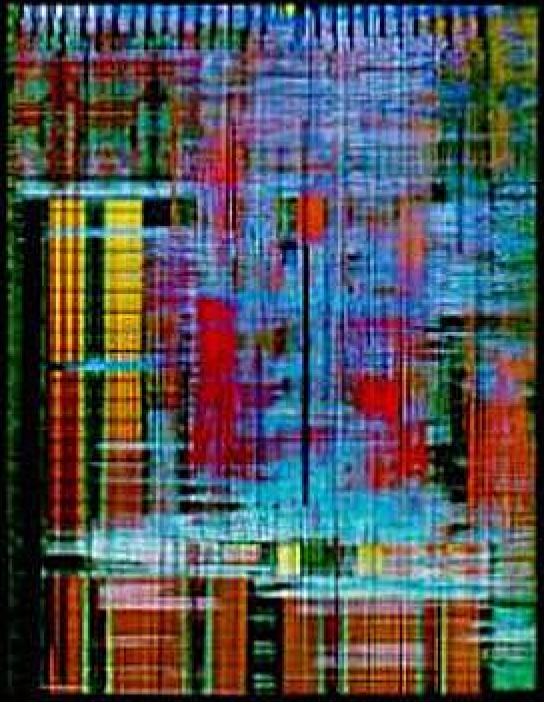
FIFTH EDITION

SOLID STATE ELECTRONIC DEVICES



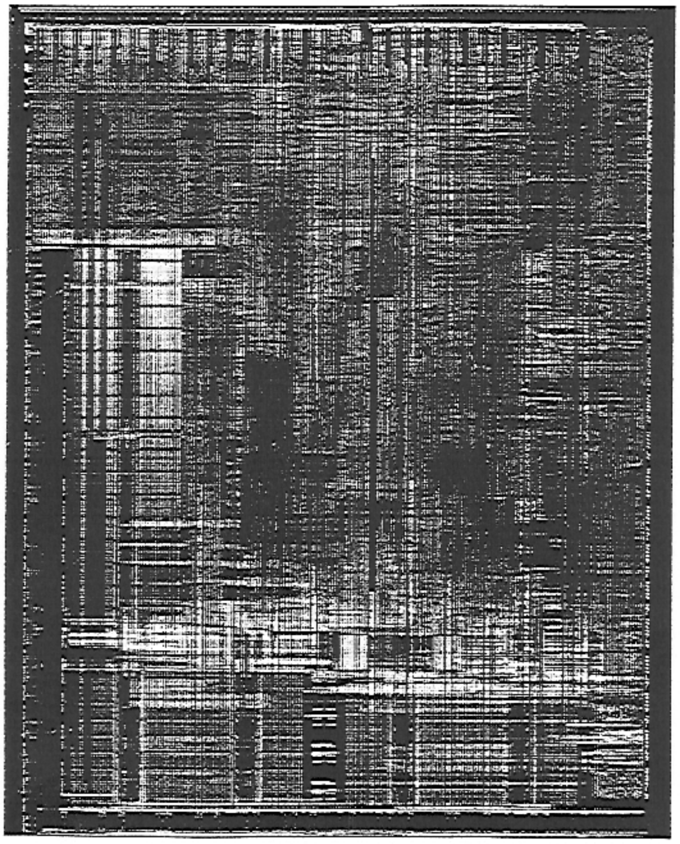
BEN G. STREETMAN ▪ SANJAY BANERJEE

Prentice Hall Series in Solid State Physical Electronics, Nick Holonyak, Jr., Series Editor

INSTRUCTOR'S MANUAL

FIFTH EDITION

SOLID STATE ELECTRONIC DEVICES



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Chapter 1

Prob. 1.1

Which semiconductor in Table 1-1 has the largest E_g? the smallest? What is the corresponding λ? How is the column III component related to E_g?

largest E_g : ZnS, 3.6 eV.

$$\lambda = \frac{1.24}{3.6} = 0.344\mu m$$

smallest E_g : InSb, 0.18 eV.

$$\lambda = 6.89\mu m$$

Note Al compounds have larger E_g than the corresponding Ga compounds, which are larger than In compounds.

Prob. 1.2

Here we need to calculate the maximum packing fraction, treating the atoms as hard spheres.

Nearest atoms are at a separation $\frac{1}{2}\times\sqrt{(5\times\sqrt{2})^2+5^2} = \mathbf{4.330\text{Å}}$

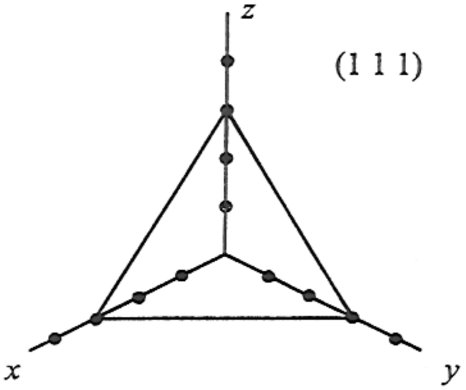
Radius of each atom $= \frac{1}{2}\times 4.330\text{Å} = \mathbf{2.165\text{Å}}$

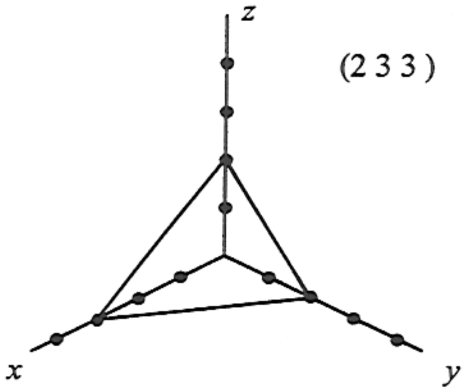
Volume of each atom $= \frac{4}{3}\pi(2.165)^3 = \mathbf{42.5\text{Å}^3}$

Number of atoms per cube $= 1+8\times\frac{1}{8} = \mathbf{2}$

Packing fraction $= \frac{42.5 \times 2}{(5)^3} = \mathbf{68\%}$

Prob. 1.3
(a) Label planes:

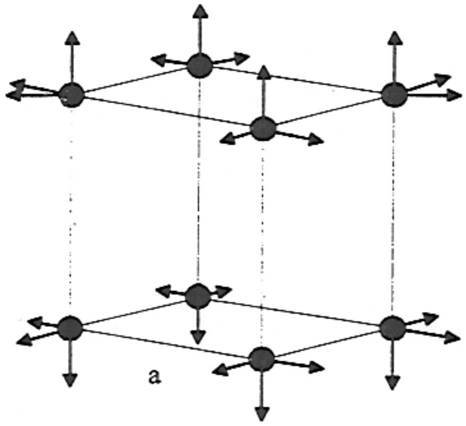




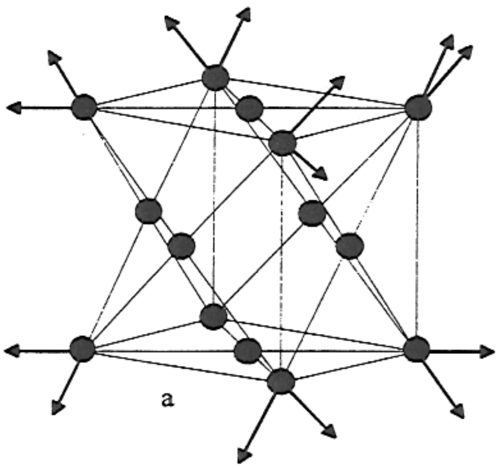
x	y	z
3	3	3
1/3	1/3	1/3
1	1	1

x	y	z
3	2	2
1/3	1/2	1/2
2	3	3

(b) Draw equivalent directions in a cube

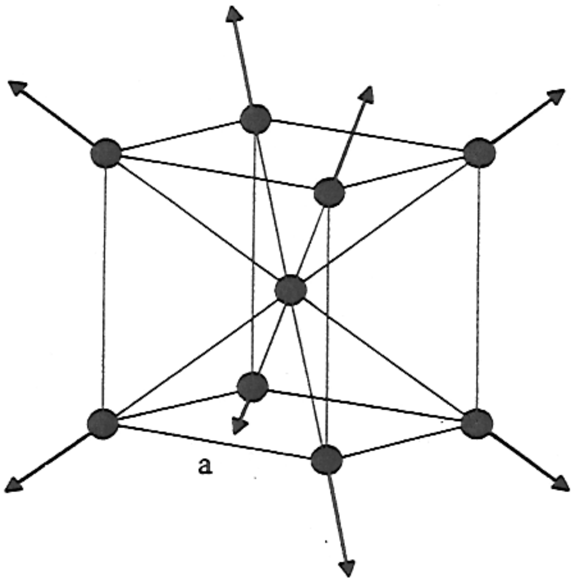


<1 0 0> all edges



<1 1 0> all face diagonals

(Need not show atoms)



<1 1 1> all body diagonals

Prob. 1.4

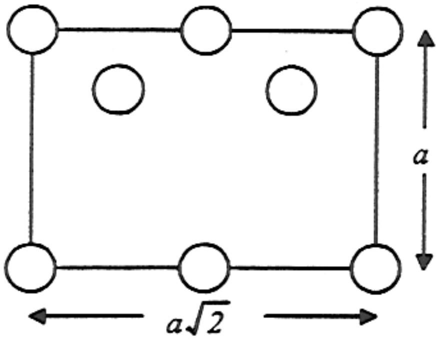
We need to calculate the volume density of Si, its density on the (110) plane and the distance between two adjacent (111) planes.

Si FCC lattice with basis of 2 atoms

$$\text{Number of atoms per cube} = \left(8\times\frac{1}{8}+\frac{1}{2}\times 6\right)\times 2 = \mathbf{8}$$

$$\text{Density} = \frac{8}{(5.43\times10^{-8})^3} = \mathbf{5.00\times10^{22}\,cm^{-3}}$$

On the (110) plane we have 4 atoms on corners, 2 on the top and bottom planes, and 2 interior (see Fig. 1-a).



(110) plane : $\dfrac{4\times\frac{1}{4}+\frac{1}{2}\times2+2}{(5.43\times10^{-8})(\sqrt{2}\times5.43\times10^{-8})}=\mathbf{9.59\times10^{14}\,cm^{-2}}$

Basis of Si crystal at 0 and $\frac{a}{4},\frac{a}{4},\frac{a}{4}$ which is along [111].

Distance $=\sqrt{3}\left(\frac{a}{4}\right)=$ **2.39Å.**

Prob. 1.5

Using the hard-sphere model, find the lattice constant of InSb, the volume of the primitive cell and the atomic density on the (110) plane.

$$\frac{\sqrt{3}a}{4}=1.44+1.36=2.8\text{Å}$$

$$a=6.47\text{Å}$$

In FCC, unit cell has 4 lattice points. Therefore, volume of primitive cell $=\frac{a^3}{4}=67.7\text{Å}^3$

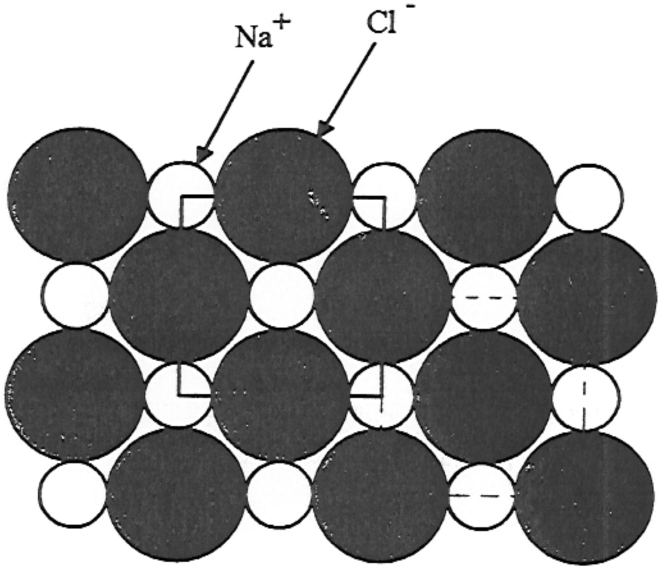
Area of (110) plane $=\sqrt{2}a^2$

Density of In atoms $=\dfrac{4\times\frac{1}{4}+2\times\frac{1}{2}}{\sqrt{2}a^2}=\dfrac{\sqrt{2}}{a^2}=\mathbf{3.37\times10^{14}\ cm^{-2}}$

Same number of Sb atoms = $\mathbf{3.37\times10^{14}\ cm^{-2}}$

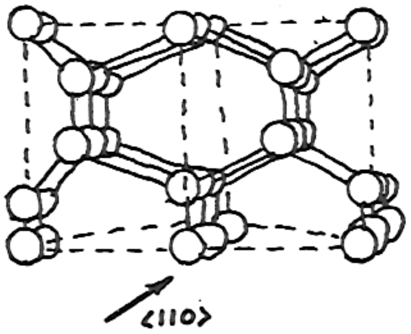
Prob. 1.6

Draw NaCl lattice (1 0 0) and unit cell.



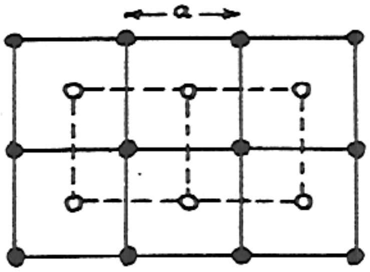
Two possible unit cells are shown, with either Na^+ or Cl^- at the corners.

Prob. 1.7

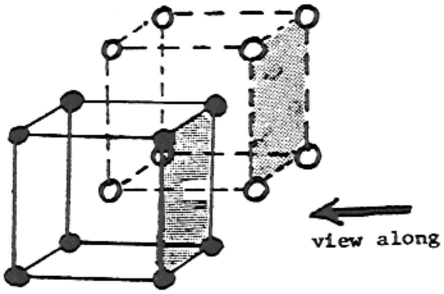


This view is tilted slightly from ⟨110⟩ to show the alignment of atoms. The open channels are hexagonal along this direction.

Prob. 1.8



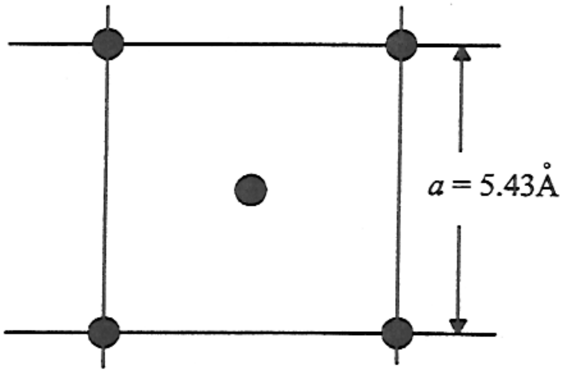
The shaded points are one sc lattice; the open points are the interpenetrating sc, located a/2 behind the plane of the front shaded points.



view along this direction

Prob. 1.9

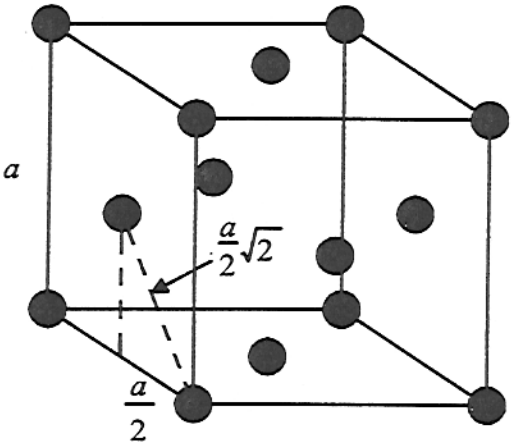
(a) Find the number of Si atoms/cm^2 on the surface of a (1 0 0) oriented Si wafer.



Each a^2 has $1+\frac{1}{4}(4) = 2$ atoms on the surface.

$$\frac{2\ \text{atoms/cell}}{(5.43\times10^{-8})^2\text{cm}^2\text{/cell}} = \mathbf{6.78\times10^{14}\,cm^{-2}}$$

(b) What is the distance (in Å) between nearest In neighbors in InP?



In atoms are in an fcc sublattice with $a = 5.87\text{Å}$, nearest neighbors are

$$\frac{a}{2}\sqrt{2} = \frac{5.87}{2}\sqrt{2} = \mathbf{4.15\text{Å}}$$

Prob. 1.10

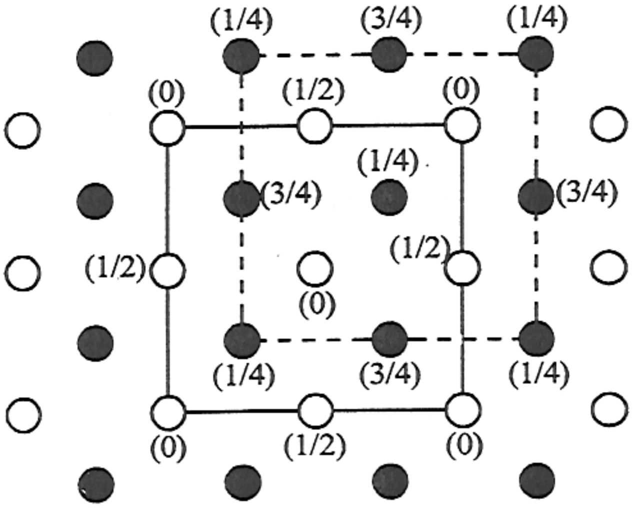
Find NaCl density.
Na^+: atomic wt. 23, radius 1 Å. Cl^-: atomic weight 35.5, radius 1.8 Å.

The unit cell contains ½ Na and ½ Cl atoms. Using the hard sphere approximation, $a = 2.8$Å.

$$\text{density} = \frac{\frac{1}{2}(23+35.5)/(6.02\times10^{23})}{(2.8\times10^{-8})^3} = \mathbf{2.2\ g/cm^3}$$

Prob. 1.11

Label atom planes in Fig. 1.8b.



Prob. 1.12

Find atoms/cell and nearest neighbor distance for sc, bcc, and fcc lattices.
(see solution to Prob. 1.5)

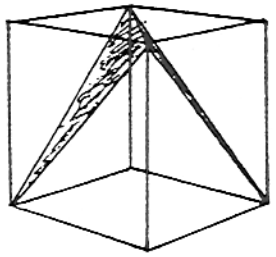
for sc
atoms/cell = $\frac{1}{8}(8) = 1$
nearest neighbor = a

for bcc
atoms/cell = $\frac{1}{8} + 1 = 2$
nearest neighbor = $\frac{a}{2}\sqrt{3}$

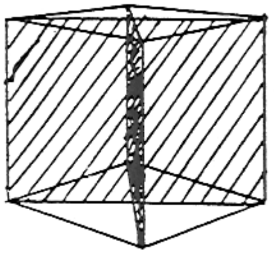
for fcc
(see Example 1-1)
atoms/cell = 4
nearest neighbor = $\frac{1}{2}a\sqrt{2}$

Prob. 1.13

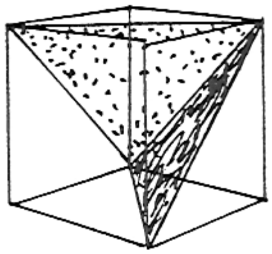
Show four {111} planes. Repeat for {110} planes.

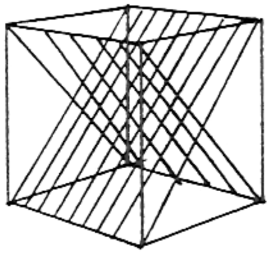


{ 111 }



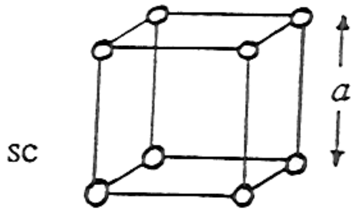
{ 110 }



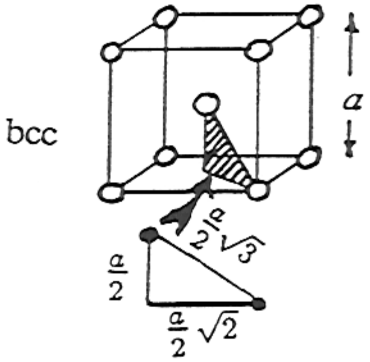


Prob. 1.14

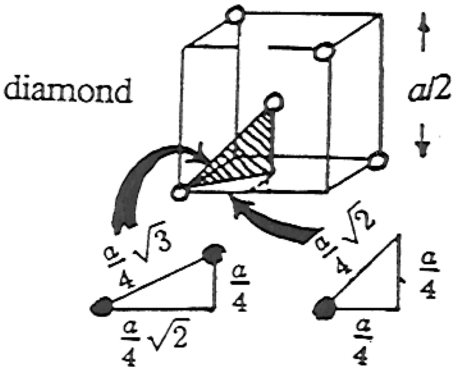
Find fraction occupied in sc, bcc, diamond.



atoms/cell = $\frac{1}{8}(8) = 1$
nearest neighbor distance = a
maximum sphere radius = $a/2$
vol. of each sphere = $\frac{4}{3}\pi(\frac{a}{2})^3$
total occupied vol.
$= 1$ atom/cell $\times \frac{\pi}{6}a^3$
vol. of unit cell = a^3
fraction occupied = $\frac{\pi}{6}$ = **0.52**



2 atoms/cell
nearest neighbor distance = $\frac{a}{2}\sqrt{3}$
$r_{max} = \frac{a}{4}\sqrt{3}$
fraction occupied
$= (\frac{4}{3}\pi(\frac{a}{4}\sqrt{3})^3 \times 2)/a^3$
$= \frac{\pi}{8}\sqrt{3} =$ **0.68**



8 atoms/cell (4 from fcc +
4 at $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ from fcc atoms)
nearest neighbor distance = $\frac{a}{4}\sqrt{3}$
$r_{max} = \frac{a}{8}\sqrt{3}$
fraction occupied
$= (\frac{4}{3}\pi(\frac{a\sqrt{3}}{8})^3 \times 8)/a^3$
$= \frac{\pi}{16}\sqrt{3} =$ **0.34**

Prob. 1.15

Find Ge and InP densities as in Example 1-3.

The atomic weight of Ge is 72.6; for In, 114.8; for P, 31.

For Ge: $a = 5.66\text{Å}$, 8 atoms per cell

$$\frac{8}{a^3} = \frac{8}{(5.66\times10^{-8})^3} = 4.41\times10^{22}\ \text{atoms/cm}^3$$

$$\text{density} = \frac{4.41\times10^{22}\times72.6}{6.02\times10^{23}} = \mathbf{5.32\ g/cm^{-3}}$$

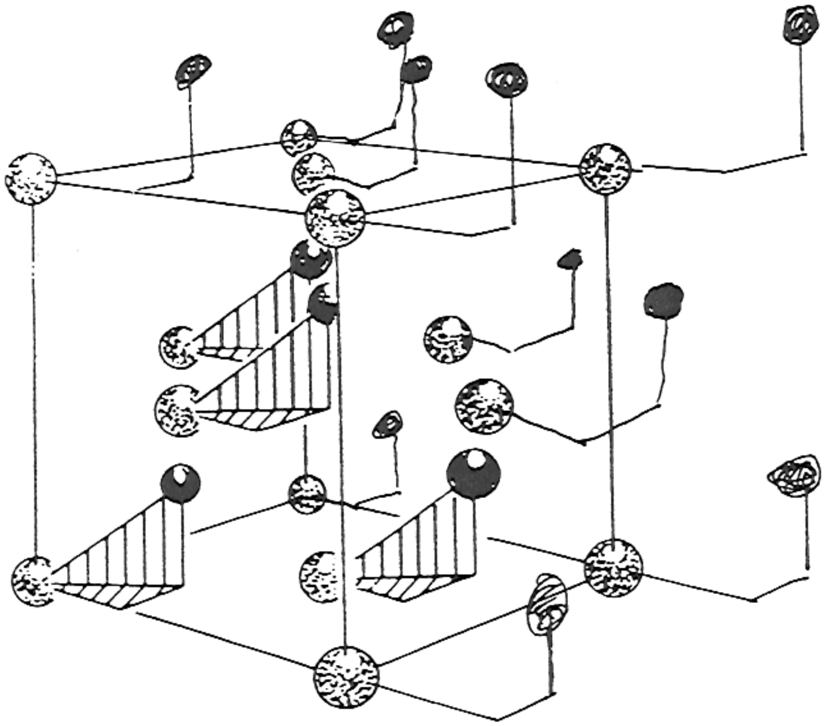
For InP: $a = 5.87\text{Å}$, 4 In + 4 P per cell

$$\frac{4}{a^3} = \frac{4}{(5.87\times10^{-8})^3} = 1.98\times10^{22}\ \text{atoms/cm}^3$$

$$\text{density} = \frac{1.98\times10^{22}\times(114.8+31)}{6.02\times10^{23}} = \mathbf{4.79\ g/cm^{-3}}$$

Prob. 1.16

Sketch diamond lattice and show only four atoms in the interpenetrating fcc are in the unit cell.



Prob. 1.17

What composition of AlSbAs is lattice matched to InP? InGaP to GaAs? What are the E_g's?

From Fig. 1-15 $AlSb_xAs_{1-x}$ ternary crosses the InP lattice constant at $x=0.43$ where $E_g = 1.9$ eV

$In_xGa_{1-x}P$ crosses the GaAs lattice constant at $x=0.48$, where $E_g = 2$ eV

Prob. 1.18

What weight of As ($k_d = 0.3$) should be added to 1 kg Si to achieve $10^{15}cm^{-3}$ doping during initial Czochralski growth?

The atomic weight of As is 74.9.

$C_s = k_d C_L$, thus $C_L = 10^{15} / 0.3 = 3.33 \text{ x } 10^{15} \text{ cm}^{-3}$

Calculating the melt volume from the weight of Si only, and neglecting the difference in density for solid and molten Si,

$$\frac{1000 \text{ g of Si}}{2.33 \text{ g/cm}^3} = 429.2 \text{ cm}^3 \text{ of Si}$$

$$3.33\times10^{15}\text{cm}^{-3}\times429.2\text{cm}^3 = 1.43\times10^{18} \text{ As atoms}$$

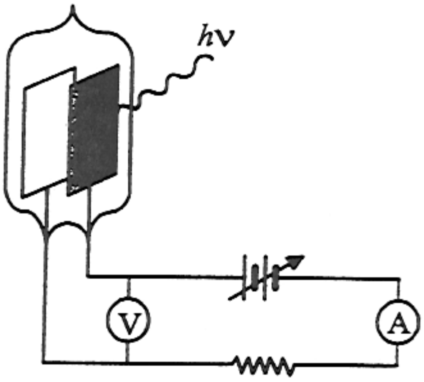
$$\frac{1.43\times10^{18}\times74.9}{6.02\times10^{23}} = 1.8\times10^{-4} \text{ g} = \mathbf{1.8\times10^{-7}} \textbf{ kg} \text{ of As.}$$

Chapter 2

Prob. 2.1

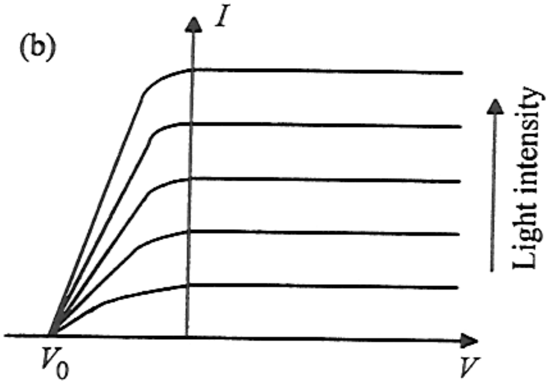
Sketch tube for photoelectric experiment and its I-V. What V_0 is required for $\lambda = 2440\text{Å}$ and Pt (4.09eV)?

(a)



This is a simplified sketch

(b)



Note the same V_0 is required for various intensities.

(c) $\lambda = 2440$ Å $= 0.244$ µm

$h\nu(\text{eV}) = 1.24/\lambda(\mu\text{m}) = 1.24/0.244 = \mathbf{5.08\ eV}$

$V_0 = h\nu - \phi = 5.08 - 4.09 \sim \mathbf{1\ V}$

Prob. 2.2

If $V_{AB} = 1\,V$, find energy and velocity of e^- moving from B to A.

The electron gains 1 eV $= 1.6 \times 10^{-19}$ J.

$$E = \frac{1}{2}m\mathrm{v}^2$$

$$\mathrm{v} = \sqrt{2E/m} = \left[\frac{2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right]^{1/2} = \mathbf{5.93 \times 10^5 m/s}$$

Prob. 2.3

Find wavelengths for hydrogen Lyman, Balmer, Paschen series.

(a)

$$\lambda = \frac{c}{\nu} = \frac{c}{cR(\frac{1}{n_1^2} - \frac{1}{n_2^2})} = \frac{1}{R}\frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)}$$

$$= \frac{1}{109{,}678}\frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)} = (911 \times 10^{-8}\,\text{cm})\frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)}$$

(b) Lyman

n	n^2	n^2-1	$n^2/(n^2$-1)	(911) $n^2/(n^2$-1)
2	4	3	1.333	**1215 Å**
3	9	8	1.125	**1025 Å**
4	16	15	1.067	**972 Å**
5	25	24	1.042	**949 Å**
				Limit: **911 Å**

Similarly, for the Balmer series the limit is 4(911) = **3644 Å**, and the wavelengths for n = 3 to n = 7 are **6559, 4859, 4338, 4100,** and **3968 Å.**
For the Paschen series the limit is 8199 Å and the wavelengths for n = 4 to n = 10 are **18741, 12811, 10932, 10044, 9541, 9224,** and **9010 Å.**

Prob. 2.4

Show Eq.(2-17) corresponds to Eq.(2-3). That is, show

$$cR = \frac{mq^4}{2K^2\hbar^2 h}$$

From the solution of Prob. 2.3, Eq.(2-17) is

$$\nu_{21} = \frac{21.7 \times 10^{-19}}{6.63 \times 10^{-34}}(\frac{1}{n_1^2} - \frac{1}{n_2^2}) = 3.27 \times 10^{15}(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

Eq.(2-3) is

$$\nu_{21} = 3 \times 10^8 \times 1.097 \times 10^7(\frac{1}{n_1^2} - \frac{1}{n_2^2}) = 3.29 \times 10^{15}(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

Prob. 2.5

(a) What is Δp_x if $\Delta x = 1$ Å?

$$\Delta p_x(\mathrm{kg\cdot m/s}) = \frac{h(\mathrm{J\cdot s})}{2\pi\Delta x(\mathrm{m})} = \frac{6.63\times10^{-34}((\mathrm{kg\cdot m^2/s^2})\mathrm{s})}{2\pi(10^{-10}\mathrm{m})}$$

$$= \mathbf{1.06\times10^{-24}\ kg\cdot m/s}$$

(b) What is Δt if $\Delta E = 1$ eV?

$$\Delta t = \frac{\hbar(\mathrm{eV\cdot s})}{\Delta E(\mathrm{eV})} = \mathbf{6.59\times10^{-16}s}$$

Prob. 2.6

Find λ for 100eV and 12 keV electrons. Comment on e- microscopes compared to visible light.

$$\mathrm{v} = \sqrt{2E/m}, \quad \lambda = \frac{h}{m\mathrm{v}} = \frac{h}{\sqrt{2Em}} = \frac{6.63\times10^{-34}}{[2\times9.11\times10^{-31}]^{1/2}}E^{-1/2}$$

For 100eV :

$$\lambda = \frac{4.9\times10^{-19}}{(1.6\times10^{-19})^{1/2}}[100]^{-1/2} = 1.23\times10^{-9}[100]^{-1/2} = 1.23\times10^{-10}\mathrm{m} = 1.23\text{Å}$$

For 12keV :

$$\lambda = 1.23\times10^{-9}[1.2\times10^4]^{-1/2} = 1.12\times10^{-11}\mathrm{m} = 0.112\text{Å}$$

Visible light is about 0.5μm = 5000Å, so the resolution of electron microscopy is much better.

Prob. 2.7

Show that τ is the average lifetime in exponential decay.

The probability of finding an atom in the unstable state at time t is $N(t)/N_0 = \exp(-t/\tau)$. This is analogous to the probability of finding a particle at x in Eq.(2-21b). Thus we have

$$\langle t\rangle = \frac{\int_0^\infty te^{-t/\tau}dt}{\int_0^\infty e^{-t/\tau}dt} = \frac{\tau^2}{\tau} = \tau$$

Alternative solution: see the approach used in calculating diffusion length, Eqs.(4-37) to (4-39).

Prob. 2.8

Calculate the expectation value for p_x^2 *and* p_z *for a plane wave* $\Psi = e^{jk_x x}$.

$$\left\langle \mathsf{p}_x^2 \right\rangle = \frac{\int_{-\infty}^{\infty} A^* e^{-jk_x x}\left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right)^2 A e^{jk_x x}\,dx}{\int_{-\infty}^{\infty} |A|^2 e^{-jk_x x} e^{jk_x x}\,dx} = (\hbar k_x)^2 \text{ after normalization}$$

$$\langle \mathsf{p}_z \rangle = \frac{\int A^* e^{-jk_x x}\left(\frac{\hbar}{j}\frac{\partial}{\partial z}\right) A e^{jk_x x}\,dx}{\int \Psi^* \Psi\,dx} = 0$$

because Ψ has no z dependence.

Prob. 2.9

Relate momentum to wave vector for a free electron described by a plane wave.

See Example 3-1.

Prob. 2.10

Calculate the expectation value for p_x, p_y *and E for a plane wave* $\Psi(x,t) = Ae^{j(10x-7t)}$.

$$\langle \mathsf{p}_x \rangle = \frac{\int_{-\infty}^{\infty} A^* e^{-jk_x x}\left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right) A e^{jk_x x}\,dx}{\int_{-\infty}^{\infty} |A|^2 e^{-jk_x x} e^{jk_x x}\,dx} = \hbar k_x \text{ after normalization}$$

$$= \left(\frac{6.63\times 10^{-34}}{2\pi}(10)\right)\mathrm{kg\cdot m\cdot s^{-1}} = \mathbf{1.055\times 10^{-33}\ kg\cdot m\cdot s^{-1}}$$

$$\langle \mathsf{p}_z \rangle = \frac{\int A^* e^{-jk_x x}\left(\frac{\hbar}{j}\frac{\partial}{\partial z}\right) A e^{jk_x x}\,dx}{\int \Psi^* \Psi\,dx} = \mathbf{0}, \text{ because } \Psi \text{ has no z dependence}$$

$$\langle E \rangle = \frac{\int A^* e^{-jk_x x}\left(-\frac{\hbar}{j}\frac{\partial}{\partial t}\right) A e^{jk_x x}\,dx}{\int \Psi^* \Psi\,dx} = \frac{-\hbar}{j}(-7j) = 7\hbar$$

$$= \left[\frac{6.63\times 10^{-34}}{2\pi}\times 7\right]\mathrm{J} = \mathbf{7.39\times 10^{-34}\ J}$$

Prob. 2.11

Calculate first three energy levels for a 10 Å *quantum well with infinite walls.*

From Eq. 2.33

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times [10^{-9}]^2}\, n^2$$

$$E_1 = 0.603 \times 10^{-19}\ J = \mathbf{0.377\ eV}$$

$$E_2 = 0.377(4) = \mathbf{1.508\ eV}$$

$$E_3 = 0.377(9) = \mathbf{3.393\ eV}$$

Prob. 2.12

Comment on the alkali metals and the halogens.

Li, Na and K have one valence e^- outside a closed shell. F, Cl and Br require one electron to fill a shell and to then have electron configurations like inert Ne, Ar, Kr.

What are the electronic configurations for Na^+ *and* $C\ell^-$ *?*

Na^+ has $1s^2\ 2s^2\ 2p^6$, which is the [Ne] configuration.
$C\ell^-$ has $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$, which is the [Ar] configuration.

Chapter 3

Prob. 3.1

Calculate Bohr radius for donor in Si ($m_n^* = 0.26\ m_0$).

From Eq.(2-10) with $n = 1$ and using $\varepsilon_r = 11.8$ for Si:

$$r = \frac{4\pi\varepsilon_r\varepsilon_0\hbar^2}{m_n^* q^2} = \frac{11.8(8.85\times10^{-12})(6.63\times10^{-34})^2}{\pi(0.26)(9.11\times10^{-31})(1.6\times10^{-19})^2}$$

$$r = 2.41\times10^{-9}\text{m} = \mathbf{24.1Å}$$

Note that this is more than four lattice spacings a (5.43Å)

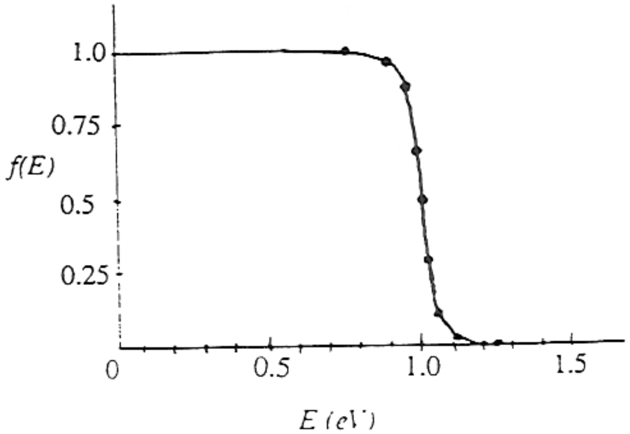
Prob. 3.2

Plot Fermi function for $E_F = 1$ eV.

$$f(E) = [1 + e^{(E-E_F)/kT}]^{-1}$$

We will choose E in eV and therefore use $kT = 0.0259$

E(eV)	$(E-E_F)/kT$	$f(E)$
0.75	-9.6525	**0.99994**
0.90	-3.8610	**0.97939**
0.95	-1.9305	**0.87330**
0.98	-0.7722	**0.68399**
1.02	+0.7722	**0.31600**
1.05	+1.9305	**0.12669**
1.10	+3.8610	**0.02061**
1.25	+9.6525	**0.00006**



Prob. 3.3

Calculate the density-of-states effective mass associated with the X minimum for the given band structure.

