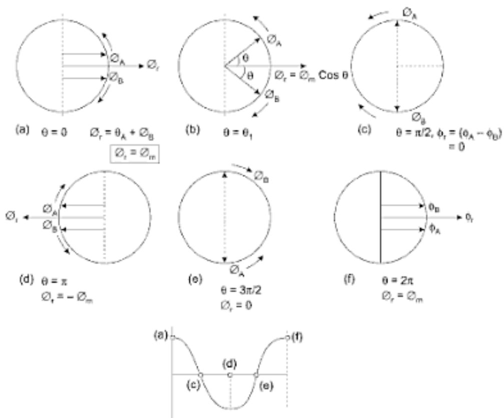
(iv) At instant (b) $\varnothing_r = \varnothing_m \cos\theta$

(v) At instant (c) $\varnothing_r = 0$

(vi) At instant (d) $\varnothing_r = -\varnothing_m$

(vii) At instant (e) $\varnothing_r = 0$

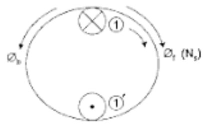
(viii) At instant (f) $\varnothing_r = \varnothing_m$



Hence, a two revolving field in opposite direction with half magnitude of $\varnothing_m$ can be represented by a pulsating flux of $\varnothing_m$ magnitude.

With the help of double field revolving theory prove that 1 – ∅ *I/M* is not self starting.

(i) According to Double field revolving theory rotor is influenced under two rotating fluxes $\varnothing_f$ (forward) and $\varnothing_b$ (backward)



(ii) So two torques (T_f, T_b) are acting on the conductor (1)-(1)′.

$$T_f = K\,I_2^2\,r_2/S_f, \quad T_b = K\,I_2^2\,r_2/S_b$$

(iii) So net or resultant torque in the rotor $\boxed{T_r = T_f \sim T_b}$

(iv) If $T_f > T_b$, then rotor rotates in forward direction.
if speed of rotor in forward direction is N rpm

(a) Slip with respect to forward rotating flux is $S_f = S$

$$S_f = S = \frac{N_S - N}{N_S} = \frac{\text{Slip speed}}{N_S}$$

(b) Slip with respect to backward rotating flux is S_b

$$S_b = \frac{N_S - (-N)}{N_S} = \frac{(N_S - N_S) + 2N_S}{N_S}$$

$$= \frac{(N - N_S)}{N_S} + 2 = (2 - S)$$

So, $\boxed{S_f = S,\ S_b = 2 - S}$

(v) At the starting $S_f = S = 1$, $S_b = 2 - S = 1$

(vi) So $T_{start} = T_f \sim T_b = 0$. Hence, 1 – ∅ induction motor is not self starting.

(vi) Because of different resistance to reactance ratio the two winding currents I_a and I_m or fluxes $\varnothing_a$ and $\varnothing_m$ are out of phase approximate 50°.

(vii) Current I_a in auxiliary winding reaches it peak value earlier than current I_m in main winding so torque

$$\boxed{T = K\, I_m\, I_a \sin\theta}$$

(viii) Direction of rotation depends on space angle of magnetic field from auxiliary winding is 90° ahead or behind the angle of main winding.

(ix) So by reversing the connection of auxiliary winding direction of rotation can be reversed.

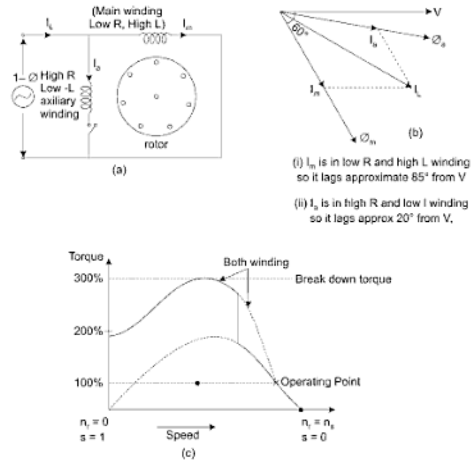
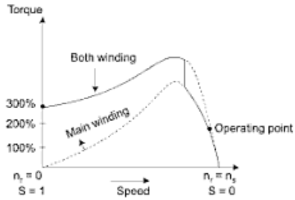


Fig. 10.1 Schematic representation of (a) Split phase motor (b) Phasor diagram (c) Torque-speed characters



10.6.3 Capacitor Start Capacitor Run Motor

(i) This motor has two capacitors in the auxiliary winding. Both the capacitors C_1 and C_2 are in the circuit during starting.

(ii) After the motor has picked up the speed the centrifugal switch opens and disconnects the capacitor C_2 (low value electrolytic capacitor and short duty cycle).

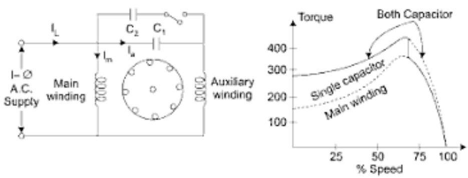
(iii) Auxiliary winding and C_1 (large value AC oil type capacitor long duty cycle) remains in the circuit during running condition too.

(iv) Thus, the motor works actually as a two-phase motor.

(v) Use of capacitor C_1 during running condition is to improve the power factor.

(vi) Thus the efficiency of a single-phase induction motor can be improved by employing another capacitor.

(vii) There are two capacitor start and run so it is also known as two value capacitor motor.

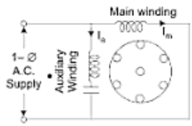


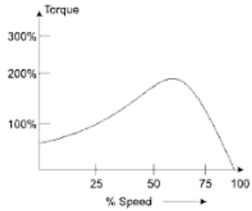
10.6.4 Permanent Split Capacitor Motor

(i) In application where motor starts at zero load, it is possible to avoid the expense of centrifugal switch as well as starting capacitor.

(ii) Same capacitor is used for starting and running so capacitor permanent in auxiliary winding.

(iii) Since, capacitor C_1 is small, the current in the auxiliary winding circuit during starting torque is small and starting torque is low.



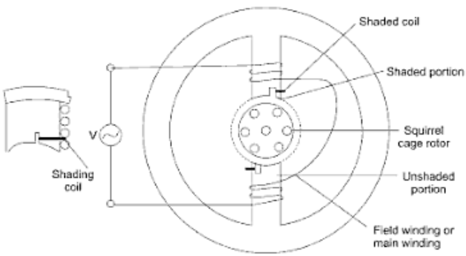


10.6.5 Shaded pole Single-phase *I/M*

(1) Necessary phase-splitting is produced by induction.

(2) These motors have salient poles on the stator and a squirrel cage type rotor.

(3) The laminated pole has a slot cut across the lamination called shaded part and a short circuited copper coil on it, is called shading coil.



(4) The two salient poles are excited by 1-phase AC current.

(5) Each pole includes a small portion that has a short circuited winding. This part of the pole is called the shaded pole.

Appendix 1

Solved Question Papers of Previous Years

FIRST/SECOND SEMESTER EXAMINATION, 2001-2002

Note: Answer all the *Five* questions.

1. Answer any *Three* parts of the following:

(a) Fig. A1-1 shows one node of an electric circuit. Find V_4 using KCL. [Given: $V_2 = 5e^{-2t}$, $V_3 = 2e^{-2t}$, $i_1 = 2e^{-2t}$]

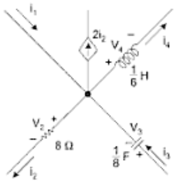


Fig. A1-1

(b) The voltage and current through a circuit element are:

$$V = 100 \sin (314t + 45°) \text{ volt}$$

$$i = 10 \sin (314t + 315°) \text{ ampere}$$

(i) Identify the circuit element.

(ii) Find its value.

(iii) Obtain the expression for power.

(c) A coil of resistance 12 Ω and inductance 0.05 H, a non-inductive resistor of 20 Ω resistance and a loss-free 40 μF capacitor are connected across a 240 volt, 50 Hz sinusoidal supply. Calculate:
   (i) The current and
   (ii) The power factor of the circuit.

(d) A series RLC circuit has $R = 10\ \Omega$, $L = 0.1$ H, and $C = 8\ \mu$F. Determine
   (i) The resonant frequency,
   (ii) Q-factor of the circuit at resonance, and
   (iii) The half power frequencies.

2. Answer any *Three* parts of the following:

(a) With the help of star/delta transformation, obtain the value of current supplied by the battery in the circuit shown in Fig. A1-2.

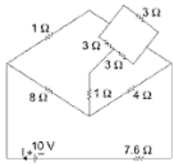


Fig. A1-2

(b) Find the value of R in the circuit of Fig. A1-3 such that the maximum power transfer takes place.

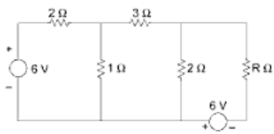


Fig. A1-3

(c) In the circuit of Fig. A1-4, find the branch current I_2 which flows through R_2, when R_2 has the following values:

5 Ω, 15 Ω and 50 Ω

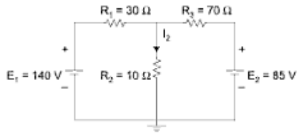


Fig. A1-4

(d) For the circuit of Fig. A1-5, find *I* using Superposition Theorem.

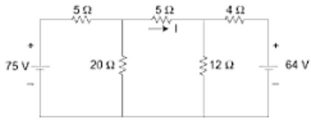


Fig. A1-5

3. Answer any *Two* of the following:

(a) Describe the operating principle of measuring instruments. What are the various torques acting on the moving mechanism of the instruments? How are these obtained and what are their roles in the operation of the instruments?

(b) Describe a PMMC instrument in detail. Also discuss its advantages and disadvantages.

(c) Attempt the following:

(i) In the two-wattmeter method of power measurement in a three-phase circuit, the readings of the wattmeters are 1000 W and 550 W. What is the power factor of the load?

(ii) A single-phase energymeter has a constant speed of 1300 revolutions /kWh. The disc revolves at 3.5 RPM when a load of 150 W is connected to it. If the load is on for 11 hours, how many units are recorded as error? What is the percentage error?

