CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Apply modified Euler's method to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$. Compute $y(0.2)$ taking $h = 0.1$. **(07 Marks)**

b. Using fourth order Runge-Kutta method, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$. **(06 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.0679$, $y(0.3) = 2.090$, find $y(0.4)$ correct to four decimal places by using Milne's predictor and corrector method. **(07 Marks)**

OR

2 a. Use Taylor's series method to find $y(4.1)$ given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$ and $y(4) = 4$. **(06 Marks)**

b. Solve $(y^2 - x^2)dx = (y^2 + x^2)dy$ in the range $0 \le x \le 0.4$ given that $y = 1$ at $x = 0$ initially by applying R-K method of fourth order. **(07 Marks)**

c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$ **(07 Marks)**

Module-2

3 a. Given $y'' - xy' - y = 0$ with the initial condition $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ and $y'(0.2)$ using fourth order R-K method. **(07 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0 \text{ if } \alpha \neq \beta.$$ **(07 Marks)**

c. Express the polynomial $4x^3 - 2x^2 - 3x + 8$ interms of Legendre polynomials. **(06 Marks)**

OR

4 a. Applying Milne's predictor and corrector formulae to compute $y(0.8)$ given that y satisfies the equation $y'' = 2yy'$ using the following data :
$y(0) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$, $y'(0) = 1$, $y'(0.2) = 1.041$, $y'(0.4) = 1.179$, $y'(0.6) = 1.468$. **(07 Marks)**

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(06 Marks)**

c. Derive Rodrigue's formula, $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$. **(07 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equation in the polar form. **(07 Marks)**

b. State and prove Cauchy's-Integral formula, $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a} dz$. **(06 Marks)**

c. Find the bilinear transformation which map the points z = 1, i − 1 into w = i, 0, −i. **(07 Marks)**

OR

6 a. Find the analytic function f(z) whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(06 Marks)**

b. Using Cauchy's residue theorem, evaluate $\int_C \frac{z\cos z}{\left(z-\frac{\pi}{2}\right)^3} dz$ where $C: |z-1| = 1$. **(07 Marks)**

c. Discuss the transformation $\omega = e^z$. **(07 Marks)**

Module-4

7 a. If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students 2.5 and $\sqrt{1.875}$. Find an estimate of the number of candidates answering correctly,

(i) 8 or more questions
(ii) 2 or less
(iii) 5 questions **(07 Marks)**

b. Derive mean and standard deviations of binomial distributions. **(06 Marks)**

c. The joint probability distribution for two random variables X and Y is as follows :

X \ Y	−3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine : (i) Marginal distribution of X and Y (ii) COV (X, Y) (iii) Correlations of X and Y. **(07 Marks)**

OR

8 a. Derive mean and standard deviations of Exponential distribution. **(06 Marks)**

b. X and Y are independent random variables. X takes values 2, 5, 7 with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively. Y take values 3, 4, 5 with the probability $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$.

(i) Find the Joint probability distribution of X and Y.
(ii) Show that the covariance of X and Y.
(iii) Find the probability distribution of Z = X + Y **(07 Marks)**

c. In 800 families with 5 childrens each how many families would be expected to have, (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls by assuming probabilities for boys and girls to be equal. **(07 Marks)**

Module-5

9 a. A survey was conducted in a slum locality of 2000 families by selecting a sample size 800. It was revealed that 180 families were illiterates. Find the probable limits of the illiterate families in the population of 2000. **(07 Marks)**

b. Ten individuals are choosen at random from a population and their heights in inches are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height of the population is 65 inches given that $t_{0.05} = 2.262$ for 9 d.f. **(07 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

(06 Marks)

OR

10 a. Four coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($x^2_{0.05} = 9.49$ for 4 d.f) **(07 Marks)**

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

b. The transition probability matrix of a Markov chain is given by,

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

and the initial probability distribution is $P^{(0)} = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$. Find $P^{(2)}_{13}$, $P^{(2)}_{23}$, $P^{(2)}$ and $P^{(2)}_{1}$.