Given that near the energy minimum along [1 0 0], the band structure is:

$$E = E_0 - A\cos(\alpha k_x) - B\{\cos(\beta k_y) + \cos(\beta k_z)\}$$

which can be Taylor expanded near the minima:

$$E \approx E_0 - A\left[1 - 2\left(\frac{\alpha k_x}{2}\right)^2\right] - B\left[2 - 2\left(\frac{\beta k_y}{2}\right)^2 - 2\left(\frac{\beta k_z}{2}\right)^2\right]$$

$$\approx (E_0 - A - 2B) + \frac{A}{2}\alpha^2 k_x^2 + \frac{B}{2}\beta^2(k_y^2 + k_z^2)$$

The effective mass is defined as: $m^* = \dfrac{\hbar^2}{\dfrac{d^2E}{dk^2}}$

Along (1 0 0) direction (longitudinal direction), the effective mass becomes:

$$m_l^* = \frac{\hbar^2}{\dfrac{d^2E}{dk_x^2}} = \frac{\hbar^2}{A\alpha^2}$$

Along the two transverse directions, the effective mass becomes:

$$m_t^* = \frac{\hbar^2}{\dfrac{d^2E}{dk_y^2}} = \frac{\hbar^2}{B\beta^2}$$

Finally, the density-of-states effective mass is given by:

$$D.O.S.\ m^* = (m_l^* m_t^{*2})^{1/3} = \frac{\hbar^2}{(AB^2\alpha^2\beta^4)^{1/3}}$$

Prob. 3.4

For the given band structure, find the temperature at which the number of electrons in the Γ minima and the X minima equal.

From Eq.(3-15), we have:

$$\frac{n_X}{n_\Gamma} = \frac{N_{cX}}{N_{c\Gamma}} e^{-\frac{0.35}{kT}}$$

Given that there are 6 X minima along the < 100 > directions, from Eq.(3-16b) we get:

$$\begin{cases} N_{cX} \propto 6 \cdot (0.30)^{3/2} \\ N_{c\Gamma} \propto (0.065)^{3/2} \end{cases}$$

$$\frac{n_X}{n_\Gamma} = 6 \times \left(\frac{0.30}{0.065} \right)^{\frac{3}{2}} e^{-\frac{0.35}{kT}}$$

When $n_\Gamma = n_X$, we obtain

$$e^{\frac{0.35}{kT}} = 6 \times \left(\frac{0.30}{0.065} \right)^{\frac{3}{2}}$$

That is: $kT = 0.0857\text{eV}$ or, $T = \mathbf{988K}$

Prob. 3.5

For the given band structure, calculate and sketch how the conductivity varies from low T to high T and find the ratio of the conductivities at 1000°C and 300°C.

$$n = n_\Gamma + n_L = n_\Gamma \left(1 + (15)^{3/2} e^{-\frac{0.30}{kT}} \right)$$

= constant, independent of temperature according to the problem

From Eq.(3-15), we have:

$$\frac{n_L}{n_\Gamma} = \frac{N_{cL}}{N_{c\Gamma}} e^{-\frac{0.30}{kT}}$$

$$n_\Gamma = N_{c\Gamma} e^{\frac{E_F - E_{c\Gamma}}{kT}}$$

$$n_L = N_{cL} e^{\frac{E_F - E_{c\Gamma}}{kT}} \cdot e^{\frac{-E_s}{kT}} = \left(\frac{15 m_\Gamma^*}{m_\Gamma^*} \right)^{3/2} n_\Gamma e^{\frac{-E_s}{kT}} = (15)^{3/2} n_\Gamma e^{\frac{-E_s}{kT}}$$

$$\sigma = q[n_\Gamma \mu_\Gamma + n_L \mu_L] = q[n_\Gamma \mu_\Gamma + n_L \frac{\mu_\Gamma}{50}]$$

$$= q n_\Gamma \mu_\Gamma \left[1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}} \right]$$

$$= q \frac{n}{1 + (15)^{3/2} e^{-\frac{0.30}{kT}}} \mu_\Gamma \left[1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}} \right]$$

$$= q n \mu_\Gamma \frac{1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}}}{1 + (15)^{3/2} e^{-\frac{0.30}{kT}}}$$

$$= q n \mu_\Gamma \left[\frac{1 + \frac{58.1}{50} e^{-\frac{0.30}{kT}}}{1 + 58.1 \times e^{-\frac{0.30}{kT}}} \right]$$

$$T << \frac{E_s}{k}, \ \sigma = \sigma_0 = q n \mu_\Gamma$$

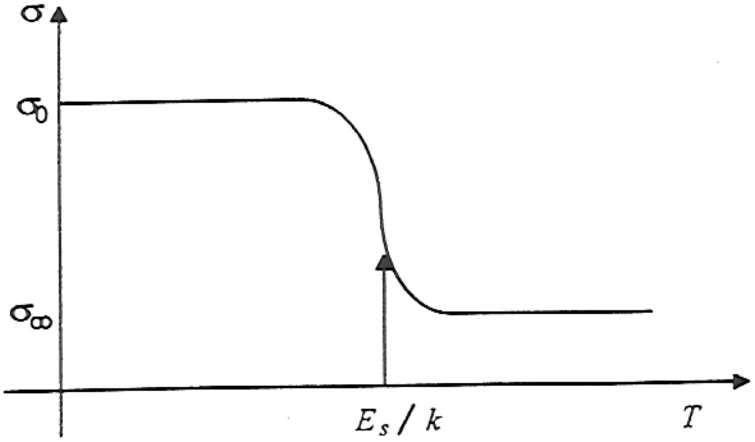
$$T >> \frac{E_s}{k}, \ \sigma = \sigma_\infty = q n \mu_\Gamma \left(\frac{1 + 1.16}{1 + 58.1} \right)$$

$$\frac{\sigma(1000°C)}{\sigma(300°C)} = \mathbf{0.254}$$

If the Γ to L separation is assumed to be 0.35 eV instead of 0.30 eV, we get:

$$\sigma = qn\mu_\Gamma \left[\frac{1+\frac{58.1}{50}e^{-\frac{0.35}{kT}}}{1+58.1\times e^{-\frac{0.35}{kT}}} \right]$$

$$\frac{\sigma(1000°C)}{\sigma(300°C)} = \mathbf{0.322}$$



Prob. 3.6

Find E_g for Si from Fig. 3-17.

$$\ln n_{i1} = \ln\sqrt{N_c N_v} - \frac{E_g}{2k}\left(\frac{1}{T_1}\right)$$

$$\ln n_{i2} = \ln\sqrt{N_c N_v} - \frac{E_g}{2k}\left(\frac{1}{T_2}\right)$$

$$\ln n_{i1} - \ln n_{i2} = \frac{E_g}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

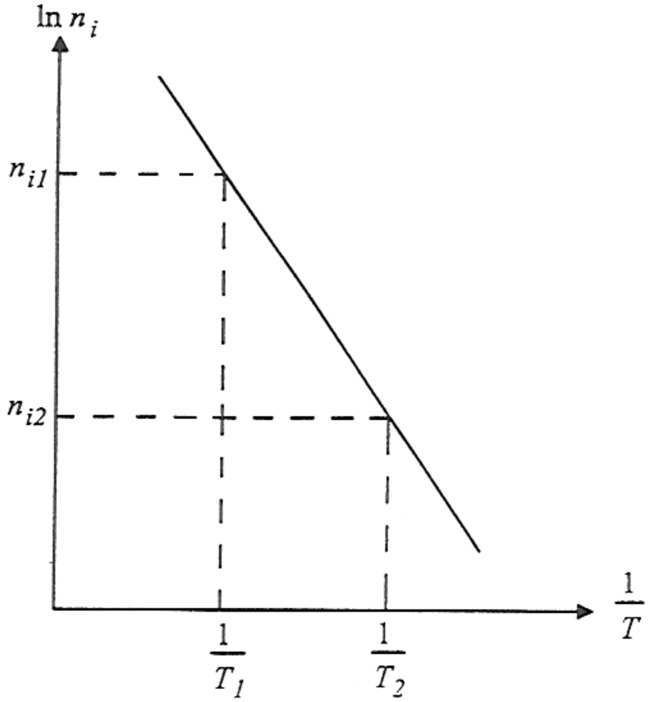
For Si, $n_{i2} = 10^8$ at $\frac{1}{T_2} = 4\times10^{-3}$

$n_{i1} = 3\times10^{14}$ *at* $\frac{1}{T_1} = 2\times10^{-3}$

$$E_g = 2k\ln(n_{i1}/n_{i2})/(\frac{1}{T_2} - \frac{1}{T_1})$$

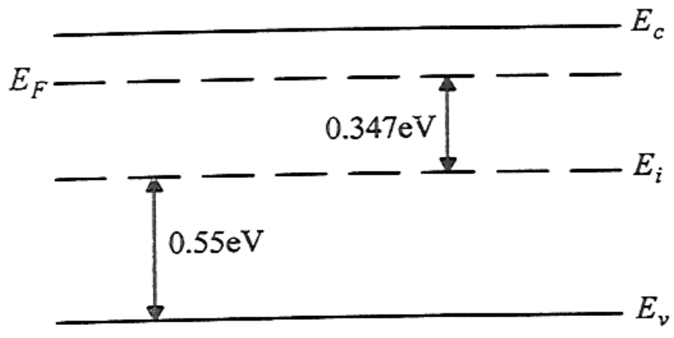
$$= \frac{2(8.62\times10^{-5})\cdot\ln(3\times10^{14}/10^8)}{(4-2)\cdot10^{-3}} = \mathbf{1.3\ eV}$$

This result is only approximate, since we neglect the temperature dependencies of N_C, N_V, and E_g.



Prob. 3.7

Show that Eq.(3-25) results from Eqs.(3-15) and (3-19) and find the position of the Fermi level relative to E_i at 300K



$$
\begin{aligned}
n_0 &= N_c e^{-(E_c-E_F)/kT} = N_c e^{-(E_c-E_i)/kT} e^{(E_F-E_i)/kT} \\
&= n_i e^{(E_F-E_i)/kT} \\
p_0 &= n_i^2 / n_0 = n_i e^{(E_i-E_F)/kT} \\
n_0 &= 10^{16} = 1.5\times10^{10} \times e^{(E_F-E_i)/0.0259}
\end{aligned}
$$

$$E_F - E_i = 0.0259\times\ln(6.667\times10^5) = \mathbf{0.347eV}$$

Prob. 3.8

Find the displacement of E_i from the middle of E_g for Si.

E_i is not exactly in the middle of the gap because the density of states N_C and N_V differ. Equating Eqs.(3-21) and (3-23).

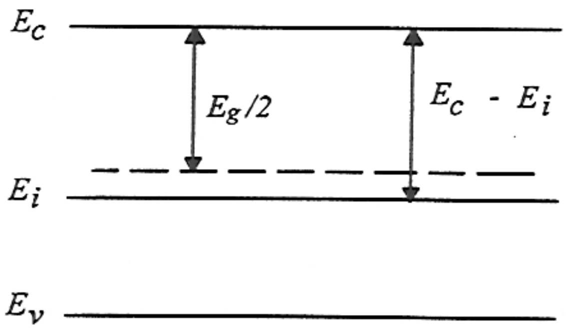
$$N_c e^{-(E_c-E_i)/kT} = \sqrt{N_c N_v} e^{-Eg/2kT}$$

$$E_g/2-(E_c-E_i) = kT\ln(N_v/N_c)^{1/2} = kT\ln(m_p^*/m_n^*)^{3/4}$$

For Si at 300K,

$$E_g/2-(E_c-E_i) = 0.0259\times\frac{3}{4}\ \ln\ (0.56/1.1) = \mathbf{-0.013\ eV}$$

Thus, E_i is about $kT/2$ below the center of the gap.



Prob. 3.9

(a) Explain why holes appear at the top of the valence band.

Electron energy is plotted "up" in band diagrams such as Fig. 3-5. Thus conduction band electrons relax to the bottom of the conduction band. Holes, having positive charge, have energies which increase oppositely to that of negatively charged electrons. That is, hole energy would be plotted "down" on an electron energy diagram such as Fig.3-5. Holes therefore relax to the lowest hole energy available to them; i.e. the "top" of the valence band.

(b) Explain why Si doped with 10^{14} cm^{-3} donors is n-type at 400K, but Ge is not.

According Fig. 3-17, the intrinsic concentration n_i at 400K is

$$n_i(400K) \sim 10^{15}\ cm^{-3} \text{ for Ge}$$
$$\sim 10^{13}\ cm^{-3} \text{ for Si}$$

Thus at this temperature, $N_d \gg n_i$ for Si, $N_d \ll n_i$ for Ge.

Prob. 3.10.

For Si with $N_d - N_a = 4 \times 10^{15}\ cm^{-3}$, find E_F and R_H.

$$n_0 = N_d - N_a = 4\times10^{15} = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT\ln(n_0/n_i)$$
$$= 0.0259\ln(4\times10^{15}/1.5\times10^{10}) = \mathbf{0.324eV}$$

$$R_H = -(qn_0)^{-1} = -(1.6\times10^{-19}\times4\times10^{15})^{-1} = \mathbf{-1562.5\ cm^3/C}$$

Prob. 3.11.

(a) Find the value of n_0 for minimum conductivity.

$$\sigma = q(n\mu_n + p\mu_p) = q(n\mu_n + \mu_p n_i^2/n)$$

$$\frac{d\sigma}{dn} = q(\mu_n - \mu_p n_i^2/n^2)$$

Setting this equal to zero and defining n_m as the electron concentration for minimum conductivity, we have

$$n_m^2 = n_i^2 \mu_p/\mu_n, \; n_m = n_i\sqrt{\mu_p/\mu_n}$$

(b) What is $\sigma_{\min}$?

$$\sigma_{\min} = qn_i(\mu_n\sqrt{\mu_p/\mu_n} + \mu_p\sqrt{\mu_n/\mu_p}) = 2qn_i\sqrt{\mu_n\mu_p}$$

(c) Calculate $\sigma_{\min}$ and σ_i for Si

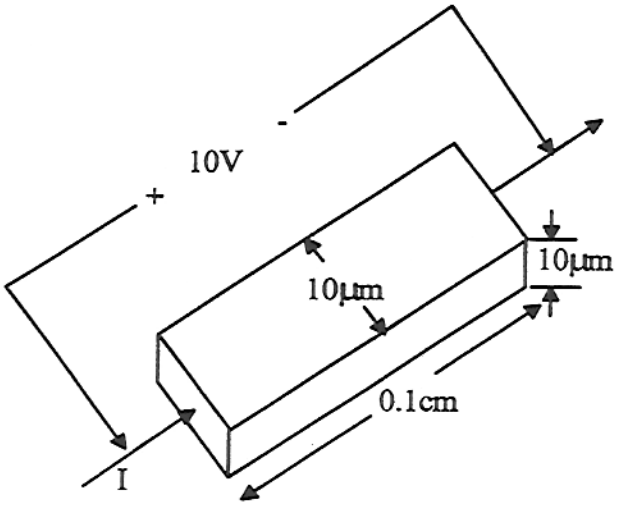
For Si,

$$\sigma_{\min} = 2(1.6\times10^{-19})(1.5\times10^{10})(1350\times480)^{1/2}$$
$$= 3.9\times10^{-6}(\Omega\cdot\text{cm})^{-1}$$
$$\sigma_i = qn_i(\mu_n + \mu_p) = (1.6\times10^{-19})(1.5\times10^{10})(1830)$$
$$= \mathbf{4.4\times10^{-6}}(\Omega\cdot\text{cm})^{-1}$$

or, take the reciprocal of ρ_i in Appendix III.

Prob. 3.12

(a) A Si bar 0.1 cm long and 100 μm² in cross sectional area is doped with 10^{17} cm⁻³ antimony. Find the current at 300K with 10V applied.



From Fig.3-23, $\mu_n = 700$ cm²/V-s

$$\sigma = q\mu_n n_0 = 1.6\times10^{-19}\times700\times10^{17} = 11.2\,(\Omega\cdot\text{cm})^{-1} = \rho^{-1}$$
$$\rho = 0.0893\,\Omega\cdot\text{cm}$$
$$R = \rho L/A = 0.0893\times0.1/10^{-6} = 8.93\times10^{3}\,\Omega$$
$$I = V/R = 10/(8.93\times10^{3}) = \mathbf{1.12\,mA}$$

Repeat for a length of 1μm.

Now $\mathcal{E}$ = 10V/10^{-4} cm = 10^5 V/cm, which is in the velocity saturation regime. From Fig.3-24, $v_s = 10^7$ cm/s

$$I = qAnv_s = (1.6\times10^{-19})(10^{-6})(10^{17})(10^{7}) = \mathbf{0.16\,A}$$

(b) How long does it take an average electron to drift 1 μm in pure Si at an electric field of 100V/cm? Repeat for 10^5 V/cm.

From Appendix III, $\mu_n = 1350$ cm²/V-s

low field: $v_d = \mu_n\mathcal{E} = 1350 \times 100 = 1.35 \times 10^5$ cm/s

$t = L/v_d = 10^{-4}/(1.35 \times 10^5) = 7.4 \times 10^{-10}$ s = **0.74 ns**

high field: scattering-limited velocity $v_s = 10^7$ cm/s (Fig. 3-24)

$t = 10^{-4}/10^7 = 10^{-11}$ s = **10 ps**

Prob. 3.13

A perfect III-V semiconductor is doped with column VI and II impurities. For the given μ_n, μ_p, calculate the energy levels introduced in the bandgap.

$$\mu_n = \frac{qt}{m_n^*} \Rightarrow m_n^* = \frac{1.6\times10^{-19}\times10^{-13}}{1000(10^{-4}\frac{m^2}{cm^2})} m_0 = 1.6\times10^{-31}\,\mathrm{kg} = 0.176 m_0$$

$$m_p^* = 1.408 m_0$$

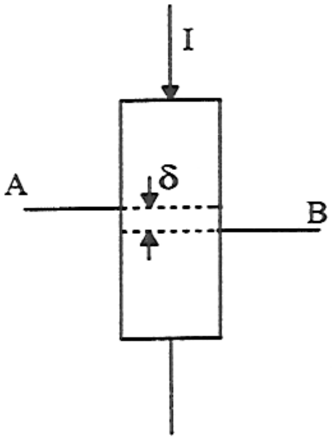
From Eq.(3 - 8) and using the value of the ground state energy of a H atom (1 Rydberg) = 13.6 eV :

$$E_D = \frac{13.6\frac{m_n^*}{m_0}}{\epsilon_r^2} = \mathbf{14.2meV} \text{ below } E_C$$

$$E_A = \mathbf{113.6meV} \text{ above } E_V$$

Prob. 3.14

Find V_H with Hall probes misaligned.



Displacement of the probes by an amount δ gives a small IR drop V_δ in addition to V_H. The Hall voltage reverses when $\mathscr{B}$ is reversed; however, V_δ is insensitive to the direction of the magnetic field. Thus,

with $\mathscr{B}$ positive: $V_{AB}^{+} = V_H + V_\delta$

with $\mathscr{B}$ negative: $V_{AB}^{-} = -V_H + V_\delta$

subtracting, $V_{AB}^{+} - V_{AB}^{-} = 2V_H$

We obtain the true Hall voltage from $V_H = \frac{1}{2}(V_{AB}^{+} - V_{AB}^{-})$.

Prob. 3.15

Find the position of the Fermi level for 11 electrons in an infinite 1-D potential well 100Å wide and the probability of exciting a carrier to the first excited state.

The energy levels are given by:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Since electrons obey the Pauli principle, only 2 electrons (opposite spin) are in each level.

Therefore, we can occupy up to $n = 6$ level $\left(\frac{11}{2} = 5 \text{ filled} + 1 \text{ half filled level}\right)$

$$E_6 = \frac{6^2\pi^2\hbar^2}{2mL^2}, \text{ where } m = \text{free electron mass, L} = 100\text{Å}$$

$$= 6^2(0.00120)\text{ eV} = 0.0432\text{ eV}$$

E_F is the highest filled level at 0K

Therefore, $E_F = E_6 = 0.0432$ eV.

First excited level is 7th level

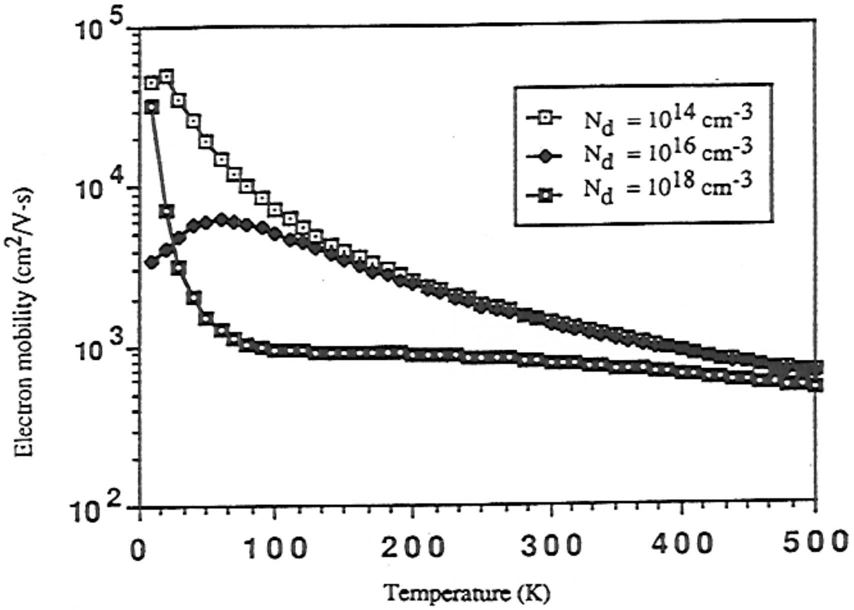
$$E_7 = \frac{7^2\pi^2\hbar^2}{2mL^2} = 0.0588\text{ eV}$$

Use Fermi - Dirac statistics for electrons :

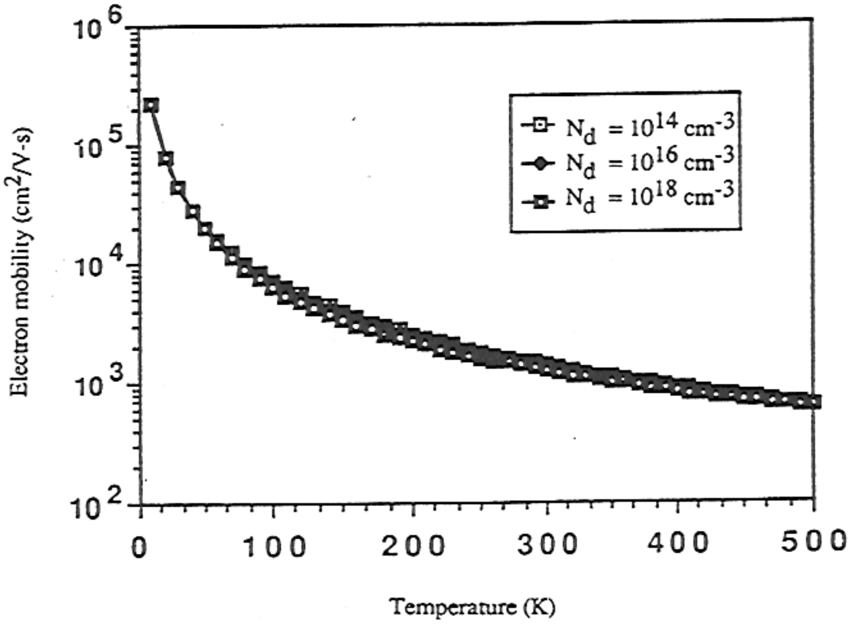
$$\text{Probability} = \frac{1}{1+e^{\frac{E_7-E_F}{kT}}} = \frac{1}{1+e^{\frac{E_7-E_6}{kT}}} = \frac{1}{1+e^{(0.0588-0.0432)/0.0259}}$$

At 300K, this is **0.354.**

Prob. 3.16



Prob. 3.17



When freeze-out occurs, ionized impurity scattering disappears, and only the phonon scattering remains. In real Si, other mechanisms, including neutral impurity scattering, contribute to mobility.

Prob. 3.18

Find the hole concentration and mobility with Hall measurement on a p-type semiconductor bar.

The voltage measured is the Hall voltage plus the ohmic drop. The sign of V_H changes with the magnetic field, but the ohmic voltage does not.

$$\text{True, } V_{Hall} = \frac{V_{H_1} - V_{H_2}}{2} = 3\,\text{mV}$$

Thus the ohmic drop is $3.2 - 3.0 = 0.2\,\text{mV}$

From Eq.(3 - 50)

$$p_0 = \frac{(3\times10^{-3}A)(10\times10^{-5}\,\text{Wb/cm}^2)}{q(20\times10^{-4}\,\text{cm})(3\times10^{-3}V)} = 3.125\times10^{17}\,\text{cm}^{-3}$$

$$\rho = \frac{\left(\dfrac{0.2\,\text{mV}}{3\,\text{mA}}\right)}{\dfrac{2\mu\text{m}}{500\mu\text{m}\times20\mu\text{m}}} = 0.033\,\Omega\cdot\text{cm} = \frac{1}{q\mu_p p_0}$$

$$\mu_p = \frac{1}{q\rho p_0} = \frac{1}{1.6\times10^{-19}(0.033)(3.125\times10^{17})} = \mathbf{600\ cm^2/(V\cdot s)}$$

Prob. 3.19

Calculate the conductivity of a hypothetical semiconductor at 600K.

The intrinsic conductivity is given as

$$\sigma_i = qn_i(\mu_n + \mu_p) = q\left(\sqrt{N_c N_v}e^{-\frac{E_g}{2kT}}\right)\times2000 = 4\times10^{-6}(\Omega\cdot\text{cm})^{-1}$$

$$4\times10^{-6} = 1.6\times10^{-19}(10^{19})(2000)e^{-\frac{E_g}{2kT}}$$

As T goes from 300K to 600K, E_g, N_c, N_v do not change, and

$e^{-\frac{E_g}{2kT_1}}$ increases to $e^{-\frac{E_g}{2k(2T_1)}}$, where $T_1 = 300K$.

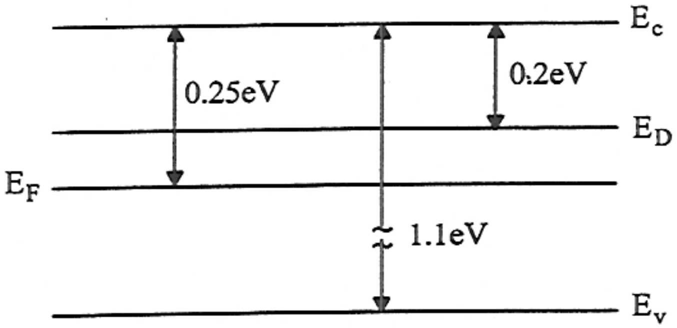
Therefore,

$$e^{-\frac{E_g}{2k(2T_1)}} = \sqrt{\frac{4\times10^{-6}}{1.6\times10^{-19}\times10^{19}\times2000}} = 3.54\times10^{-5}$$

$$\sigma = \frac{\sigma_i}{3.54\times10^{-5}} = \mathbf{0.113(\Omega\cdot cm)^{-1}}$$

Prob. 3.20

Calculate the number of electrons, holes, and n_i in the unknown semiconductor with E_F 0.25eV below E_c.



Incomplete ionization:

$$f(E_d) = \frac{1}{1+e^{\frac{0.05}{0.0259}}} = 0.1267$$

$$n = (1-f)N_d = 8.733\times10^{14}\,\text{cm}^{-3}$$

Also, $n = N_c e^{-\frac{E_c-E_F}{kT}}$

$$N_c = ne^{\frac{E_c-E_F}{kT}} = 8.733\times10^{14}\times e^{0.25/0.0259}$$

$$= 1.359\times10^{19}\,\text{cm}^{-3} = N_v$$

$$p = N_v e^{-\frac{E_F-E_v}{kT}} = 1.359\times10^{19}\times e^{-(1.1-0.25)/0.0259} = 7.591\times10^{4}\,\text{cm}^{-3}$$

$$n_i = \sqrt{np} = \mathbf{8.142\times10^{9}\,cm^{-3}}$$

Prob. 3.21

Referring to Fig. 3.25, find the type, concentration and mobility of the majority carrier.

Given,

$B_z = 10^{-4}\,\text{Wb/cm}^2$

From the sign of V_{AB}, we can see the majority carriers are electrons.

$$n_0 = \frac{I_x B_z}{qt(-V_{AB})} = \frac{(10^{-3})(10^{-4})}{1.6\times10^{-19}(10^{-3})(2\times10^{-3})} = 3.125\times10^{17}\,\text{cm}^{-3}$$

$$\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_x}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01\times10^{-3}} = 0.002\,\Omega\cdot\text{cm}$$

$$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6\times10^{-19})(3.125\times10^{17})} = \mathbf{10{,}000\ cm^2(V\cdot s)^{-1}}$$

Chapter 4

Prob. 4.1

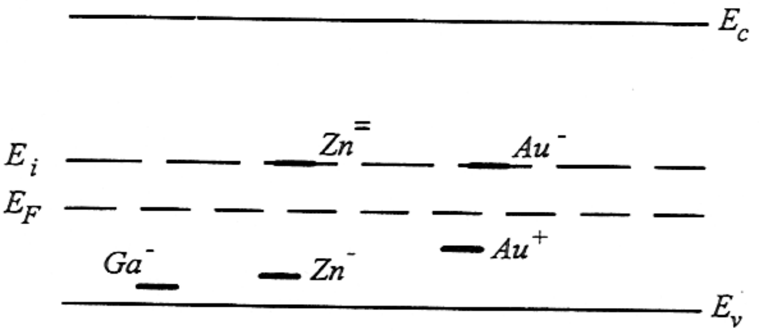
With E_F located 0.4 eV above the valence band in a Si sample, what charge state would you expect for most Ga, Zn, Au atoms in the sample?

From Fig.4-9, we have

Ga$^-$: $E_F >>$ Ga$^-$: singly negative

Zn$^-$: $E_F >$ Zn$^-$ but below Zn$^=$: singly negative

Au^0: Au$^+ <$ E$_F <$ Au$^-$: neutral.



Prob. 4.2

How much Zn must be added to exactly compensate a Si sample doped with 10^{16} cm^{-3} Sb?

$E_i = E_F \approx E_{Zn^-}$

$$\left.\begin{array}{l}\text{All Zn}^{-}\text{ state are filled}\\ \frac{1}{2}\text{ of Zn}^{=}\text{ states are filled}\end{array}\right\}\frac{3}{2}N_{Zn}\text{ electrons on Zn atoms}$$

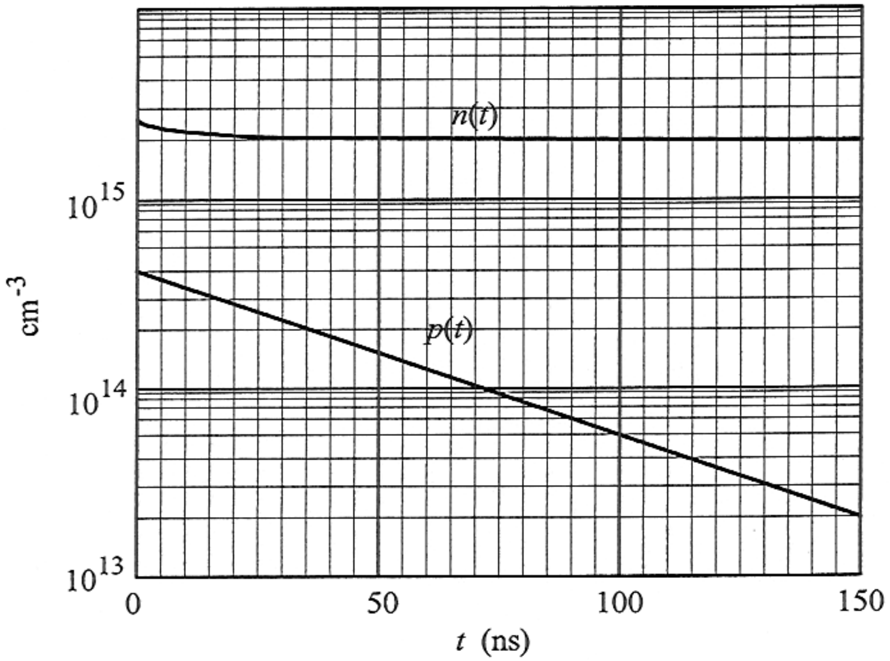
For compensation :

$$\frac{3}{2}N_{Zn} = N_d \;\Rightarrow\; N_{Zn} = \frac{2}{3}N_d = \mathbf{0.667 \times 10^{16} cm^{-3}}$$

Prob. 4.3

Draw a semilogarithmic plot such as Fig. 4-7 for the given GaAs.

For the given GaAs,
$n_0 = 2 \times 10^{15}$ cm^{-3}, $p_0 = n_i^2/n_0 = 0.002$ (negligible)
At $t = 0$, $\Delta n = \Delta p = 4 \times 10^{14}$ cm^{-3}.



Prob. 4.4

Calculate the recombination coefficient for the low-level excitation in Prob. 4.3. Find the steady state excess carrier concentration.

$\alpha_r = 1/(\tau n_0) = [50 \times 10^{-9} \times 2 \times 10^{15}]^{-1} = 10^{-8}$
From Eq. (4-12), $g_{op} = \alpha_r[n_0\delta n + \delta n^2] = 10^{-8}\,[2 \times 10^{15}\,\delta n + \delta n^2]$
$\delta n^2 + 2 \times 10^{15}\,\delta n - 10^{28} = 0$ and $\delta n \sim 5 \times 10^{12}$ cm^{-3} $= \Delta n$
or, since the low-level lifetime is valid, $\Delta n = g_{op}\tau = 5 \times 10^{12}$ cm^{-3}.

Prob. 4.5

If $n_0 = Gx$, find $\mathcal{E}(x)$ for $n_0 >> n_i$. We also assume E_F remains below E_c.

At equilibrium:

$$J_n = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx} = 0$$

$$\mathcal{E}(x) = -\frac{D_n}{\mu_n}\frac{dn/dx}{n}$$

$$= -\frac{kT}{q}\frac{G}{Gx} = -\frac{kT}{q}x^{-1}$$

Prob. 4.6

Find the separation of the quasi-Fermi levels and the change of conductivity upon shining light on a Si sample.

The light induced electron-hole pair concentration is determined by:

$$\delta n = \delta p = g_{op}\tau = (10^{19})(10^{-5}) = 10^{14}\,\text{cm}^{-3}$$

$$<< \text{dopant concentration of } 10^{15}\,\text{cm}^{-3} \quad \Rightarrow \text{ low level}$$

$$n = 10^{15} + 10^{14} = 1.1\times10^{15}\,\text{cm}^{-3}$$

$$p_0 + \delta p = \frac{n_i^2}{n_0} + \delta p = \frac{(1.5\times10^{10})^2}{10^{15}} + 10^{14} \approx 10^{14}\,\text{cm}^{-3}$$

$$F_n - F_p = kT\ln\left(\frac{np}{n_i^2}\right) = 0.0259\ln\left(\frac{1.1\times10^{29}}{2.25\times10^{20}}\right) = 0.518\,\text{eV}$$

$$\mu_n = 1300\,\text{cm}^2/(\text{V}\cdot\text{s}) \text{ from Fig. 3.23}$$

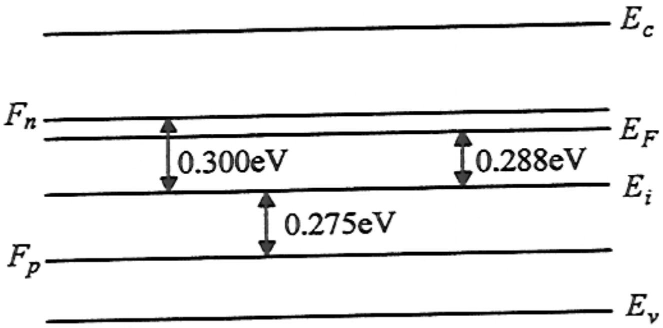
$$\mu_p = \frac{D_p}{kT/q} = \frac{12}{0.0259} = 463\,\text{cm}^2/(\text{V}\cdot\text{s})$$

$$\Delta\sigma = \text{q}(\mu_n\delta n + \mu_p\delta p) = 1.6\times10^{-19}(1300+463)(10^{14})$$

$$= \mathbf{0.0282\,(\Omega\cdot cm)^{-1}}$$

Prob. 4.7

Calculate the separation in the quasi-Fermi levels and draw a band diagram for an n-type Si being steadily illuminated.



The induced electron concentration is

$\delta n = g_{op}\tau = (10^{21})(10^{-6}) = 10^{15}\,\mathrm{cm}^{-3}$

which is comparable with $N_d = 10^{15}\,\mathrm{cm}^{-3}$.

$\Rightarrow$ this is not low level and δn^2 cannot be neglected.