4. Answer any *Two* parts out of the following:

(a) For the AC excited magnetic circuit of Fig. A1-6, calculate the excitation current and induced emf of the coil to produce a core flux of 0.6 sin 314*t* mWb.

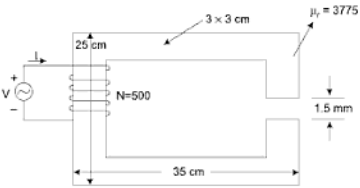


Fig. A1-6

(b) Attempt the following:

(i) Develop the equivalent circuit of a single-phase transformer. Mention the physical significance of all its parameters.

(ii) A 200/100 volt, 50 Hz transformer has an impedance of $0.3 + j\,0.8$ Ω in the 200 volt winding and an impedance of $0.1 + j\,0.25$ Ω in the 100 volt winding. What are the currents on the high and low sides, if a short circuit occurs on the 100 volt side with 200 volt applied to the high side?

(c) Why is three-phase system popular over single-phase system? What do you understand by Single Line Diagram of a power system? Draw and explain the various components for a simple power system having one transmission line and two bus bars connected to a generator at one end, and two rotating loads and one static load at the other.

5. Answer any *Three* parts out of the following:

(a) Attempt the following:

(i) Explain the motor and the generator actions of a DC machine.

(ii) What are the various losses occurring in rotating machines? Mention the methods to reduce them.

(b) Describe the open circuit characteristics of DC generators. How do you obtain the internal characteristics using external characteristics of a DC shunt generator?

(c) Compare a three-phase Induction motor with a single-phase Induction motor on the basis of the following points:

(i) Starting torque

(ii) Slip torque characteristics

(iii) Magnetic field

(iv) Power factor and efficiency

(v) Applications

(d) A 27 hp, 230 volt, 1200 rpm shunt motor has four poles, four parallel armature paths, and 868 armature conductors. The armature circuit resistance is 0.18 Ω. At rated speed and rated output the armature current is 70 ampere and the field current is 1.5 ampere.

Calculate:

(i) The electromagnetic torque,

(ii) The flux per pole,

(iii) The rotational losses, and

(iv) The efficiency.

(e) Explain a synchronous machine on the basis of the following points:

(i) Construction,

(ii) Operation as a motor and as a generator,

(iii) Application for power factor control.

FIRST SEMESTER EXAMINATION, 2002-2003

1. Answer any *Four* of the following:

(a) Show that the average power demand, in a purely inductive AC circuit, is zero.

(b) In an AC circuit, the current is given by—

$i = 22 \sin\left(314t - \frac{\pi}{6}\right)$ A. If the voltage of the circuit is the reference quantity, determine:

(i) Power factor, (ii) rms value of the current,

(iii) Frequency of the current.

(c) A resistance and an inductance are connected in series across a voltage: $v = 238 \sin 314t$. The current expression is found to be $4 \sin\left(314t - \frac{\pi}{4}\right)$. Find the values of resistance, inductance and power factor.

(d) Two impedances, $z_1 = (10 + j15)\ \Omega$ and $z_2 = (6 - j8)\ \Omega$ are connected in parallel. The total current supplied is 15 A. What is the power taken by each impedance?

(e) A 20 Ω resistor is connected in series with an inductor and a capacitor, across a variable frequency 25 V supply. When the frequency is 400 Hz, the current is at its minimum value of 0.5 A and the potential difference across the capacitor is 150 V. Calculate the resistance and inductance of the inductor.

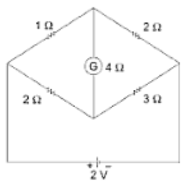
(f) An inductor of resistance R ohms and inductance of L henries is connected in parallel with a capacitor of C farads. Derive an expression for resonant frequency.

2. Answer any *Four* of the following:

(a) Three resistances r, $2r$ and $3r$ are connected in delta. Determine the resistances for an equivalent star connection.

(b) Calculate current through the galvanometer in the following bridge:

Fig. A1-12



(c) Using 'Superposition Theorem' or 'Norton's Theorem', find the current *I* in the following:

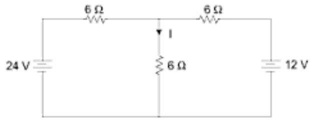


Fig. A1-13

(d) Using Thevnin's Theorem, find the current *I* in the following:

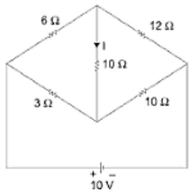


Fig. A1-14

(e) State and prove maximum power transfer theorem.

(f) Using Loop current method, find the currents I_1 and I_2 in the following:

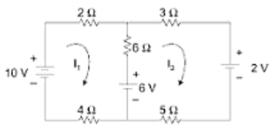


Fig. A1-15

3. Answer any *Two* of the following:

(a) Explain the principle and construction of attraction type moving iron instruments. Discuss their merits and demerits.

(b) With appropriate mathematical derivations, explain the principle of electrodynamic wattmeter.

(c) Three identical resistors of 20 Ω each are connected in star to a 415 V, 50 Hz, three-phase supply. Calculate:

(i) The total power consumed,

(ii) The total power consumed, if they are connected in delta,

(iii) The power consumed, if one of the resistors is opened.

4. Answer any *Two* of the following:

(a) Derive the e.m.f. equation of a single-phase transformer.

(b) The following results were obtained on a 50 kVA, 2400/120 V single-phase transformer:

(1) O.C. test on the l.v. side 396 W, 9.65 A, 120 V.

(2) S.C. test on the h.v. side 810 W, 20.8 A, 92 V.

Calculate:

(i) The equivalent circuit parameters,

(ii) Efficiency at full-load, 0.8 power factor lagging.

(c) Using Double-revolving Field Theory, explain the principle of operation of a single-phase motor.

(d) What are the causes of low power factor in a supply system? Discuss its harming effect. How can the power factor be improved?

5. Answer any *Four* of the following:

(a) A DC generator has an armature e.m.f. of 100 V when the useful flux per pole is 20 mWb, and the speed is 800 rpm. Calculate the generated e.m.f.

(i) with the same flux and a speed of 1000 rpm,

(ii) with a flux per pole of 24 mWb and a speed of 900 rpm.

(b) Draw and explain the load characteristics of:

(i) Shunt generator,

(ii) Compound generator,

(iii) Series generator.

(c) Derive efficiency equation of a DC motor.

(d) A 250 V DC shunt motor having an armature resistance of 0.25 Ω carries an armature current of 50 A and runs at 750 r.p.m. If the flux is reduced by 10%, find the speed. Assume that the torque remains the same.

(e) Describe the principle of operation of three-phase induction motor.

(f) Why are the synchronous motors not self-starting? Explain the various methods of starting these motors.

(d) For the two circuits given below, calculate the current *A*.

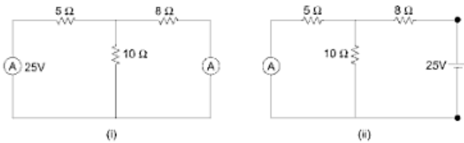


Fig. A1-17

What theorem does the two results signify?

(e) Describe the following in case of measuring instruments:

(i) Deflecting torque (ii) Controlling torque (iii) Damping torque.

(f) Two ammeters, one with a current scale of 10 A and resistance of 0.01 ohm and the other with a current scale of 15 A and resistance of 0.005 ohm are connected in parallel. What can be the maximum current carried by this parallel combination so that no meter reading goes out of the scale?

3. Answer any *Two* parts of the following:

(a) A balanced star-connected load of $(8 + j6)\ \Omega$ per phase is connected to a three-phase 400 V supply. Find the line current, power factor and three-phase volt-amperes.

(b) Describe the power losses that take place in a transformer. On what factors and how do each of these losses depend?

(c) Following results were obtained on a 100 kVA, $\dfrac{11000}{220}$ V single-phase transformer:

(i) O.C. Test (L.V. side) 220 V, 45 A, 2 kW

(ii) S.C. test (H.V. side) 500 V, 9.09 A, 3 kW

Determine equivalent circuit parameters of the transformer referred to low-voltage side.

4. Answer any *Three* parts of the following:

(a) Derive an equation for EMF in a DC machine.

(b) Enumerate all the parts of a DC machine. State the material and function of each part.

(c) A 4-pole DC shunt motor working on 220 V DC supply takes a line current of 3 A at no-load while running at 1500 rpm. Determine, of

SECOND SEMESTER EXAMINATION, 2003-2004

1. Answer any *Three* parts of the following:

(a) Two AC voltages are represented by—

$$v_1(t) = 30 \sin (314t + 45°)$$
$$v_2(t) = 60 \sin (314t + 60°)$$

Calculate the resultant voltage $v(t)$ and express in the form—

$$v(t) = v_m \sin (314t + \phi)$$

(b) Calculate the average value, effective value and form factor of the output voltage wave of a half wave rectifier as shown in Fig. A1-18.

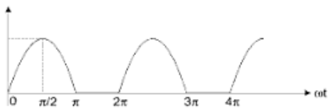


Fig. A1-18

(c) The voltage applied to a circuit is $v = 100 \sin (\omega t + 30°)$ and the current flowing in the circuit is $i = 15 \sin (\omega t + 60°)$. Determine the impedance, resistance, reactance, power and the power factor of the circuit.

(d) If the bandwidth of a resonant circuit is 10 kHz and the lower half power frequency is 120 kHz, find out the value of the upper half power frequency and the quality factor of the circuit.

(e) Differentiate between—

(i) Statically and Dynamically induced emf

(ii) Electric and magnetic circuits.

2. Answer any *Three* parts of following:

(a) Derive the expression for torque produced in a moving coil type of instrument and explain, briefly, its working.

(b) A 40 amperes, 230 volts energymeter on full-load test makes 60 revolutions in 46 seconds. If the normal disc speed is 500 revolutions per kWh, find the percentage error with proper sign by assuming the load to be purely resistive.

(c) For the circuit shown in Fig. A1-19, find voltages of nodes *B* and *C*, and determine current in 8 Ω resistor.

(d) Draw the Norton's equivalent circuit across *AB*, and determine current flowing through 12 Ω resistor for the network shown in Fig. A1-20.

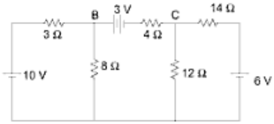


Fig. A1-19

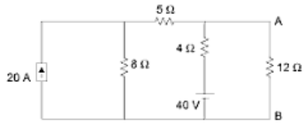


Fig. A1-20

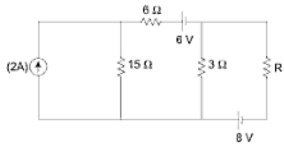


Fig. A1-21

(e) Find the value of resistance '*R*' to have maximum power transfer in the circuit as shown in Fig. A1-21. Also obtain the amount of maximum power.

3. Answer any *Two* parts of the following:

(a) A balanced 3-phase star-connected load of 120 kW takes a leading current of 85 amp, when connected across a 3-phase 1100 volts, 50 Hz supply. Obtain the values of the resistance, impedance and capacitance of the load per phase and also calculate the power factor of the load.

(b) A $\frac{250}{500}$ volts, single-phase transformer gave the following test results:

Short circuit test : 20 volts, 12 A, 100 watts
(L.V. short circuited)

Open circuit test : 250 volts, 1 A, 80 watts
(Low voltage side)

Determine the efficiency of the transformer when the output is 12 Amp, 500 volts at 0.85 power factor lagging.

FIRST SEMESTER EXAMINATION, 2004-2005

1. Answer any *Four* of the following:

(a) In a purely inductive circuit, prove from first principles, that the current lags behind applied voltage by quarter of a cycle and also show that the average power demand, is zero.

(b) An alternating current is given by $i = 20 \sin 600t$ amperes. Find the (i) frequency, (ii) peak value of current and (ii) the time taken from $t = 0$ for the current to reach a value of 10 A.

(c) For a LCR series circuit derive an expression for resonant frequency, bandwidth and quality factor.

(d) Two impedances $Z_1 = (10 + j5)$ and $Z_2 = (8 + j6)$ are connected in parallel across a voltage of $V = 200 + j0$. Calculate the circuit current, power factor and reactive power.

(e) An alternating current of 1.5 A flows in a circuit when applied voltage is 300 V. The power consumed is 225 W. Find resistance and reactance of circuit.

(f) A 100 V, 80 W lamp is to be operated on 230 V, 50 Hz AC Calculate the inductance to be connected in series with the lamp. Lamp can be taken as a pure resistance.

2. Answer any *Four* of the following:

(a) Out of Mesh and Nodal analysis, which method will require least number of equations to solve the network in Fig. A1-22. Write the equations in matrix form following the method requiring least effort.

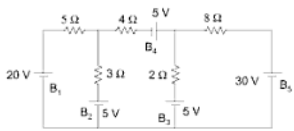


Fig. A1-22

(b) By Superposition Theorem find the current in R.

(c) State Thevnin theorem. For the circuit shown in Fig. A1-24, find the Thevnin's equivalent voltage source. Also give the value of load resistance to be connected across terminals A and B to get maximum power delivered to it.

(c) In a 50 KVA, 1-phase $\frac{3300}{230}$ V, transformer, the iron losses and full-load copper losses were found to be as 500 W and 650 W, respectively. Calculate efficiency at half load and 0.6 power factor. Determine maximum efficiency and corresponding load.

(d) If in a transformer core, the hysteresis and eddy current losses are 80 W and 50 W at normal voltage and frequency, calculate the losses when voltage and frequency are increased by 20%.

4. Answer any *Four* of the following

(a) The armature of a DC separately excited machine has a resistance of 0.1Ω and is connected to a 230 V supply. Calculate the generated emf when it is running (i) As a generator giving 80 A. (ii) As a motor taking 60 A.

(b) Discuss in detail the load characteristics of

(i) shunt generator (ii) series generator

(c) Derive an expression for torque of a DC motor. Also draw torque V_s speed characteristics for DC shunt and series motor.

(d) What are the various methods of speed control of DC shunt motor? Explain them in brief with relative advantages and disadvantages.

(e) Discuss the advantages and disadvantages of synchronous motors.

5. Answer any *Four* of the following:

(a) Differentiate the constructional details of squirrel cage and slip ring (wound rotor) 3-phase induction motor and also give their relative merits or demerits.

(b) A 3-phase, 4-pole induction motor is supplied from diesel-generator set running at 600 rpm. The generator has 10 poles. Find the synchronous speed of the induction motor and also the actual speed for a slip of 4%.

(c) Draw typical speed-torque characteristics of 3-phase squirrel cage induction motor, mark the starting torque, maximum torque and normal operating region.

(d) Explain why do pure, 1-phase induction motors have no starting torque. What modifications are being done to have starting torque in normal 1-phase fan motors? Give brief explanations.

(e) Discuss in detail the advantages and disadvantages of star-delta starter, after giving due importance for the need of starter in 3-phase induction motor.

FIRST SEMESTER EXAMINATION, 2004-2005

1. Attempt any *Four* parts of the following:

(a) Derive an expression for the instantaneous current drawn by a pure inductive circuit on application of a sinusoidal voltage and show that the current lags behind applied voltage by quarter of a cycle.

(b) Discuss the effects of varying the frequency upon the current drawn and the power factor in a RLC series circuit. A series RLC circuit with $R = 10\ \Omega$, $L = 0.02$ H, $C = 2\ \mu$F is connected to 100 V variable frequency source. Find the frequency for which the current is maximum.

(c) A coil connected to 100 V DC supply draws 10 Amp and the same coil when connected 100 V, AC voltage of frequency 50 Hz draws 5 Amp. Calculate the parameters of the coil and power factor.

(d) For the circuit shown in Fig. A1-26, find the current and power drawn from the source.

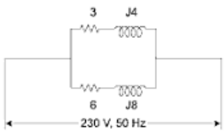


Fig. A1-26

(e) Define bandwidth and quality factor of a parallel resonance circuit. Derive the expression of resonance frequency of the circuit shown in Fig. A1-27.

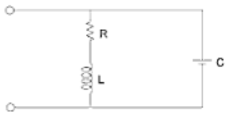


Fig. A1-27

(f) Compare electric and magnetic circuits for similarities and dissimilarities. Also give the relation between mmf, flux and reluctance.

2. Attempt any *Four* part of the following:

(a) Distinguish the following terms:

(i) Active and passive elements

(ii) Linearity and non-linearity

(iii) Unilateral and bilateral elements.

(b) Give the basis of star-delta transformation.

For the circuit given in Fig. A1-28, find current drawn from the source.

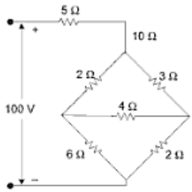


Fig. A1-28

(c) Using nodal analysis, find the current I through 10 W resistor in Fig. A1-29.

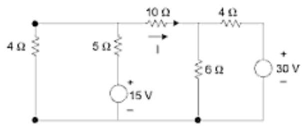


Fig. A1-29

(d) State and explain Thevnin's theorem. Find Thevnin's equivalent of the circuit shown in Fig. A1-30.

(e) Explain the principle of operation of attraction type of moving iron intruments and explain how the 'controlling' and 'damping' forces are obtained.

(f) A moving coil instrument gives a full-scale defection of 20 mA when a potential difference of 50 mv is applied. Calculate the series resistance to measure 500 V on full scale.

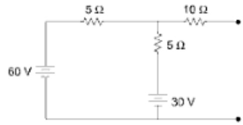


Fig. A1-30

3. Attempt any *Two* parts of the following:

(a) Explain why the hysteresis and eddy current loss occur in a transformer. On what factor do they depend for a fixed frequency supply? A 25 kVA, $\frac{3300}{230}$ V, 50 Hz, 1-phase transformer draws no load current of 15 A when excited on low-voltage side and consumes 350 watts. Calculate two components of current.

(b) Define the efficiency of a transformer. Obtain the expression of efficiency in terms of its VA rating, given power factor, iron losses and full load copper losses for a given fraction 'x' of the load. Also deduce the condition for maximum efficiency.

(c) (i) Show that power in 3-phase balanced system is constant at every instant and is given by $3\ V_P I_P \cos\phi$ where V_P, I_P and $\cos\phi$, are phase values of voltages and currents and power factor.

(ii) Derive the relation between phase and line voltage and currents for a balanced 3-phase star-connected system.

4. Attempt any *Two* parts of the following:

(a) Derive the expression for the generated emf in a DC machine. Explain the term back emf when applied to DC motor. Briefly explain the role back emf plays in starting and running of the motor.

(b) (i) Give the various methods of speed control of DC shunt motor. Give merits and demerits of them.

(ii) A 200 V, 8-pole, lap-connected DC shunt generator supplies sixty, 40 W, 200 V lamps. It has armature and field circuit resistances of 0.2 Ω and 200 Ω, respectively. Calculate the generated emf, armature current and current in each armature conductor.

(c) State the advantages of having rotating field system rather than a rotating armature system in synchronous machine. What is the effect of change of excitation of a synchronous motor on its power factor for different loads?

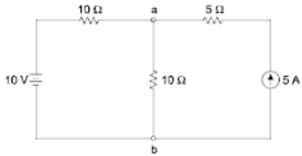


Fig. A1-32

(d) Using mesh equation method find the current in resistance R_1 of network given below.

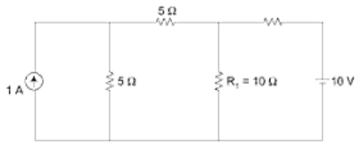


Fig. A1-33

(e) An energymeter revolves 10 revolutions of disc for one unit of energy. Find the number of revolutions made by it during an hour when connected across load which takes 20 A at 210 V and 0.8 power factor leading. If energymeter revolves 350 revolutions, find the percentage error.