$g_{op} = \alpha_r n_0 \delta n + \alpha_r \delta n^2$

$$\alpha_r = \frac{1}{\tau_n n_0} = \frac{1}{(10^{-6})(10^{15})} = 10^{-9}\,\mathrm{cm}^3 s^{-1}$$

$\Rightarrow 10^{21} = (10^{-9})(10^{15})\delta n + 10^{-9}\delta n^2$

Solve for δn $\quad\Rightarrow\quad \delta n = 6.18\times10^{14}\,\mathrm{cm}^{-3} = \delta p$

$$F_n - E_i = kT\ln\left(\frac{n_0+\delta n}{n_i}\right) = 0.0259\ln\left(\frac{10^{15}+6.18\times10^{14}}{1.5\times10^{10}}\right) = \mathbf{0.300eV}$$

$$E_i - F_p = kT\ln\left(\frac{\delta n}{n_i}\right) = 0.0259\ln\left(\frac{6.18\times10^{14}}{1.5\times10^{10}}\right) = \mathbf{0.275eV}$$

$$E_F - E_i = kT\ln\left(\frac{n_0}{n_i}\right) = 0.0259\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = \mathbf{0.288eV}$$

Prob. 4.8

Calculate the current in a long Si bar as described.

$$\alpha_r = \frac{1}{\tau n_0} = \frac{1}{(10^{-4})(10^{16})} = 10^{-12}\, cm^3 s^{-1}$$

$$g_{op} = \alpha_r n_0 \delta n + \alpha_r \delta n^2 \qquad \Rightarrow$$

$$10^{20} = 10^{-12}[(10^{16})\delta n + \delta n^2] \qquad \Rightarrow$$

$$10^{-32}\delta n^2 + 10^{-16}\delta n - 1 = 0$$

$(\overline{\delta n})^2 + \overline{\delta n} - 1 = 0$, where $\overline{\delta n} = 10^{-16}\delta n$

Solve for $\overline{\delta n}$ to get $\delta n \quad \Rightarrow$

$$\delta n = 10^{16}\frac{-1+\sqrt{1+4}}{2}$$

$$\delta n = 6.18\times 10^{15}\, cm^{-3} = \delta p$$

Assume the α_r is the low level, even if the calculation may require high level injection assumption.

No light:

$$\mathcal{E} = \frac{10\text{V}}{2\text{cm}} = 5\text{V/cm}$$

$\mu_n = 1070\ \text{cm}^2/(\text{V}\cdot\text{s})$ from Fig. 3.23, $\mu_p = 500\ \text{cm}^2/(\text{V}\cdot\text{s})$

$$I = Aqn_0\mu_n\mathcal{E} = (0.05)(1.6\times 10^{-19}\times 10^{16}\times 1070\times 5) = \mathbf{0.428A}$$

With light:

$$I = A\cdot\left(q\left\{(n_0+\delta n)\mu_n + \delta p\mu_p\right\}\right)\cdot\mathcal{E}$$

$$= 0.05(1.6\times 10^{-19}(1.73\times 10^{19} + 3.09\times 10^{18}))\cdot 5$$

$$= \mathbf{0.816A}$$

High field + light:

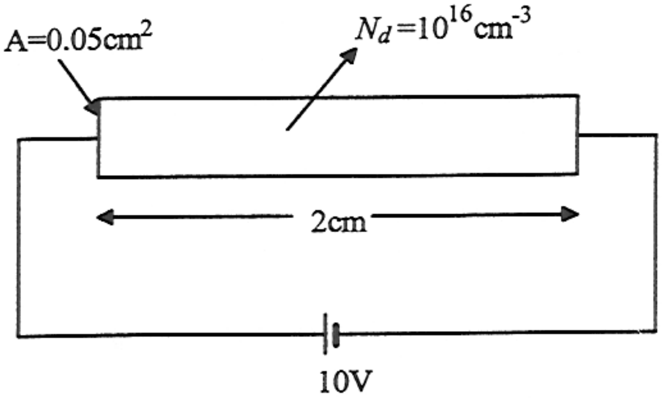
For electrons, saturation velocity $\text{v}_s = 10^7$ cm/s

For holes, assume μ_p is same as for low field, $\mathcal{E} = \frac{100{,}000}{2} = 50{,}000\text{V/cm}$

$$I = Aq[(n_0+\delta n)\text{v}_s + \delta p\mu_p\mathcal{E}]$$

$$= (0.05)(1.6\times 10^{-19})(1.618\times 10^{16}\times 10^7 + 6.18\times 10^{15}\times 500\times 5\times 10^4)$$

$$= \mathbf{2.53\times 10^3\,A}$$



Prob. 4.9

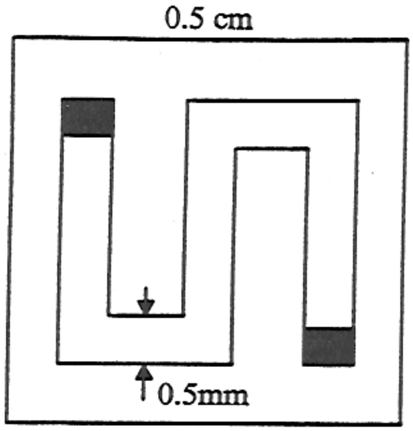
Design a 5-μm CdS photoconductor with 10 MΩ dark resistance, 0.5 cm square. Assume $\tau = 10^{-6}$ s and $N_d = 10^{14}$ cm^{-3}.

In the dark, $\sigma = q\mu_n n_0$, neglecting p_0

$\rho = \sigma^{-1} = [1.6 \times 10^{-19} \times 250 \times 10^{14}]^{-1} = 250$ Ω-cm
$R = 10^7\ \Omega = \rho L/wt$, thus $L = 10^7\ (5\times10^{-4})w/250$

Since this is a design problem, there are many solutions. For example, choosing w = 0.5mm, L = 1 cm:
with the light on, $g_{op} = 10^{21}$ EHP/cm^3-s

$$\sigma = q[(n_0 + \Delta n)\mu_n + \Delta p\mu_p]$$
$$= 1.6\times10^{-19}[(1.1\times10^{15})250 + 10^{15}\times15]$$
$$= 4.64\times10^{-2}(\Omega\cdot\text{cm})^{-1}$$
$$R = \frac{\rho L}{wt} = [(4.64\times10^{-2})(0.05)(5\times10^{-4})]^{-1}$$
$$R = 8.62\times10^5\Omega$$
$$\Delta R = 10^7 - 8.62\times10^5 = \mathbf{9.14M\Omega}$$



Prob. 4.10

Calculate the steady state separation between F_P and E_C at $x = 1000\text{Å}$ in a very long p-type Si bar with steady state excess hole concentration. Also find the hole current there and the excess stored hole charge.

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 500 = 12.95\ \text{cm}^2/\text{s}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5}\text{cm}$$

$$p = p_0 + \Delta p e^{-\frac{x}{L_p}} = 10^{17} + 5 \times 10^{16} e^{-\frac{10^{-5}}{3.6 \times 10^{-5}}}$$

$$= 1.379 \times 10^{17} = n_i e^{(E_i - F_p)/kT} = (1.5 \times 10^{10}\text{cm}^{-3}) e^{(E_i - F_p)/kT}$$

$$E_i - F_p = \left(\ln \frac{1.379 \times 10^{17}}{1.5 \times 10^{10}}\right) \cdot 0.0259 = 0.415\text{eV}$$

$$E_c - F_p = 1.1/2\text{eV} + 0.415\text{eV} = \mathbf{0.965eV}$$

Hole current :

$$I_p = -qAD_p \frac{dp}{dx} = qA\frac{D_p}{L_p}(\Delta p)e^{-\frac{x}{L_p}}$$

$$= 1.6\times10^{-19}\times0.5\times\frac{12.95}{3.6\times10^{-5}}\times5\times10^{16}e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$

$$= \mathbf{1.09\times10^{3}\,A}$$

$$Q_p = qA(\Delta p)L_p$$

$$= 1.6\times10^{-19}(0.5)(5\times10^{16})(3.6\times10^{-5})$$

$$= \mathbf{1.44\times10^{-7}\,C}$$

Prob. 4.11

Find the photocurrent ΔI in terms of τ_n and τ_t for a sample dominated by μ_n.

$$\Delta\sigma \approx q\mu_n\Delta n = q\mu_n g_{op}\tau_n$$

$$\Delta I = V/\Delta R = VA\Delta\sigma\,/\,L = VAq\mu_n g_{op}\tau_n/L$$

The transit time is

$$\tau_t = \frac{L}{v_d} = \frac{L}{\mu_n V/L} = \frac{L^2}{\mu_n V}$$

$$\Delta I = qALg_{op}\tau_n/\tau_t$$

Prob. 4.12

Find $F_p(x)$ for an exponential excess hole distribution.

For $\delta p \gg p_0$,

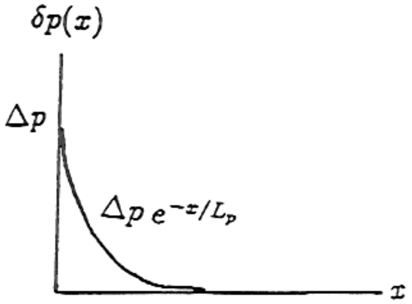
$$p(x) \simeq \delta p(x) = \Delta p\, e^{-x/L_p}$$

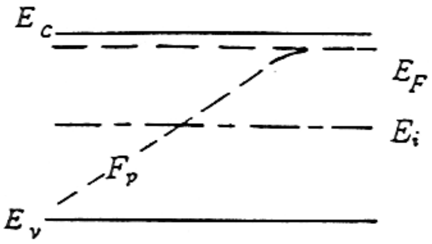
$$= n_i\, e^{(E_i - F_p)/kT}$$

$$E_i - F_p = kT \ln \frac{\delta p}{n_i}$$

$$= kT \ln \frac{\Delta p}{n_i} e^{-x/L_p}$$

$$= kT \left[\ln \frac{\Delta p}{n_i} - \frac{x}{L_p} \right]$$

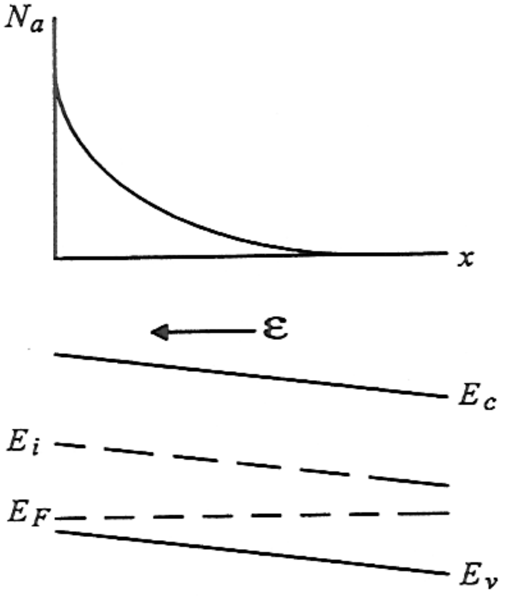


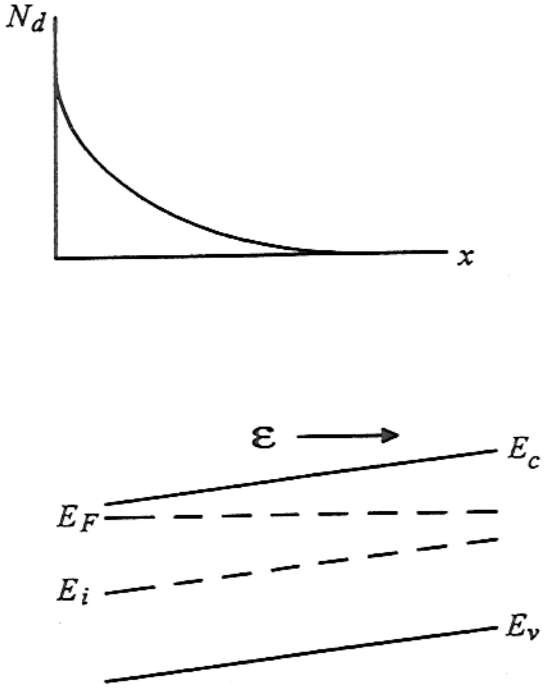


We assume the excess minority hole concentration is small compared to n_0 throughout, so no band bending is observable on this scale.

Prob. 4.13

Sketch the equilibrium bands and field in an exponential acceptor distribution. Repeat for donors.



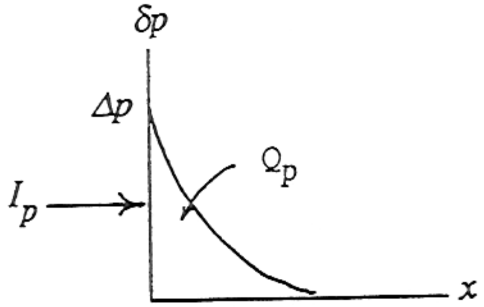


Prob. 4.14

Show the hole current feeding an exponential $\delta p(x)$ can be found from Q_p/τ_p.

From Fig. 4-17,

$$
\begin{aligned}
Q_p &= qA\int_0^\infty \Delta p\, e^{-x/L_p}dx \\
&= qAL_p\Delta p \\
I_p &= \frac{Q_p}{\tau_p} = qAL_p\Delta p/\tau_p = qAD_p\Delta p/L_p
\end{aligned}
$$



The charge distribution Q_p disappears by recombination and must be replaced by injection on the average every τ_p seconds. Thus the current injected is Q_p/τ_p.

Prob. 4.15

Include recombination in the Haynes-Shockley experiment and find τ_p if the peak is 4 times as large for $t_d = 50$ µs as it is for 200 µs.

To include recombination, let the peak value vary as *exp(-t/τ_p)*

$$\delta p(x,t) = \frac{\Delta p e^{-t/\tau_p}}{\sqrt{4\pi D_p t}} \exp(-x^2/4D_p t)$$

At the peak ($x = 0$),

$$V_p = \text{peak} = B\frac{\Delta p e^{-t/\tau_p}}{\sqrt{4\pi D_p t}},$$ where B is a proportionality constant.

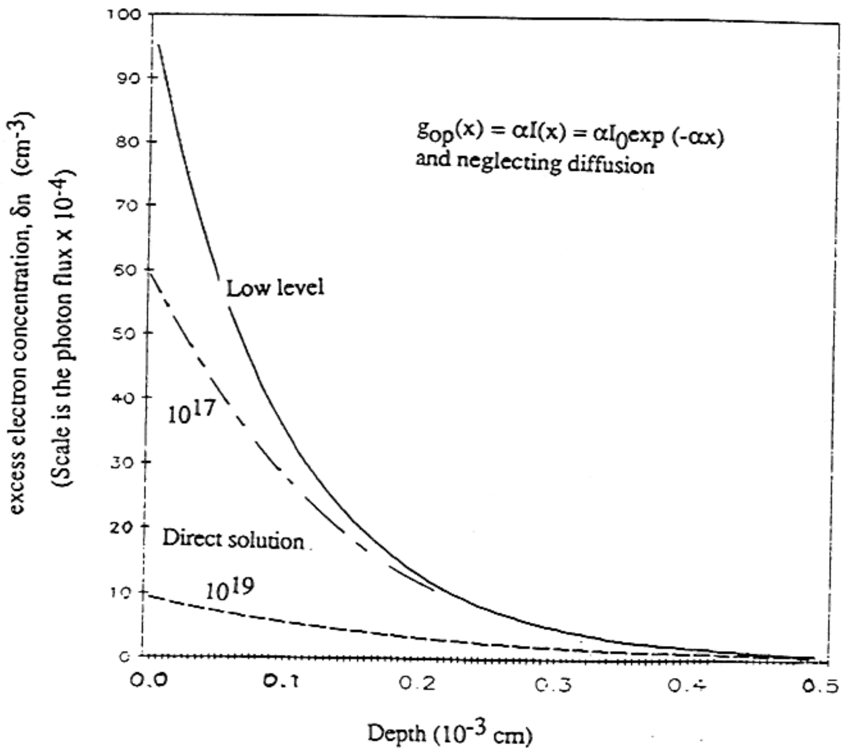
$$\frac{V_{p1}}{V_{p2}} = \sqrt{\frac{t_2}{t_1}}\,\frac{e^{-t_1/\tau_p}}{e^{-t_2/\tau_p}} = \sqrt{\frac{t_2}{t_1}}\,e^{(t_2-t_1)/\tau_p}$$

$$\frac{80}{20} = \sqrt{\frac{200}{50}}\,e^{150/\tau_p}$$

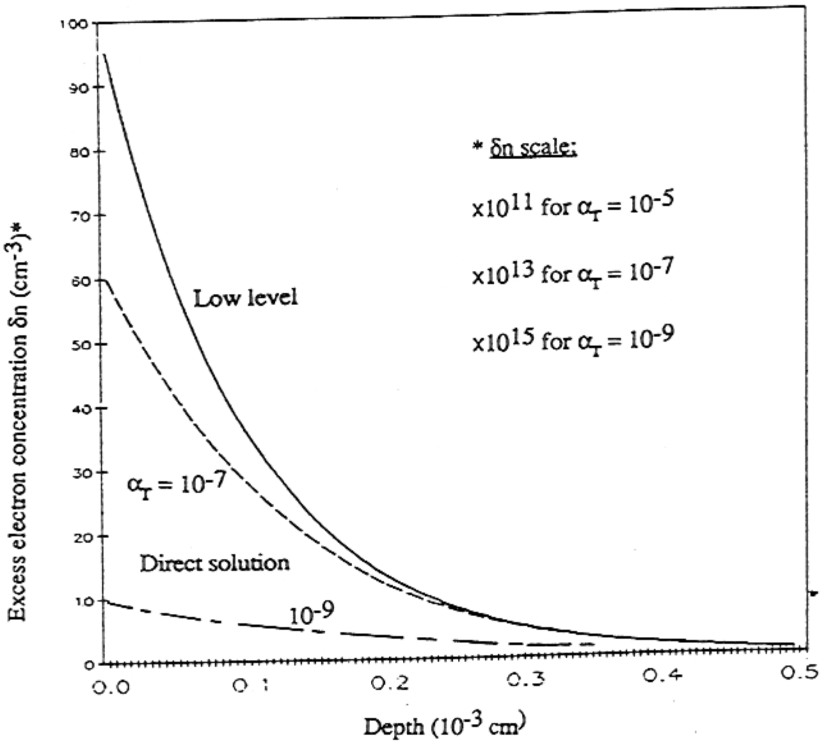
$$\frac{150}{\tau_p} = \ln\frac{4}{\sqrt{4}}$$

$$\tau_p = \frac{150}{\ln 2} = \mathbf{216.4\mu s}$$

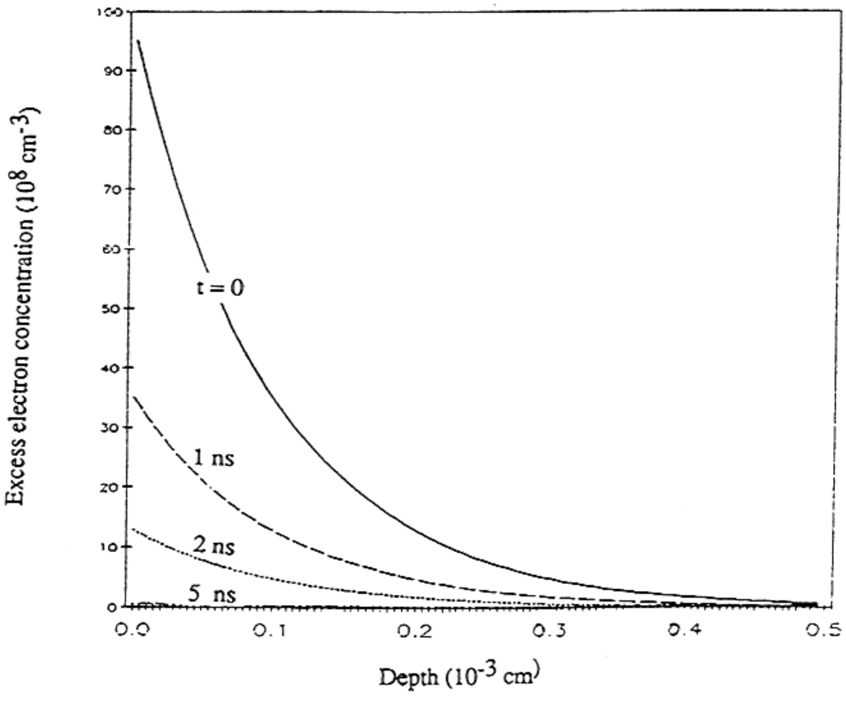
Prob. 4.16



Prob. 4.17

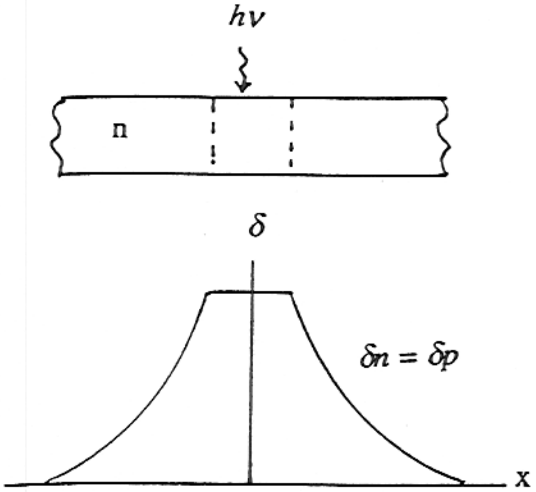


Prob. 4.18

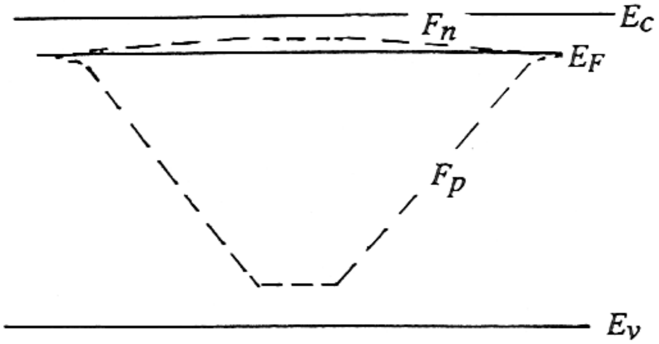


Prob. 4.19

Sketch the quasi-Fermi levels in an n-type sample illuminated in a narrow region.



Excess carriers diffuse and recombine, decaying exponentially away from the illuminated region

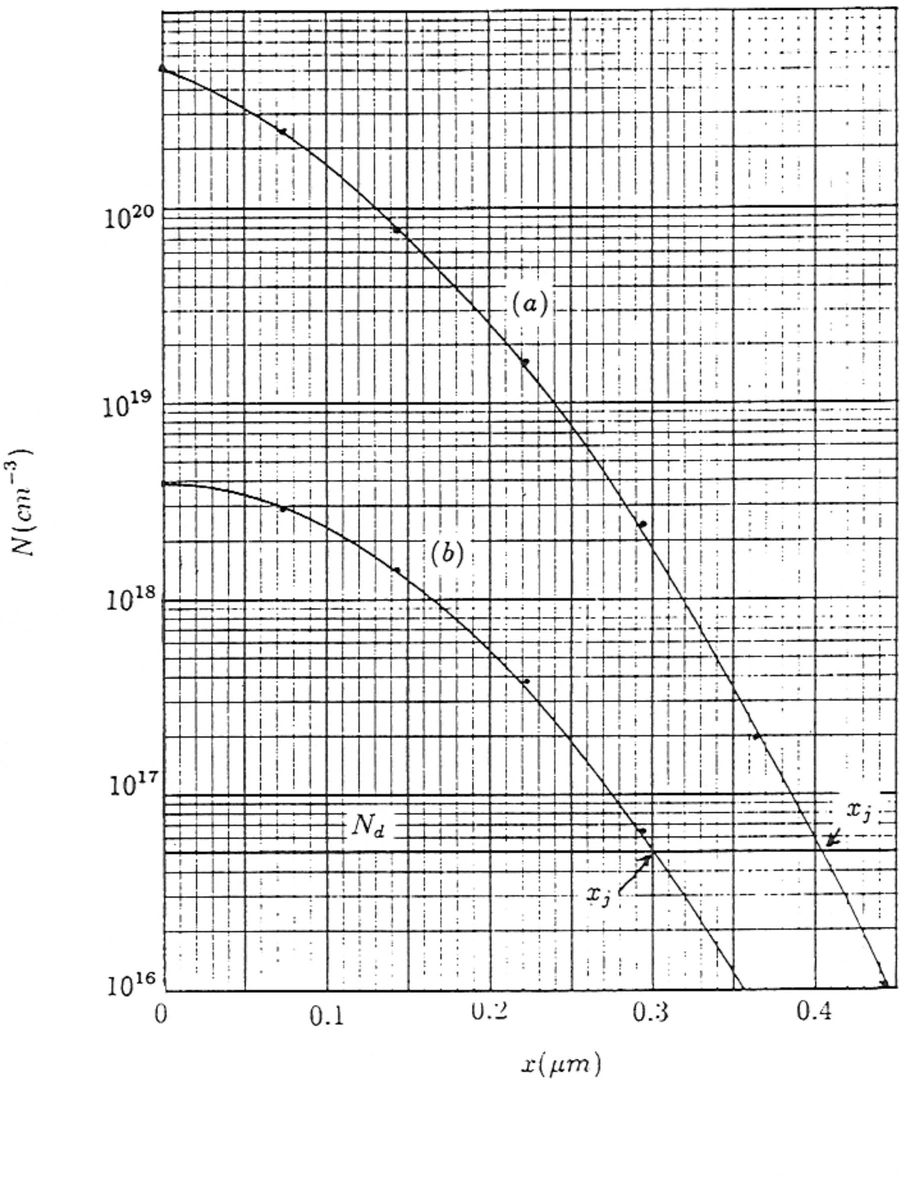


As in prob. 4.12, the quasi-Fermi levels vary linearly outside the excitation region while $\delta p \gg p_0$

(b) After diffusion. $N_0 = N_s/\sqrt{\pi Dt} = \dfrac{5 \times 10^{13}}{0.1302 \times 10^{-4}} = 3.84 \times 10^{18}$

$x(\mu m)$	u	$\exp(-u^2)$	$N(x)$
0.0735	0.5	0.78	3.0×10^{18}
0.1470	1.0	0.37	1.4×10^{18}
0.2205	1.5	0.105	4.0×10^{17}
0.2940	2.0	0.018	6.9×10^{16}
0.3675	2.5	0.0019	7.3×10^{15}

$x_j = 0.3\ \mu m.$



Chapter 5

Prob. 5.1

Design an oxide mask to block P diffusion at 1000°C for 30 minutes, calculate how long to grow it and the total number of Si atoms from the wafer that are consumed.

Diffusion coefficient at T = 1273K :

$$D = D_0 e^{-E_A/kT}, \text{ where } D_0 = 5.3\times10^{-8}\,\text{cm}^2/\text{s},\ \text{E}_\text{A} = 1.46\text{eV}$$

$$= 8.83\times10^{-14}\,\text{cm}^2/\text{s}$$

$$t_{mask} = 8\sqrt{\text{Dt}} = 8\sqrt{D\times30\times60} = 1.009\mu\text{m}$$

Time at 1100°C = 2 hours, using Appendix VI.

$$t_{Si} = 0.44(1.009\times10^{-4})\text{cm} = 4.44\times10^{-5}\,\text{cm}$$

$$\text{Volume of Si consumed} = \frac{\pi(20)^2}{4} t_{Si} = 0.01394\text{cm}^3$$

Number of Si atoms = $1.394\times10^{-2}\,\text{cm}^3\times5\times10^{22}$ atoms/cm^3 = $\mathbf{6.97\times10^{20}}$ **atoms.**

Prob. 5.2

Plot the distributions for B diffused into Si ($N_d = 5\times10^{16}\,cm^{-3}$) at 1000°C for 30 minutes, for which $D = 3\times10^{-14}\,cm^2/s$, with (a) constant source $N_0 = 5\times10^{20}\,cm^{-3}$, (b) limited source $N_s = 5\times10^{13}\,cm^{-2}$ on the surface prior to diffusion.

The Gaussian distribution differs from Eq.(4-44) because all atoms are assumed to diffuse into the sample (i.e., there is no diffusion in the $-x$ direction).

$$Dt = 3\times10^{-14}\times30\times60 = 0.54\times10^{-10}, \quad \sqrt{Dt} = 0.0735\,\mu m$$

$$2\sqrt{Dt} = 0.147\,\mu m, \quad \sqrt{\pi Dt} = 0.1302\,\mu m$$

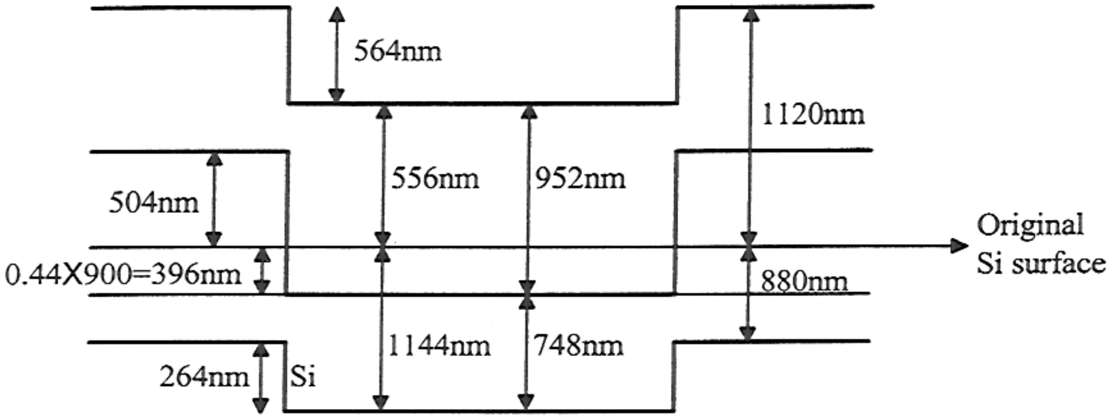
(a)

$x(\mu m)$	u	erfc u	$N(x) = N_0$ erfc u
0.0735	0.5	0.47	2.4×10^{20}
0.1470	1.0	0.16	8.0×10^{19}
0.2205	1.5	0.033	1.7×10^{19}
0.2940	2.0	0.0048	2.4×10^{18}
0.3675	2.5	0.0004	2.0×10^{17}
0.4410	3.0	0.000023	1.2×10^{16}

From the plot below, $x_j = 0.4\,\mu m$.

Prob. 5.3

Find the time to grow first 200nm. the next 300nm and the final 400nm for a 900nm oxide grown on (100) Si in wet oxygen at 1100°C. Draw a cross section diagram including step heights if a square window is etched in this oxide and the wafer is re-oxidized.



Using wet oxidation curve in Appendix VI, the time required to grow 200 nm is t_1 = 0.13h.

To determine the time needed to grow the next 300 nm (for a total thickness of 200 + 300 = 500 nm), we cannot simply read off the time needed for 300 nm because we already have 200 nm oxide grown. The total time needed to achieve 500 nm thickness is 0.6 h. Therefore, the additional time required for the increase of thickness by 300 nm is 0.6 – 0.13 = **0.47 h.**

Similarly, the time needed for the final 400 nm is 1.8 – 0.6 = **1.2 h.**

When a window is etched in the oxide and we re-oxidize the wafer, we have to start from an oxide thickness = 0 inside the window, but oxide thickness = 900 nm outside the window. So, the time required to increase the oxide thickness outside the window is 6 – 1.8 = 4.2 h. In 4.2 h, the oxide in the window starting from zero oxide thickness = **1700 nm.** Keeping in mind that for every micron of oxide, we consume 0.44 micron of Si, we get:

step in oxide = **564 nm**
step in Si = **264 nm**

Prob. 5.4

Find the implant parameters for an As implant into Si with the peak being at the interface.

For an implanted dose of ϕ ions/cm^2, the ion distribution is given by Eq. (5 - 1a):

$$N(x) = \frac{\phi}{\sqrt{2\pi}\,\Delta R_p}\exp\left[-\frac{1}{2}\left(\frac{x-R_p}{\Delta R_p}\right)^2\right]$$

Peak value lies at R_p, straggle is ΔR_p.

From Appendix IX, $R_p = 0.1\mu\text{m} \Rightarrow \text{Energy} = 180\,\text{keV}$

$\Delta R_p = 0.035\mu\text{m}$

From Eq.(5 - 1a), $N_{peak} = 5\times10^{19} = \dfrac{\phi}{\sqrt{2\pi}\,\Delta R_p} \Rightarrow$

$$\phi = 4.39\times10^{14}\text{cm}^{-2} = \frac{It}{qA} = \frac{I(20)}{(1.6\times10^{-19})200}$$

$\Rightarrow\ \mathbf{I = 7.024\times10^{-4}\,A}$

Prob. 5.5

Use a singly and doubly charged B implant machine to achieve B implant into Si with required conditions. Find out how long this implant will take.

For $R_p = 0.5\mu\text{m}$, we need 200 keV implant machine (from Appendix IX).

Therefore, use 100 keV doubly ionized boron

$I = 0.1\text{mA}$ for doubly ionized boron

$$\phi = \frac{It}{qA} \Rightarrow 5\times10^{14} = \frac{6.25\times10^{18}(0.1\times10^{-3})t}{2(100)q}$$

$\Rightarrow$ $\mathbf{t = 160\,s}$

For 1000 higher current, t = **0.16 s**. This is too short for uniformity of dose.

Prob. 5.6

Assuming a constant (unlimited) source diffusion of P at 1000°C into 1Ω-cm p-type Si, calculate the time required to achieve a junction depth of 1μm.

For 1Ω-cm p-type Si, we have 1.7×10^{16} atoms/cm^3.
At 1000°C, $N_{\text{solid sol.}} = 10^{21}\,\text{cm}^{-3}$.

We have: $10^{21} erfc(\frac{x}{2\sqrt{Dt}}) = 1.7\times10^{16}$

where

$D = D_0 \exp(-E_a / kT)$

$$= 10.5\exp\left(\frac{-3.69}{8.62\times10^{-5}(1273)}\right)\text{cm}^2/\text{s}$$

$$= 2.61\times10^{-14}\,\text{cm}^2/\text{s}$$

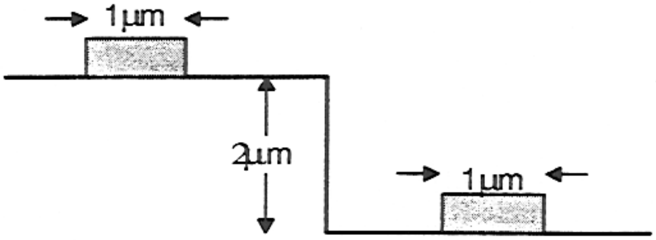
From Table, $erfc^{-1}\left(\frac{1.7\times10^{16}}{10^{21}}\right) = 3.1 = \frac{x_j}{2\sqrt{Dt}}$

$$\Rightarrow \quad 2\sqrt{2.61\times10^{-14}t} = \frac{1.0\times10^{-4}}{3.1}$$

$$\Rightarrow \quad t = \mathbf{9.97\times10^3\ s}$$

Prob. 5.7

In patterning the structure shown in the question, design the mask aligner optics in terms of numerical aperture of the lens and the wavelength of the source.



$$\frac{0.8\lambda}{NA} = 1, \quad \frac{\lambda}{2(NA)^2} = 2$$

$$\left(\frac{1}{NA}\right)\left(\frac{1}{2}\right)\left(\frac{1}{0.8}\right) = 2$$

$NA = \mathbf{0.3125}$

$\lambda = \mathbf{0.39\mu m}$

Prob. 5.8

Find the electron diffusion and drift currents at x_n in a p^+-n junction.

$$I_p(x_n) = qA\frac{D_p}{L_p}p_n e^{qV/kT}e^{-x_n/L_p} \quad \text{for } V >> kT/q$$

$$I = I_p(x_n = 0) = qA\frac{D_p}{L_p}p_n e^{qV/kT}$$

Assuming space charge neutrality, the excess hole distribution is matched by an excess electron distribution $\delta n(x_n) = \delta p(x_n)$.

$$I_n(x_n)_{diff.} = qAD_n\frac{d\delta p}{dx_n} = -qA\frac{D_n}{L_p}p_n e^{qV/kT}e^{-x_n/L_p}$$

$$I_n(x_n)_{drift} = I - I_n(x_n)_{diff.} - I_p(x_n)$$

$$= qA\frac{p_n}{L_p}[D_p(1-e^{-x_n/L_p}) + D_n e^{-x_n/L_p}]e^{qV/kT}$$

Prob. 5.9

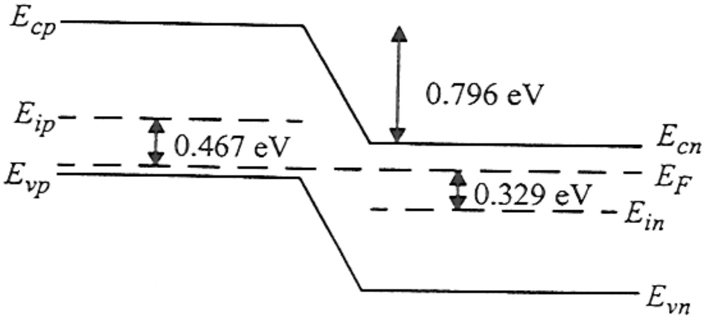
A Si junction has $N_a = 10^{18}$ and $N_d = 5 \times 10^{15}$ cm^{-3}. Find (a) E_F on each side, (b) band diagram, (c) V_0.

$$(a)\;\; E_{ip} - E_F = kT\ln\frac{p_p}{n_i} = 0.0259\ln\frac{10^{18}}{(1.5\times10^{10})} = \mathbf{0.467eV}$$

$$E_F - E_{in} = kT\ln\frac{n_n}{n_i} = 0.0259\ln\frac{5\times10^{15}}{(1.5\times10^{10})} = \mathbf{0.329eV}$$

$$(b)\;\; qV_0 = 0.467 + 0.329 = \mathbf{0.796eV}$$

$$(c)\;\; qV_0 = kT\ln\frac{N_aN_d}{n_i^2} = 0.0259\ln\frac{5\times10^{33}}{2.25\times10^{20}} = \mathbf{0.796eV}$$



Prob. 5.10

For the junction of Prob. 5.9, with circular area of diameter 10 μm, calculate parameters and sketch as in Fig. 5.12.

$$A = \pi(5\times10^{-4})^2 = 7.85\times10^{-7}\,\text{cm}^2$$

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.8)(8.85\times10^{-14})(0.796)}{1.6\times10^{-19}}\left(10^{-18}+2\times10^{-16}\right)\right]^{1/2} = \mathbf{0.457\mu m}$$

$$x_{n_0} = \frac{W}{1+N_d/N_a} = \frac{0.457}{1+5\times10^{-3}} = \mathbf{0.455\mu m}$$

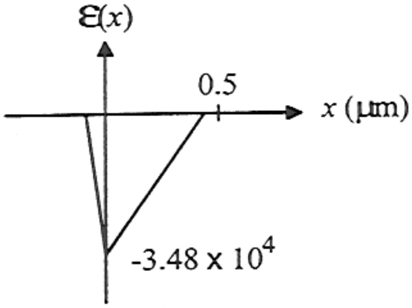
$$x_{p_0} = \frac{0.457}{1+200} = \mathbf{2.27\times10^{-3}\,\mu m}$$

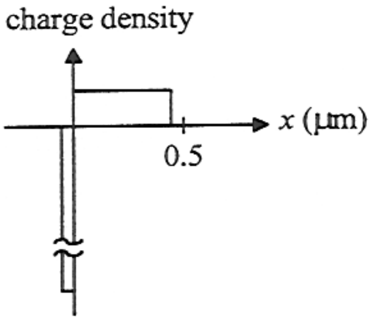
$$Q_+ = qAx_{n_0}N_d = qAx_{p_0}N_a = (1.6\times10^{-19})(7.85\times10^{-7})(2.27\times10^{11})$$

$$= \mathbf{2.85\times10^{-14}\,C}$$

$$\mathcal{E}_0 = -\frac{q}{\epsilon}x_{n_0}N_d = -\frac{q}{\epsilon}x_{p_0}N_a = \frac{1.6\times10^{-19}}{(11.8)(8.85\times10^{-14})}(2.27\times10^{11})$$

$$= \mathbf{-3.48\times10^{4}\ V/cm}$$





Prob. 5.11

Find the electron injection efficiency I_n/I.