3. Attempt any *two* parts of the following:

(a) Calculate the reading of each wattmeter in the circuit shown below (Fig. A1-34). The load impedance $z = 40\angle{-30°}\ \Omega$

(b) An open circuit (OC) test and short circuit (SC) test are performed on a single phase transformer in standard manner with results in table. Quantities not measured are noted as "NM".

	OC	**OC**	**SC**
Quantity	Side 1	Side 2	Side 3
Voltage, V	7500	220	210
Current, I	NM	3.00	2
Power, W	NM	250	200

(i) What is voltage rating of transformer?

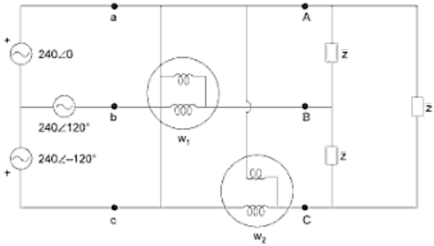


Fig. A1-34

(ii) What is transformer's apparent power rating?

(iii) Draw an equivalent circuit referred to the HV side with impedance.

(iv) Find the magnetising current, if excited from HV side.

(c) Explain the terms: Magnetomotive force (mmf), Intensity of magnetisation, reluctance and susceptibility in magnetic circuit.

A wrought iron bar 30 cm long and 2 cm in diameter is bent into a circular shape as given in Fig. A1-35(c). It is then wound with 500 turns of wire. Calculate the current required to produce a flux of 0.5 mWb in magnetic circuit with an air gap of 1 mm; μ_r (iron) = 4000 (assume constant).

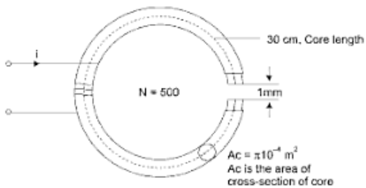


Fig. A1-35

4. Attempt any two parts of the following:

(a) A 10 kW, 200 V, 1200 rpm series DC generator has armature resistance of 0.1 Ω, field winding resistance of 0.3 Ω. The frictional and windage loss of the machine is 200 W and brush contact drop is 1 volt per brush. Find the efficiency of the machine and the load current at which this machine has maximum efficiency.

(b) Give the constructional details of salient pole synchronous machine. Explain '*V*' curve give the applications of synchronous motor.

(c) Explain the following:

(i) Synchronous motor always runs at synchronous speed.

(ii) Field winding of synchronous generators is placed on rotor.

5. Attempt any *Three* parts of the following:

(a) Induction motor cannot operate between slip 1 and slip at max torque. Explain it.

(b) Single phase induction motor is not a self-starting. Explain it.

(c) What is the use of starter? Explain various methods of starting of 3-phase induction motor.

(d) A 3-phase, 440 V, 50 hp, 50 Hz induction motor runs at 1450 rpm when it delivers rated output power. Determine:

(i) Number of poles in the machine

(ii) Speed of rotating air gap field

(iii) Rotor induced voltage if stator to rotor turns ratio is 1 : 0.80. Assume the winding factors are the same.

(iv) Frequency of rotor current.

(e) Discuss various methods of starting of single-phase induction motors.

FIRST SEMESTER EXAMINATION, 2005-2006

1. Attempt any *Four* of the following questions:

(a) An AC Voltage $e(t) = 141.4 \sin 120t$ is applied to a series R-C circuit. The current through the circuit is obtained as $i(t) = 14.14 \sin 120t + 7.07 \cos(120t + 30)$

Determine: (i) Values of resistance and capacitance

(ii) Power factor

(iii) Power delivered by the source

(b) A non-inductive resistance of 10 ohms is connected in series with an inductive coil across 200 V, 50 Hz AC supply. The current drawn by the series combination is 10 amperes. The resistance of the coil is 2 ohms. Determine

(i) Inductance of the coil (ii) Power factor (iii) Voltage across the coil.

(c) The following Fig. shows a series-parallel circuit. Find

(i) Admittance of each parallel branch

(ii) Total circuit impedance

(iii) Supply current and power factor

(iv) Total power supplied by the source.

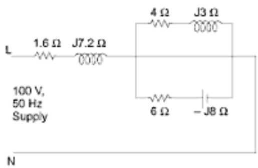


Fig. A1-36

(d) Discuss why?

(i) At resonance the current is maximum in a series circuit and minimum in a parallel circuit.

(ii) In a series RLC circuit, the voltages across L and C at resonance may exceed even the supply voltage.

(iii) The shape of resonance curve depends on Q of the coil.

(e) For the current shown below, determine

(i) Resonant frequency

(ii) Total impedance of the circuit at resonance

(iii) Bandwidth

(iv) Quality factor.

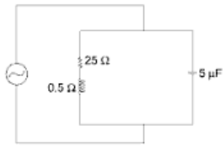


Fig. A1-37

(f) What is hysteresis loop and what information is obtained from this loop? Draw hysteresis loop for

(i) Permanent magnet

(ii) Transformer core

(iii) Ferrite

2. Attempt any *Four* of the following questions:

(a) Using mesh current method, determine current I_x in the following circuit:

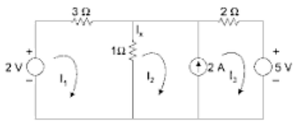


Fig. A1-38

(b) Using Superposition theorem, determine currents in all the resistances of the following network:

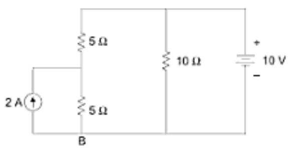


Fig. A1-39

SECOND SEMESTER EXAMINATION, 2005-2006

1. Attempt any *Four* parts of the following:

(a) The equation of an alternating current is $i = 42.42 \sin 628t$.

Determine—

(i) Its maximum value

(ii) Frequency

(iii) rms value

(iv) Average value

(v) Form factor

(b) Draw a phasor diagram showing the following voltages:

$$V_1 = 100 \sin 500t,\ V_2 = 200 \sin\left(500t + \frac{\pi}{3}\right)$$

$$V_3 = -50 \cos 500t,\ V_4 = 150 \sin\left(500t - \frac{\pi}{4}\right)$$

Find RMS value of resultant voltage.

(c) A 120 V, 60 W lamp is to be operated on 220 V, 50 Hz supply mains. In order that lamp should operate on correct voltage. Calculate value of—

(i) Non-inductive resistance

(ii) Pure inductance

(d) A series AC circuit has a resistance of 15 Ω and inductive reactance of 10 Ω. Calculate the value of a capacitor which is connected across this series combination so that system has unity power factor. The frequency of AC supply is 50 Hz.

(e) A series RLC circuit has $R = 10\ \Omega$, $L = 0.1$ H and $C = 8\ \mu$F.

Determine

(i) Resonant frequency

(ii) Q-factor of the circuit at resonance

(iii) The half power frequencies

(f) Explain parallel resonance. Why is parallel resonance called the current resonance? Show the graphical representation of current in parallel resonance.

2. Attempt any *Four* parts of the following:

(a) Using loop current method, find the current I_1 and I_2, shown in Fig. A1-41.

(b) Use nodal analysis to find the currents in various resistors of the circuit shown (Fig. A1-42).

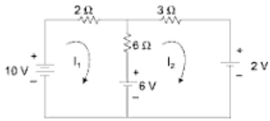


Fig. A1-41

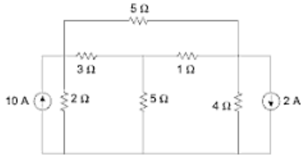


Fig. A1-42

(c) Find Thevnin's equivalent circuit across *AB* shown in Fig. A1-43.

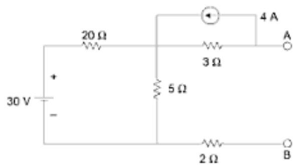


Fig. A1-43

(d) Find the resistance between A B of the circuit shown in Fig. A1-44 use *Y*-Δ (star-delta) transformation.

(e) A 50 A, 230 V meter on full-load test makes 61 revolutions in 37 seconds. If the normal disc speed is 500 revolutions per kWh, find the percentage error.

(f) Explain the working principle of induction type wattmeter with the help of a diagram.

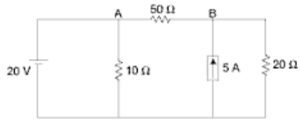


Fig. A1-50

(e) Differentiate amongst indicating, recording and integrating type of instruments giving examples of each type.

(f) Briefly discuss the working of a dynamometer-type wattmeter with the help of a neat diagram.

3. Answer any *Two* of the following:

(a) Two wattmeters are used to measure power supplied to a balanced 3-phase load and each of them are indicating 60 kW. If the power factor of the load is changed to 0.8666 (leading), determine the readings of the two wattmeters with the total power still remaining the same. Draw the phasor diagram for the second condition of the load.

(b) (i) From first principle, derive the EMF equation of a single-phase transformer.

(ii) Derive the condition for the maximum efficiency of a single-phase transformer.

(c) A $\frac{230}{460}$ volts, single-phase transformer has primary winding resistance of 0.2 Ω and reactance of 0.5 Ω. The corresponding values for the secondary winding are 0.75 Ω and 1.8 Ω, respectively. Find the secondary terminal voltage while supplying 10 A load current at 0.8 p.f. lagging.

4. Answer any *Two* of the following

(a) Discuss methods by which the speed of a DC shunt motor can be controlled. State the merits and demerits of each method.

(b) A 4 pole DC generator has 564 conductors on its armature and is driven at 800 rpm. The flux per pole being 20 milli-webers and current per parallel path is 60 amp's. Calculate the total armature current, emf induced and power generated in the armature when the armature is (i) wave-wound (ii) lap-wound.

(c) Explain the working principle of a synchronous motor and enlist at least 02 of its applications.

5. Answer any *Two* of the following:

(a) Draw and explain the torque-slip characteristics of a 3-phase induction motor showing clearly the starting torque, maximum torque and normal operating region.

(b) (i) Explain why an induction motor cannot run at synchronous speed.