(a)

$$\frac{I_n}{I} = \frac{qA(D_n/L_n)n_p\,(e^{qV/kT}-1)}{qA\left[(D_p/L_p)p_n + (D_n/L_n)n_p\right](e^{qV/kT}-1)}$$

$$= \left[1 + \frac{D_p/D_n}{L_p/L_n}\,\frac{p_n}{n_p}\right]^{-1}$$

(b) Since

$$\frac{D_p^n}{D_n^p} = \frac{\mu_p^n}{\mu_n^p} \quad \text{and} \quad \frac{p_n}{n_p} = \frac{p_p}{n_n}$$

we can write

$$\frac{D_p^n\ p_n}{D_n^p\ n_p} = \frac{\mu_p^n\ p_p}{\mu_n^p\ n_n}$$

then

$$\frac{I_n}{I} = \left[1 + \frac{L_n^p\ \mu_p^n\ p_p}{L_p^n\ \mu_n^p\ n_n}\right]^{-1}$$

to increase I_n/I, make $n_n \gg p_p$ (i.e., use n^+-p).

Prob.5.12

A Si p^+-n junction with $N_d = 5 \times 10^{16}$ cm^{-3} and $A = 10^{-3}$ cm^2 has $\tau_p = 1\mu s$ and $D_p = 10$ cm^2/s. Find I for $V_f = 0.5V$.

$$I = qA\frac{D_p}{L_p}p_n e^{qV/kT}$$

$$p_n = n_i^2/n_n = (2.25\times10^{20})/(5\times10^{16}) = 4.5\times10^3\,\text{cm}^{-3}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10^{-5}} = 3.16\times10^{-3}\,\text{cm}$$

$$I = 1.6\times10^{-19}(10^{-3})\frac{10}{3.16\times10^{-3}}(4.5\times10^3)e^{0.5/0.0259} = \mathbf{0.55\mu A}$$

Prob. 5.13

(a) Why is C_s negligible in reverse bias?

For reverse bias of more than a few tenths of a volt, $\Delta p_n \sim -p_n$. Changes in the reverse bias do not appreciably alter the (negative) excess hole distribution. The primary variation is in the width of the depletion region, giving rise to the junction capacitance.

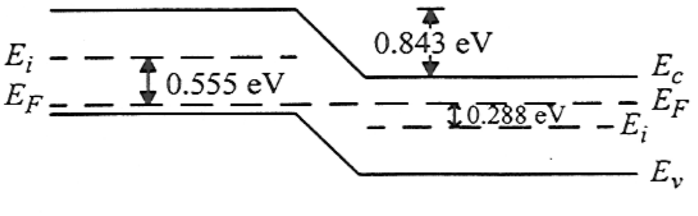
(b) With equal doping, which carrier dominates injection in a GaAs junction?

Electron injection dominates since $\mu_n \gg \mu_p$. With $n_n = p_p$ it is clear that a carrier with higher mobility will determine the injection.

Prob. 5.14

(a) Find C_j for $V = -10V$ for a Si p^+-n junction 10^{-2} cm^2 in area with $N_d = 10^{15}$ cm^{-3}.

On the n-side:



$$E_F - E_i = kT\ln\frac{N_d}{n_i} = 0.0259\ln\frac{10^{15}}{1.5\times10^{10}} = 0.288\,\text{eV}$$

$$qV_0 = 0.555 + 0.288 = \mathbf{0.843\,eV}$$

$$C_j = \frac{A}{2}\left[\frac{2q\epsilon N_d}{V_0 - V}\right]^{1/2}$$

$$= \frac{10^{-2}}{2}\left[\frac{2\times1.6\times10^{-19}\times11.8\times8.85\times10^{-14}\times10^{15}}{10.843}\right]^{1/2}$$

$$= \mathbf{2.78\times10^{-11}F}$$

(b) What is W just prior to avalanche?

From Fig. 5-22, $V_{br} = 300$ V for a p^+-n with $N_d = 10^{15}$ cm^{-3}.

$$W = \left[\frac{2\epsilon_s V_{br}}{qN_d}\right]^{1/2} = \left[\frac{2\times11.8\times8.85\times10^{-14}\times300}{1.6\times10^{-19}\times10^{15}}\right]^{1/2}$$

$$= 1.98\times10^{-3}\text{cm} \approx \mathbf{20\mu m}$$

Prob. 5.15

Show $\mathscr{E}_0$ depends on doping of the lightly-doped side.

$$\mathscr{E}_0 = -\frac{q}{\epsilon}N_d x_{n_o} = -\left[\frac{2qV_0}{\varepsilon}\left(\frac{N_a N_d}{N_a + N_d}\right)\right]^{1/2}$$

$$= -\left[\frac{2qV_0}{\epsilon}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)^{-1}\right]^{1/2}$$

the lightly doped side dominates (the doping variation of V_0 has minor effect). For example, for a p^+-n:

$$\mathscr{E}_0 = -\left[\frac{2qV_0 N_d}{\epsilon}\right]^{1/2}$$

Prob. 5.16

A Si p-n junction has $p_p = 10^{17}$ and $n_n = 10^{15}$ cm^{-3}, $\mu_p^n = 450$ and $\mu_n^p = 700$ cm^2/V-s, $\tau_n^p = 10^{-7}$ and $\tau_p^n = 10^{-5}$ s and $A = 10^{-4}$ cm^2.

Draw an equilibrium band diagram and find V_0.

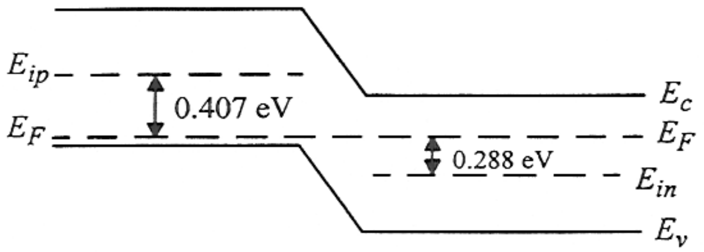
$$E_{ip} - E_F = kT\ln\frac{p_p}{n_i} = 0.0259\ln\frac{10^{17}}{1.5\times10^{10}} = 0.407\text{eV}$$

$$E_F - E_{in} = 0.0259\ln\frac{10^{15}}{1.5\times10^{10}} = 0.288\text{eV}$$

$$V_0 = 0.407 + 0.288 = \mathbf{0.695V}$$

or

$$V_0 = kT\ln\frac{N_a N_d}{n_i^2} = 0.0259\ln\frac{10^{32}}{2.25\times10^{20}} = \mathbf{0.695V}$$



Prob. 5.17

The current in a long p^+-n diode is tripled at t = 0.

(a) What is the slope of $\delta p(x_n=0)$?

The slope triples at $t = 0$:

$$3I = -qAD_p \left.\frac{d\delta p}{dx_n}\right|_{x_n=0}$$

The slope is

$$\left.\frac{d\delta p}{dx_n}\right|_{x_n=0} = -3I/qAD_p$$

(b)

Relate $V(t=\infty)$ to $V(t=0^-)$.

Call V^- the voltage before $t=0$. Call V^∞ the voltage at $t=\infty$.

at $t=0^-$:

$$I = \frac{qAD_p}{L_p} p_n e^{qV^-/kT}$$

at $t=\infty$:

$$3I = \frac{qAD_p}{L_p} p_n e^{qV^\infty/kT}$$

Taking the ratio:

$$3 = e^{q(V^\infty - V^-)/kT}$$

$$V^\infty = V^- + \frac{kT}{q}\ln 3$$

or $V^- + 0.0285$

Prob. 5.18

If $p_p = n_n$, *find* $I_p(x_p)$.

The total current is

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1)$$

On the p side this is composed of

$$I_n(x_p) = qA\frac{D_n}{L_n}n_p e^{-x_p/L_n}(e^{qV/kT} - 1)$$

and

$$I_p(x_p) = I - I_n(x_p)$$

$$= qA\left[\frac{D_n}{L_n}(1 - e^{-x_p/L_n})n_p + \frac{D_p}{L_p}p_n\right](e^{qV/kT} - 1)$$

Since $N_a = N_d$, $n_p = p_n = n_i^2/N_a$

$$I_p(x_p) = qA\left[\frac{D_n}{L_n}(1-e^{-x_p/L_n}) + \frac{D_p}{L_p}\right]\frac{n_i^2}{N_a}(e^{qV/kT}-1)$$

Prob. 5.19

For the given p-n junction, calculate the contact potential, zero-bias space-charge width and the current with a forward bias of 0.5V.

(a) The contact potential is given by

$$V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = 0.0259\ln\frac{10^{15}}{n_i^2/10^{17}} = \mathbf{0.695V}$$

(b) Calculate total width of space charge region :

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\cdot V_0}$$

$$= \sqrt{\frac{2(11.8)(8.85\times10^{-14})}{1.6\times10^{-19}}\left(10^{-15}+10^{-17}\right)\cdot 0.695}$$

$$= \mathbf{9.57\times10^{-5}cm}$$

(c) Given : $\mu_n = 1500\text{cm}^2/\text{Vs}$, $\mu_p = 1500\text{cm}^2/\text{Vs}$, $\tau = 2.5\times10^{-6}\text{s}$,

and $n_i = 1.5\times10^{10}\text{cm}^{-3}$, we have :

$$D_n = \mu_n\left(\frac{kT}{q}\right) = 450(0.0259) = 11.66\text{cm}^2/\text{s}$$

$$D_p = 38.85\text{cm}^2/\text{s}$$

$$L_n = \sqrt{D_n\tau} = 5.4\times10^{-3}\text{cm},\ L_p = \sqrt{D_p\tau} = 9.86\times10^{-3}\text{cm}$$

$$J_0 = (qn_i^2)\left(\frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n}\right)$$

$$= \left[(1.6\times10^{-19})(1.5\times10^{10})^2\right]\left(\frac{38.85}{10^{17}\times9.86\times10^{-3}} + \frac{11.66}{10^{15}\times5.4\times10^{-3}}\right)$$

$$= 7.92\times10^{-11} C/(cm^2 s)$$

$$I = AJ\left(e^{\frac{qV}{kT}} - 1\right) = (0.001)(7.92\times10^{-11})\left[e^{0.5/0.0259} - 1\right] = \mathbf{1.918\times10^{-5}\,A}$$

Most of the current is carried by electrons because N_d is greater than N_a.

To double the electron current, halve the acceptor doping.

Prob. 5.20

For the given n^+-p junction, calculate the electric field in the p-region far from the junction.

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = 0.0259V \Rightarrow D_n = 0.0259\times\mu_n = 25.9\text{cm}^2/\text{s}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{(25.9)(2\times10^{-6})} = 7.20\times10^{-3}\text{cm}$$

$$\frac{D_p}{\mu_p} = \frac{kT}{q} = 0.0259V \Rightarrow \mu_p = 502\text{cm}^2/\text{V}\cdot\text{s}$$

The total current is $\approx$ the electron current crossing the junction.

Deep in the p - region the current is hole drift.

$$J \approx J_n = q\left[\frac{D_n}{L_n}\left(\frac{n_i^2}{N_a}\right)\right]\left(e^{qV/kT} - 1\right) = J_p \text{ in p region far from junction}$$

$$= 1.6\times10^{-19}\left(\frac{25.9}{7.20\times10^{-3}}\right)\left(\frac{(1.5\times10^{10})^2}{10^{16}}\right)\left(e^{0.7/0.0259} - 1\right)$$

$$= 7.08\text{ A/cm}^2 = q\mu_p N_a \varepsilon$$

$$\text{thus } \varepsilon = \frac{J}{q\mu_p N_a} = \frac{7.08}{1.6\times10^{-19}\times500\times10^{16}} = \mathbf{8.85V/cm}$$

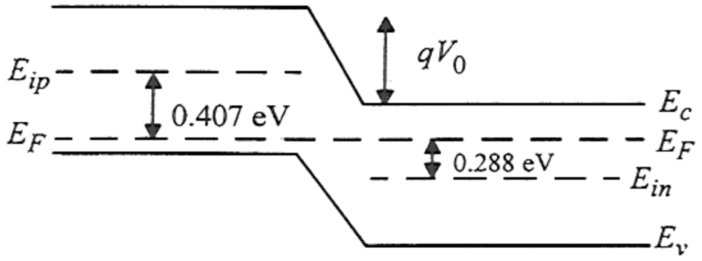
Prob. 5.21

For the diode in Prob. 5.16, draw the band diagrams qualitatively under forward and reverse bias showing the quasi-Fermi levels.

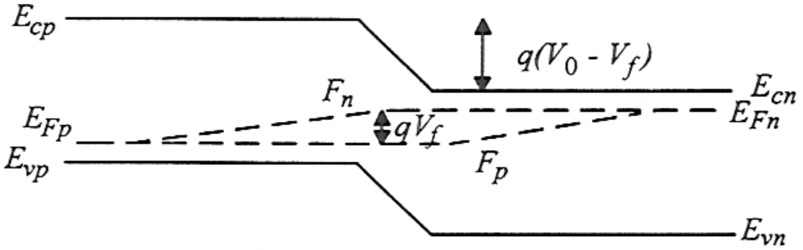
$$E_{ip} - E_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\ln\frac{10^{17}}{1.5\times10^{10}} = 0.407\text{eV}$$

$$E_F - E_{in} = \frac{kT}{q}\ln\frac{N_d}{n_i} = 0.0259\ln\frac{10^{15}}{1.5\times10^{10}} = 0.288\text{eV}$$

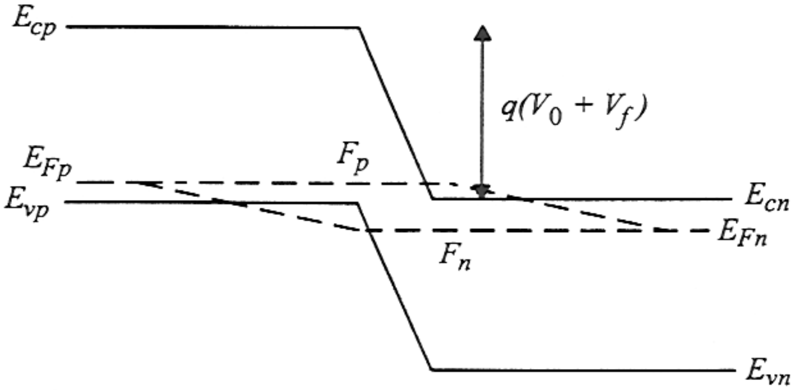
$$V_0 = E_{ip} - E_{in} = 0.407 + 0.288 = \mathbf{0.695eV}$$



(without bias voltage)



(with forward bias voltage)



(with reverse bias voltage)

Prob. 5.22

In a p^+-n junction with N-doping being changed from N_d to $2N_d$, describe how do junction capacitance, built-in potential, break-down voltage, ohmic losses.

(a) Increases
(b) Increases
(c) Decreases
(d) Decreases

Prob. 5.23

Find the forward current at a forward bias of 0.5V and the current at a reverse bias of –0.5V for the junction in Prob. (5-16).

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$$

$$p_n = \frac{n_i^2}{n_n} = \frac{(1.5\times10^{10})^2}{10^{15}} = 2.25\times10^5\,\text{cm}^{-3}$$

$$n_p = \frac{n_i^2}{p_p} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3\,\text{cm}^{-3}$$

For minority carriers :

$$D_p = \frac{kT}{q}\mu_p = 0.0259\times450 = 11.66\text{cm}^2/\text{s} \text{ on n - side}$$

$$D_n = \frac{kT}{q}\mu_n = 0.0259\times700 = 18.13\text{cm}^2/\text{s} \text{ on p - side}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{11.66\times10\times10^{-6}} = 1.08\times10^{-2}\,\text{cm}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{18.13\times0.1\times10^{-6}} = 1.35\times10^{-3}\,\text{cm}$$

$$I_0 = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)$$

$$= 1.6\times10^{-19}\times0.0001\left(\frac{11.66}{0.0108}2.25\times10^5 + \frac{18.13}{0.00135}2.25\times10^3\right)$$

$$= 4.370\times10^{-15}\,\text{A}$$

$I = I_0(e^{0.5/0.0259} - 1) \approx \mathbf{1.058\times10^{-6}\,A}$ in forward bias.

$I = -I_0 = \mathbf{-4.37\times10^{-15}\,A}$ in reverse bias.

Prob. 5.24

In the junction of Prob. (5-16), calculate the total depletion capacitance at –4V.

$$C_j = \sqrt{\epsilon} A \left[\frac{q}{2(V_0 - V)} \frac{N_d N_a}{N_d + N_a} \right]^{1/2}$$

$$= \sqrt{(8.85\times10^{-14}\times11.8)}(10^{-4})\left[\frac{1.6\times10^{-19}}{2(0.695+4)} \left(\frac{10^{15}\times10^{17}}{10^{15}+10^{17}} \right) \right]^{1/2}$$

$$= \mathbf{4.198\times10^{-13}\,F}$$

Prob. 5.25

For the given p^+-n diode, explain whether avalanche breakdown or punchthrough breakdown occurs.

$V_{avalanche} = 13\text{V}$ from Fig. 5.22.

$$W = \left[\frac{2\epsilon_s (V_0 + V_{Br})}{qN_d} \right]^{1/2} = \left[\frac{2(11.8)(8.85\times10^{-14})(0.956+13)}{1.6\times10^{-19}\times10^{17}} \right]^{1/2}$$

$$= \mathbf{4.27\times10^{-5}\,cm}$$

which is less than the width of the n - region.

Therefore, avalanche breakdown occurs.

Prob. 5.26

For the given n^+-p junction, calculate the capacitance.

$$\text{C} = \text{Capacitance} = \frac{\epsilon_s}{W} A$$

$$= A\sqrt{\frac{qN_a\epsilon_s}{2(V_0 + V_R)}}$$

Assume $E_F \approx E_C$ for the n^+ material.

$$V_0 = 0.0259 \ln \frac{N_a}{n_i} + 0.55$$

For $N_a = 10^{15}\text{cm}^{-3}$, $V_0 = 0.84\text{V}$

For $N_a = 10^{17}\text{cm}^{-3}$, $V_0 = 0.94\text{V}$

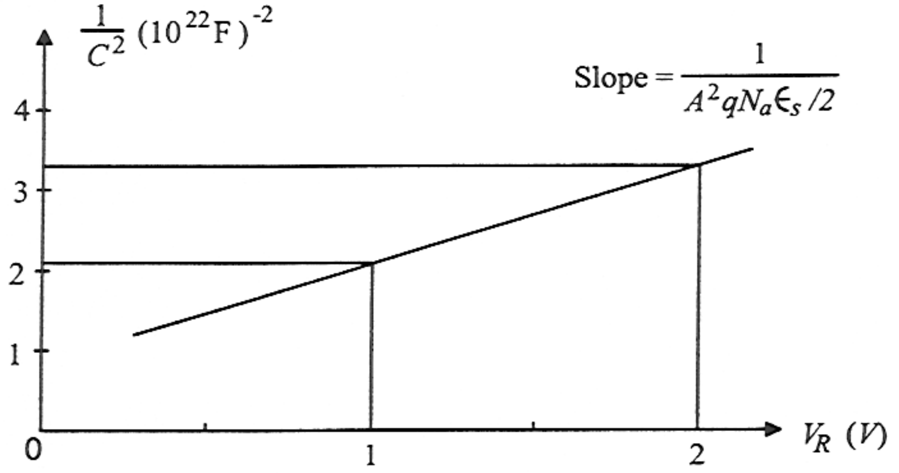
For $N_a = 10^{15}\text{cm}^{-3}$,

$$\frac{1}{C^2} = \frac{1}{A^2}\frac{(V_0 + V_R)}{qN_a\epsilon_s/2} = \frac{0.84 + V_R}{(0.001)^2(1.6\times10^{-19})(10^{15})(11.8\times8.85\times10^{-14})/2}$$

$$= 1.197\times10^{22}(V_R + 0.84)$$

which is linearly proportional to V_R with the slope being $1/(A^2qN_a\epsilon_s/2)$ which in turn yields N_a.

The plot of $\frac{1}{C^2}$ as a function of V_R is given below.



Prob. 5.27

Calculate the Debye length in Si for $N_d = 10^{14}$. 10^{16}, and 10^{18} cm^{-3} and compare with W in junctions having $N_a = 10^{18}$ cm^{-3}.

$$L_D = \left[\frac{\epsilon_s kT}{q^2 N_d}\right]^{\frac{1}{2}} = \left[\frac{11.8\times8.85\times10^{-14}\times0.0259}{1.6\times10^{-19}}\right]^{\frac{1}{2}} N_d^{\frac{1}{2}} \quad \left(\text{using } \frac{kT}{q} = 0.0259V\right)$$

$$= 411/N_d^{\frac{1}{2}}$$

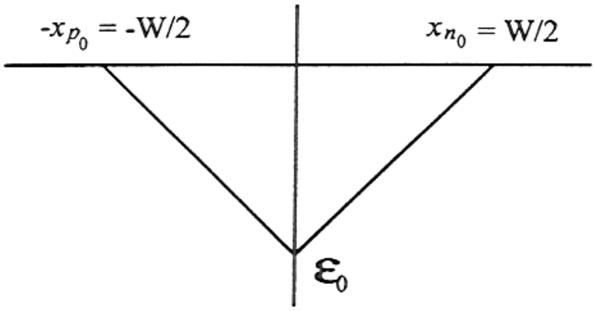
$$W = \left[\frac{2\epsilon_s kT}{q^2}\left(\ln\frac{N_aN_d}{n_i^2}\right)\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{\frac{1}{2}} = 581\left[\left(\ln\frac{N_d}{225}\right)\left(10^{-18}+\frac{1}{N_d}\right)\right]^{\frac{1}{2}}$$

N_d (cm^{-3})	L_D (cm)	W (cm)
10^{14}	4.11×10^{-5}	3×10^{-4}
10^{16}	4.11×10^{-6}	3.3×10^{-5}
10^{18}	4.11×10^{-7}	4.9×10^{-6}

We notice that L_D is about 14 % of W for the lightest doping and 8 % for the heaviest doping

Prob. 5.28

For the given symmetric p-n Si junction, find the reverse breakdown voltage.



$$N_a = N_d \;\Rightarrow\; x_{n_0} = x_{p_0} = \frac{W}{2}$$

$$\mathcal{E}_0 = \frac{qN_a}{\epsilon} x_{n_0} = \frac{qN_a}{\epsilon}\frac{W}{2}$$

The breakdown voltage is :

$$\int_{-x_{p0}}^{x_{n0}} \mathcal{E} \cdot dx = \text{the area under the triangle.}$$

$$V_{Br} = \mathcal{E}_0 x_{n_0} = \mathcal{E}_0 \frac{W}{2} = \mathcal{E}_0 \frac{1}{2}\left(\frac{2\epsilon\mathcal{E}_0}{qN_a}\right) = \frac{\epsilon\mathcal{E}_0^2}{qN_a}$$

$$= \frac{11.8 \times 8.85 \times 10^{-14} \times (5 \times 10^5)^2}{1.6 \times 10^{-19} \times 10^{17}} = \mathbf{16.32V}$$

Prob. 5.29

For the given parameters, design a p⁺-n diode and determine the width and doping of the n-region.

$$\mathcal{E}_{\max} = 1\,\text{MV/cm} = \frac{q}{\epsilon}N_d W \text{ (here W is used because of p}^{+}\text{ - n)}$$

$$V_{pt,total} = \frac{1}{2}\frac{qN_d W^2}{\epsilon} = V_0 + V_{pt}$$

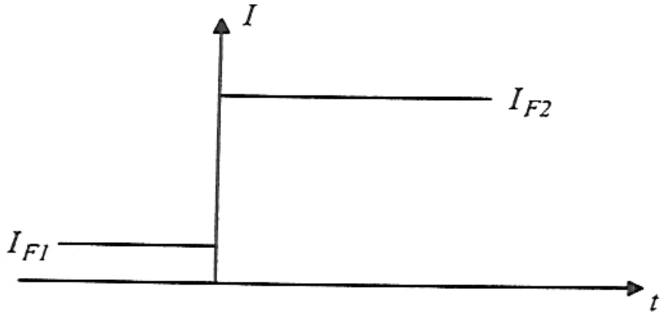
$$\frac{V_{pt,total}}{\mathcal{E}_{\max}} = \frac{W}{2} = \frac{15+0.5}{10^6} \quad \Rightarrow \quad \mathbf{W = 3.1\times10^{-5}\,cm}$$

$$\mathcal{E}_{\max} = \frac{q}{\epsilon}N_d W \quad \Rightarrow$$

$$N_d = \frac{\epsilon\mathcal{E}_{\max}}{qW} = \frac{10\times10^6\times8.85\times10^{-14}}{1.6\times10^{-19}\times3.1\times10^{-5}} = \mathbf{1.784\times10^{17}\,cm^2}$$

Prob. 5.30

Find the stored charge Q_p as a function of time in the n-region if a long p⁺-n forward bias current is switched from I_{F1} to I_{F2} at t = 0.



$$Q_p(0) = I_{F1}\cdot\tau_p$$

$$Q_p(\infty) = I_{F2}\cdot\tau_p$$

$$I_{F2} = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p}{dt}$$

Taking the Laplace transform

$$\frac{I_{F2}}{s} = \frac{Q_p(s)}{\tau_p} + sQ_p(s) - I_{F1}\cdot\tau_p$$

$$Q_p(s) = \left(\frac{I_{F2}}{s} + I_{F1}\tau_p\right)\frac{1}{s+\frac{1}{\tau_p}}$$

Transforming back to time domain

$$Q_p(t) = I_{F2}\tau_p\left(1-e^{-t/\tau_p}\right) + I_{F1}\tau_p e^{-t/\tau_p}$$

Prob. 5.31

Find v(t) in the quasi-steady state approximation.

$$2I = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$$

the solution is in the form

$$Q_p(t) = A + Be^{-t/\tau_p}$$

$$\begin{aligned} &\text{at } t = 0: \quad Q_p(0) = I\tau_p = A + B \\ &\text{at } t = \infty: \quad Q_p(\infty) = 2I\tau_p = A, \qquad B = -I\tau_p \end{aligned}$$

$$Q_p(t) = I\tau_p(2 - e^{-t/\tau_p})$$

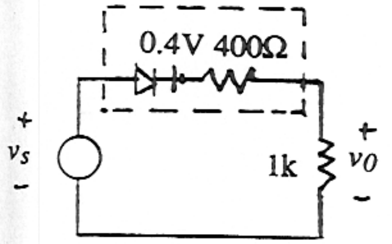
$$\Delta p_n(t) = p_n e^{qv(t)/kT} = \frac{I\tau_p}{qAL_p}\left(2 - e^{-t/\tau_p}\right)$$

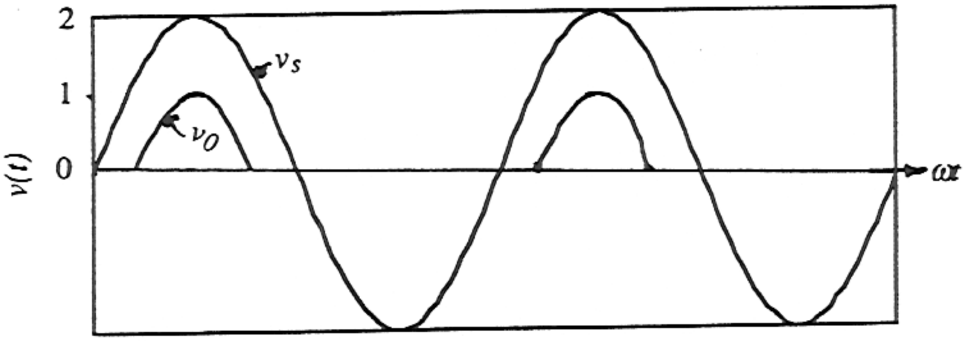
$$v(t) = \frac{kT}{q}\ln\left(\frac{I\tau_p}{qAL_p p_n}\right)\left(2 - e^{-t/\tau_p}\right)$$

Prob. 5.32

Sketch the voltage across a 1 kΩ resistor in series with a diode having an offset of 0.4 V and resistance of 400 Ω, and a source of 2 sin ωt

While the input $v_s > 0.4V, v_0 = \frac{(v_s - 0.4)\times 1k\Omega}{1.4k\Omega}$
when $v_s = 2, v_0 = 1.14V$





Prob. 5.33

What n-region thickness ensures avalanche rather than punch-through in a Si p^+-n with $N_d = 10^{15}$ cm^{-3} ?

From Fig. 5-22, $V_{br} \simeq 300V$. From Eq. (5-23b)
with $V_0 - V \simeq V_r = 300V$ and $N_a \gg N_d$,

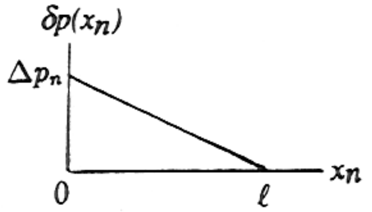
$$x_{no} \simeq W = \left[\frac{2\epsilon V_r}{qN_d}\right]^{\frac{1}{2}} = \left[\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 300}{1.6 \times 10^{-19} \times 10^{15}}\right]^{\frac{1}{2}}$$

$$= 2 \times 10^{-3} cm = 20\ \mu m$$

Thus, if the n region is 20 μm or thicker, avalanche will occur before punch-through.

Prob. 5.34

Find Q_p and I when holes are injected from p^+ into a short n-region of length l, if δp varies linearly.



$$Q_p = qA\int_0^l \delta p\, dx_n = qA\frac{l\Delta p_n}{2}$$

$$I = I_p(x_n = 0) = \frac{Q_p}{\tau_p} = \frac{qAl\Delta p_n}{2\tau_p}$$

Prob. 5.35

Find (a) the hole distribution, and (b) the total current in a narrow-base diode.

$$(1)\quad \frac{d^2\delta p(x_n)}{dx_n^2} = \frac{\delta p(x_n)}{L_p^2}$$

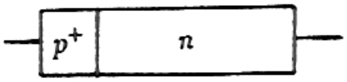
$$(2)\quad \delta p(x_n) = C\, e^{-x_n/L_p} + D\, e^{x_n/L_p}$$

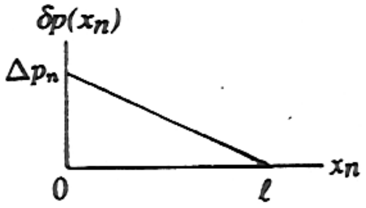
$$(3)\quad \text{At } x_n = 0\,,\ \delta p = \Delta p_n = C + D$$

$$(4)\quad \text{At } x_n = \ell\,,\ \delta p = 0 = C\, e^{-\ell/L_p} + D\, e^{\ell/L_p}$$

Putting (3) into (4) : $\quad 0 = C\, e^{-\ell/L_p} + (\Delta p_n - C)\, e^{\ell/L_p}$

$$(5)\quad C = \frac{\Delta p_n\, e^{\ell/L_p}}{e^{\ell/L_p} - e^{-\ell/L_p}}\,,\quad D = \Delta p_n - C = \frac{-\Delta p_n\, e^{-\ell/L_p}}{e^{\ell/L_p} - e^{-\ell/L_p}}$$





Thus, from (5) and (2) we have

$$(6)\quad \delta p(x_n) = \frac{\Delta p_n \left[e^{(\ell - x_n)/L_p} - e^{(x_n - \ell)/L_p}\right]}{e^{\ell/L_p} - e^{-\ell/L_p}} \qquad \text{(a)}$$

$$I = -qAD_p \left.\frac{d\delta p(x_n)}{dx_n}\right|_{x_n=0} = -\frac{qAD_p\Delta p_n}{L_p}\left[-\frac{e^{\ell/L_p} + e^{-\ell/L_p}}{e^{\ell/L_p} - e^{-\ell/L_p}}\right]$$

$$I = \left[\frac{qAD_p}{L_p} p_n \ \mathrm{ctnh}\ \frac{\ell}{L_p}\right](e^{qV/kT} - 1) \qquad \text{(b)}$$

Prob. 5.36

For the narrow-base diode, find the current components due to (a) recombination in n, and (b) recombination at the ohmic contact.

The steady state charge stored in the excess hole distribution is

$$Q_p = qA\int_0^{\ell} \delta p(x_n)dx_n = qA\int_0^{\ell}\left[C\ e^{-x_n/L_p} + D\ e^{x_n/L_p}\right]dx_n$$

$$= qAL_p\left[-C(e^{-\ell/L_p}-1) + D(e^{\ell/L_p}-1)\right]$$

$$= qAL_p\Delta p_n \frac{e^{\ell/L_p}+e^{-\ell/L_p}-2}{e^{\ell/L_p}-e^{-\ell/L_p}}$$

Thus, the current due to recombination in n is

(a) $$\frac{Q_p}{\tau_p} = \frac{qAL_p p_n}{\tau_p}\left[\text{ctnh}\ \frac{\ell}{L_p} - \text{csch}\ \frac{\ell}{L_p}\right](e^{qV/kT}-1)$$

$$= qA\frac{D_p}{L_p}p_n\left[\tanh\frac{\ell}{2L_p}\right](e^{qV/kT}-1)$$

(b) The current due to recombination at $x_n = \ell$ is

$$I - \frac{Q_p}{\tau_p} = \left[\frac{qAL_p p_n}{\tau_p}\text{csch}\ \frac{\ell}{L_p}\right](e^{qV/kT}-1)$$

(note that $L_p/\tau_p = D_p/L_p$)

These correspond to the base recombination and collector currents in the *p-n-p* BJT with $V_{CB} = 0$, given in Eqs.(7-20).

Prob. 5.37

If the n region of a graded p^+-n has $N_d = Gx^m$, find $\mathcal{E}_0$, $\mathcal{E}(x)$, Q, and C_j.

(a) $$\frac{d\mathcal{E}}{dx} = \frac{q}{\epsilon}Gx^m\ , \quad \int_{\mathcal{E}_0}^{0} d\mathcal{E} = \frac{q}{\epsilon}G\int_0^W x^m dx$$

$$-\mathcal{E}_0 = \frac{q}{\epsilon}G\frac{W^{m+1}}{m+1}$$

Prob. 5.38

Find $\mathcal{E}(x)$, W, and C_j for a linearly graded junction.

(a)

$$\int_0^{\mathcal{E}(x)} d\mathcal{E} = \frac{q}{\epsilon} G \int_{-W/2}^{x} x \, dx$$

or use a dummy variable.

$$\mathcal{E}(x) = \frac{qG}{2\epsilon}\left[x^2 - (\frac{W}{2})^2\right]$$

(b)

$$\mathcal{E} = -dV/dx \text{ , or } V_0 - V = -\int_{-W/2}^{W/2} \mathcal{E}(x)\, dx$$

$$V_0 - V = -\frac{qG}{2\epsilon}\left[\frac{(W/2)^3}{3} - (W/2)^3 + \frac{(W/2)^3}{3} - (W/2)^3\right]$$

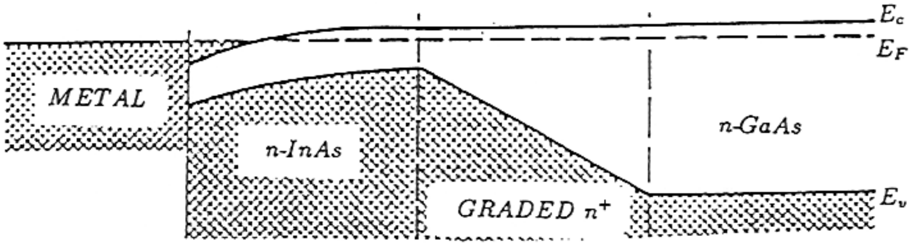
$$V_0 - V = \frac{qGW^3}{12\epsilon}, \quad W = [12\epsilon(V_0 - V)/qG]^{1/3}$$

(c)

$$\begin{aligned} C_j &= \frac{dQ}{d(V_0 - V)}, \quad Q = qA\int_0^{W/2} Gx\, dx = qAGW^2/8 \\ C_j &= \frac{qAG}{8}\,\frac{d}{d(V_0 - V)}\,[12\epsilon(V_0 - V)/qG]^{2/3} \\ &= A\left[\frac{qG\epsilon^2}{12(V_0 - V)}\right]^{1/3} \end{aligned}$$

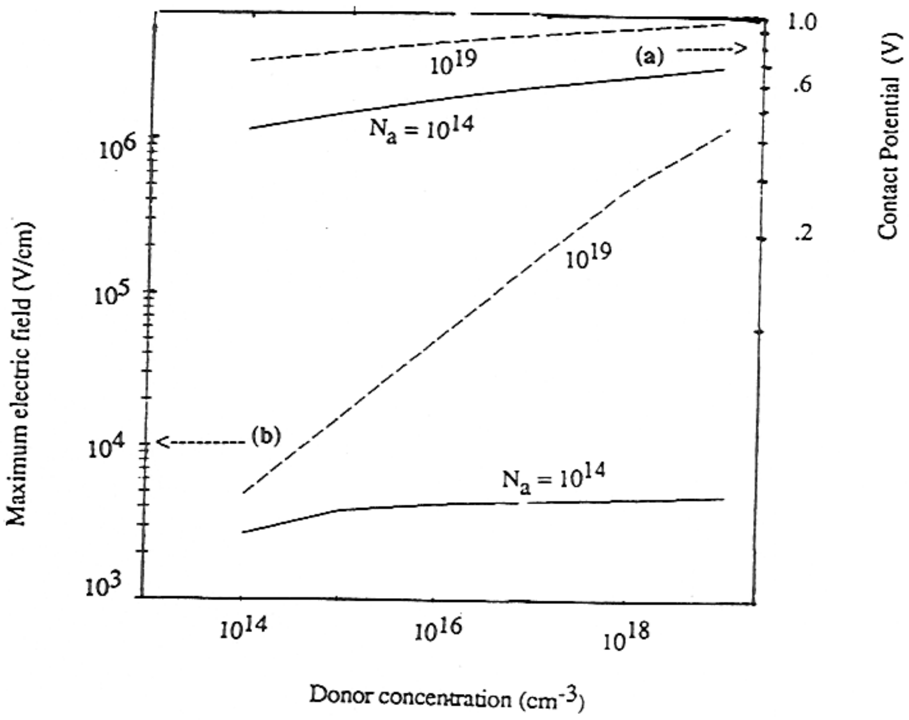
Prob. 5.39

Use InAs to make an ohmic contact to GaAs.



For further discussion, see *Woodall, et al., J. Vac. Sci. Technol. 19, 626(1981).*

Prob. 5.40



Prob. 5.41

A Schottky barrier is formed between a metal having $\Phi_m = 4.3$ *V and p-type Si* ($\chi = 4$ *V). The acceptor doping in the Si is* $N_a = 10^{17}$ cm^{-3}.

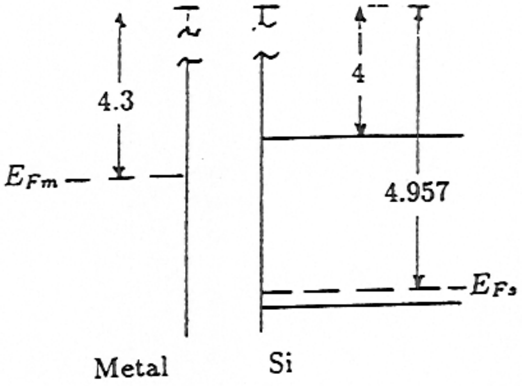
(a) Draw the equilibrium band diagram, showing a numerical value for qV_0.

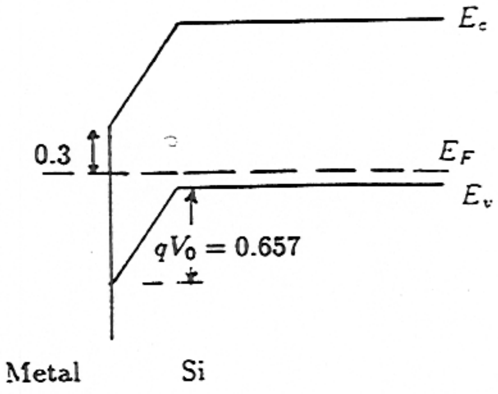
$$E_i - E_F = kT\ln\frac{p_0}{n_i}$$

$$= 0.0259\ln\frac{10^{17}}{1.5\times10^{10}}$$

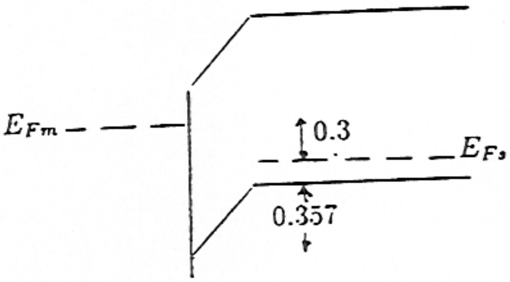
$$= 0.407\text{eV}$$

$\Phi_s = 4 + 0.55 + 0.407 = \mathbf{4.957\ eV}$

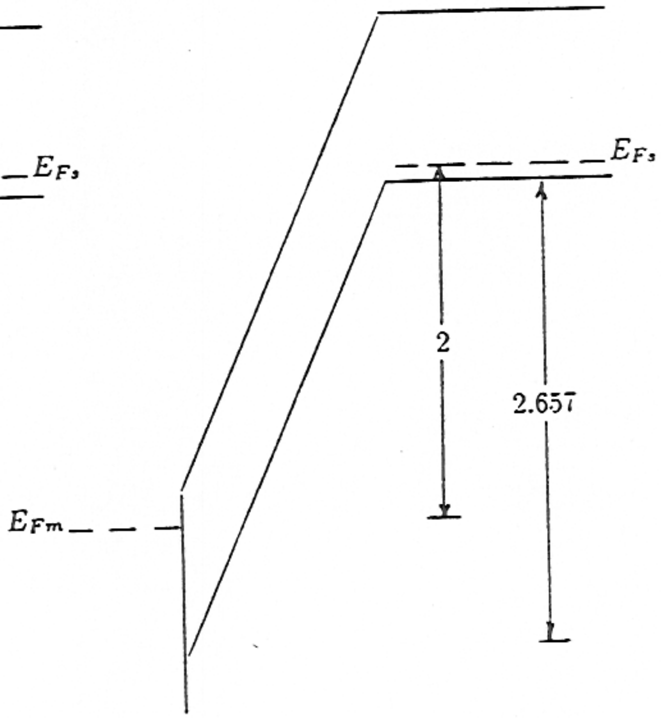




(b) *Draw the band diagram with* 0.3 *V forward bias. Repeat for* 2 *V reverse bias.*



Forward bias $V = 0.3\ V$
metal negative



Reverse bias $V = -2\ V$
metal positive

Prob. 5.42

Under the given parameters for a piece of Ge, find its conductivity, work function difference. Explain whether it is a Schottky barrier or an ohmic contact.

$$n+N_a=p+N_d=\frac{n_i^2}{n}+N_d$$

$$n+2.5\times10^{13}=\frac{(2.5\times10^{13})^2}{n}+5\times10^{13},\ \text{ solve for } n \Rightarrow$$

$$n=4.04\times10^{13}\text{cm}^{-3},\ p=1.54\times10^{13}\text{cm}^{-3}$$

$$\sigma=q(n\mu_n+p\mu_p)=\frac{q}{kT/q}(nD_n+pD_p)$$

$$=\frac{1.6\times10^{-19}}{0.0259}(4.04\times10^{13}\times100+1.54\times10^{13}\times50)=0.0297(\Omega\cdot\text{cm})^{-1}$$

$$\phi_F=E_F-E_i=kT\ln\frac{n}{n_i}=0.0259\ln\left(\frac{4.04\times10^{13}}{2.5\times10^{13}}\right)=0.0124\text{eV}$$

For n - type semiconductor, the Fermi level is above the intrinc Si Fermi level by the Fermi potential ϕ_F.

$$\Phi_{ms}=\Phi_m-(\chi+\frac{E_g}{2}-\phi_F)=4.5-(4.0+\frac{0.67}{2}-0.012)=\mathbf{4.338eV}$$

Electrons move from Ge to the metal. Therefore, we lose majority carriers in the semiconductors, making this a Schottky barrier.

Chapter 6

Prob. 6.1

Modify Eqs. (6-2) - (6-5) to include V_0 and a new pinch-off V_T.

Including the contact potential.

$$W(x=\mathrm{L}) = \left[\frac{2\epsilon(V_0 - V_{GD})}{qN_d}\right]^{1/2}$$

$$h(x=\mathrm{L}) = a - \left[\frac{2\epsilon}{qN_d}\right]^{1/2}(V_0 - V_{GD})^{1/2}$$

at pinch − off, $h(x=\mathrm{L}) = 0$ and $V_0 - V_{GD} = \dfrac{qa^2N_d}{2\epsilon}$

$$-V_{GD}\,(\text{pinch}-\text{off}) = \frac{qa^2N_d}{2\epsilon} - V_0 \equiv V_T$$

$$V_T = -V_G + V_D \text{ at pinch} - \text{off} = V_P - V_0$$

where $V_P = \dfrac{qa^2N_d}{2\epsilon}$

Prob. 6.2

Modify Eqs.(6-7) – (6-10) to include V_0 and V_T from **Prob. 6.1**

$$W(x) = \left[\frac{2\epsilon(V_0 - V_{Gx})}{qN_D}\right]^{1/2}, \quad h(x) = a\left[1 - \left(\frac{V_0 + V_x - V_G}{V_P}\right)^{1/2}\right]$$

$$I_D = G_0V_P\left[\frac{V_D}{V_P} + \frac{2}{3}\left(\frac{V_0 - V_G}{V_P}\right)^{3/2} - \frac{2}{3}\left(\frac{V_0 + V_D - V_G}{V_P}\right)^{3/2}\right]$$

Saturation occurs when $-V_{GD} = V_T = V_P - V_0 = V_D - V_G$

i.e.. when $V_0 + V_D - V_G = V_P$

thus $$I_D(\text{sat.}) = G_0 V_P \left[\frac{V_G - V_0}{V_P} + \frac{2}{3}\left(\frac{V_0 - V_G}{V_P}\right)^{3/2} + \frac{1}{3} \right]$$

Prob. 6.3

An n-channel Si JFET with $N_a = 10^{18} cm^{-3}$ in the p^+ gate regions and $N_d = 10^{16} cm^{-3}$ in the channel has $a = 1\ \mu m$. Find V_0, V_P, and V_T. Find V_D (sat.) if $V_G = -3$ V.

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = 0.0259 \ln \frac{10^{34}}{2.25 \times 10^{20}} = \mathbf{0.814V}$$

$$V_P = \frac{qa^2 N_d}{2\epsilon} = \frac{1.6 \times 10^{-19} \times 10^{-8} \times 10^{16}}{2 \times 11.8 \times 8.85 \times 10^{-14}} = \mathbf{7.66V}$$

$$V_T = V_P - V_0 = \mathbf{6.85V} = V_D\,(\text{sat.}) - V_G$$

$$V_D\ (\text{sat.}) = V_T + V_G = 6.85 - 3 = \mathbf{3.85V}$$

Prob. 6.4

For the JFET in Prob. 6.3, $Z/L = 10$ and $\mu_n = 1000 cm^2/Vs$. Plot I_D (sat.) vs. V_D (sat.) for $V_G = 0, -2, -4, -6$ V.

$$G_0 = 2aq\mu_n n Z/L = 2\times10^{-4}\times1.6\times10^{-19}\times10^{3}\times10^{16}\times10 = 3.2\times10^{-3} S$$

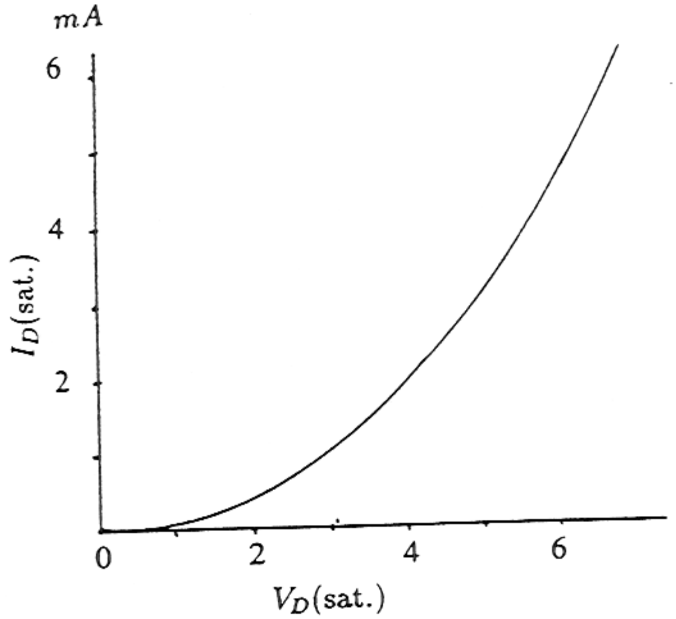
From Prob. 6.2

$$I_D(\text{sat.}) = G_0 V_P \left[\frac{V_G - V_0}{V_P} + \frac{2}{3}\left(\frac{V_0 - V_G}{V_P}\right)^{3/2} + \frac{1}{3} \right] \text{ See Prob. 6.3 for } V_0 \text{ and } V_P$$

$$I_D(\text{sat.}) = 3.2\times10^{-3}\times7.66 \left[\frac{V_G - 0.814}{7.66} + \frac{2}{3}\left(\frac{0.814 - V_G}{7.66}\right)^{3/2} + \frac{1}{3} \right]$$

We can plot this vs. V_D (sat.) $= 6.85 + V_G$

V_G	V_D(sat.)	I_D(sat.)
0 V	6.85 V	6.13 mA
−1	5.85	4.25
−2	4.85	2.80
−3	3.85	1.707
−4	2.85	0.907
−5	1.85	0.372
−6	0.85	0.076



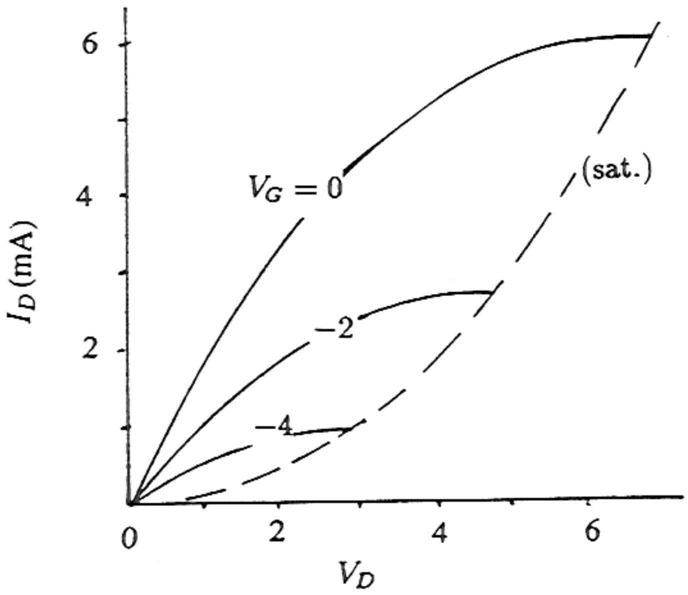
Prob. 6.5

For the JFET of Prob. 6.4, plot I_D vs. V_D up to pinch-off.

From Prob. 6.2,

$$I_D = G_0 V_P \left[\frac{V_D}{V_P} + \frac{2}{3}\left(\frac{V_0 - V_G}{V_P}\right)^{3/2} - \frac{2}{3}\left(\frac{V_0 + V_D - V_G}{V_P}\right)^{3/2}\right]$$

$$I_D = 3.2\times10^{-3}\times7.66 \left[\frac{V_D}{7.66} + \frac{2}{3}\left(\frac{0.814 - V_G}{7.66}\right)^{3/2} - \frac{2}{3}\left(\frac{0.814 + V_D - V_G}{7.66}\right)^{3/2}\right]$$



Prob. 6.6

The current I_D varies almost linearly with V_D in a JFET for low values of V_D.

(a) Use the binomial expansion with $V_D/(-V_G) < 1$ to rewrite Eq.(6-9) as an approximation to this case.

(b) Show that I_D/V_D in the linear range is the same as $g_m(sat.)$.

(c) What value of V_G turns the device off?

Eq.(6-9) can be rewritten as

$$I_D = G_0\left[V_D + \frac{2}{3}\frac{(-V_G)^{3/2}}{V_p^{1/2}} - \frac{2}{3}\frac{(-V_G)^{3/2}}{V_p^{1/2}}\left(\frac{V_D}{-V_G}+1\right)^{3/2}\right]$$

using $(1+x)^{3/2} \approx 1+\frac{3}{2}x$ for small x, where $x = V_D/(-V_G)$:

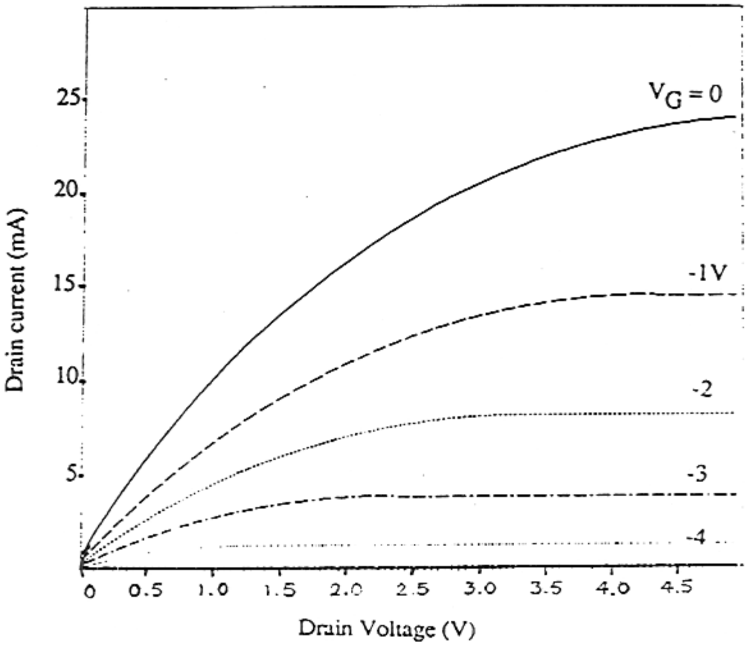
$$I_D \approx G_0\left[V_D + \frac{2}{3}\frac{(-V_G)^{3/2}}{V_p^{1/2}} - \frac{2}{3}\frac{(-V_G)^{3/2}}{V_p^{1/2}} - \frac{(-V_G)^{3/2}}{V_p^{1/2}}\frac{V_D}{(-V_G)}\right]$$

(a) $I_D = G_0\left[1-\left(\frac{-V_G}{V_p}\right)^{1/2}\right]V_D$

(b) $I_D/V_D = G_0\left[1-\left(\frac{-V_G}{V_p}\right)^{1/2}\right]$

(c) This is zero when $-V_G = V_p$.

Prob. 6.7



Prob. 6.8

Show that the width of the depletion region in Fig. 6-15 is given by Eq.(6-30).

Use the mathematics leading to Eq. (5-23b), with ϕ_s for the potential difference across the depletion region, which is contained in $x_{p0} = W$.

Prob. 6.9

An n^+-polysilicon gate n-channel MOS transistor is made on a p-type Si substrate with $N_a = 5 \times 10^{15}$ cm^{-3}. The SiO$_2$ thickness is 100Å in the gate region, and the effective interface charge Q_i is 4×10^{10} qC/cm^2. Find W_m, V_{FB}, and V_T.

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\ln\frac{5\times10^{15}}{1.5\times10^{10}} = 0.329\text{ eV}$$

$$W_m = 2\left[\frac{\epsilon_s\phi_F}{qN_a}\right]^{1/2} = 2\left[\frac{11.8\times8.85\times10^{-14}\times0.329}{1.6\times10^{-19}\times5\times10^{15}}\right]^{1/2}$$

$$= 4.15\times10^{-5}\text{cm} = \mathbf{0.415\mu m}$$

From Fig.6−17: $\Phi_{ms} \approx -0.95$V

$$Q_i = 4\times10^{10}\times1.6\times10^{-19} = 6.4\times10^{-9}\text{ C/cm}^2$$

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9\times8.85\times10^{-14}}{0.1\times10^{-5}} = 3.45\times10^{-7}\text{ F/cm}^2$$

$$V_{FB} = \Phi_{ms} - Q_i/C_i = -0.95 - 6.4\times10^{-9}/3.45\times10^{-7} = \mathbf{-0.969\text{ V}}$$

$$Q_d = -qN_aW_m = -1.6\times10^{-19}\times5\times10^{15}\times4.15\times10^{-5}$$

$$= -3.32\times10^{-8}\text{C/cm}^2$$

$$V_T = V_{FB} - \frac{Q_d}{C_i} + 2\phi_F = -0.969 + \frac{3.32\times10^{-8}}{3.45\times10^{-7}} + 0.658 = \mathbf{-0.215\text{ V}}$$

$$C_d = \frac{\epsilon_s}{W_m} = \frac{11.8\times8.85\times10^{-14}}{4.15\times10^{-5}} = 2.5\times10^{-8}\text{F/cm}^2$$

$$C_{\min} = \frac{C_iC_d}{C_i+C_d} = \frac{3.45\times10^{-7}\times2.5\times10^{-8}}{3.45\times10^{-7}+2.5\times10^{-8}} = \mathbf{2.33\times10^{-8}\text{F/cm}^2}$$

Prob. 6.10

An n^+-polysilicon gate p-channel MOS transistor is made on an n-type Si substrate with $N_d = 5 \times 10^{16}$ cm^{-3}. The SiO$_2$ thickness is 100Å in the gate region, and the effective interface charge Q_i is 2×10^{11} qC/cm^2. Find W_m, V_{FB}, and V_T, and sketch the C-V curve.

$$\phi_F = -\frac{kT}{q}\ln\frac{N_d}{n_i} = -0.0259\ln\frac{5\times10^{16}}{1.5\times10^{10}} = -0.389V$$

$\Phi_{ms} \approx -0.2$ V from Fig.6 − 17

$$W_m = 2\left[\frac{\epsilon_s(-\phi_F)}{qN_d}\right]^{1/2} = 2\left[\frac{(11.8)(8.85\times10^{-14})(0.389)}{(1.6\times10^{-19})(5\times10^{16})}\right]^{1/2}$$

$$= \mathbf{0.143\mu m}$$

$$Q_d = +qN_dW_m = (1.6\times10^{-19})(5\times10^{16})(0.143\times10^{-4})$$

$$= 1.144\times10^{-7}\text{C/cm}^2$$

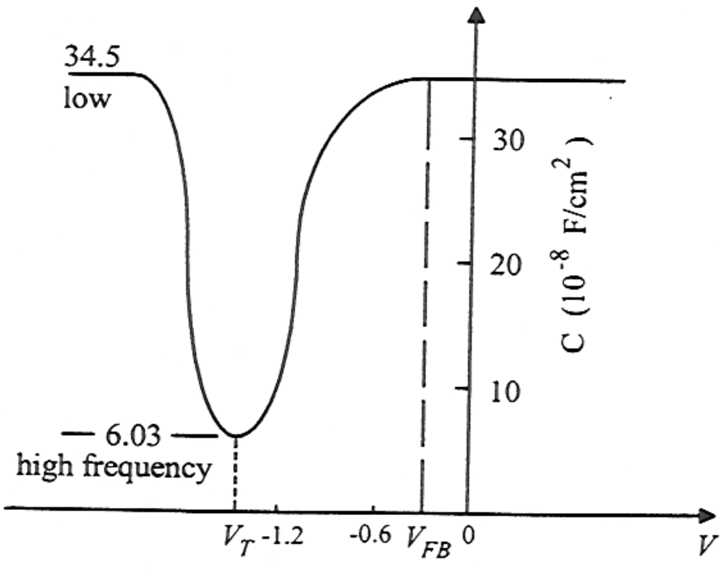
$$C_i = \frac{\epsilon_i}{d} = \frac{(3.9)(8.85\times10^{-14})}{10^{-6}} = \mathbf{3.45\times10^{-7}\,F/cm^2}$$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -0.2 - \frac{(2\times10^{11})(1.6\times10^{-19})}{3.45\times10^{-7}} = \mathbf{-0.293V}$$

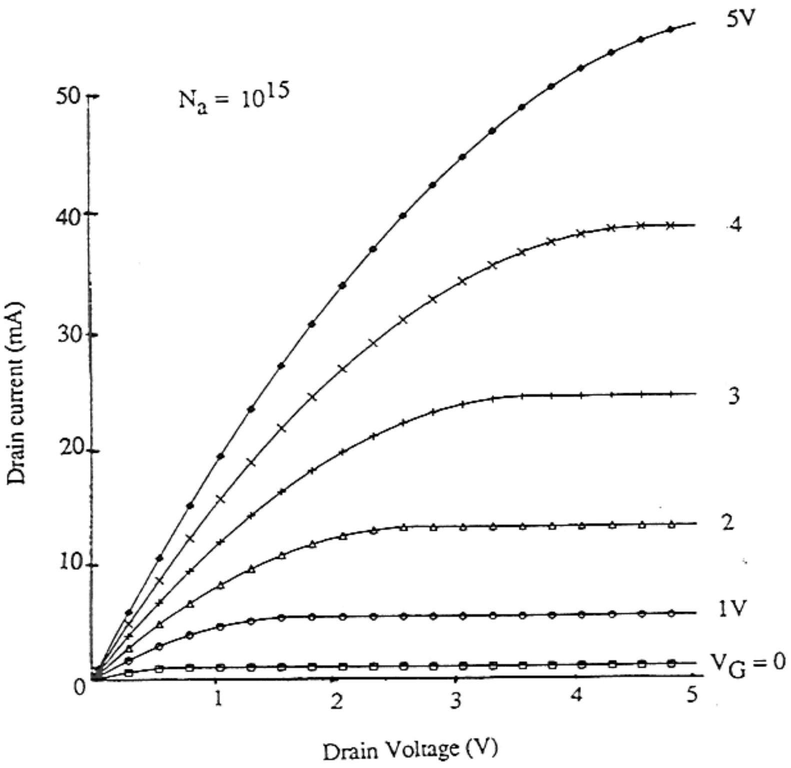
$$V_T = 2\phi_F - \frac{Q_d}{C_i} + V_{FB} = -0.778 - \frac{1.144\times10^{-7}}{3.45\times10^{-7}} - 0.293 = \mathbf{-1.402V}$$

$$C_d = \frac{\epsilon_s}{W_m} = \frac{11.8\times8.85\times10^{-14}}{1.43\times10^{-5}} = 7.30\times10^{-8}\,\text{F/cm}^2$$

$$C_{\min} = \frac{C_iC_d}{C_i + C_d} = \frac{3.45\times10^{-7}\times7.30\times10^{-8}}{3.45\times10^{-7} + 7.30\times10^{-8}} = \mathbf{6.03\times10^{-8}\,F/cm^2}$$



Prob. 6.11


$N_a = 10^{15}$
50
40
30
20
10
0
Drain current (mA)
1
2
3
4
5
Drain Voltage (V)
5V
4
3
2
1V
$V_G = 0$

Prob. 6.12

For the given Si-MOS, calculate V_T, B dose requirement to change it to zero. Discuss whether it is an enhancement or depletion mode device.

From Fig. 6-17, we have $\Phi_{ms} \approx -0.1\,\text{V}$

$$C_i = \frac{\varepsilon_i}{d} = \frac{3.9\times 8.85\times 10^{-14}}{50\times 10^{-8}} = 6.903\times 10^{-7}\,\text{F/cm}^2$$

$$\phi_F = -kT\ln\frac{N_d}{n_i} = -0.0259\cdot\ln\left(\frac{10^{18}}{1.5\times 10^{10}}\right) = -0.467\,\text{V}$$

$$W = 2\sqrt{\frac{\varepsilon_s(-\phi_F)}{qN_d}} = 2\sqrt{\frac{11.8\times 8.85\times 10^{-14}\times 0.467}{1.6\times 10^{-19}\times 10^{18}}} = 3.492\times 10^{-6}\,\text{cm}$$

$V_T = \Phi_{ms} + 2\phi_F - \frac{Q_d}{C_i} - \frac{Q_i}{C_i}$ (for p-channel, where $Q_d = qN_dW$, Q_i is given as 2×10^{10} qC/cm^2).

$$V_T = -0.1 - 2(0.467) - \frac{(1.6\times 10^{-19})(10^{18})(3.492\times 10^{-6})}{6.903\times 10^{-7}} - \frac{(2\times 10^{10})(1.6\times 10^{-19})}{6.903\times 10^{-7}}$$

$= -1.848\,\text{V}$

$\Rightarrow$ Enhancement mode p-channel

$$\Delta V_T = 0 - (-1.848) = 1.848\text{V} = \frac{Q_{Boron}}{C_i}$$

$$\Rightarrow\ Q_{Boron} = (1.848)C_i = (1.848)(6.903\times 10^{-7}) = 1.276\times 10^{-6}\ \text{C/cm}^2$$

$$\text{Boron dose} = \frac{Q_{Boron}}{q} = \frac{1.276\times 10^{-6}}{1.6\times 10^{-19}} = \mathbf{7.975\times 10^{12}\ atoms/cm^2}$$

Prob. 6.13

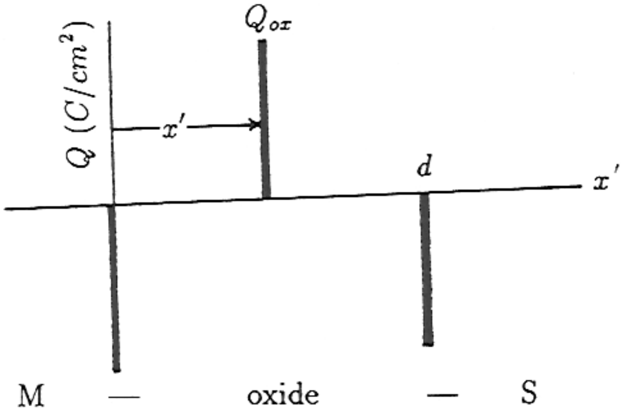
Find V_{FB} for a positive oxide charge Q_{ox} located x' below the metal. Repeat for an arbitrary distribution $\rho(x')$.

(a) At flat band the induced charge in the semiconductor is zero. Thus the field between the metal and Q_{ox} is $\mathcal{E} = -Q_{ox}/\epsilon_i$. The voltage required is

$$V = V_{FB} = x'\,\mathcal{E} = -x'\,\frac{Q_{ox}}{\epsilon_i}$$

(b) For a distributed charge, we sum up discrete charges such as Q_{ox} over the entire oxide

$$V_{FB} = -\frac{1}{\epsilon_i}\int_0^d x'\rho\,dx' = -\frac{1}{C_i}\int_0^d \frac{x'}{d}\,\rho\,dx'$$



Prob. 6.14

Calculate the change in the surface bandbending at 100°C if the bias on a Si MOS capacitance with the substrate doping of $10^{16}cm^{-3}$ is changed from inversion to accumulation mode.

$$\Delta\phi = 2\phi_F = -2\frac{kT}{q}\ln\frac{N_d}{n_i}$$

$T = 373\text{K}$

From Fig 3 - 17, we obtain :

$n_i = 2\times10^{12}\text{cm}^{-3}$ (at 373K)

Thus,

$$\Delta\phi = 2\frac{kT}{q}\ln\frac{N_d}{n_i} = 2\frac{kT_{room}}{q}\frac{T}{T_{room}}\ln\frac{N_d}{n_i}$$

$$= 2(0.0259)\frac{373}{300}\ln\frac{10^{16}}{2\times10^{12}} = \mathbf{0.549V}$$

Prob. 6.15

C_i = Measured capacitance/area in strong accumulation

$$C_i = \frac{37.85\times10^{-12}F}{0.001\text{cm}^2} = 37.85\times10^{-9}\text{F/cm}^2$$

Oxide thickness

$$d = \frac{\epsilon_i}{C_i} = \frac{3.9\times8.85\times10^{-14}}{37.85\times10^{-9}} = 9.12\times10^{-6}\text{cm}$$

To determine the doping density :

From experimental C - V curve the normalized minimum capacitance corresponding to quasi - equilibrium can be determined.

$$C'_{min} = \frac{C_{min}}{C_i} = 0.2$$

where C_{min} is the total capacitance of C_i in series connection with $C_{d_{min}}$.

$$C_{min} = \frac{C_iC_{d_{min}}}{C_i + C_{d_{min}}}$$

Now, solve for $C_{d_{min}}$.

$$C_{d_{min}} = C_i\left[\frac{C_{min}}{C_i - C_{min}}\right] = C_i\left[\frac{C'_{min}}{1 - C'_{min}}\right]$$

$$= 37.85\times10^{-9}\left[\frac{0.2}{1-0.2}\right] = 9.46\times10^{-9}\,\text{F/cm}^2$$

Also :

$$C_{d_{min}} = \frac{\epsilon_s}{W_m} = \frac{\epsilon_s}{\sqrt{\frac{2\varepsilon_s}{qN_A}(2\phi_F)}}$$

From Eq.(6 - 39)

$$N_a = 10^{[30.388+1.683\times\log(C_{d_{min}})-0.03177(\log(C_{d_{min}}))^2]}$$

$$= 10^{[30.388+1.683(-8.02)-0.03177(64.39)]} = \mathbf{6.88\times10^{14}\,cm^{-3}}$$

<u>Prob. 6.16</u>

For the Prob. (6.15), determine the initial flatband voltage.

To determine the initial (pre - stressed) flatband voltage V_{FB_0}

First calculate C_{FB} from the previously determined doping density :

$$C_{FB} = \frac{C_i C_{debye}}{C_i + C_{debye}} \Rightarrow$$

$$\frac{C_{FB}}{C_i} = \frac{C_{debye}}{C_i + C_{debye}} = C'_{FB}$$ (C'_{FB} is the normalized flatband capacitance)

$$C_{debye} = \frac{\sqrt{2}\epsilon_s}{L_D} = \frac{\sqrt{2}\epsilon_s}{\sqrt{\frac{2kT}{q}\frac{\epsilon_s}{qN_a}}} = \sqrt{\frac{q}{2kT}\epsilon_s qN_a}$$

$$= \sqrt{\left(\frac{1}{2(0.0259)}\right)(11.8)(8.85\times10^{-14})(1.6\times10^{-19})(6.88\times10^{14})}$$

$$= 4.71\times10^{-8}\,\text{F/cm}^2$$

$$C'_{FB} = \frac{C_{debye}}{C_i + C_{debye}} = \frac{4.71\times10^{-8}}{3.785\times10^{-8} + 4.71\times10^{-8}} = \mathbf{0.55}$$

From Fig. 6-15, the V_{EB0} = -1.0V

Prob. 6.17

For the Prob. (6.15), determine the fixed oxide charge, Q_i and the mobile ion concentration.

On the experimental C - V curve we can determine the gate voltage at which the normalized capacitance prior to bias - temperature stress is 0.636.

$$V_{FB_0} = V_G\big|_{C'_{FB}=0.55} = -1.0\text{V}$$

$$V_{FB_0} = \Phi_{\text{ms}} - q\frac{Q_i}{C_i},$$

where $\Phi_{\text{ms}} \approx -0.35\text{V}$

$$\frac{Q_i}{q} = (\Phi_{\text{ms}} - V_{FB_0})\frac{C_i}{q} = [-0.35-(-1.0)]\frac{3.785\times10^{-8}}{1.6\times10^{-19}}\text{cm}^{-2} = \mathbf{1.53\times10^{11}\,cm^{-2}}$$

To determine mobile ion concentration

1. Determine V_{FB+} corresponding to the flatband voltage after the MOS capacitance had been positively biased at 300°C.

(Note: Flatband capacitance has not changed)

From C-V curve measured after positive bias-temperature stress

$$V_{FB+} = V_G\big|_{C'=C'_{FB}=0.55} = \mathbf{-1.5V}$$

2. Determine V_{FB-} the flatband voltage after negative bias temperature stress. From the C-V curve measured after bias temperature stress

$$V_{FB-} = V_G\big|_{C'=C'_{FB}=0.55} = -1\text{V}$$

$$\frac{Q_i}{q} = (V_{FB-} - V_{FB+})\frac{C_i}{q} = [-1.0-(-1.5)]\frac{3.785\times10^{-8}}{1.6\times10^{-19}} = \mathbf{1.2\times10^{11}\,ions/cm^2}$$

Prob. 6.18

Derive the drain conductance $g'_D = \partial I'_D / \partial V_D$ *beyond saturation in terms of the effective channel length* $L - \Delta L$*, and then in terms of* V_D.

Using L' in Eq.(6-53),

$$I'_D = \frac{1}{2}\,\mu_n C_i \frac{Z}{L'}\,(V_G - V_T)^2 = I_D(\text{sat.})\,\frac{L}{L'} = I_D(\text{sat.})\,\frac{L}{L - \Delta L}$$

$$g'_D = \frac{\partial I'_D}{\partial V_D} = I_D(\text{sat.})\,\frac{\partial}{\partial V_D}\left(\frac{L}{L - \Delta L}\right)$$

$$\frac{\partial}{\partial V_D}\left(1 - \frac{\Delta L}{L}\right)^{-1} = (-1)\left(1 - \frac{\Delta L}{L}\right)^{-2}\left(-\frac{1}{L}\right)\frac{\partial \Delta L}{\partial V_D}$$

$$= \frac{1}{L}\left(1 - \frac{\Delta L}{L}\right)^{-2}\frac{\partial}{\partial V_D}\left[\frac{2\epsilon_s(V_D - V_D(\text{sat.}))}{qN_a}\right]^{1/2}$$

$$g'_D = \frac{I_D(\text{sat.})\;L\;(2\epsilon_s/qN_a)^{1/2}}{2[L - (2\epsilon_s/qN_a)^{1/2}(V_D - V_D(\text{sat.}))^{1/2}]^2\;(V_D - V_D(\text{sat.}))^{1/2}}$$

Prob. 6.19

For the given Si n-channel MOSFET, calculate V_T*. Repeat for a substrate bias of* $-2.5V$*.*

$$V_T = V_{FB} + 2\phi_F - \frac{Q_d}{C_i}$$

$$V_{FB} = 2\phi_F - \frac{Q_i}{C_i}$$

$$C_i = \frac{\epsilon_i}{d} = \frac{8.85\times10^{-14}\times3.9}{100\times10^{-8}} = \mathbf{3.452\times10^{-7}\ F/cm^2}$$

According to Fig. (6 - 17), for $N_a = 10^{18}\text{cm}^{-3}$

$\Rightarrow \ \Phi_{ms} = -1.5\text{V}$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -1.5 - \frac{5\times10^{10}\times1.6\times10^{-19}}{3.452\times10^{-7}} = -1.523\text{V}$$

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\cdot\ln\left(\frac{10^{18}}{1.5\times10^{10}}\right) = 0.467\text{V}$$

$$W = \sqrt{\frac{2\varepsilon_s(2\phi_F)}{qN_a}} = \sqrt{\frac{2(11.8)(8.85\times10^{-14})(2\times0.347)}{1.6\times10^{-19}\times10^{18}}} = 3.49\times10^{-6}\text{cm}$$

Note : Here we used dielectric constant of Si.