(ii) A 3-phase, 6-pole, 50 Hz, induction motor has a slip of 1% at no-load and 3% at full-load. Determine (a) synchronous speed (b) no-load speed (c) change in speed from no-load to full-load.

(c) Why are single-phase induction motors not self-starting? Discuss any two methods of its starting.

(c) Classify moving iron instruments. Explain working principle of electrodynamometer instruments.

(d) A moving coil instrument gives full-scale deflection wtih 15 mA. The resistance of coil is 5 Ω. It is desired to convert this instrument into an ammeter to read upto 2 A. How to achieve it? Further how to make this instrument to read upto 30 volts.

(e) A constant voltage E is applied to N groups of rheostats in series, where each group has *M* identical rheostats in parallel. If one rheostat burns out in one group, find the percentage increase of current in each rheostat of the faulty group and percentage decrease of current in each rheostat of the sound group.

(f) A 20 V battery with an internal resistance of 5 Ω is connected to a resistor of *x* Ω. If an additional 6 Ω resistance is connected across the battery terminals, find the value of *x* so that external power supplied by the battery remains the same.

3. Answer any *Two* parts of the following:

(a) Three inductive coils each with a resistance of 10 Ω and an inductance of 0.04 H are connected (i) in star, (ii) in delta to a 3-phase 400 V, 50 Hz supply. Calculate for each of the above case (i) phase current and line current and (ii) total power absorbed.

(b) (i) Deduce the relationship between the phase and line voltages in a three-phase star-connected circuit.

(ii) Show that the power intake by a three-phase circuit can be measured by two wattmeters connected properly in the circuit.

(c) A 40 kVA transformer has iron loss of 500 watts and full load copper-loss of 800 watts. If the power factor of the load is 0.6 lagging, calculate (i) full-load efficiency (ii) the load at which maximum efficiency occurs and (iii) the maximum efficiency.

4. Answer any *Two* parts of the following:

(a) (i) Explain with suitable diagrams circuit resistance of the field circuit of DC shunt generator and critical speed of generator.

(ii) List various conditions for the build-up of voltage in a DC shunt generator.

(b) A 500 V DC shunt motor running at 750 rpm takes an armature current of 50 A. Effective armature resistance is 0.5 Ω. What resistance must be placed in series with the armature to reduce the speed to 700 rpm, the torque remaining constant?

(c) Derive an expression for voltage induced in an alternator phase consisting of a number of full pitch coils joined in series. Assume the air gap flux to have sinusoidal distribution.

5. Answer any *Two* parts of the following:

(a) Explain with the help of diagrams how a rotating magnetic field is produced in the air gap of a 3-phase induction motor. Describe concept of slip and expression for rotor frequency.

(b) Draw a neat diagram showing the connections of a 3-phase induction motor with star-delta starter. Explain how the above starter reduces the starting current.

(c) (i) A 4-pole, 50 Hz induction motor has a slip of 2% at no load. When at full load the slip increases to 3%. Find the change in speed of the motor from no load to full-load.

(ii) There are number of applications of induction motors of two different kinds, these kinds are squirrel cage induction motor and slip ring motors. Give name where these are specifically used. Name the induction motors typically used for the following:

(1) Lathe

(2) Cranes

(3) Conveyors

(4) Agricultural and industrial pumps

(5) Drilling machines

(6) Compressors

(7) Industrial drive

(8) Lifts

CARRY OVER PAPER EXAMINATION, 2006

1. Answer any *Four* parts of the following:

(a) Two sinusoidal phasers have the form of A sin ($wt + \phi$). Their addition is $7\sqrt{2}$ sin ($wt + 45°$) and difference is $\sqrt{2}$ sin ($wt - 45°$). Determine both the phasers.

(b) An inductive coil is connected to a single-phase sinusoidal AC supply. Derive an expression for instantaneous current, average power drawn by the coil.

(c) A generator supplies power to an electric heater, an inductor and a capacitor as shown in the Fig. A1-53.

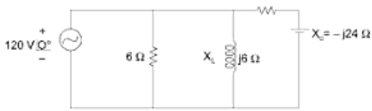


Fig. A1-53

(i) Determine real and reactive power for each load.

(ii) Total real and reactive power supplied by the generator.

(iii) Draw the power triangle for combined loads and determine total apparent power.

(iv) Generator supply current and power factor.

(d) In the series-parallel circuit shown in Fig. A1-54, the effective value of the voltage across the parallel part of the circuit is 100 volts. Determine the magnitude of the supply voltage V_s and the real power supplied by it.

(e) A coil of resistance 10 ohm and inductance 10 mH is connected in parallel with a 25 μF capacitor. Calculate frequency at resonance and effective impedance of the circuit.

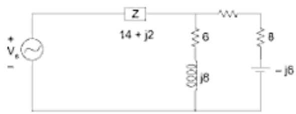


Fig. A1-54

(f) An iron ring of mean diameter 15 cm is uniformly wound with 450 turns of wire. A current of 2 amperes when passed through the coil, establishes a flux density of 1.2 wb/m^2 in the iron ring. Determine the relative permeability of iron.

2. Answer any *Four* parts of the following:

(a) Determine voltages V_A, V_B and V_C in the Fig. A1-55 shown below:

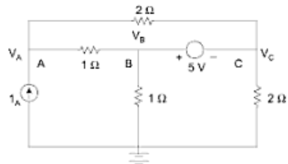


Fig. A1-55

(b) Determine current in 4 ohm resistance in the following Fig. A1-56, using Norton's theorem.

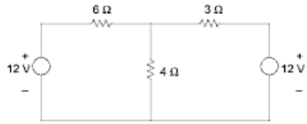


Fig. A1-56

(c) In the network shown in Fig. A1-57.

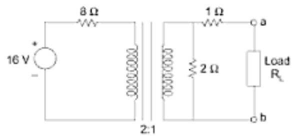


Fig. A1-57

(i) Determine the value of the load resistance for maximum power transfer.

(ii) Find the power delivered to the load.

(d) Show that in delta to star conversion the each element of star network is the product of the adjacent two elements of the delta network divided by the sum of all the three elements of the delta network.

(e) Explain how following torques are produced in permanent magnet moving coil instrument and attracted type moving iron instrument:

(i) Deflecting torque

(ii) Control torque

(iii) Damping torque

(f) Explain in brief why energymeter reads energy while wattmeter does not.

An energymeter has a registration constant of 100 rev/kWh. If the meter is connected to a load drawing 20 amperes at 230 volt and 0.8 power factor for 5 hours, find the number of revolutions should be made by it. If it actually makes 1800 revolutions, find the percentage error and explain it from the consumer's point of view.

3. Answer any *Two* parts of the following:

(a) A 3-phase, 440 V, 50 Hz supply is connected to a star-connected balanced load. Each phase of the load consists of series connected resistance and capacitance. The readings of two wattmeters connected to measure the total power supplied are 810 W and 2100 W. Calculate (i) power factor of the circuit (ii) line current (iii) values of resistance and capacitance.

(b) Describe open circuit and short circuit tests on a single-phase transformer. Explain how the equivalent circuit can be derived from these two tests.

(c) The following readings were obtained from OC and SC tests on 8 kVA $\frac{400}{120}$ V, 50 Hz transformer.

OC test (L.V. side) : 120 V, 4 A, 75 W

SC test (H.V. side) : 9.5 V, 20 A, 100 W

Calculate voltage regulation and efficiency at full load and 0.8 power factor lagging.

FIRST SEMESTER EXAMINATION, 2006-2007

1. Attempt any *Four* parts of the following:

(a) An alternating voltage is given by $v = 141.4 \sin 314t$. Find

(i) Frequency

(ii) r.m.s. value

(iii) Average value

(iv) The instantaneous value of voltage when 't' is 3 m sec.

(v) The time taken for the voltage to reach 100 V for the first time after passing through zero value.

(b) A single-phase sinusoidal AC voltage supply is fed to a series *R-C* circuit. Determine

(i) The instantaneous expression of current flowing in the circuit

(ii) The impedance

(iii) The power factor,

(iv) The power consumed

(v) The reactive volt-ampere.

(c) The parallel circuit shown in Fig. A1-58 is connected across a single-phase 100 V, 50 Hz AC supply. Calculate:

(i) The branch currents

(ii) The total current

(iii) The supply power factor

(iv) The active and reactive power supplied by the supply.

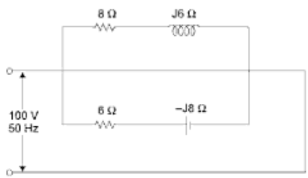


Fig. A1-58

(d) Derive an expression for parallel resonance and mention its salient features.

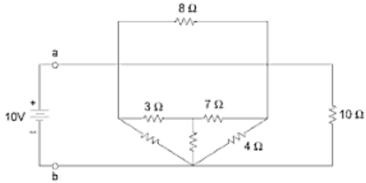


Fig. A1-61

(e) Explain the construction and principle of operation of dynamometer instrument. Why is this instrument suitable for both DC and AC measurements?

(f) Why shunt is usually used in voltmeter and ammeter?

A moving coil instrument has a resistance of 5 ohm and gives a full-scale deflection of 100 mV. Show how the instrument may be used to measure:

(i) voltage upto 50 V

(ii) currents upto 10 A.

3. Attempt any *Two* parts of the following:

(a) Derive the relationship between the line and phase voltages of an alternator.

Three similar coils each having a resistance of 8 ohm and an inductance of 0.0191 H in series in each phase is connected across a 400 V, three-phase, 50 Hz supply. Calculate the line current, power input kVA and kVAR taken by the load.

(b) Draw exact equivalent circuit and corresponding phasor diagram of a single-phase transformer on load and explain them. Why no load current is kept small and how it is reduced?

(c) List various losses occurring in a transformer and mention the condition for the maximum efficiency.

In a 25 kVA, $\frac{2000\text{ V}}{200\text{ V}}$ transformer the iron and copper losses are 200 W and 400 W, respectively. Calculate the efficiency at half load and 0.8 power factor lagging. Determine also the maximum efficiency and the corresponding load.