$Q_d = -qN_aW_m$

$$V_T = V_{FB} + 2\phi_F - \frac{Q_d}{C_i} = -1.523 + 2(0.467) + \frac{1.6\times10^{-19}\times10^{18}\times3.49\times10^{-6}}{3.452\times10^{-7}} = \mathbf{1.03V}$$

With V_B = -2.5V, depletion charge increases. Instead of bandbending of $2\phi_F$, now have bandbending of $(2\phi_F + V_B)$. The new width will be :

$$W_m = \sqrt{\frac{2\varepsilon_s(2\phi_F + V_B)}{qN_a}} = \sqrt{\frac{2(11.8)(8.85\times10^{-14})(2\times0.467+2.5)}{1.6\times10^{-19}\times10^{16}}}$$

$$= 6.695\times10^{-6}\text{cm} \ \Rightarrow$$

$$Q_d = -qN_aW_m = -1.6\times10^{-19}\times10^{18}\times6.695\times10^{-6} = -1.071\times10^{-6}\text{C}$$

$$-\frac{Q_d}{C_i} = \frac{1.071\times10^{-6}}{3.452\times10^{-7}} = 3.103\text{V}$$

$$V_T = -1.523 + 0.934 + 3.103 = \mathbf{2.514V}$$

Prob. 6.20

For the MOSFET in Prob. (6.19), calculate the drain current at $V_G = 5V$, $V_D = 0.1V$. Repeat for $V_G = 3V$, $V_D = 5V$.

For $V_G = 5\text{V},\ V_D = 0.1\text{V}$, since $V_T = 1.03\text{V},\ V_D < (V_G - V_T)$ and we are in the linear region.

$$I_D = \frac{Z}{L}\bar{\mu}_n C_i\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

$$= \frac{50}{2}(200)(3.452\times10^{-7})\left[(5-1.03)\times0.1 - \frac{1}{2}(0.1)^2\right] = \mathbf{6.77\times10^{-4}A}$$

For $V_G = 3\text{V},\ V_D = 5\text{V},\ V_D(\text{sat}) = V_G - V_T = 3 - 1.03 = 1.97\text{V}$

$$I_D = \frac{Z}{L}\bar{\mu}_n \mathrm{C}_i\left[(V_G - V_T)V_D(\text{sat}) - \frac{1}{2}V_D^2(\text{sat})\right]$$

$$= \frac{50}{2}(200)(3.452\times10^{-7})\left[(1.97)^2 - \frac{1}{2}(1.97)^2\right] = \mathbf{3.35\times10^{-3}\,A}$$

Prob. 6.21

Find the parameters of the given implant machine to use to make a $V_T = 2V$ n-channel MOSFET with a 400Å gate.

$$\Delta V_T = 2V = \frac{\Delta \mathrm{Q}_i}{\mathrm{C}_i} \Rightarrow$$

$$\frac{\Delta \mathrm{Q}_i}{q\mathrm{C}_i} = \frac{\epsilon_i}{d}\frac{2}{q} = \frac{(3.9)(8.85\times10^{-14})}{4\times10^{-6}}\frac{2}{1.6\times10^{-19}} = 1.08\times10^{12}\,\text{ions/cm}^2$$

Any n - type ion is okay, but based on projected range, use p - type.

$R_p = 400\text{Å} \Rightarrow E = \mathbf{33keV}$ (from Appendix IX).

Half of the dose is wasted in oxide.

$$Dose = \frac{\Delta \mathrm{Q}_i}{q\mathrm{C}_i}\times2 = 1.08\times10^{12}\times2 = \mathbf{2.16\times10^{12}\,ions/cm^2}$$

$$Dose = \frac{I\,t}{qA} \Rightarrow$$

$$I = \frac{(Dose)(qA)}{t} = \frac{2.16\times10^{12}\times1.6\times10^{-19}\times200}{20} = \mathbf{3.456\times10^{-6}\,A}$$

Prob. 6.22

For the given MOSFET, calculate the linear V_T and k_N, saturation V_T and k_N.

1a. Choose $V_D \ll V_D(sat)$ to ensure that I_D-V_D curve is in the linear regime. e.g., choose $V_D = 0.2$V

(1) $V_G = 4$ V $\quad V_D = 0.2$ V $\quad I_D = 0.35$ mA

(2) $V_G = 5$ V $\quad V_D = 0.2$ V $\quad I_D = 0.62$ mA

In linear regime

(3) $I_D = k_N[(V_G - V_T)V_D - V_D^2/2]$

From equation (3), inserting the values from (1) and (2)

$0.35 \times 10^{-3} = k_N [(4 - V_T)(0.2)]$
$0.62 \times 10^{-3} = k_N [(5 - V_T)(0.2)]$
$0.35/0.62 = (4 - V_T) / (5 - V_T)$
$1.75 - 0.35V_T = 2.48 - 0.62V_T$
$V_T =$ **2.71V**, therefore, $k_N =$ **1.36×10^{-3} A/V^2**

1b. Choose $V_D >> V_D(sat)$ to ensure that I_D-V_D curve is in the saturation regime, e.g. choose $V_D = 3$V

(4)	$V_G = 4$ V	$V_D = 3$ V	$I_D = 0.74$ mA
(5)	$V_G = 5$ V	$V_D = 3$ V	$I_D = 1.59$ mA

In saturation regime
(6) $I_D = (1/2)\, k_N (V_G - V_T)^2$

2. Insert values from (4) and (5) into (6)

$$0.74\times10^{-3} = \frac{k_N}{2}(4 - V_T)^2$$

$$1.59\times10^{-3} = \frac{k_N}{2}(5 - V_T)^2$$

$$\Rightarrow \frac{0.74}{1.59} = \frac{(4 - V_T)^2}{(5 - V_T)^2} \Rightarrow$$

$$V_T = \mathbf{1.85V}, \quad k_N = \mathbf{3.20\times10^{-4}\,A/V^2}$$

Prob. 6.23
For Prob. (6.22), calculate the gate oxide thickness and the substrate doping either graphically or iteratively.

(a) $k_N = \frac{Z}{L}\bar{\mu}_n C_i$

Use k_N from Prob. 6.22 and $\bar{\mu}_n = 500\text{cm}^2/(\text{V}\cdot\text{s})$

1.36×10^{-3} A/V^2 = (100/2) 500 C_i
$C_i = 5.42 \times 10^{-8}$ F/cm^2 = $\varepsilon_i / d = 3.9 \times 8.85 \times 10^{-14} / d$
$d = 6.36 \times 10^{-6}$ cm = **636 Å**

(b) $V_T = V_{FB} + 2\,\phi_F - Q_d / C_i$

$$2.71\text{V} = 2\phi_F - \frac{Q_d}{C_i} = 2\phi_F - \frac{\sqrt{q\epsilon_s N_a \phi_F}}{C_i}$$

As a first guess, we can start from $\phi_F = 0.3$ V

(Note: since $V_T = 2.71$ V, it cannot be PMOS)

Step 1:

$$2.71 = 0.6 + \frac{2\sqrt{1.6\times10^{-19}\times11.8\times8.85\times10^{-14}\times N_a\times0.3}}{5.42\times10^{-8}}$$

$$N_a = 6.523\times10^{16}\text{cm}^{-3} \Rightarrow$$

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\ln\frac{6.37\times10^{16}}{1.5\times10^{10}} = 0.395\text{ V}$$

Step 2:

$$2.71 = 0.792 + \frac{2\sqrt{1.6\times10^{-19}\times11.8\times8.85\times10^{-14}\times N_a\times0.395}}{5.42\times10^{-8}}$$

$$N_a = 4.08\times10^{16}\text{cm}^{-3} \Rightarrow$$

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\cdot\ln\frac{4.08\times10^{16}}{1.5\times10^{10}} = 0.384\text{V}$$

Step 3:

$$2.71 = 0.767 + \frac{2\sqrt{q\epsilon_s N_a \phi_F}}{C_{OX}}$$

$N_a = 4.22\times10^{16}\text{cm}^{-3} \Rightarrow \phi_F = \mathbf{0.385\ V}$

Now, we have a self - consistent set of values.

n - channel MOSFET, $N_a = \mathbf{4.22\times10^{16}\,cm^{-3}}$

Prob. 6.24

For the given Si MOSFET, calculate the inversion charge per unit area. Also sketch the dispersion relation for the first three subbands.

For 2-D situation, the density of states is given by

$$N(E) = \frac{m^*}{\pi\hbar^2} \text{ for x and y plane.}$$

In *k* space :

$$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2), \text{ where } k_z = \frac{n\pi}{L_z}$$

$$= \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2) + \underbrace{\frac{\hbar^2\pi^2 n^2}{2m^* L_z^2}}_{E_n}$$

$$E_n = \frac{\hbar^2\pi^2 n^2}{2m^* L_z^2} = \frac{\left(\dfrac{6.63\times10^{-34}}{2\pi}\right)^2 \pi^2 n^2}{2[0.2\times(9.11\times10^{-31})](10^{-8})^2}$$

$$= 3.016\times10^{-21} n^2\, J = 0.01885 n^2 eV = 18.85 n^2 meV$$

$$\Rightarrow E_1 = 18.85meV,\ E_2 = 75.4meV,\ E_3 = 170meV$$

Let units be $\dfrac{m^*}{\pi\hbar^2}$

The number of electrons per unit area is given by :

$$n = \int_{E_1}^{E_F} N(E)f(E)dE = \frac{m^*}{\pi\hbar^2}(E_2 - E_1) + \frac{2m^*}{\pi\hbar^2}\left(\frac{E_3 - E_2}{2}\right)$$

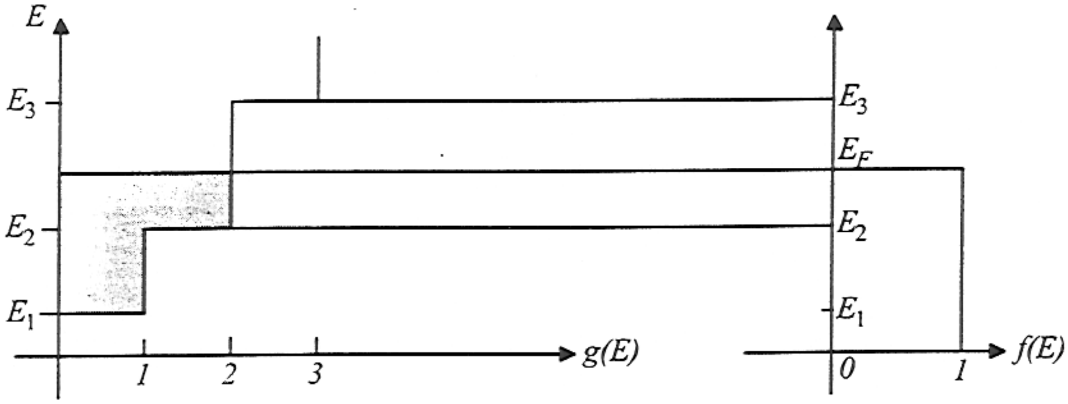
(since the Fermi probability is 1 below E_F and E_F is in the middle between E_2 and E_3.)

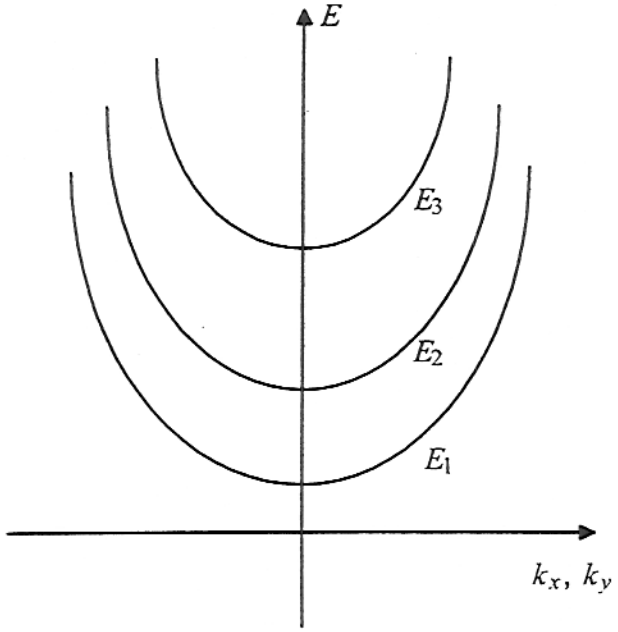
Charge per unit area by simplifying the above expression :

$$qn = \frac{qm^*}{\pi\hbar^2}(E_3 - E_1) = 4\pi\frac{qm^*}{h\cdot h}[(9-1)\times18.85\times10^{-3}]$$

$$= 4\pi\frac{1.6\times10^{-19}(0.2\times9.11\times10^{-31})}{(4.14\times10^{-15})(6.63\times10^{-34})}[(9-1)\times18.85\times10^{-3}]$$

$$= \mathbf{20.12C/m^2}$$


E
E3
E2
E1
1
2
3
g(E)
E3
EF
E2
E1
0
1
f(E)

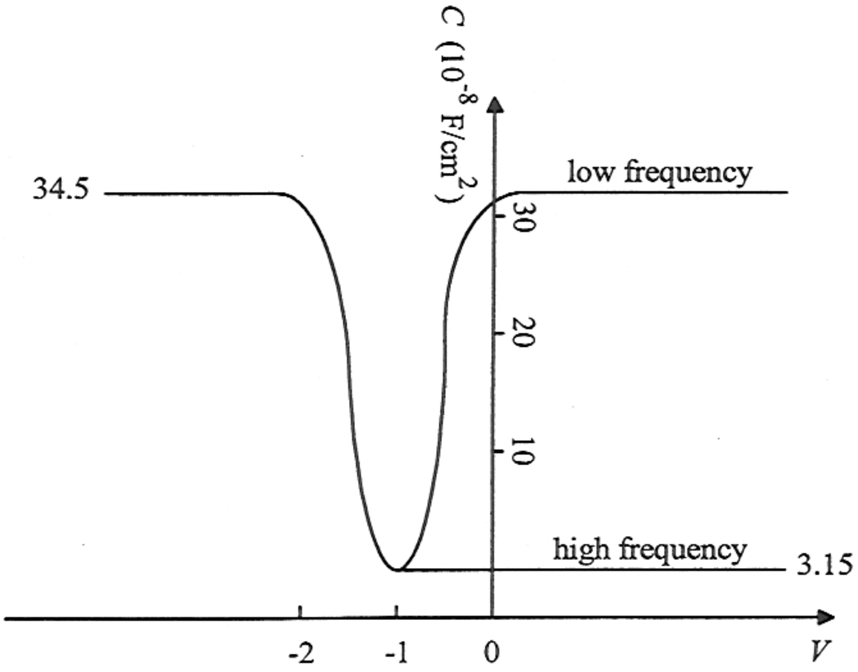

E
E3
E2
E1
kx, ky

Prob. 6.25

The ideal capacitor of Example 6-2 has $V_{FB} = -2\ V$. *Redraw Fig. 6-16 and find* Q_i.

$$\begin{aligned} V_{FB} &= \Phi_{ms} - \frac{Q_i}{C_i} \\ -2 &= -0.95 - \frac{Q_i}{3.45 \times 10^{-7}} \\ Q_i &= 1.05(3.45 \times 10^{-7}) = \mathbf{3.6 \times 10^{-7}\ C/cm^2} \end{aligned}$$

The figure is shown with V_{FB} = -2V.



Prob. 6-26

Plot the drain characteristics for an n^+-polysilicon-SiO2-Si p-channel transistor with $N_d = 10^{16}$, $Q_i = 5 \times 10^{10}\ q$, $d = 100Å$, $\mu_p = 200$, and $Z = 10L$.

$$-I_D = \frac{\mu_p Z C_i}{L}\left[(V_G - V_T) - \frac{1}{2}V_D\right] V_D$$

where $V_T = -1.1\ V$ and $\mu_p Z C_i / L = (200)(10)(3.45\times10^{-7}) = 6.9\times10^{-4}$

For $V_G = -3$, $V_D(\text{sat.}) = -1.9$

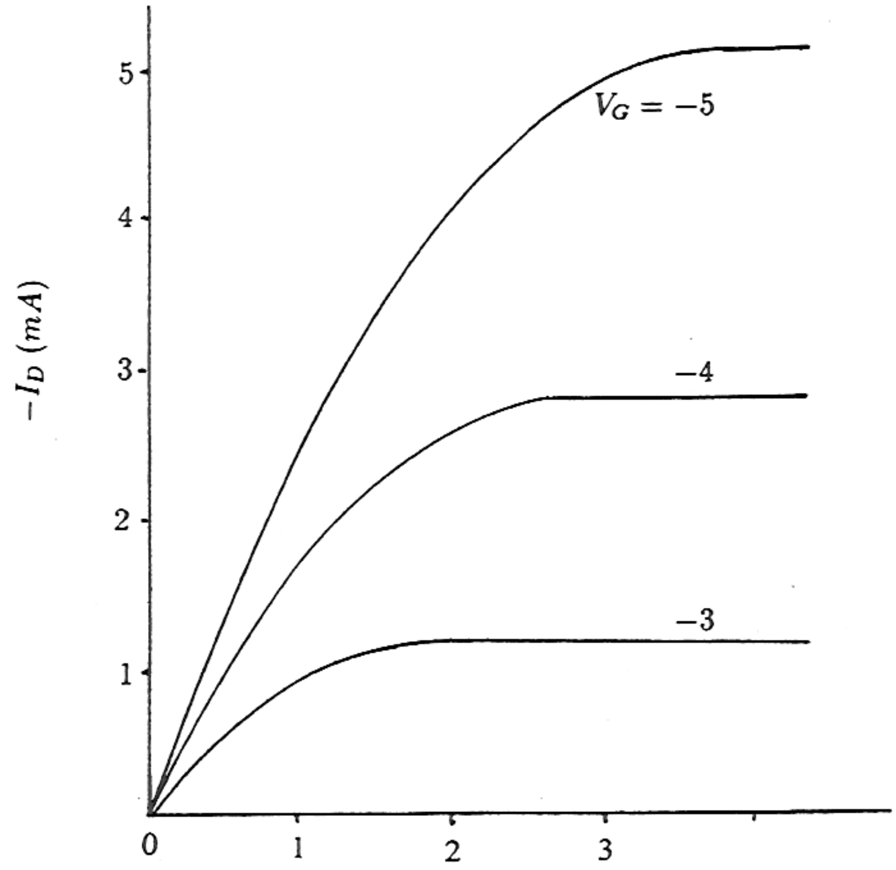
$-V_D$:	0.3	0.5	0.8	1.0	1.5
$-I_D\ (mA)$:	0.36	0.57	0.83	0.96	1.2

For $V_G = -4$, $V_D(\text{sat.}) = -2.9$

$-V_D$:	0.5	1	1.5	2	2.5
$-I_D\ (mA)$:	0.9	1.7	2.2	2.6	2.8

For $V_G = -5$, $V_D(\text{sat.}) = -3.9$

$-V_D$:	1	1.5	2	2.5	3	3.9
$-I_D\ (mA)$:	2.3	3.3	4	4.6	5	5.2



Prob. 6.27

For the transistor of Prob. 6-26 with $L = 1\ \mu m$, calculate the cutoff frequency above pinch-off.

$$f_c \simeq \frac{g_m}{2\pi C_i L Z}$$

For p-channel we must include a minus sign in Eq.(6-54) for positive g_m.

$$f_c = \frac{1}{2\pi C_i L Z}\,\frac{Z}{L}\,\mu_p\, C_i\,(V_T - V_G) = \frac{\mu_p}{2\pi L^2}\,(V_T - V_G)$$

For $V_G = -5, \quad f_c = \dfrac{200}{2\pi(10^{-4})^2}\,(3.9) =$ **12.4 GHz**

For $V_G = -3, \quad f_c =$ **6 GHz**

Prob. 6.28

An n-channel MOSFET has a $1\ \mu m$ - long channel with $N_a = 10^{16}$, and $N_d = 10^{20}$ in the source and drain. What V_D causes punch-through ?

$$V_0 = 0.0259 \ln \frac{10^{36}}{2.25 \times 10^{20}} = 0.933V$$

We have two depletion regions to calculate, at the source and drain ends of the channel. Note $N_d \gg N_a$, so most of W is in the p-side (channel).

At the (zero – bias) source end, $x_{pS} = \left[\dfrac{2\epsilon V_0}{qN_a}\right]^{\frac{1}{2}}$

$$x_{pS} = \left[\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 0.993}{1.6 \times 10^{-19} \times 10^{16}}\right]^{\frac{1}{2}} = 0.35\ \mu m$$

At the drain end, $x_{pD} = \left[\dfrac{2\epsilon(V_0 + V_D)}{qN_a}\right]^{\frac{1}{2}}$

Punch-through occurs when $x_{pD} = L - x_{pS} = 0.65\ \mu m$

$$0.933 + V_D = \frac{(0.65 \times 10^{-4})^2(1.6 \times 10^{-3})}{2 \times 11.8 \times 8.85 \times 10^{-14}}, \quad \mathbf{V_D = 2.3V}$$

Prob. 6.29

For the n-channel MOSFET of Example 6-3 what substrate bias will shift V_T to +0.5 V ?

$$\text{Eq. (6-62)} \quad \Delta V_T = \frac{\sqrt{2\epsilon_s q N_a}}{C_i}\left[(2\phi_F - V_B)^{\frac{1}{2}} - (2\phi_F)^{\frac{1}{2}}\right]$$

In example 6.3 $V_T = -0.14$, $N_a = 10^{16}$, $\phi_F = 0.347$, $C_i = 3.45 \times 10^{-7}$

$$\begin{aligned} \Delta V_T = 0.64 &= \frac{[2 \times 11.8 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{16}]^{\frac{1}{2}}}{3.45 \times 10^{-7}}[(0.694 - V_B)^{\frac{1}{2}} - (0.694)^{\frac{1}{2}}] \\ 0.64 &= 0.1676[(0.694 - V_B)^{\frac{1}{2}} - 0.833] \\ V_B &= \mathbf{-20.95V} \end{aligned}$$

Using Eq. (6-63) $V_B = -\left(\frac{0.64}{0.1676}\right)^2 = \mathbf{-14.6V}$

Since C_i is large for a thin gate oxide such as this, a large substrate bias is required to make a modest change in threshold voltage. This method of affecting V_T is not practical for thin-oxide transistors.

Chapter 7

Prob. 7.1

For the bipolar junction transistor whose fabrication is described in the question, find where the peaks and widths of the boron and phosphorus profiles prior to annealing are emitter, and where the peaks and widths are after annealing, and what the collector junction depths are after annealing.

(*a*)

From Appendix IX :

$R_p = \mathbf{0.16\mu m}$ for 50 keV boron.

$\Delta R_p = \mathbf{0.05\mu m}$.

width $= 2\Delta R_p = \mathbf{0.1\mu m}$

$R_p = \mathbf{0.04\mu m}$ for 30 keV P.

$\Delta R_p = \mathbf{0.018\mu m}$

width $= 2\Delta R_p = \mathbf{0.036\mu m}$

(*b*)

Annealing causes diffusion. From Eq.(5 - 1b),

Variance $\sigma^2 = (\Delta R_p^2 + 2Dt)$

For boron, from Appendix IX we have

$$\sigma^2 = [(0.05)^2 + 2(10^{-12} \times 10 \times 10^8)]\mu\text{m}^2$$

$$= 0.0025 + 0.002 = 0.0045\mu\text{m}^2$$

$\Rightarrow \sigma = 0.067\mu\text{m}$.

The width of Gaussian distribution is

$2\sigma = \mathbf{0.134\mu m}$

Peak locations are the same before and after annealing.

(*c*)

Since the Base - Collector junction is far from the emitter peak, we can ignore the phosphorus concentration at the collector junction.

$$N(\text{Boron}) = \frac{\phi}{\sqrt{2\pi}\sigma} e^{-y^2} \quad (\text{ where y}^2 = \frac{(x-R_p)^2}{2(\Delta R_p^2 + 2Dt)})$$

$$= \frac{5\times10^{14}}{\sqrt{2\pi}\,[0.067\times10^{-4}]} e^{-y^2} = 2\times10^{15}\text{cm}^{-3}$$

$$\text{Solve for } y^2 \;\Rightarrow\; y^2 = -\ln\left(\frac{2\times10^{15}\times\sqrt{2\pi}\,[(0.067)\times10^{-14}]}{5\times10^{14}}\right)$$

$$\Rightarrow\; y = 3.1$$

$$\text{Since y}^2 = \frac{(x-R_p)^2}{2(\Delta R_p^2 + 2Dt)}, \text{ solve for } x:$$

$$x = y\sqrt{2(\Delta R_p^2 + 2Dt)} + R_p = 3.1\sqrt{2\times0.0045} + 0.16 = 0.45\mu\text{m}$$

$\Rightarrow$ Collector junction = **0.45μm.**

$$\text{Peak concentration} = \frac{\phi}{\sqrt{2\pi}\sigma} \Rightarrow$$

$$\text{Boron peak} = \frac{5\times10^{14}}{\sqrt{2\pi}\,[(0.067)\times10^{-4}]} = 2.988\times10^{19}\text{cm}^{-3}$$

$$\text{Phosphorus peak} = \frac{5\times10^{14}}{\sqrt{2\pi}\,[(0.0364)\times10^{-4}]} = 1.096\times10^{20}\text{cm}^{-3}$$

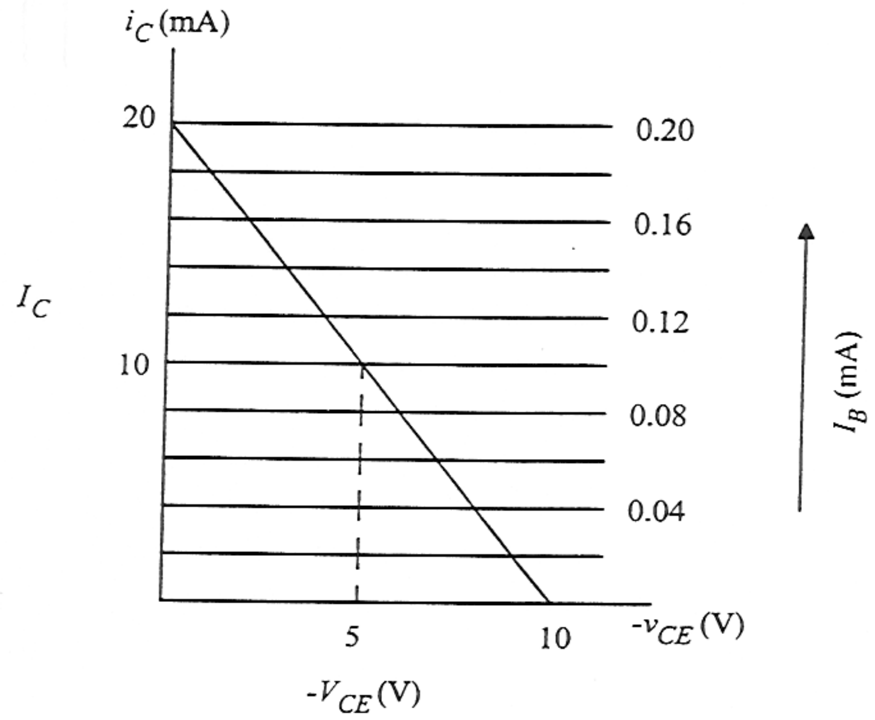
At $x = 0$,

$$\text{Phosphorus concentration} = 1.096\times10^{20} e^{-\frac{1}{2}\left(\frac{0.04}{0.0364}\right)^2} = 5.99\times10^{19}\text{cm}^{-3}$$

Graphically, emitter junction = **0.108μm.**

Prob. 7.2

Sketch i_C vs. $(-v_{CE})$ for the BJT of Fig. 7-4, and find $-V_{CE}$ for $I_B = 0.1$ mA.



Prob. 7.3

Plot δp across the base of a p-n-p with $W_b/L_p = 0.5$.

Refer to Fig. 7-7.

$$\frac{\delta p}{\Delta p_E} = M_1 e^{-x_n/L_p} - M_2 e^{x_n/L_p}$$

$e^{1/2} = 1.65, \ e^{-1/2} = 0.606. \ M_1 = 1.58, \ M_2 = 0.58.$

$$\text{At } x_n/L_p = 0, \ \frac{\delta p}{\Delta p_E} = 1.58 - 0.58 = 1.0.$$

$$\text{At } x_n/L_p = 0.5, \ \frac{\delta p}{\Delta p_E} = 1.58(0.606) - 0.58(1.65) = 0.$$

Values can be filled in to obtain a plot such as Fig. 7-7, with normalized axes.

Prob. 7.4

Derive I_B from Q_p/τ_p.

$$Q_p = qA\int_0^{W_b} \delta p(x_n)dx_n$$

$$= qAL_p\left[C_1(e^{W_b/L_p}-1) - C_2(e^{-W_b/L_p}-1)\right]$$

$$I_B = \frac{Q_p}{\tau_p}$$

$$= \frac{qAL_p}{\tau_p}\times\left[\frac{(\Delta p_C - \Delta p_E e^{-W_b/L_p})(e^{W_b/L_p}-1)}{e^{W_b/L_p}-e^{-W_b/L_p}} - \frac{(\Delta p_E e^{W_b/L_p} - \Delta p_C)(e^{-W_b/L_p}-1)}{e^{W_b/L_p}-e^{-W_b/L_p}}\right]$$

$$= \frac{qAD_p}{L_p}\left[\frac{(\Delta p_C + \Delta p_E)[(e^{W_b/L_p}+e^{-W_b/L_p})-2]}{e^{W_b/L_p}-e^{-W_b/L_p}}\right]$$

$$= \frac{qAD_p}{L_p}\left[(\Delta p_E + \Delta p_C)(\mathrm{ctnh}\frac{W_b}{L_p} - \mathrm{csch}\frac{W_b}{L_p})\right]$$

Prob. 7.5

Extend Eq. (7-20a) to include the effects of nonunity emitter injection efficiency (γ<1). Derive Eq. (7-25) for γ.

Eq.(7 – 20a) is actually I_{Ep}.

$$I_{En} = \frac{qAD_n}{L_n}n_p e^{qV_{EB}/kT} \quad \text{for } V_{EB} >> kT/q$$

Thus, the total emitter current is:

$$I_E = I_{Ep} + I_{En} = qA\left[\frac{D_p}{L_p}p_n \mathrm{ctnh}\frac{W_b}{L_p} + \frac{D_n}{L_n}n_p\right]e^{qV_{EB}/kT}$$

$$\gamma = \frac{I_{Ep}}{I_E} = \left[1 + \frac{I_{En}}{I_{Ep}}\right]^{-1} = \left[1 + \frac{\frac{D_n^p}{L_n^p}}{\frac{D_p^n}{L_p^n}}\frac{n_p}{p_n}\tanh\frac{W_b}{L_p^n}\right]^{-1}$$

Using $\frac{n_p}{p_n} = \frac{n_n}{p_p}, \frac{D_n^p}{D_p^n} = \frac{\mu_n^p}{\mu_p^n}$, and $\frac{D}{\mu} = \frac{kT}{q} \Rightarrow$

$$\gamma = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tan\frac{W_b}{L_p^n}\right]^{-1}$$

Apply the charge control approach to the distribution in Fig. 7-8b to find the base transport factor B.

$$i_{Ep} = Q_p(\tau_t^{-1} + \tau_p^{-1}),\ i_C = Q_p\tau_t^{-1},\ i_B = Q_p\tau_p^{-1}$$

$$B = \frac{i_C}{i_{Ep}} = \frac{\tau_t^{-1}}{\tau_t^{-1} + \tau_p^{-1}} = \left[1 + \frac{\tau_t}{\tau_p}\right]^{-1} = \left[1 + \frac{W_b^2}{2D_p\tau_p}\right]^{-1}$$

Using $(1+x)^{-1} \approx 1 - x$ for small x, we have:

$$B \approx \left[1 + \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2\right]^{-1} \approx 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2$$

Eq.(7 – 26) is

$$B = sech\frac{W_b}{L_p} \approx 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2 \quad \text{from Table 7.1.}$$

Prob. 7.6

A symmetrical Si p^+-n-p^+ BJT with area 10^{-4} cm^{-2} and base width of one micron has an emitter with $N_a = 10^{17}$ cm^{-3}, $\tau_n = 0.1$ μs, and $\mu_n = 700$. In the base $N_d = 10^{15}$ cm^{-3}, $\tau_p = 10$μs, and $\mu_p = 450$. Find I_{ES}, and I_B with $V_{EB} = 0.3$ V and large reverse bias on the collector junction.

In the base:

$$p_n = n_i^2/n_n = (1.5\times10^{10})^2/10^{15} = 2.25\times10^5$$

$$D_p = 450(0.0259) = 11.66,\ \ L_p = (11.66\times10^{-5})^{1/2} = 1.08\times10^{-2}$$

$$W_b/L_p = 10^{-4}/1.08\times10^{-2} = 9.26\times10^{-3}$$

$$I_{ES} = I_{CS} = qA(D_p/L_p)\,p_n\ \text{ctnh}(W_b/L_p)$$

$$= (1.6\times10^{-19})(10^{-4})(11.66/1.08\times10^{-2})(2.25\times10^5)\ \text{ctnh}\ 9.26\times10^{-3}$$

$$= 4.2\times10^{-13}\,\text{A}$$

$$\Delta p_E = p_n e^{qV_{EB}/kT},\ \ \Delta p_C \approx 0$$

$$\Delta p_E = 2.25\times10^5\times e^{(0.3/0.0259)} = 2.4\times10^{10}$$

$$I_B = qA(D_p/L_p)\Delta p_E\ \tanh(W_b/2L_p)$$

or

$$I_B = \frac{Q_b}{\tau_p} = qAW_b\Delta p_E/2\tau_p = \mathbf{1.9\times10^{-12}\,A}$$

Find γ and β for a long emitter of the BJT.

In the emitter,

$$D_n = 700\ (0.0259) = 18.13$$

$$L_n = (18.13\times10^{-7})^{1/2} = 1.35\times10^{-3}$$

$$I_{En} = \frac{qAD_n^E}{L_n^E}\, n_p^E\, e^{qV_{EB}/kT}$$

$$I_{Ep} = \frac{qAD_p^B}{L_p^B}\, p_n^B\ \text{ctnh}\frac{W_b}{L_p^B}\, e^{qV_{EB}/kT}$$

$$\gamma = \frac{I_{Ep}}{I_{En}+I_{Ep}} = \left[1+\frac{I_{En}}{I_{Ep}}\right]^{-1}$$

$$= \left[1+\frac{D_n^E/L_n^E}{D_p^B/L_p^B}\frac{n_p^E}{p_n^B}\tanh\frac{W_b}{L_p}\right]^{-1} \quad (\text{use } \frac{n_p^E}{p_n^B} = \frac{n_n^B}{p_p^E})$$

$$\gamma = \left[1+\frac{18.13\times1.08\times10^{-2}\times10^{15}}{11.66\times1.35\times10^{-3}\times10^{17}}\tanh 9.26\times10^{-3}\right]^{-1} = \mathbf{0.99885}$$

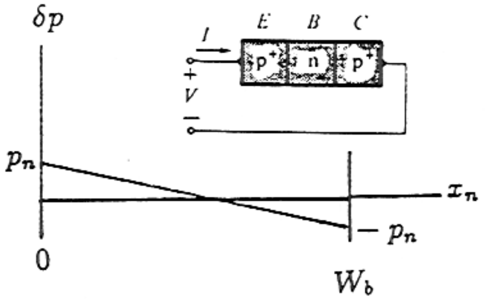
$$B = \text{sech}\frac{W_b}{L_p} = sech\ 9.26\times10^{-3} = \mathbf{0.99996}$$

$$\alpha = B\gamma = (0.99885)(0.99996) = \mathbf{0.9988}$$

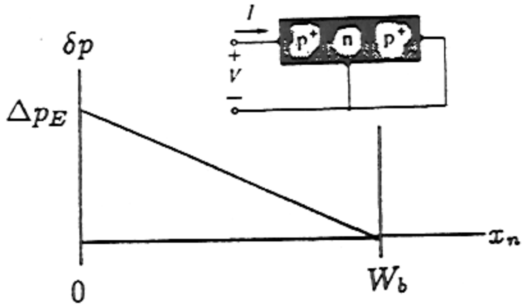
$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9988}{0.0012} = \mathbf{832}$$

Prob. 7.7

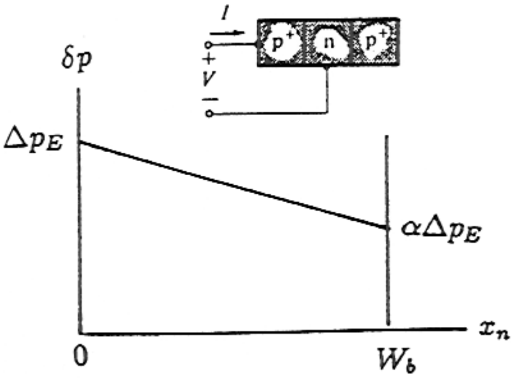
For the diode connections shown, sketch δp in the base. Which is the best diode ?