Appendix 2

Objective Questions for Practical Quiz

1. In the circuit shown in Fig. A2-1, the current '*I*' marked is given by

   (a) 1/3 (b) 1/5 (c) 1 (d) 5/3

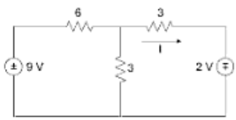


Fig. A2-1

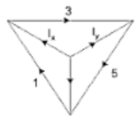


Fig. A2-2

2. In Fig. A2-2, currents in some branches are given. The value of I_x and I_y respectively are:

   (a) –2, 2 (b) 2, –2 (c) 2, 2 (d) –2, –2

3. The voltage labelled *V* in Fig. A2-3 is given by

   (a) 2 (b) 0 (c) 9 (d) 3

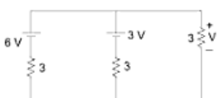


Fig. A2-3

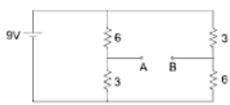


Fig. A2-4

4. The Thevnin's equivalent across point *AB* in Fig. A2-4 is given by V_{th} and R_{th}. Their values are:
   (a) $V_{th} = 3\ R_{th} = 4$ (b) $V_{th} = -3\ R_{th} = 4$
   (c) $V_{th} = -3\ R_{th} = 4.5$ (d) $V_{th} = 6\ R_{th} = 4$
5. The Norton's equivalent across *AB* in Fig. A2-5 is given by:
   (a) $I_{sc} = V_s/3\ R_{th} = 3/2$ (b) $I_{sc} = V_s/6\ R_{th} = 3/2$
   (c) $I_{sc} = V_s/4\ R_{th} = 6$ (d) $I_{sc} = V_s/2\ R_{th} = 2/3$
6. $V = \sqrt{2}\ 200 \cos 500\ t$, $P_{av} = 250$ watt, power factor = 0.7 lagging. The reactive power is given by
   (a) 250 VAR (b) 0 VAR (c) 500 VAR (d) 175 VAR
7. The dimension of L/CR is
   (a) Volt/Amp (b) Amp/Volt Sec^2
   (c) Secs (d) None of these
8. The Thevnin's equivalent across *AB* in the Fig. A2-6 is given by
   (a) −16, 10/3 (b) −12, 10 (c) 4, 16/5 (d) 4, 10/3

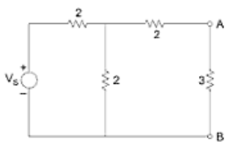


Fig. A2-5

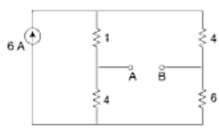


Fig. A2-6

9. In Electronics the term IC denotes:
   (a) Industrial control (b) Integrated circuits
   (c) Internal combustion (d) Indian calculations
10. The forbidden energy gap in semiconductors:
    (a) Lies just below the valance band
    (b) Is the same as the valance band
    (c) Lies just above the conduction band
    (d) Lies between the valance band and the conduction band
11. A zener diode has a
    (a) High forward voltage rating
    (b) Negative resistance

(c) High amplification
(d) Sharp breakdown voltage at low reverse voltage

12. When a reverse bias is applied across a crystal diode, it
(a) Raises of potential barrio
(b) Lowers the potential barrio
(c) Increases the majority-carrier current greatly

13. Under normal operating voltage, the reverse current in a silicon diode is about
(a) 10 mA (b) 1 μA
(c) 1000 μA (d) None of these

14. In a single phase AC motor, condenser uses
(a) For splitting the phase
(b) Minimizing the radio interference
(c) Minimizing the losses
(d) Minimizing the current

15. What is called the electromotive force (emf) of a voltage source?
(a) Terminal voltage when load is applied
(b) Internal voltage when no load is applied.
(c) Product of internal resistance and load current
(d) Electric pressure provided to the load.

16. The condition in ohm's law is that
(a) The temperature should remain constant
(b) The ratio of V/I should be constant
(c) The temperature should vary
(d) Current should be proportional to voltage.

17. Conductance is analogous to
(a) Flux (b) Reluctance
(c) Permeance (d) None of these

18. With the increase of frequency, the capacitive reactance or the circuit
(a) Decreases (b) Increases
(c) Remains same (d) None of these

19. The form factor is
(a) The ratio of rms value to average value.
(b) The ratio of peak value to rms value.
(c) The ratio of average value to rms value.

20. The maximum and minimum value of power factor can be
    (a) 1 and 0 (b) + 1 and – 1 (c) + 1 and – 5 (d) + 5 and – 5
21. A series *RLC* circuit is fed from 230 volts variable frequency supply. If the frequency is adjusted to create resonance, the voltage across the resistance is
    (a) 230 V (b) Less than 230 V
    (c) 0 V (d) Greater than 230 V
22. A series circuit has $R = 10$ ohms, $L: = 0.01$ H and $C = 10$ µF, the *Q*-factor is
    (a) 100 (b) 10 (c) 115 (d) 10.1
23. In a *RLC* circuit, the current at resonance is
    (a) Maximum in series circuit and minimum in parallel circuit
    (b) Maximum in parallel circuit and minimum in series circuit
    (c) Maximum in both the circuits
    (d) Minimum in both the circuits
24. The star transformation of the following circuit shown in Fig. A2-7 is
    (a) 1, 2, 3 (b) 1/2, 1/6, 1/3
    (c) 3/11, 1/11, 2/11 (d) 1, 1/3, 1/2

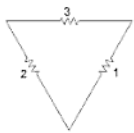


Fig. A2-7

25. If the speed of the DC generator is increased, the generated emf will
    (a) Increase (b) Decrease
    (c) Remain constant (d) Decrease and then increase
26. In a DC motor the iron losses occur in
    (a) The yoke (b) The armature
    (c) The field (d) None of these
27. If the current in the armature of a DC series motor is reduced by 5%, the torque of the motor will become
    (a) 50% of the previous value (b) 25% of the previous value
    (c) 150% of the previous value (d) 125% of the previous value

28. The armature of DC motor is laminated
    (a) To reduce the hysteresis losses
    (b) To reduce the eddy current losses
    (c) To reduce the inductivity of the armature
    (d) To reduce the mass of the armature
29. What happens if the field winding of a running shunt motor suddenly breaks open?
    (a) Its speed slow down
    (b) Its speed becomes dangerously high
    (c) It gives out spark (d) It stops at once
30. A synchronous motor can run at a
    (a) Leading power factor
    (b) Unity power factor
    (c) Lagging or leading or unity power factor
    (d) Zero power factor
31. The power factor of the input power to a synchronous motor is adjusted by varying
    (a) Number of poles
    (b) Magnitude of excitation
    (c) Magnitude of armature reaction
    (d) None of these
32. If a synchronous motor is switched onto 3-phase supply with its rotor winding short circuited, it will
    (a) Start (b) Not start
    (c) Start and continue to run as induction motor
    (d) Start and continue to run as synchronous motor
33. Alternator generates
    (a) DC (b) AC
    (c) DC and AC both (d) Pulsating DC
34. The rotor of the alternator requires
    (a) DC (b) AC
    (c) Pulsating DC (d) None of the above
35. The rotor of the alternator has
    (a) 4 slip rings (b) 3 slip rings
    (c) 2 slip rings (d) No slip rings

46. The efficiency in a transformer is maximum at
    (a) Full-load (b) Half the full load
    (c) One-third the full load (d) No-load
47. The primary reason why open circuit test is performed on the low-voltage winding of the transformer is that it
    (a) Draws sufficiently large no-load current for convenient reading
    (b) Requires least voltage to perform the test
    (c) Needs minimum power input
    (d) Has become a universal custom
48. During short circuit test, the iron-loss of a transformer is negligible because
    (a) The entire input is just sufficient to meet copper losses only
    (b) Voltage applied across the h.v. side is a small fraction of the rated voltage and so is the flux
    (c) Iron core becomes fully saturated
    (d) Supply frequency is held constant
49. The main purpose of using core in a transformer is to
    (a) Decrease iron-loss
    (b) Prevent eddy current loss
    (c) Eliminate magnetic hysteresis
    (d) Decrease reluctance of the common magnetic flux path
50. A transformer has a percentage resistance of 1% and percentage reactance of 4%. Its regulation of power factor 0.8 lagging and 0.8 leading are respectively
    (a) 3.2% & –1.6% (b) 6% & –4%
    (c) 4.8% & –3.2% (d) None of these
51. Eddy current loss in a transformer depends upon
    (a) 80th voltage and frequency (b) Voltage alone
    (c) Load current alone (d) Thickness of core
52. Hysteresis loss in a transformer depends upon
    (a) Frequency (b) Supply voltage
    (c) Square of frequency alone (d) Square of the voltage alone
53. The iron losses in a 10 kVA, 240/100V single-phase transformer are 100 watts. The maximum efficiency occurs at full-load. When the secondary current is 50A, the copper losses in the transformer will be
    (a) 100 W (b) 50 W (c) 25 W (d) l0 W

54. During a short circuit test, the iron losses are negligible because
    (a) The mutual flux is small (b) The power factor is low
    (c) The current is high (d) None of these
55. If DC supply is given to a transformer, it will
    (a) Work (b) Not work
    (c) Give lower voltage than the rated voltage on secondary side.
    (d) Burn the winding
56. A synchronous can run at
    (a) A leading power factor (b) Unity power factor
    (c) Lagging or leading or unity power factor
    (d) Zero power factor
57. An induction motor is coupled to a synchronous motor. In order to bring up the synchronous motor to synchronous speed at starting, the pair of pole of induction motor should be
    (a) Less than that of synchronous motor
    (b) Equal to that of synchronous motor
    (c) Greater than that of synchronous motor
    (d) None of these
58. A 10 pole, 25 Hz alternator is directly coupled to, and is driven by a 60 Hz synchronous motor than the number of poles in a synchronous motor are
    (a) 48 poles (b) 12 poles
    (c) 24 poles (d) None of these
59. In a synchronous generator
    (a) The field mmf lags the air gap flux and the air gap flux lags the armature mmf.
    (b) The field mmf leads the air gap flux and the air gap flux leads the armature mmf
    (c) The field mmf leads the air gap flux lags the armature mmf
    (d) The field mmf lags the air gap flux and the air gap flux leads the armature mmf.
60. The synchronous motor runs at
    (a) Less than synchronous speed (b) Synchronous speed
    (c) More than synchronous speed (d) None of the above
61. The construction of synchronous motor is similar to
    (a) DC compound motor (b) Slip ring induction motor
    (c) DC shunt generator (d) Alternator