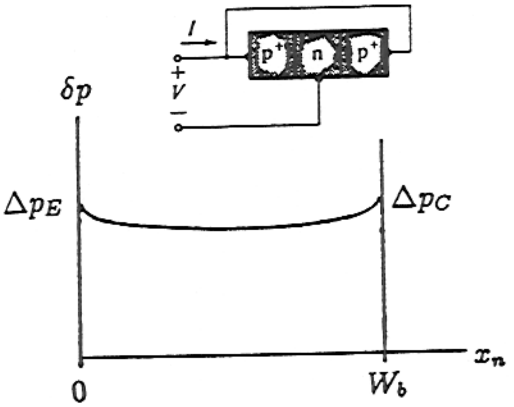
(a) $I_E = I_C$, $I_B = 0$. Since V is large, the collector is strongly reverse biased, $\Delta p_C = -p_n$. Since $I_E = I_C$, $\Delta p_E = -\Delta p_C$ $= p_n$ from Eq.(7-34). The area under $\delta p(x_n)$ is zero.



(b) $V_{CB} = 0$, thus $\Delta p_C = 0$. Notice that this is the narrow-base diode distribution.



(c) *Since $I_C = 0$, $\Delta p_C = \alpha\Delta p_E$ from Eq.(7-34b).*



(d) *$V_{EB} = V_{CB} = V$. Thus $\Delta p_C = \Delta p_E$*

Connection (b) gives the best diode since the stored charge is least; (a) is not a good diode since the current is small and symmetrical about $V = 0$.

Prob. 7.8

For the transistor connection in Fig. P7-7 a, (a) show that $V_{EB} = (kT/q)\ln 2$; (b) find the expression for I when $V \gg kT/q$ and sketch I vs. V.

While $V \gg kT/q$, $\Delta p_E = -\Delta p_C = p_n$

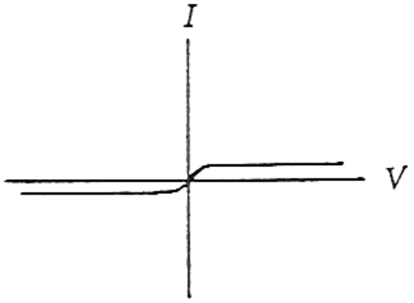
(a) $\Delta p_E = p_n(e^{qV_{EB}/kT} - 1) = p_n$

$$p_n\, e^{qV_{EB}/kT} = 2p_n\,, \quad V_{EB} = \frac{kT}{q}\ln 2$$

(b) $I = I_E = I_C = I_{ES}(1+\alpha)$ since $I_{ES} = I_{CS}$

or $I = \dfrac{qAD_p}{L_p}\, p_n \left(\text{ctnh}\,\dfrac{W_b}{L_p} + \text{csch}\,\dfrac{W_b}{L_p}\right)$

The current is small and independent of V except when V drops below kT/q.



Prob. 7.9

(a) Find I for the transistor connection of Fig. P7-7b; compare with the narrow base diode.

Since $\Delta p_C = 0$, $I = \dfrac{qAD_p}{L_p}\Delta p_E \operatorname{ctnh}\dfrac{W_b}{L_p}$

(b) How does I divide between the base lead and the collector lead?

$$I_C = \frac{qAD_p}{L_p}\Delta p_E \operatorname{csch}\frac{W_b}{L_p}$$

$$I_B = \frac{qAD_p}{L_p}\Delta p_E \tanh\frac{W_b}{2L_p}$$

where I_C and I_B are the components in the collectors and base lead, respectively. Note that these results are analogous to those of Probs. 5.35 and 5.36.

Prob. 7.10

Suppose that V is negative in Fig. P7-7c, (a) Find I from the Ebers-Moll equations; (b) find the expression for V_{CB}; (c) sketch $\delta p(x_n)$ in the base.

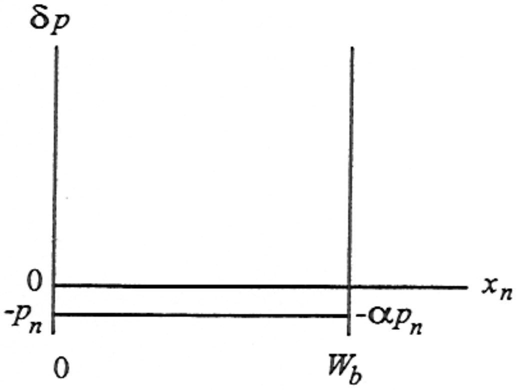
With $V_{EB} << -kT/q$ and $I_C = 0$:

(a) From Eq.(7-36), $I_E = (1-\alpha^2)\, I_{ES}\,(-1) = I_{ES}\,(\alpha^2 - 1)$.
Since $\alpha_N = \alpha_I = \alpha$ for a symmetrical device.

(b) From Eq. (7 - 32), $e^{qV_{CB}/kT}\text{-}1 = \alpha(-1)$

$$\text{Thus } V_{CB} = \frac{kT}{q}\ln(1\text{-}\alpha)$$

(c) From Eq. (7-34), $\Delta p_C = \alpha\Delta p_E$
Since $\Delta p_E = -p_n$, $\Delta p_C = \alpha(-p_n)$.



Prob. 7.11

For the transistor connection of Fig. P7-7d, (a) find $\delta p(x_n)$ in the base; (b) find the current I.

$V_{EB} = V_{CB} = V$, $\Delta p_E = \Delta p_C$

(a) In Eq. (7-13), $C_2 = e^{W_b/L_p} C_1$

$$\delta p(x_n) = \Delta p_E\left(e^{x_n/L_p} + e^{(W_b\text{-}x_n)/L_p}\right)\frac{1 - e^{\text{-}W_b/L_p}}{e^{W_b/L_p} - e^{\text{-}W_b/L_p}}$$

(b) $I = 2qA\dfrac{D_p}{L_p}p_n(\tanh\dfrac{W_b}{2L_p})(e^{qV/kT} - 1)$ from Eq. (7 - 19).

Prob. 7.12

(a) Show from Eq. (7-32) that I_{EO} and I_{CO} are the saturation currents of the emitter and collector junctions, respectively, with the opposite junction open circuited.

For $I_C = 0$, Eq. (7-32b) gives

$$I_{CS}(e^{qV_{CB}/kT}-1)=\alpha_N I_{ES}\,(e^{qV_{EB}/kT}-1)$$

Substituting this into Eq. (7 - 32a) we obtain

$$I_E = I_{ES}(1-\alpha_I\alpha_N)(e^{qV_{EB}/kT}-1)$$

This is a diode equation with saturation current $I_{ES}(1-\alpha_I\alpha_N) = I_{EO}$.
Similarly, for $I_E = 0$, Eq. (7-32a) gives

$$I_{ES}\,(e^{qV_{EB}/kT}-1)\;=\;\alpha_I I_{CS}(e^{qV_{CB}/kT}-1)$$

Substituting this into Eq. (7 - 32b) we obtain

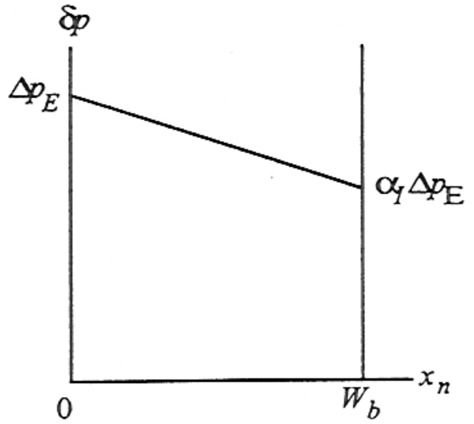
$$-I_C = I_{CS}(1-\alpha_I\alpha_N)(e^{qV_{CB}/kT}-1) = I_{CO}(e^{qV_{CB}/kT}-1)$$

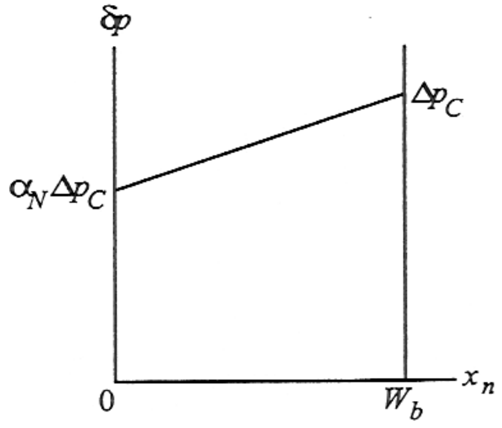
where the minus sign arises from the choice of I_C defined in the reverse direction through the collector junction.

(b) Find expressions for: Δp_C with the emitter junction forward biased and the collector open; Δp_E with the collector junction forward biased and the emitter open.

From Eq. (7-34): $\Delta p_C = \alpha_I \Delta p_E$ for $I_C = 0$
and $\Delta p_E = \alpha_N \Delta p_C$ for $I_E = 0$.

(c) Sketch $\delta p(x_n)$ in the base for the two cases of part (b).





Prob. 7.13

(a) Show that the definitions of Eq.(7-40) are correct; what does q_N represent?

$$\alpha_N = \frac{I_C}{I_E} = \frac{Q_N/\tau_{tN}}{Q_N(1/\tau_{tN} + 1/\tau_{pN})} = \frac{\tau_{pN}}{\tau_{tN} + \tau_{pN}}$$

$$I_{EN} = Q_N(1/\tau_{tN} + 1/\tau_{pN}) = I_{ES}\frac{\Delta p_E}{p_n}$$

thus $I_{ES} = q_N(1/\tau_{tN} + 1/\tau_{pN})$, where $q_N = Q_N \dfrac{p_n}{\Delta p_E}$

and similarly for the inverted mode.

$q_N \sim \frac{1}{2} qAW_b p_n$ is the magnitude of the charge stored in the base when the emitter junction is reverse biased and the collector junction is shorted.

(b) Show that Eqs.(7-39) correspond to Eqs.(7-34).

$$Q_N = I_{ES}\frac{\Delta p_E}{p_n}\frac{\tau_{tN}\tau_{pN}}{\tau_{tN} + \tau_{pN}}, \quad Q_I = I_{CS}\frac{\Delta p_C}{p_n}\frac{\tau_{tI}\tau_{pI}}{\tau_{tI} + \tau_{pI}}$$

Thus Eq.(7 - 39a) is

$$I_E = I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n}\frac{\tau_{pI}}{\tau_{tI} + \tau_{pI}} = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n}$$

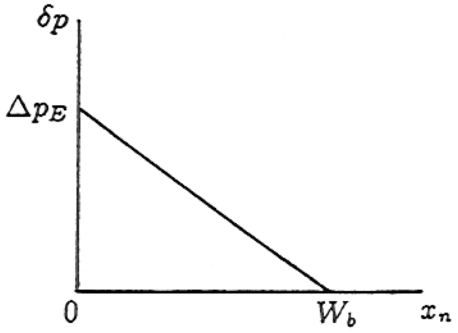
Similarly, Eq.(7 – 39b) is

$$I_C = I_{ES}\frac{\Delta p_E}{p_n}\frac{\tau_{pN}}{\tau_{tN} + \tau_{pN}} - I_{CS}\frac{\Delta p_C}{p_n} = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n}$$

Prob. 7.14

(a) *How can the transit time across the base τ_t be shorter than the hole lifetime in the base τ_p ?*

$\delta p(x_n)$ is a *steady state* distribution and is replaced on average every τ_p seconds. However, the distribution is made up of indistinguishable holes in transit across the base, each spending on average τ_t seconds in transit.

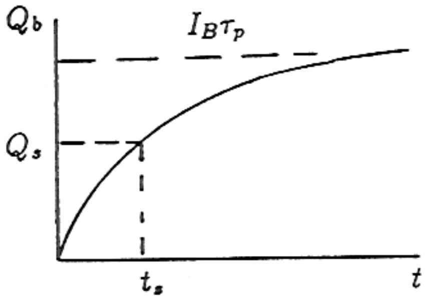


(b) *Explain why the turn-on transient of a BJT is faster when the device is driven into oversaturation.*

Saturation ($Q_b = Q_s$) is reached earlier in the exponential rise:

$$\begin{aligned} Q_b &= I_B\,\tau_p\,(1 - e^{-t/\tau_p}) \\ t_s &= \tau_p\,\ln\frac{1}{1 - I_C/\beta I_B} \end{aligned}$$

use $\beta I_B \gg I_C(sat.)$

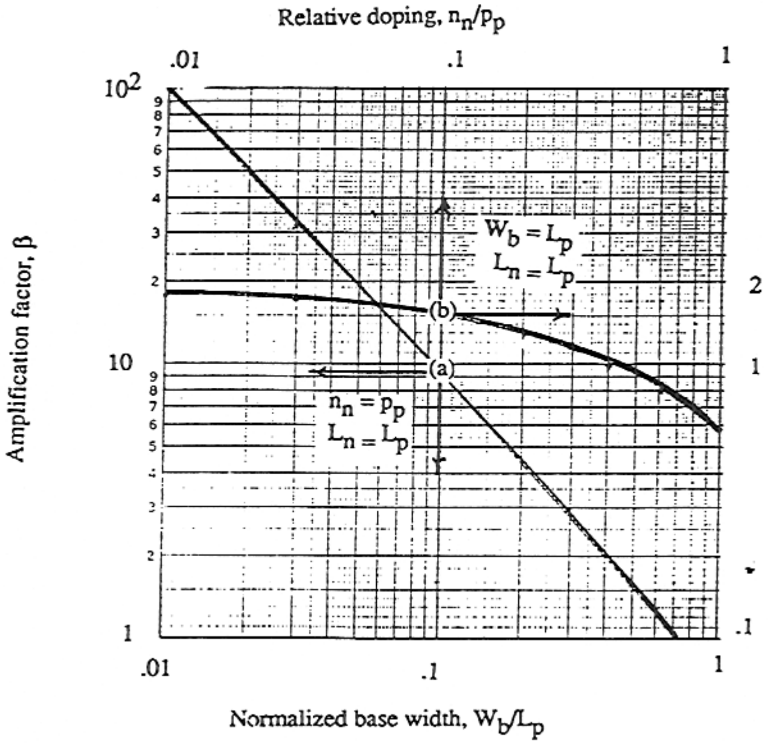


Prob. 7.15

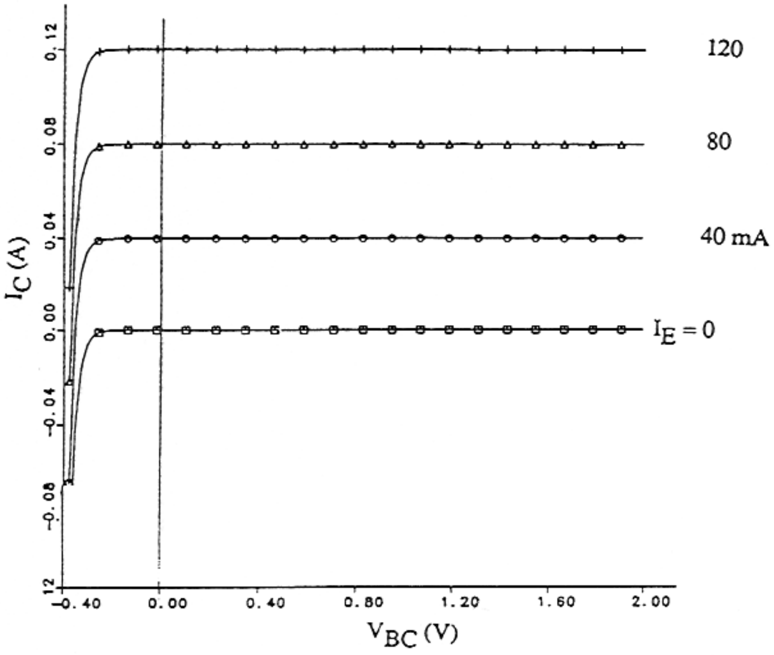
Design an n-p-n HBT with reasonable γ and base resistance.

Since this is an open-ended design problem, there is no unique solution. Students should use the results of Eq.(7-81) with the band gap difference of 0.42 eV between GaAs and $Al_{0.3}Ga_{0.7}As$, to conclude that the base doping can be considerably higher than the emitter doping while maintaining a good emitter injection efficiency for electrons. It is possible to estimate the base spreading resistance with the higher doping concentration. This will require using Fig.3-23 to estimate the GaAs electron mobility at that concentration. Note that App. III only gives the value for light doping. Clearly, much important information will be lost in these estimates, because of the sparse information the students have to work with. For example, real HBTs using AlGaAs/GaAs suffer from surface recombination problems, and scaling to small dimensions is inhibited. Some students will be interested enough to read current articles on HBTs and will therefore provide comments to this effect. In fact, a good answer to this problem might be "I wouldn't use AlGaAs/GaAs at all. What I would do instead is ..."

Prob. 7.16



Prob. 7.17



Prob.7.18

(a) What is Q_b in Fig. 7-4 at the d-c bias?

$Q_b = I_B \tau_p = (10^{-4}\ \text{A})\ (10^{-5}\ \text{s}) = 10^{-9}\ \text{C}$
also $I_C \tau_t = (10^{-2})\ (10^{-7}) = 10^{-9}\ \text{C}$

(b) Why is B different in the normal and inverted mode of a diffused BJT?

The base transport factor is affected by the built-in field resulting from the doping gradient in the base. This field assists transport in the normal mode, but opposes transport in the inverted mode.

Prob. 7.19

For the given p-n-p transistor, calculate the neutral base width W_b.

The emitter doping is so high that $E_{Fp} \sim E_{vp}$. Therefore,

The built - in potential at the base - emitter junction is approximately given by :

$$V_{0_{BE}} \approx \frac{E_g}{2} + \frac{kT}{q} \ln \frac{N_B}{n_i} = 0.55 + 0.0259 \ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.897\text{V}$$

The built - in potential at the collector - base junction is given by :

$$V_{0_{BC}} \approx \frac{kT}{q}\left[\ln\frac{N_B}{n_i} + \ln\frac{N_C}{n_i}\right]$$

$$= 0.0259\left[\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) + \ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)\right] = \mathbf{0.695V}$$

Next calculate the width of the base - emitter and base - collector space charge regions :

$$W_{EB} = \sqrt{\frac{2\epsilon_s}{qN_b}\left(V_{0_{BE}} - V_{EB}\right)}$$

Since $N_E >> N_B$ and the base - emitter junction is forward biased,

$$W_{EB} = \sqrt{\frac{2(11.8)(8.85\times10^{-14})}{1.6\times10^{-19}\times10^{16}}(0.897 - V_{EB})}$$

For $V_{EB} = 0.2$ V, $W_{EB} = 3.02\times10^{-5}$ cm

For $V_{EB} = 0.6$ V, $W_{EB} = 1.97\times10^{-5}$ cm

The width of the collector - base space charge region is given by :

$$W_{CB} = \sqrt{\frac{2\epsilon_s}{q}\frac{N_C + N_B}{N_C N_B}\Phi_T}$$

where $\Phi_T = V_0 + V_{CB}$ is the voltage drop at the base - collector junction.

Note : One - sided step junction cannot be assumed since for this problem $N_B = N_C$

Given : $V_{CB} = 0,\ \Phi_T = V_{0_{B-C}} = 0.695\text{V}$

Hence : $W_{CB} = 4.26\times10^{-5}\text{cm}$

Calculate width of neutral base region :

Given : W = metallurgical base width = 1.5μm

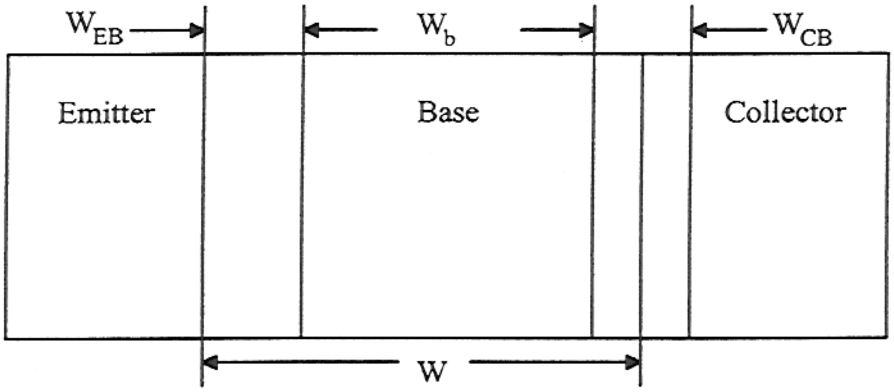
$$W_b = W - W_{EB}\frac{N_C}{N_C+N_B}W_{CB}$$

For $V_{EB} = 0.2\text{V}$

$$W_b = 1.5 - 0.302 - \frac{0.426}{2} = \mathbf{0.985}\boldsymbol{\mu}\mathbf{m}$$

For $V_{EB} = 0.6\text{V}$

$$W_b = 1.5 - 0.197 - \frac{0.426}{2} = \mathbf{1.09}\boldsymbol{\mu}\mathbf{m}.$$



Prob. 7.20

For the BJT in Prob. (7.19), calculate the base transport factor and the emitter injection efficiency for V_{EB} = 0.2 and 0.6 V.

At first, we determine the electrons and hole difusion lengths :

Given $\tau_n = \tau_p = \tau_o = 10^{-7}\text{s}$

and $D_n = D_p = 10\ \text{cm}^2/\text{s}$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{10\times10^{-7}} = 10^{-3}\ \text{cm}$$

$$L_p = L_n = L = 10\mu\text{m}$$

Calculate the base transport factor, B.

For $\frac{W_b}{L_p} << 1,\ B = \frac{1}{\cosh\left(\frac{W_b}{L_p}\right)} \approx 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2$

For $V_{EB} = 0.2$V,

$$B = 1 - \frac{1}{2}\left(\frac{0.985}{10}\right)^2 = \mathbf{0.995}$$

For $V_{EB} = 0.6V$:

$$B = 1 - \frac{1}{2}\left(\frac{1.09}{10}\right)^2 = \mathbf{0.994}$$

Calculate the emitter efficiency γ :

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

where

I_{Ep} is the current for holes injected from the emitter to the base;

I_{En} is the current for electrons injected from the base to the emitter.

Calculate I_{Ep} and I_{En} as a function of V_{EB}.

I_{Ep} = Diffusion current injected across B - E junction by the emitter (holes for the p - n - p transistor).

For the given p - n - p :

$$I_{Ep} = Aq\frac{D_p n_i^2}{N_B W_b}\exp(\frac{qV_{EB}}{kT}) \quad \text{(hole current)}$$

At $V_{EB} = 0.2\text{V}$,

$$I_{Ep} = \frac{10^{-5}\times 1.6\times 10^{-19}\times 10\times(1.5\times 10^{10})^2}{10^{16}\times 0.985\times 10^{-4}}\exp\left(\frac{0.2}{0.0259}\right) = 8.251\times 10^{-12}\,\text{A}$$

Similarly, at $V_{EB} = 0.6\text{V}$, $I_{Ep} = 3.8\times 10^{-5}\,\text{A}$

I_{E_n} = Diffusion current injected across B - E junction by the base (electrons for the p - n - p transistor).

$$I_{En} = Aq\frac{D_n n_i^2}{N_E W_E}\exp(\frac{qV_{EB}}{kT}) \quad \text{(electron current)}$$

Here we have W_E rather than L_n in the denorminator because $W_E << L_n$.

For $V_{EB} = 0.2\text{V}$,

$$I_{En} = \frac{10^{-5}\times 1.6\times 10^{-19}\times 10\times(1.5\times 10^{10})^2}{10^{19}\times 3\times 10^{-4}}\exp\left(\frac{0.2}{0.0259}\right) = 2.709\times 10^{-15}\,A$$

Similarly, at $V_{EB} = 0.6\text{V}$, $I_{En} = 1.38\times 10^{-8}\,A$

Now that the various current components are known, we can calculate the emitter injection efficiency,

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

For $V_{EB} = 0.2\text{V}$,

$$\gamma = \frac{8.251\times 10^{-12}}{8.251\times 10^{-12} + 2.709\times 10^{-15}} = \mathbf{0.9997}.$$

For $V_{EB} = 0.6\text{V}$,

$$\gamma = \frac{3.8\times 10^{-5}}{3.8\times 10^{-5} + 1.38\times 10^{-8}} = \mathbf{0.9996}.$$

Prob. 7.21

For the BJT in Prob. (7.19), calculate α, β, I_E, I_B, I_C *and the base Gummel number.*

To calculate the common base current gain α :

$\alpha = B\gamma$.

For $V_{EB} = 0.2\text{V}$, $\alpha = (0.995)(0.9997) = \mathbf{0.9947}$.

Similarly, for $V_{EB} = 0.6\text{V}$, $\alpha = (0.994)(0.9996) = \mathbf{0.9936}$.

To calculate β : $\beta = \dfrac{\alpha}{1-\alpha}$

For $\text{V}_{\text{EB}} = 0.2\text{V}$, $\beta = \dfrac{0.9947}{1-0.9947} = \mathbf{187.7}$.

For $\text{V}_{\text{EB}} = 0.6\text{V}$, $\beta = \dfrac{0.9936}{1-0.9936} = \mathbf{155.3}$.

To calculate current $\text{I}_\text{E}, \text{I}_\text{B}$, and I_C for $\text{V}_{\text{EB}} = 0.2$ and 0.6V, the emitter current is given by :

$I_E = I_{Ep} + I_{En}$

where I_{Ep} and I_{En} are the hole and electron currents, respectively, injected across the base - emitter junction.

At $V_{EB} = 0.2\text{V}$,

$I_E = 8.251\times10^{-12} + 2.709\times10^{-15} = 8.254\times10^{-12}\text{A} = \mathbf{8.254pA}$

At $V_{EB} = 0.6\text{V}$,

$I_E = 3.8\times10^{-5} + 1.38\times10^{-8} = 3.8\times10^{-5}\text{A} = \mathbf{38\mu A}$

The collector and base current can be determined by

$$I_C = \alpha I_E \text{ or } I_C = BI_{Ep} = BAq\frac{D_p n_i^2}{N_B W_b}e^{qV_{EB}/kT}$$

and $I_B = (1-\alpha)I_E = I_E - I_C$

For $V_{EB} = 0.2\text{V}$, $\alpha = 0.9947$ and $I_E = 8.254\text{pA}$,

$I_C = 0.9947\times8.254 = \mathbf{8.21pA}$

$I_B = 8.254 - 8.21 = \mathbf{0.044pA}$

For $V_{EB} = 0.6\text{V}$, $\alpha = 0.9936$ and $I_E = 38\mu\text{A}$,

$I_C = 0.9936\times38 = \mathbf{37.8\mu A}$

$I_B = 38 - 37.8 = \mathbf{0.2\mu A}$

Base Gummel number $= N_B W_b = 10^{16}\times1.09\times10^{-4} = \mathbf{1.09\times10^{12}\,cm^{-2}}$ (for $V_{EB} = 0.2\text{V}$)

For $V_{EB} = 0.6\text{V}$,

Base Gummel number $= N_B W_b = 10^{16}\times0.985\times10^{-4} = \mathbf{9.85\times10^{11}\,cm^{-2}}$

Prob. 7.22

For the given Si p-n-p BJT, calculate the β of the transistor in terms of B and γ, and using the charge control model. Comment on the results.

In emitter,

$$L_n^E = \sqrt{D_n \tau_n} = \sqrt{\mu_n \frac{kT}{q} \tau_n} = \sqrt{(150)(0.0259)(1\times10^{-6})} = 1.97\times10^{-3}\text{cm} = 19.7\mu\text{m}$$

In base,

$$L_p^B = \sqrt{D_p \tau_p} = \sqrt{\mu_p \frac{kT}{q} \tau_p} = \sqrt{(400)(0.0259)(25\times10^{-6})} = 1.61\times10^{-2}\text{cm} = 161\mu\text{m}$$

Assuming the emitter width is much greater than L_n^E,

$$\gamma = \left[1 + \frac{\mu_n^E N_d^B W_b}{\mu_p^B N_a^E L_n^E}\right]^{-1} = \left[1 + \frac{(150)(10^{16})(0.2)}{(400)(5\times10^{18})(19.7)}\right]^{-1} = 0.999992$$

$$B = 1 - \frac{W_b^2}{2L_p^2} = 1 - \frac{(0.2)^2}{2(161)^2} = 0.9999992$$

$$\beta = \frac{B\gamma}{1 - B\gamma} = \frac{(0.9999992)(0.999992)}{1-(0.9999992)(0.999992)} = \mathbf{1.136\times10^5}$$

Charge control approach

$$\tau_t = \frac{W_b^2}{2D_p} = \frac{(0.2\times10^{-4})^2}{2(\mu_p \frac{kT}{q})} = \frac{(0.2\times10^{-4})^2}{2(400\times0.0259)} = 1.93\times10^{-11}\text{s}$$

$$\beta = \frac{\tau_p}{\tau_t} = \frac{25\times10^{-6}}{1.93\times10^{-11}} = \mathbf{1.3\times10^6}$$

By the two methods, the values of β calculated are different because of the different approximations made in the two approaches. For the second (charge control) approach we erroneously assume $\gamma = 1$ which, therefore, gives a higher value of β. In the first approach, we have a better approximation for γ ($\gamma < 1$).

Prob. 7.23

For the BJT in Prob.(7.22), calculate the charge stored in the base when $V_{CB} = 0V$ and $V_{EB} = 0.7V$. Find f_T, if the base transit time is the dominant delay component for this BJT.

$$Q_b \approx \frac{1}{2} qAW_b p_n \left(e^{qV_{EB}/kT}\right)$$

$$= \frac{1}{2}\left(1.6\times10^{-19}\right)\left(10^{-4}\right)\left(0.2\times10^{-4}\right)\left(\frac{\left(1.5\times10^{10}\right)^2}{10^{16}}\right)\left(e^{0.7/0.0259}\right)$$

$$= \mathbf{1.968\times10^{-12}C}$$

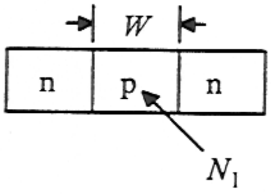
$$\tau_t = \frac{W_b^2}{2D_p} = \frac{(0.2\times10^{-4})^2}{2(400\times0.0259)} = 1.93\times10^{-11}\text{s}$$

$$f_T = \frac{1}{2\pi\tau_t} = \frac{1}{2\times3.14\times1.93\times10^{-11}} = 8.25\times10^{9}\text{Hz}$$

Prob. 7.24

Given 3 npn transistors which are identical except that transistor #2 has a base region twice as long as transistor #1 and transistor #3 has a base region doped twice as heavily as transistor #1. All other dopings and lengths are identical for transistors #1, #2 and #3. Give the number (s) of the transistors or transistors which have the largest value of each parameter, and give clear mathematical reasons for each: (a) emitter injection efficiency (b) Base transport factor (c) punchthrough voltage (d) collector junction capacitance with V_{CB} reverse biased at 10V (e) common emitter current gain.

1



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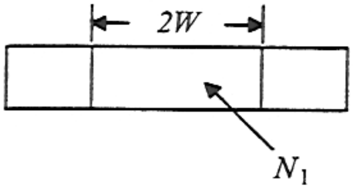

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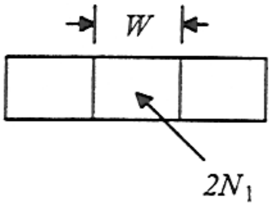


(a) $\gamma = \dfrac{I_{En}}{I_{En} + I_{Ep}}$

$\gamma_{\#1} > \gamma_{\#3}$ because base doping is higher in #3 $\Rightarrow$ I_{En} less is in 3.

$\gamma_{\#1} > \gamma_{\#2}$ because base width is larger in #2 $\Rightarrow$ I_{En} less is in 2.

(b) $B = \dfrac{i_{En}\big|_{x=0}}{i_{En}\big|_{x=W_b}}$

$B_{\#1} > B_{\#2}$ because base width is larger in #2 $\Rightarrow$ cause more recombination.

$B_{\#1} > B_{\#3}$ because base doping is higher in #3 $\Rightarrow$ cause more recombination.

(c) $V_{pt} = \dfrac{qN_B W_b^2}{2\varepsilon_s}$

$\Rightarrow$ #2 > #1 because $W_b \rightarrow 2W_b$ (four times more)

#3 > #1 because $N_a \rightarrow 2N_a$ (two times more). Therefore, #2 highest.

(d) $C_i \propto (N_{doping})^{1/2}$ $\Rightarrow$ #3 highest.

(e) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{B\gamma}{1-B\gamma}$, Since (a) and (b) show #1 to be higher, #1 has highest β.

Prob. 7.25

$\tau_t = 100$ ps in the base of an n-p-n, and electrons drift at v_s through the 1 μm collector depletion region. The emitter junction charges in 30 ps and the collector has $C_c = 0.1$ pF and $r_c \doteq 10\Omega$. Find f_T.

The total delay time for the parameters given is

$$\begin{aligned}\tau_d &= 100\text{ps} + \frac{10^{-4}}{10^7}s + 30\text{ps} + 10(0.1)\text{ps} = 141\text{ps} \\ f_T &= \frac{1}{2\pi\tau_d} = 1.1\text{GHz}\end{aligned}$$

Prob. 7.26

An n-p-n Si BJT has $N_d^E = 10^{18}$ *and* $N_a^B = 10^{16}$ cm^{-3}. *At what* V_{BE} *is* $\Delta n_E = N_a^B$ *? Comment on* γ.

$\Delta n_E = n_p e^{qV_{BE}/kT}$ Set this equal to N_a^B

This occurs for $V_{BE} = \frac{kT}{q} \ln N_a^B / n_p = 0.0259 \ln \frac{10^{16}}{2.25\times 10^4} = \mathbf{0.695V}$

With $\frac{N_E}{N_B} = 100$, high injection is not reached until the emitter junction is biased to nearly 0.7 V. Since the contact potential $V_0 = 0.0259 \ln \frac{10^{34}}{2.25\times 10^{20}} = 0.81V$, this is a very high bias. Thus γ rolloff due to high injection is not likely in the normal operating range.

Prob. 7.27

Find $\Delta p_E(t)$ *if the emitter has an applied voltage* $v_{EB}(t) = V_{EB} + v_{eb}(t)$ *where* $V_{EB} \gg kT/q$ *and* $v_{eb} \ll kT/q$.

$$\Delta p_E(t) = p_n(e^{qv(t)/kT} - 1) \simeq p_n\, e^{qv(t)/kT}$$

using $v(t) = v_{EB} = V_{EB} + v_{eb}$ we have

$$\Delta p_E(t) = p_n \exp[(V_{EB}+v_{eb})q/kT] = p_n[\exp(qV_{EB}/kT)][\exp(qv_{eb}/kT]$$

$$= \Delta p_E(\mathrm{dc}) \exp(qv_{eb}/kT)$$

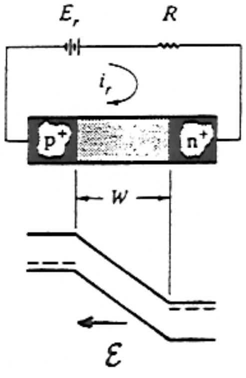
If $v_{eb} \ll kT/q$,

$$\Delta p_E(t) = \Delta p_E(\mathrm{dc})[1 + \frac{qv_{eb}}{kT}]$$

Chapter 8

Prob. 8.1

For the p-i-n photodiode of Fig. 8-7



(a) Explain why this photodetector does not have gain .

An EHP created within W by absorption of a photon is collected as the e^- is swept to n and the h^+ to p. Since only one EHP is collected per photon, there is no gain.

(b) Explain how making the device more sensitive to low light levels degrades its speed

If W is made wider to receive more photons, the transit time to collect the EHP is longer (degraded response speed).

If W is kept the same and the area A is increased to receive more photons, the capacitance is increased and again the response speed is degraded.

(c) If this photodiode is to be used to detect light with $\lambda = 0.6\ \mu m$, what material would you use ? What substrate would you grow this material on?

$E_{ph} = 1.24/0.6 = 2.07eV$

From Fig. 1-13 :

$In_{.5}Ga_{.5}P$ grown on GaAs has $E_g \approx 2\ eV$
$AlAs_{.55}Sb_{.45}$ grown on InP has $E_g \approx 1.95\ eV$

Both of these have E_g slightly smaller than the photon energy.

Prob. 8.2

A Si solar cell has $A = 4\ cm^2$, $I_{th} = 32\ nA$ *and* $W = 1\ \mu m$. *If* $g_{op} = 10^{18} EHP/cm^3 s$ *within* $L_p = L_n = 2\ \mu m$ *of the junction, find* I_{sc} *and* V_{oc}.

From Eq. (8-1),

$$I_{sc} = I_{op} = qAg_{op}(L_p + L_n + W) = 1.6 \times 10^{-19} \times 4 \times 10^{18} (5 \times 10^{-4}) = \mathbf{0.32\ mA}$$

From Eq. (8-3),

$$\begin{aligned} V_{oc} &= \tfrac{kT}{q} \ln(1 + I_{op}/I_{th}) = 0.0259 \ln(1 + \tfrac{0.32 \times 10^{-3}}{32 \times 10^{-9}}) \\ &= \mathbf{0.24V} \end{aligned}$$

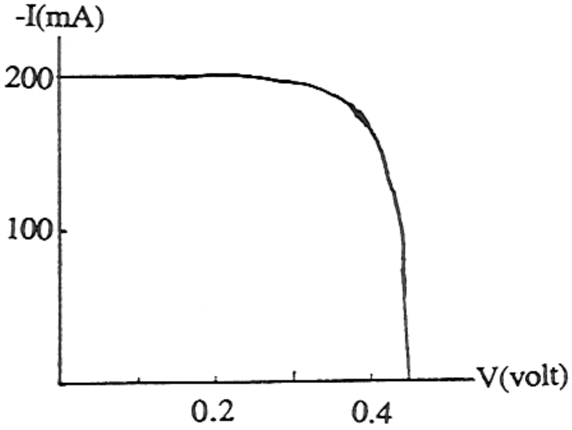
Prob. 8.3

Plot I_r*-*V *for a Si solar cell with* $I_{th} = 5\ nA$ *and* $I_{sc} = 200 mA$.