62. The synchronous motor runs on
    (a) 3-phase AC supply (b) 3-phase AC and DC Supply
    (c) DC Supply only
    (d) 3-phase AC and single-phase AC.
63. If a synchronous motor is switch on 3-phase supply with its rotor winding short circuited, it will
    (a) Start (b) Not start
    (c) Start and continue to run as induction motor
    (d) Start and continue to run as synchronous motor
64. The magnitude of the EMF induced in the stator due to revolving flux depends upon the
    (a) Speed of the motor (b) DC Excitation current
    (c) Load on the motor (d) Speed and rotor flux
65. Under no-load running condition, the angle between the induced voltage and supply voltage will be
    (a) 180 degree (b) 90 degree
    (c) Between 90 degree and 180 degree
    (d) Zero
66. Under running condition on load, the angle between the induced voltage and supply voltage will be
    (a) Zero (b) 180 degree
    (c) 90 degree (d) None of these
67. If the field of synchronous motor under excited, the power factor will be
    (a) More than unity (b) Unity
    (c) Lagging (d) Leading
68. When the synchronous motor is started, the field winding is initially
    (a) Short circuited (b) Open circuited
    (c) Excited by a DC source (d) None of these
69. V-curve of a synchronous motor show the relation between
    (a) Armature current and field current
    (b) Applied voltage and field current
    (c) Applied voltage and armature current
    (d) None of these
70. For a given load, the armature current of a synchronous motor is minimum when the power factor is
    (a) Unity
    (b) Lightly less than unity and lagging

80. If the speed of a induction motor be synchronous speed, then the relative speed between the rotating flux and the rotor will be
    (a) Maximum and hence, torque will be maximum
    (b) Maximum and hence, torque will be zero
    (c) Zero and hence, torque will be maximum
    (d) Zero and hence, torque will be zero
81. If the rotor winding of a wound rotor 3-phase induction motor is connected to balance 3-phase supply and the stator winding is short circuited, then the rotor will
    (a) Not run at all
    (b) Run at sub-synchronous speed in the direction of rotating flux
    (c) Run at sub-synchronous speed against the direction of rotating flux
    (d) Run synchronously with rotating flux
82. In terms of air gap power p_g the rotor copper loss and the mechanical power developed are given by
    (a) sp_g and $(I\text{-}s)\, p_g$ (b) $(I\text{-}s)\, P_g$ and SP_g
    (c) p_g and p_g/s (d) p_g/s and $p_g\, (I\text{-}s)$
83. In a 3-phase induction motor, internal developed torque, T_e in terms of supply voltage. V_1 is proportional to:
    (a) V_1 (b) $\sqrt{V_1}$
    (c) V_1^2 (d) none of these
84. In a 3-phase induction motor, slip for maximum torque, in terms of rotor resistance r_2
    (a) Independent of r_2 (b) Directly proportional to r_2
    (c) Inversely proportional to r_2 (d) None of these
85. In a 3-phase induction motor, the current is produced in the rotor conductors by
    (a) Giving AC supply (b) Giving DC supply
    (c) Induction effect (d) Pulsating DC supply
86. The direction of rotation of 3-phase revolving field can changed by interchanging
    (a) *R* and *Y* phase only (b) *B* and *Y* phase only
    (c) *R* and *B* phase only (d) Any two phases
87. Different types of 3-phase induction motors are
    (a) Squirrel cage (b) Slip ring
    (c) Commutator (d) All of these

(c) Supply voltage
(d) Supply voltage and line current both

99. Megger is used for measuring
    (a) Low resistance (b) High resistance
    (c) Medium resistance (d) Very low resistance

100. Sensitivity of a voltmeter is expressed as
    (a) Volt/ohms (b) ohms/volt (c) ohms volt (d) l/ohms.volt

ANSWERS

1. (c)	2. (a)	3. (d)	4. (b)	5. (b)	6. (a)
7. (a)	8. (d)	9. (b)	10. (d)	11. (d)	12. (a)
13. (b)	14. (a)	15. (b)	16. (a)	17. (c)	18. (a)
19. (a)	20. (a)	21. (a)	22. (b)	23. (a)	24. (d)
25. (a)	26. (a)	27. (b)	28. (b)	29. (b)	30. (c)
31. (b)	32. (c)	33. (b)	34. (a)	35. (c)	36. (c)
37. (a)	38. (d)	39. (c)	40. (d)	41. (b)	42. (d)
43. (c)	44. (d)	45. (d)	46. (a)	47. (a)	48. (b)
49. (d)	50. (a)	51. (d)	52. (a)	53. (c)	54. (a)
55. (d)	56. (c)	57. (a)	58. (c)	59. (b)	60. (b)
61. (d)	62. (d)	63. (c)	64. (b)	65. (d)	66. (d)
67. (c)	68. (a)	69. (a)	70. (a)	71. (b)	72. (b)
73. (b)	74. (a)	75. (a)	76. (c)	77. (b)	78. (c)
79. (b)	80. (d)	81. (c)	82. (a)	83. (c)	84. (b)
85. (c)	86. (d)	87. (d)	88. (b)	89. (a)	90. (d)
91. (c)	92. (d)	93. (c)	94. (d)	95. (a)	96. (c)
97. (d)	98. (c)	99. (b)	100. (b)		

Appendix 3

Electromagnetism

1. BRIEF HISTORY

Oersted discovered in 1819 that whenever an electric current flows through a conductor, a magnetic field is created in the space around the conductor. This was explained by saying that the flow of current through a conductor causes movement of the electrons. The motion of the electrons produces magnetic field. The research was carried out to find out the converse *i.e.*, if conductor is brought in a magnetic field does an emf induce or movement of electrons take place. Thus, the work advanced in the direction of conversion of magnetism into electricity. In 1831, Michael Faraday discovered that if a closed conductor is moved in a magnetic field in a certain manner there will be an induced current in the moving conductor. This phenomenon is known as electromagnetic induction. Further, Faraday enunciated the basic laws[1] of electromagnetic induction upon which is based the working of most of the commercial apparatus like motors, generators and transformers, etc. This was proved experimentally and then analytically that when a magnetic field linking with a closed conductor moves relative to the conductor, it produces a flow of electrons. The flow of electrons induces a current in the conductor and hence the conversion of electricity into magnetism and *vice versa* was fully established.

2. MAGNETIC LINES OF FORCE

The magnetic field of a magnet is the entire space around the magnet in which magnetic force is felt. A magnetic line of force is a line along which an isolated north pole would travel if free to move in the magnetic field, and a tangent to any point on the line gives the direction of the resultant force at that point.

The lines of force due to a bar magnet are shown in Fig. A3-1.

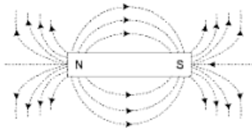


Fig. A3-1

3. LINES OF INDUCTION

The lines of force are external to the magnet and they pass from north pole through the field to the south pole. But the lines within the magnetic material are called as lines of induction as shown in Fig. A3-2(a).

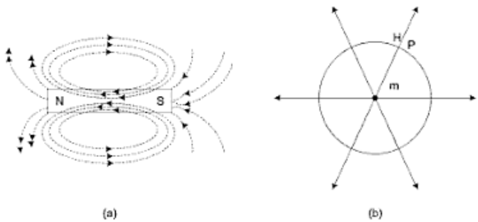


Fig. A3-2

3.1 Tubes of Force

If the field intensity at a point is H Ampere turns/metere[1], then there are H tubes of force per square meter of cross-section taken at right angles to the direction of the field.

Consider a point pole of strength m units in free space, then a unit positive pole at P, r meter from m will experience a force of $\frac{m}{\mu r^2}$ Newton.

$\therefore$ Field intensity at $P, H = \frac{m}{r^2}$ (since, $\mu = 1$ for air).

This field intensity will be same in magnitude at all points of the surface of the sphere of radius r metere as shown in Fig. A3-2(b) and hence H tubes of force will cross each square metere of the surface of the sphere.

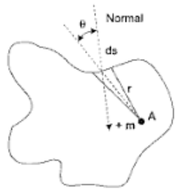


Fig. A3-3

4. GUASS THEOREM

Guass showed that the total number of lines of induction that emanate from a pole are independent of shape or size of the surface enclosing it.

Consider a point pole of strength m units at A. Let ds be an elementary surface and θ be the angle between the normal and the line $A\ ds$ as shown in Fig. A3-3.

If r is the distance of ds from A, then,

Flux density over the elementary area = $\frac{m}{r^2}$ we ber/metre^2,

Flux normal to elementary surface ds

$$= \frac{m}{r^2}\ ds \cos\theta$$

$$\therefore \qquad \text{Total flux} = \int \frac{m}{r^2}\ ds \cos\theta$$

$$= m \int \frac{ds \cos\theta}{r^2}$$

5. RULES FOR FINDING POLARITY

When a current flows through a conductor, a magnetic field is produced around it. The polarity or the direction of the flux can be found out by the following rules:

5.1 Ampere's Rule

This rule gives the relationship between the direction of the deflection of a magnetic needle and the direction of current passing through a wire placed above or below it, parallel to the axis of the needle.

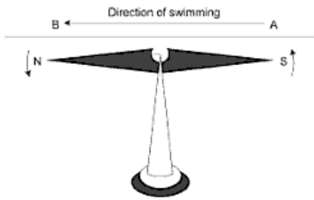


Fig. A3-4

Consider a wire *AB* placed over a pivoted magnetic needle parallel to its axis. When a current passes through it in the direction from *A* to *B* shown in Fig. A3-4, the needle is found to deflect in the direction represented by arrow head. If the direction of the current through the wire be reversed, the needle is found to deflect in the opposite direction. Thus Ampere Law may be stated as follows:

Imagine a man swimming in the direction of the current with his face towards the needle, then the north pole of the needle will be deflected towards his left hand.

5.2 Right Hand Rule

Hold the wire or conductor in the right hand with the thumb pointing in the direction of the current. The fingers then encircle the conductor in the direction of the magnetic field as shown in Fig. A3-5.

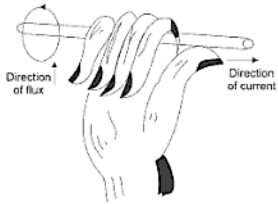


Fig. A3-5

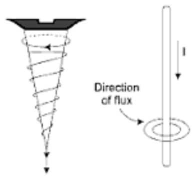


Fig. A3-5

5.3 Maxwell's Corkscrew rule

Imagine a right-handed corkscrew to be along the conductor and pointing in the direction of current. Then the direction of the magnetic lines of force will be the same as that in which the corkscrew has got to be rotated to cause it to advance in the direction of the current.

5.4 Magnetic Fields due to Straight Current Carrying Conductors

If a magnetic needle is brought near a current carrying conductor it never points towards the conductor indicating that the conductor itself does not become magnetized but is surrounded by magnetic lines of force which are circular in nature and can be traced by passing the conductor through a cardboard and plotting the field with the help of a magnetic compass. The direction of the magnetic field is shown in Fig. A3-6 when the current flowing through a conductor is downward in (a) and upward in (b).

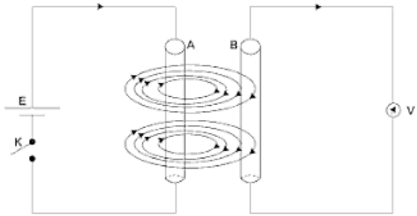


Fig. A3-13

15.1 Mutually Induced emf

Consider two parallel conductors *A* and *B* lying close to each other. Let conductor *A* be connected to a battery through a key *K* and a voltmeter is connected across *B*.

At the instance of opening or closing, there is a momentary deflection in the voltmeter. This is due to the production of mutually induced emf in *B*.

15.2 Self-Induced emf

It is the emf induced in a coil due to the change of its own flux linked with it. If a current through coil A changes direction, then the flux linked with its own turns will also change and will produce a self-induced emf.

The direction of this induced emf would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter emf of self-induction.

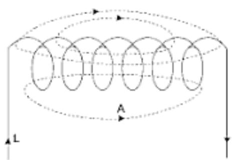


Fig. A3-14

The ratio $\frac{M}{\sqrt{L_1 L_1}}$ is called the coefficient of coupling and may be denoted by K, so

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Close coupled: If the coefficient of coupling is nearly unity, the circuits said side to be close coupled.

Loosely coupled: If the coefficient of coupling is sufficiently less than unity, the circuits are said to be loosely coupled.

19. MUTUAL INDUCTANCE OF CONCENTRIC SOLENOIDS

Consider two concentric solenoids as shown in Fig. A3.17.

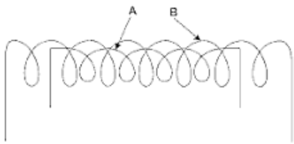


Fig. A3.16

Let N_1 be the number of turns, l_1 be the length and a_1 be the cross-sectional area of A. If a current I_1 amperes flows through it, then the flux through A, will be:

$$\phi_1 = \mu . a_1 \frac{0.4\pi N_1 I_1}{l_1}$$

If the number of turns on the inner solenoid be N_2 and it lies wholly within the outer solenoid, then it would lie in a uniform field.

Let a_2 be the area of cross-section of B,

$$\therefore \text{ Flux passing through } \quad B = \frac{\phi_1}{a_1} a_2$$

$$= \mu \frac{0.4\pi N_1 I_1}{l_1} a_2$$

Flux linkage with the coil B

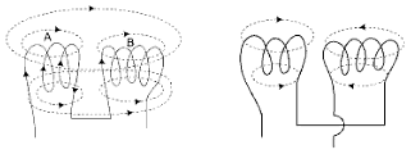
$$= \mu . \frac{0.4\pi N_1 I_1}{l_1} a_2 . N_2$$

Mutual inductance of the two coils

= Flux linkages of B due to per unit current in A.

$$\therefore \qquad M = \mu \frac{0.4\pi N_1 . N_2 . a_2}{l_1} \times 10^{-8} H$$

20. SELF-INDUCTANCES IN SERIES



(a) Cumulative (b) Opposition

Fig. A3.17 Self-inductance in series

Let L_1 = Self-inductance of coil A

L_2 = Self-inductance of coil B

M = Mutual inductance between coils A and B.

Let two coils A and B be connected in series in such a way, that the flux produced by coil B, links with the coil A in the same direction as shown in Fig. A3-17(a).

Then, effective self-inductance of coil $A = L_1 + M$

And, effective self-inductance of coil $B = L_2 + M$

$\therefore$ Total self-inductance of the circuit

$$L = L_1 + L_2 + 2M$$

The value of leakage coefficient in electrical machines usually varies from 1.1 to 1.25.

21. METHODS OF MINIMISING LEAKAGE

The magnetic leakage can be minimised in the following two ways:-

(i) By placing the magnetisting coils as closely as possible to the air gap or to the place in the magnetic circuit where flux is required for the useful purposes as shown in Fig. A3-18(a).

(ii) By placing the copper bands around the air gap or at the place in the magnetic circuit where flux is required as shown in Fig. A3-18(b).

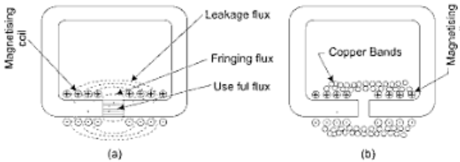


Fig. A3-18

22. FRINGING

From the Fig. A3-19, it may also be seen that there is spreading of the lines of force at the edges of air gap. This is known as fringing and due to it, the effective area of the air gap increases.

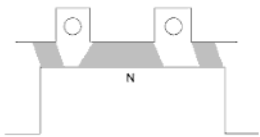


Fig. A3-19

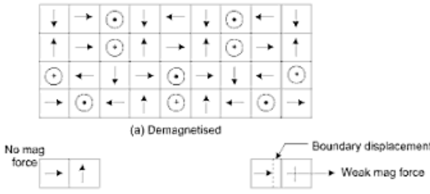
The phenomenon of fringing may be illustrated by the diagram of Fig. A3-19. Due to the effect of fringing the area occupied by the flux in the air gap is more than in iron at tooth.

Cast steel, wrought iron, dynamo sheet steel and good forged steel are shown in Fig. A3-19(b) and for cast iron and dynamo sheet steel are shown in Fig. A3-19.

24. ATOM-DOMAIN THEORY OF MAGNETISATION

The metals are composed of crystals or grains which are so small that they can hardly be seen by the naked eye. The domains are small local regions which are magnetised to saturation and have a tendency to group themselves into the crystals or grains. These domains align themselves in any one of the six equivalent directions of the crystal axes.

In Fig. A3-21(a), a portion of a crystal is shown and the domains are represented by small cubes. The direction of magnetisation of the domains is represented by the arrows. *A*. represents the direction of magnetisation upwards and a + represents downwards.



(a) Demagnetised

(b) Magnetisation by boundary displacement

Fig. A3-21

The probability of domain orientation is same in all the six equivalent directions of the crystal axes. Under these conditions, the crystal as a whole will be unmagnetised. As a small magnetising force is applied, the crystal begins to indicate a week magnetic orientation. This initial effect is due to slight boundary displacement between two domains as shown in Fig. A3.21(b).

H-Increasing:

In Fig. A3-22, the flux density *B* is plotted against magnetising force *H*. The lowest part *A* of the magnetisation curve is due to the boundary displacement and is slightly concave upwards.

When the magnetising force is further increased the domains become progressively aligned in the direction of *H* and the rate of this alignment is almost uniform. So, the portion B of the magnetisation curve is nearly linear and is said to be due to sudden change in orientation.

When the magnetising force becomes sufficiently strong, the direction of orientation of all the domains become in the direction of the magnetising force.

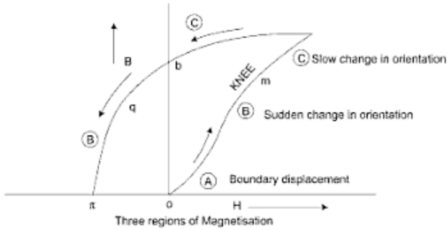


Fig. A3-22

Thus, when all the domains are oriented in the same direction each crystal or grain acts like one very large domain. This state is represented by the 'knee' of the curve and the iron is near saturation.

When the magnetising force is further increased and becomes very strong, the magnetisation is very slow and takes place due to electron spins. During electron spins the domains tend to rotate out of their stable direction along the direction of the magnetising force. The iron is now fully saturated and this process is called the 'rotation of domain' and the portion *C* of the magnetisation curve is obtained.

Thus, the normal saturation curve omn is obtained.

H-Decreasing:

If the magnetising force is decreased from the point '*n*' the curve shall not retrace along the normal saturation curve. This curve will follow a path *np* lying above the normal curve. This happens because orientation of domains tends to persist along the direction of their last orientation. Thus, when *H* reaches zero, there exists a flux density *op* due to the above reasons. The flux density is called remanence and the amount of magnetism existing is called residual magnetism.

If at this instant the magnetising force is reversed in direction, the curve follows the path *pq*. The portion *q* corresponds to the knee.

When there is further negative increase in the magnetising force *H*, the flux density drops rapidly and the portion *qr* of the curve is obtained. This rapid drop in flux density is due to the domains in the crystals which become progressively deoriented. The magnetising force or at which the flux density becomes zero is called coercive force.

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