From Eq.(8-2),

$$V = \frac{kT}{q} \ln(1 + \frac{I + I_{sc}}{I_{th}}) = 0.0259 \ln(1 + \frac{I + 0.2}{5 \times 10^{-9}})$$

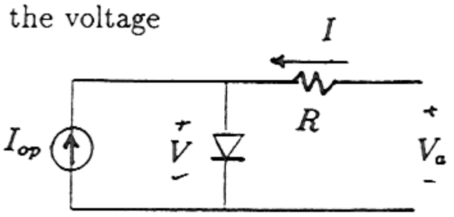
I (mA)	V (volt)
−200	0
−190	0.376
−180	0.39
−160	0.41
−120	0.43
−80	0.44
−40	0.448
0	0.453



Prob. 8.4

Repeat Prob. 8.3 with a 1 Ω series resistance.

For a given current I and terminal voltage V_a, the voltage across the diode is reduced by IR

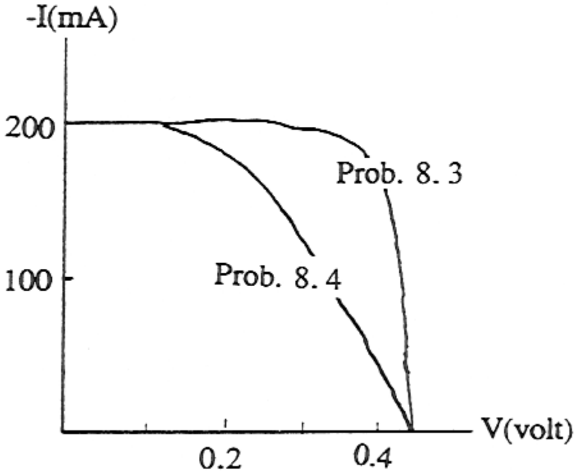


$$V_a = V + IR$$

so

$$I = I_{th}(e^{q(V_a-IR)/kT} - 1) - I_{op} = 5\times 10^{-9}(e^{(V_a-I)/0.0259} - 1) - 0.2$$

I	V_a
0	0.453
−0.04	0.408
−0.08	0.36
−0.12	0.31
−0.16	0.25
−0.18	0.21



Prob. 8.5

How can several semiconductors be used in a solar cell ?

In a GaAs cell ($E_g \sim 1.4$ eV), a top layer of AlGaAs ($E_g \sim 2$ eV) can be grown lattice-matched to keep generated carriers from the surface, reducing surface recombination. One might also use a second cell below with $E_g < 1.4$ eV (e.g., Si) to absorb the light passed through the GaAs band gap,

Prob. 8.6

Find the photocurrent ΔI in terms of τ_n and τ_t for a sample dominated by μ_n.

$$\Delta\sigma \simeq q\mu_n\Delta n = q\mu_n g_{op}\tau_n$$

$$\Delta I = V/\Delta R = VA\Delta\sigma/L = VAq\mu_n g_{op}\tau_n/L$$

The transit time is

$$\tau_t = \frac{L}{v_d} = \frac{L}{\mu_n V/L} = \frac{L^2}{\mu_n V}$$

$$\Delta I = qALg_{op}\tau_n/\tau_t$$

Prob. 8.7

What ternary alloys produce $\lambda = 680$ *nm ?*

$$E_g = \frac{1.24}{0.680} = 1.82eV$$

From Fig. 3-6(c),
$Al_xGa_{1-x}As$: x = 0.32

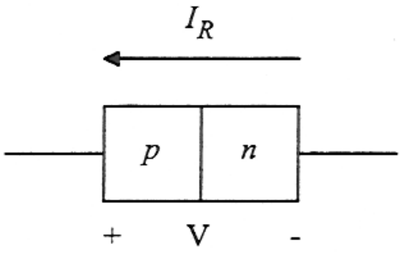
From Fig. 8-11,
$GaAs_{1-x}P_x$: x = 0.32

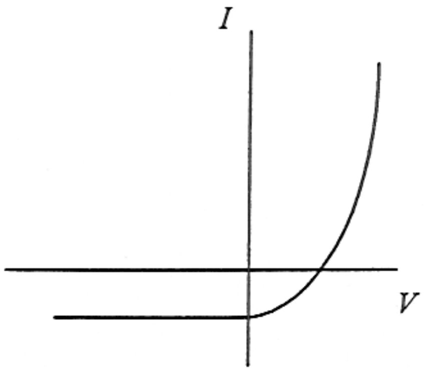
From Fig. 1-13,
$In_xGa_{1-x}P$: x = 0.4

Prob. 8.8

(a) Why must a solar cell be operated in the 4th quadrant of the junction I-V characteristic?

Power is consumed in 1st (+I, +V) and 3rd (-I, -V) quadrants. Power is generated in 4th (-I, +V) quadrant.





(b) What is the advantage of a quaternary alloy in fabricating LEDs for fiber optics?

Bandgap (E_g) and therefore wavelength (λ) can be chosen while lattice constant (a) is adjusted for epitaxial growth on convenient substrates.

(c) Why is a reverse-biased GaAs p-n junction not a good photodetector for light of λ = 1μm?

$$E_{ph.} = \frac{hc}{\lambda} = \frac{4.14\times10^{-15}\times3\times10^{10}}{10^{-4}} = 1.24eV$$

Since E_g = 1.43 eV for GaAs, the photon is not absorbed.

Prob. 8.9

For a uniformly illuminated p^+-n diode with g_{op} in steady state, find (a) $\delta p(x_n)$, (b) $I_p(x_n)$ and $I_p(0)$.

$$\frac{d^2\delta p}{dx_n^2} = \frac{\delta p}{L_p^2} - \frac{g_{op}}{D_p}$$

$$\delta p(x_n) = Be^{-x_n/L_p} + \frac{g_{op}L_p^2}{D_p}$$

at $x_n = 0$, $\delta p(0) = \Delta p_n$. Thus $B = \Delta p_n - \dfrac{g_{op}L_p^2}{D_p}$

(a) $\delta p(x_n) = [p_n(e^{qV/kT} - 1) - g_{op}L_p^2 / D_p]e^{-x_n/L_p} + g_{op}L_p^2 / D_p$

$$\frac{d\delta p}{dx_n} = -\frac{1}{L_p}[\Delta p_n - g_{op}L_p^2 / D_p]e^{-x_n/L_p}$$

(b) $I_p(x_n) = -qAD_p\dfrac{d\delta p}{dx_n} = \dfrac{qAD_p}{L_p}[\Delta p_n - g_{op}L_p^2 / D_p]e^{-x_n/L_p}$

$$I_p(x_n = 0) = \frac{qAD_p}{L_p}p_n(e^{qV/kT} - 1) - qAL_pg_{op}$$

This corresponds to Eq.(8-2) for $n_p << p_n$ except that the component due to generation on the p side is not included.

Prob. 8.10

An illuminated Si solar cell has a short-circuit current of 100 mA and an open-circuit voltage of 0.8V, and a fill factor of 0.7. What is the maximum power delivered to a load by this cell?

$P_{max} = (f.f.)\, I_{sc}V_{oc} = (0.8)(100)(0.7) =$ **56 mW**

Prob. 8.11

For a solar cell with I_{sc} under illumination and I_{th} in the dark, find an expression for the voltage at maximum power and solve graphically for I_{th} = 1.5 nA and I_{sc} = 100mA to get V_{mp} and the maximum power.

Eq.(8-2) can be written as

$$I = I_{th}(e^{qV/kT} - 1) - I_{sc}$$

$$IV = I_{th}(e^{qV/kT} - 1)V - I_{sc}V$$

$$\frac{d(IV)}{dV} = I_{th}(e^{qV/kT} - 1) + \frac{q}{kT} I_{th} V e^{qV/kT} - I_{sc} \overset{set}{=} 0$$

(a) $(1 + \frac{q}{kT} V_{mp}) e^{qV_{mp}/kT} = 1 + \frac{I_{sc}}{I_{th}}$

For $I_{sc} >> I_{th}$ and $V_{mp} >> kT/q$,

$$\frac{qV_{mp}}{kT} e^{qV_{mp}/kT} = \frac{I_{sc}}{I_{th}}$$

(b) $\ln(qV_{mp}/\text{kT}) + qV_{mp}/kT = \ln(I_{sc}/I_{th})$

$$\frac{I_{sc}}{I_{th}} = \frac{100 \times 10^{-3}}{1.5 \times 10^{-9}} = 6.67 \times 10^{7}$$

$$\ln \frac{I_{sc}}{I_{th}} = 18$$

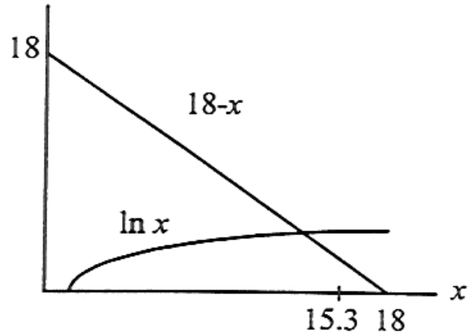
$\ln x = 18 - x$, where $x = qV_{mp}/kT$

(c) The solution is $x = 15.3$

$V_{mp} = 15.3 \times 0.0259 = \mathbf{0.396V}$

$I = 10^{-9} e^{15.3} - 10^{-1} = \mathbf{-96mA}$

(d) $P = -IV = \mathbf{37.9mW}$

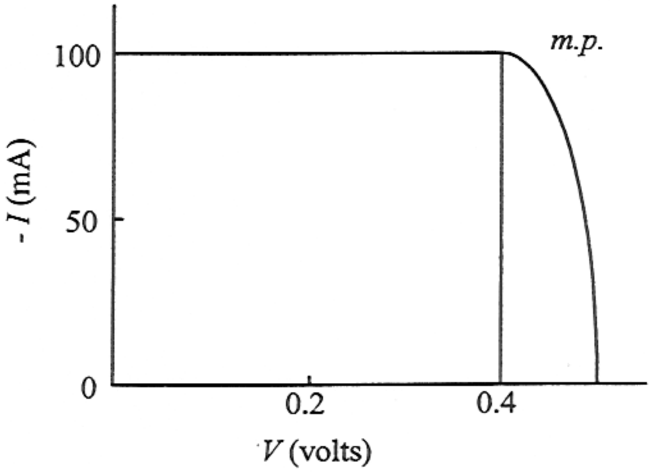


Prob. 8.12

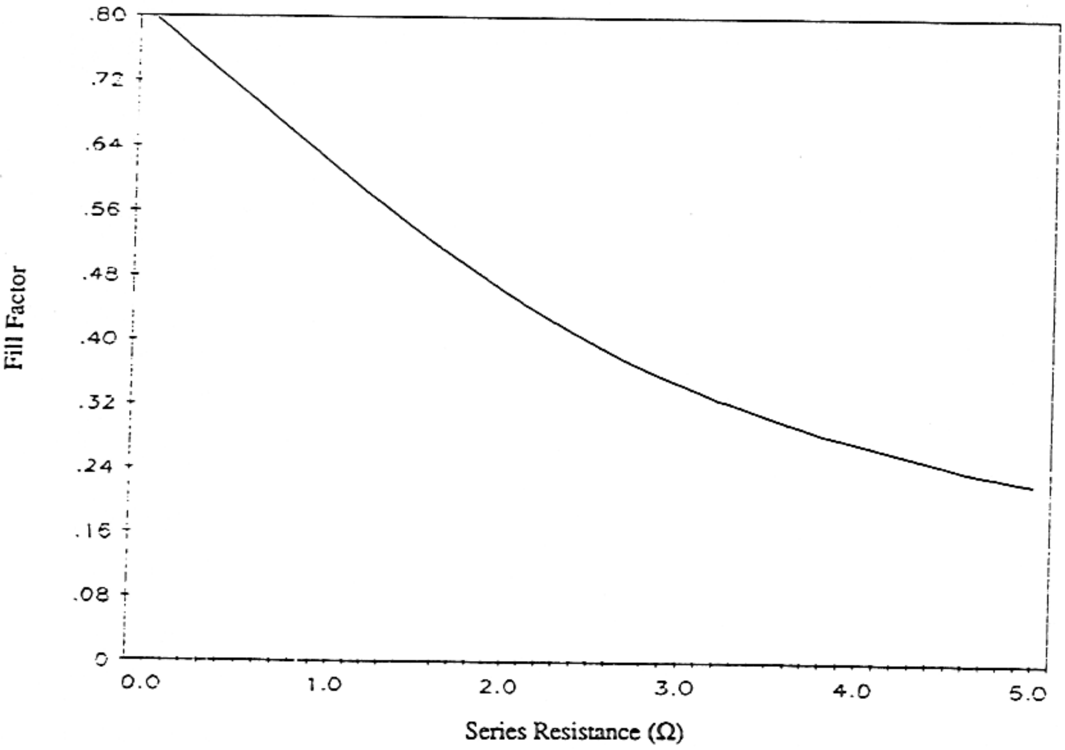
For the solar cell of Prob. 8.11, plot the I-V curve and draw the maximum power rectangle.

$$V = 0.0259 \ln\left(1 + \frac{0.1 + I}{1.5 \times 10^{-9}}\right)$$

I (mA)	V (mV)
0	**467**
-25	**459**
-50	**449**
-75	**431**
-90	**407**
-95	**389**
-98	**365**
-100	**0**



Prob. 8.13



Prob. 8.14

From Fig. 1-13, what epitaxial layer/substrate combination would you choose for an LED with λ = 1.55 μm? Repeat for 1.3 μm.

Assume the lattice constant varies linearly with composition between the binary limits.
For λ = 1.55 μm, use $GaAs_{0.5}Sb_{0.5}$ on an InP substrate.
For λ = 1.3 μm, use InGaAsP on InP.

Prob. 8.15

How does degeneracy prevent absorption of the emission wavelength ?

Since absorption requires promotion of an electron from a filled state in the valence band to an empty state in the conduction band, Fig. 8-19 shows that photons with $h\nu > (F_n - F_p)$ are absorbed. On the other hand, emitted photons have $h\nu < (F_n - F_p)$. This is true only in the inversion region, and absorption becomes important away from the neighborhood of the junction.

Prob. 8.16

Show $B_{12} = B_{21}$ *at high temperature.*

$$B_{12} n_1 \rho(\nu_{12}) = A_{21} n_2 + B_{21} n_2 \rho(\nu_{12})$$

$$B_{12} = \left[\frac{A_{21}}{\rho(\nu_{12})} + B_{21}\right] e^{-(E_2-E_1)/kT}$$

As $T \rightarrow \infty, \quad e^{-(E_2-E_1)/kT} \rightarrow 1$

Thus as $\rho(\nu_{12}) \rightarrow \infty, \quad B_{12} = B_{21}$

Prob. 8.17

Use Planck's radiation law to find A_{21}/B_{12}.

$$\rho(\nu_{12}) = \frac{A_{21}}{B_{12}} \frac{n_2}{n_1} + \frac{n_2}{n_1} \rho(\nu_{12}) = \left[\frac{A_{21}}{B_{12}} + \rho(\nu_{12})\right] \exp\left(-h\nu_{12}/kT\right)$$

$$\frac{A_{21}}{B_{12}} = \rho(\nu_{12}) \left[\exp\left(h\nu_{12}/kT\right) - 1\right] = \frac{8\pi h \nu_{12}^3}{c^3}$$

Prob. 8.18

Estimate minimum $n = p$ *for population inversion in GaAs.*

$$F_n - F_p = E_g = 1.43\, eV$$

For $n = p, \quad F_n - E_i = E_i - F_p \simeq 0.715$

$$n = p = n_i\, e^{(F_n - E_i)/kT} = 10^6\, e^{0.715/0.0259}$$

$$\simeq \mathbf{10^{18}\ cm^{-3}}$$

Chapter 9

Prob. 9.1

Relate the sheet resistance of a diffused layer to $N_a(x)$ *and* x_j*.*

$$R_s = \frac{\langle\rho\rangle}{x_j} = \frac{1}{\langle\sigma\rangle} x_j$$

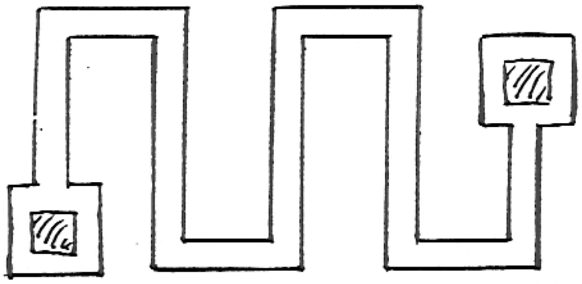
$$\langle\sigma\rangle = \frac{q}{x_j}\int_0^{x_j} \mu_p N_a \, dx\,, \quad \text{if } N_a(x) \gg N_d \text{ over most of the profile}$$

$$R_s = \left[q \int_0^{x_j} \mu_p(x) N_a(x)\, dx\right]^{-1}$$

Prob. 9.2

For a 200 Ω*/square diffusion, find the aspect ratio for a* 10 $k\Omega$ *resistor, and draw a diffusion pattern for a width of* $5\mu m$*.*

$$R = R_s L/W, \text{ thus } L/W = 10^4/200 = 50$$



For $W = 5\ \mu m$, $L = 250\ \mu m$.

Prob. 9.3

A boron isolation diffusion ($D = 2.5 \times 10^{-12}$, $N_0 = 10^{20}$) penetrates a $3\mu m$ epitaxial layer with $N_d = 10^{16}$. (a) What diffusion time is required ? (b) How far would an Sb-doped buried layer ($D = 2 \times 10^{-13}$, $N_0 = 10^{20}$) move during this diffusion ?

(a) $N(x,t) = N_0 \text{ erfc } \dfrac{x}{2\sqrt{Dt}}$. We want $N(5 \times 10^{-4}, t) = 10^{16}\ cm^{-3}$.

$$10^{16} = 10^{20} \text{ erfc } \frac{3 \times 10^{-4}}{2\sqrt{2.5 \times 10^{-12} t}}$$

$$\text{erfc } u = \frac{10^{16}}{10^{20}} = 10^{-4}, \quad u \simeq 2.75 \text{ from Fig. P5} - 2$$

$$2.75 = \frac{300}{3.16\ \sqrt{t}}, \quad \sqrt{t} = \frac{300}{8.69}, \quad t = \mathbf{1192\ seconds}$$

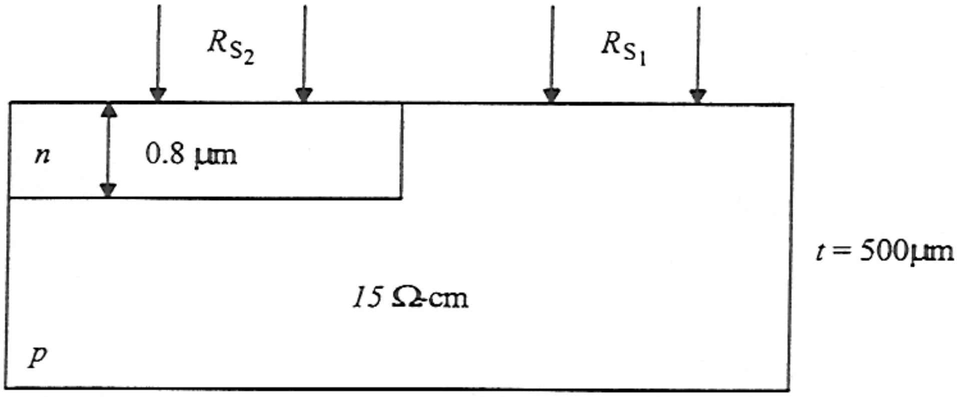
(b) For $D = 2 \times 10^{-13}$, $u = \dfrac{x}{2\ \sqrt{2.38 \times 10^{-10}}} = 3.24 \times 10^4\ x$

$$\text{erfc } u = \frac{10^{16}}{10^{20}} = 10^{-4}, \quad u = 2.75$$

Thus, $x = \dfrac{2.75}{3.24 \times 10^4} = \mathbf{0.85\ \mu m}$

Prob. 9.4

For the given p-type wafer with 4-point probe measurements on the doped and undoped part of the wafer being used, find the measured sheet resistance in the undoped part. Calculate the average resistivity if we have a sheet resistance of 90 ohms/square in the doped part. Find the temperature at which the P diffusion was done.



$N_{sub} = 10^{15} \text{cm}^{-3}$,

R_{S_1} in undoped part $= \dfrac{\rho}{t} = \dfrac{15}{500\times10^{-4}} = 300\Omega/\text{square}$

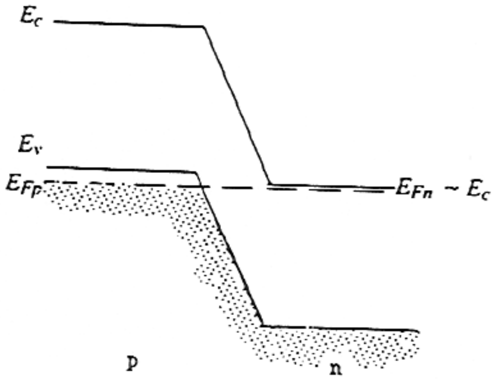
In undoped part, $\bar{\rho} = R_{S_2} \cdot x = 90\,(0.8\times10^{-4}) = 0.0072\Omega\text{cm} = 72\Omega\mu\text{m}$.

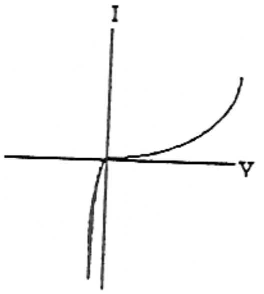
From solid solubility curve for P (App. VII), if P concentration = 6×10^{20} cm^{-3}, T = 900°C.
If P concentration = 6×10^{19} cm^{-3}, extrapolating the curve, T = 700°C.

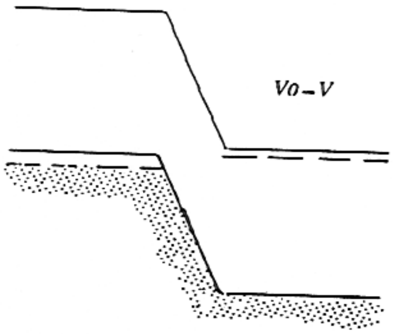
Chapter 10

Prob. 10.1

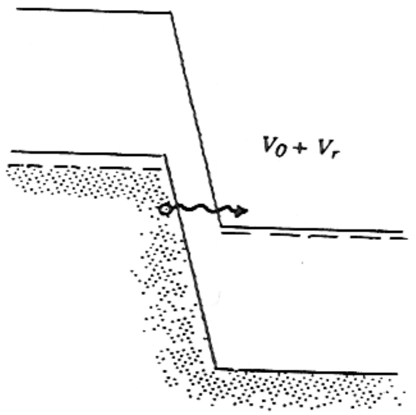
Sketch band diagrams for a junction with a degenerate p side and $E_F = E_c$ *on the n side.*







Forward bias

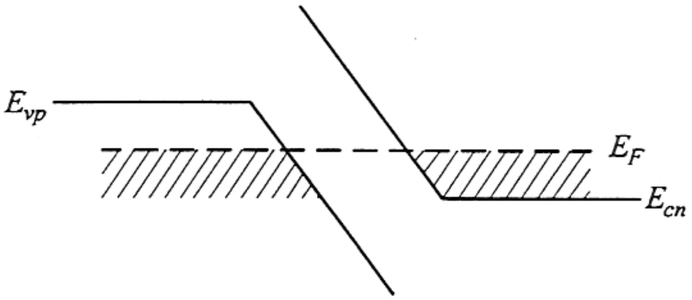


Reverse bias

This is called a backward diode because it conducts freely in the reverse direction (due to tunneling), but the current remains small for low voltage forward bias.

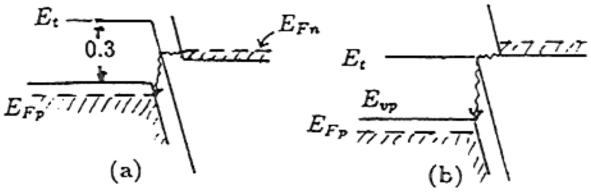
Prob. 10.2

Explain what determines the peak tunneling voltage V_p of a tunnel diode. For the given tunnel diode, (a) calculate the minimum forward bias at which tunneling through E_t occurs. (b) Calculate the maximum forward bias for tunneling via E_t. (c) Sketch the I-V curve for this diode.

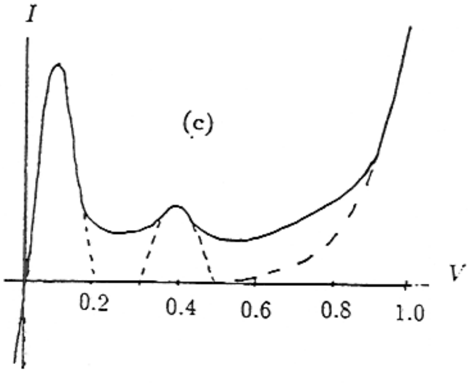


The sizes of $E_{vp} - E_F$ and $E_F - E_{cn}$ determine the voltage required to align the most filled states opposite empty states.

(a) Tunneling through the level begins when $E_{Fn} - E_{Fp} = E_t$ i.e., at a forward bias of 0.3 V.



(b) Tunneling through the level ends when $E_{cn} - E_{vp} = E_t$ i.e., when $E_{Fn} - E_{Fp} = V = 0.3 + 0.1 + 0.1 = 0.5$ V.
(c) Band-to-band tunneling is maximum when $E_{Fn} - E_{Fp} = 0.1$ V and is essentially zero when $E_{Fn} - E_{Fp} = 0.2$ V.



Prob. 10.3

(a) *Relate $\partial\rho/\partial t$ (where ρ is space charge density) to σ and ϵ, neglecting recombination.*

(b) *Show space charge $\rho(t)$ decays exponentially with time constant τ_d.*

(c) *Find the RC time constant of a sample.*

$$J = \sigma\,\mathcal{E} = -\sigma\,\nabla V$$

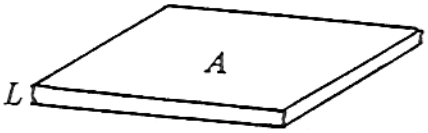
$$\nabla . J = -\frac{\partial\rho}{\partial t}\,, \quad \nabla^2 V = -\frac{\rho}{\epsilon}$$

(a) $\nabla . J = -\sigma\,\nabla^2 V$; thus $-\dfrac{\partial\rho}{\partial t} = \sigma\,\rho/\epsilon$

(b) $\rho = \rho_0\,e^{-t/\tau_d}$, where $\tau_d = \epsilon/\sigma$

(c) $R = \dfrac{L}{\sigma A}\,, \quad C = \dfrac{\epsilon A}{L}$

$$RC = \left(\frac{L}{\sigma A}\right)\left(\frac{\epsilon A}{L}\right) = \frac{\epsilon}{\sigma} = \tau_d$$



Prob. 10.4

Find the criterion for negative conductivity in terms of mobilities and electron concentrations in the Γ and L bands of GaAs.

$$J = \sigma\,\mathcal{E} = q\,[\mu_\Gamma n_\Gamma + \mu_L n_L]\,\mathcal{E} = q\,[\mu_\Gamma n_\Gamma + \mu_L(n_0 - n_\Gamma)]\,\mathcal{E}$$

$$\frac{dJ}{d\mathcal{E}} = q\,[\mu_\Gamma n_\Gamma + \mu_L n_L] + q\mathcal{E}\left[(\mu_\Gamma - \mu_L)\frac{dn_\Gamma}{d\mathcal{E}} + n_\Gamma\frac{d\mu_\Gamma}{d\mathcal{E}} + n_L\frac{d\mu_L}{d\mathcal{E}}\right]$$

since $\dfrac{dn_0}{d\mathcal{E}} = 0.$

This is negative when

$$\frac{\mathcal{E}\,(\mu_\Gamma - \mu_L)\,dn_\Gamma/d\mathcal{E} \;+\; \mathcal{E}\,(n_\Gamma\,d\mu_\Gamma/d\mathcal{E} + n_L\,d\mu_L/d\mathcal{E})}{\mu_\Gamma n_\Gamma \;+\; \mu_L n_L} < -1$$

If we assume $\mu_\Gamma = A/\mathcal{E}$ and $\mu_L = B/\mathcal{E}$,

$$\frac{d\mu_\Gamma}{d\mathcal{E}} = -A/\mathcal{E}^2 \text{ and } \frac{d\mu_L}{d\mathcal{E}} = -B/\mathcal{E}^2, \text{ and the condition is}$$

$$\frac{(A-B)\,dn_\Gamma/d\mathcal{E} \;-\; \mathcal{E}^{-1}(n_\Gamma A + n_L B)}{\mathcal{E}^{-1}n_\Gamma A \;+\; \mathcal{E}^{-1}n_L B} < -1$$

$$\frac{\mathcal{E}(A-B)\,dn_\Gamma/d\mathcal{E}}{n_\Gamma A \;+\; n_L B} - 1 < -1$$

B is less than A, since $\mu_L \ll \mu_\Gamma$. Thus $dn_\Gamma/d\mathcal{E}$ must be negative. That is, the conductivity is negative only while electrons are being transferred from the lower lying Γ valley into the upper L valley.

Prob. 10.5

Estimate d-c power dissipated per unit volume in a 5 μm GaAs Gunn diode biased just below threshold.

(a) $n_0 L \simeq 10^{12}, \quad n_0 \simeq 10^{12}/(5\times10^{-4}) = 2\times10^{15}\ cm^{-3}$

$\tau_t = L/v_s = 5\times10^{-4}/10^7 = 5\times10^{-11}\ s$

(b) $P = IV = (qn_0 v_d A)(\mathcal{E}L)$

$$\frac{P}{AL} = 1.6\times10^{-19} \times 2\times10^{15} \times 2\times10^{7} \times 3\times10^{3}$$

$$\simeq \mathbf{2 \times 10^7\ W/cm^3}$$

For a device with higher frequency, τ_t must be smaller. Thus, L must be smaller and n_0 correspondingly larger. As a result, the power dissipation P, which is proportional to $n_0 L$, is about the same according to this simple analysis. However, the power density, which is proportional to n_0, is larger for higher frequency devices. This is important, since P/AL is critical to heat dissipation requirements.

Prob. 10.6

(a) *Calculate the ratio of the densities of states in the Γ and L conduction bands in GaAs.*

(b) *Assuming a Boltzmann distribution, find the ratio of electron concentrations in these bands at 300 K.*

(c) *What is the equivalent temperature of an electron in the L minima ?*

(a) $N_c = 2(2\pi kTm_n^*/h^2)^{3/2}$

$$\frac{N_L}{N_\Gamma} = \left[\frac{m_n^*(L)}{m_n^*(\Gamma)}\right]^{3/2} = \left[\frac{0.55}{0.067}\right]^{3/2} = 8.2^{3/2} = \mathbf{23.5}$$

(b) $\dfrac{n_L}{n_\Gamma} = 23.5\, e^{-0.30/0.0259} = 23.5\, e^{-11.6} = \mathbf{2.2 \times 10^{-4}}$

We notice that the upper (L) valley is essentially empty in equilibrium at 300 K, compared with the lower (Γ) valley.

(c) $T = k^{-1}(0.0259 + 0.30) = 0.326/(8.62\times10^{-5}) = \mathbf{3782\ K}$

Chapter 11

Prob. 11.1

Explain why two separate BJTs cannot be connected to make a p-n-p-n switch.

The p-n-p-n switching action depends on injection of carriers across both base regions (n_1 and p_2 in Fig. 11-3) and collection into the base regions of the opposite transistor. Transistor action in the p-n-p feeds majority carrier holes to the base of the n-p-n, for example. This cannot occur with separate transistors, so the p-n-p-n switching effect does not occur.

Prob. 11.2

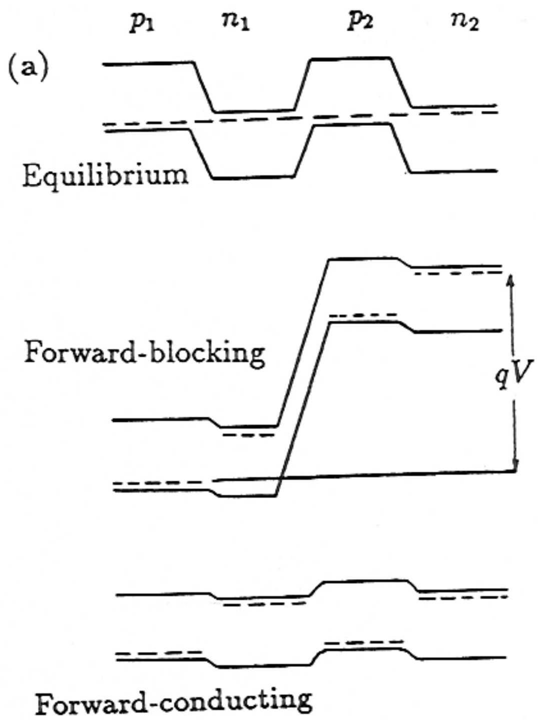
How does gate bias provide switching in an SCR ?

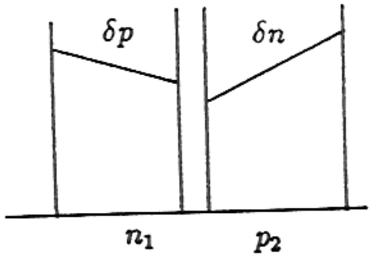
Switching in the SCR of Fig. 11-4 occurs when holes are supplied to p_2 at a sufficient rate. Although j_3 is forward biased with $i_G = 0$, transistor action does not begin until hole injection by i_G reaches the critical value for switching.

Prob. 11.3

(a) *Sketch the energy band diagrams for the p-n-p-n diode in equilibrium; in the forward-blocking state; and in the forward-conducting state.*

(b) *Sketch the excess minority carrier distributions in regions* n_1 *and* p_2 *when the p-n-p-n diode is in the forward-conducting state.*



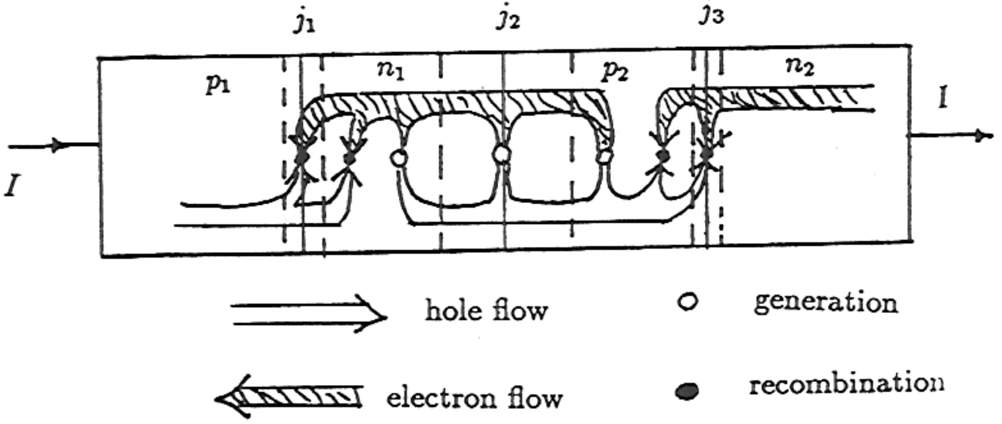


(b) Each equivalent transistor is in saturation. Thus the minority carrier distribution in each base resembles Fig. 7-14b.

Prob. 11.4

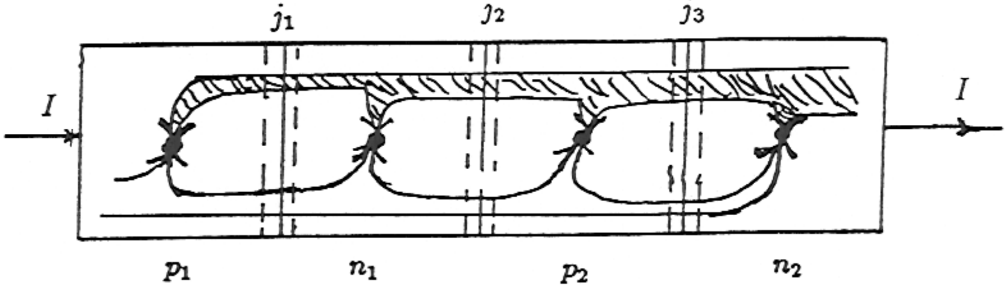
Draw diagrams such as Fig. 7-3 for the forward-blocking and forward- conducting states of a p-n-p-n.

Forward-blocking state



In the simplified diagram above we neglect minority carrier transport across each base region. Electrons generated thermally in and about j_2 recombine in n_1 and j_1 with injected holes. Similarly, generated holes feed recombination with injected electrons in p_2 and j_3. In the absence of transistor action, I is limited to essentially the reverse saturation current of j_2. In the figure below we neglect generation compared with transport due to transistor action. Recombination takes place in n_1 and p_2, but many injected carriers are transported through the device by transistor action. More complete diagrams can be found in the book by *Gentry et al.* (Chapter 11 reading list), p. 72 and 76.

Forward-conducting state



Prob. 11.5

Include avalanche in j_2 in the coupled transistor model.

Referring to Fig. 11-2,

$$i_{C1} = \alpha_1 i M_p + I_{C01} M_p$$

$$i_{C2} = \alpha_2 i M_n + I_{C02} M_n$$

$$i = i_{C1} + i_{C2} = i(\alpha_1 M_p + \alpha_2 M_n) + I_{C01} M_p + I_{C02} M_n$$

$$i = \frac{I_{C01} M_p + I_{C02} M_n}{1 - (\alpha_1 M_p + \alpha_2 M_n)}$$

The current becomes large as $\alpha_1 M_p + \alpha_2 M_n$ approaches unity.