(06 Marks)

c. A man's smoking habits are as follows. If he smokes filters cigarettes one week, he switches to non filter cigarettes the next week with probability 0.2. On the other hand if he smokes nonfilter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes? **(07 Marks)**

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CBCS SCHEME

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17MATDIP41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Additional Mathematics - II

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Rank of the Matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(06 Marks)**

b. Test for consistency and solve $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$. **(07 Marks)**

c. Solve the system of equations by Gauss Elimination Method $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$. **(07 Marks)**

OR

2 a. Find the Eigen values and Eigen vectors of the Matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(06 Marks)**

b. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. **(07 Marks)**

c. Find the Rank of the Matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(07 Marks)**

Module-2

3 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 5e^{-2x}$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = \cos 2x$. **(07 Marks)**

OR

4 a. Solve $\frac{d^2y}{dx^2} + 4y = \sin^2 x$. **(06 Marks)**

b. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)\,y = 0$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$. **(07 Marks)**

Module-3

5 a. Find the Laplace Transform of the function $\sin 5t \cos 2t$. **(06 Marks)**

b. Find the $L\left[\frac{\cos at - \cos bt}{t}\right]$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Find the Laplace Transform of the Periodic function defined by $f(t) = \frac{Kt}{T}$, $0 < t < T$, $f(t+T) = f(t)$. **(07 Marks)**

OR

6 a. Find Laplace Transform of $[(3t+4)^3 + 5^t]$. **(06 Marks)**

b. Find $L[t \cos at]$. **(07 Marks)**

c. Express the following function interms of Unit step function and hence find its Laplace Transform, where

$$f(t) = \begin{cases} t , & 0 < t < 4 \\ 5 , & t > 4 \end{cases}.$$

(07 Marks)

Module-4

7 a. i) Find $L^{-1}\left[\frac{s}{s^2-16}\right]$ ii) Find $L^{-1}\left[\frac{(s+2)^3}{s^6}\right]$. **(06 Marks)**

b. Find $L^{-1}\left[\frac{2s^2+5s-4}{s(s-1)(s+2)}\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\frac{2s-1}{s^2+4s+29}\right]$. **(07 Marks)**

OR

8 a. Find $L^{-1}\left[\frac{3}{s^2}+2\frac{e^{-s}}{s^3}-3\frac{e^{-2s}}{s}\right]$. **(06 Marks)**

b. Find $L^{-1}\left[\frac{3s+2}{(s-2)(s+1)}\right]$. **(07 Marks)**

c. Solve by using Laplace Transform, $\frac{d^2y}{dt^2}+k^2y=0$, given that $y(0)=2$, $y'(0)=0$. **(07 Marks)**

Module-5

9 a. State and prove Addition Theorem of probability
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. The probability that an integrated circuit chip will have defective etching is 0.12. The probability that it will have a crack defect is 0.29 and the probability that it will have both defects is 0.07. What is the probability that a newly manufactured chip will have
i) an etching of crack defect? ii) neither defect? **(07 Marks)**

c. If A and B are events with $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{4}$, $P(A \cap \overline{B}) = \frac{1}{3}$. Find P(A), P(B) and $P(\overline{A} \cap B)$. **(07 Marks)**

OR

10 a. State and prove Baye's Theorem. **(06 Marks)**

b. In a certain college 4% of Men students and 1% of Women students are taller than 1.8m. Further more 60% of the students are Women. If a student is selected at random and is found taller than 1.8m, what is the probability that the student is a Women? **(07 Marks)**

c. The probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and high selectivity is 0.18. Find the probability that a system will have high selectivity, given it has high fidelity. **(07 Marks)**

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CBCS SCHEME

USN

17CS43

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Design and Analysis of Algorithms

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Discuss the sequential search algorithm along with its best, average and worst case efficiency. **(08 Marks)**

b. Write an algorithm to check whether all the elements in a given array are distinct. Discuss its worst case efficiency. **(06 Marks)**

c. Discuss the following:

i) Connected graph and connected component.

ii) Adjacency list and adjacency matrix representation of a graph. **(06 Marks)**

OR

2 a. Discuss any three types of Asymptotic notations with suitable example. **(08 Marks)**

b. Give the algorithm to solve towers of Hanoi problem. Solve the recurrence relation to find the numbers of moves. **(06 Marks)**

c. What is an Algorithm? What are the five criteria that all algorithms must satisfy? **(06 Marks)**

Module-2

3 a. Give recursive algorithm for binary search. Discuss its complexity. **(06 Marks)**

b. Write an algorithm to sort 'n' numbers using Quick sort. Discuss the best, average and worst case time complexity. **(08 Marks)**

c. Discuss how to multiply two matrices using Strassen's matrix multiplication. How this algorithm is better than Brute force matrix multiplication? **(06 Marks)**

OR

4 a. Write recursive algorithm to find maximum and minimum element in an array. Discuss its complexity. **(06 Marks)**

b. Write an algorithm to sort 'n' numbers using merge sort. Trace the algorithm for the input 15, 45, 30, 10, 20, 25, 18, 60. **(08 Marks)**

c. Apply DFS based algorithm and source removal algorithm to solve the following topological sorting problem. **(06 Marks)**

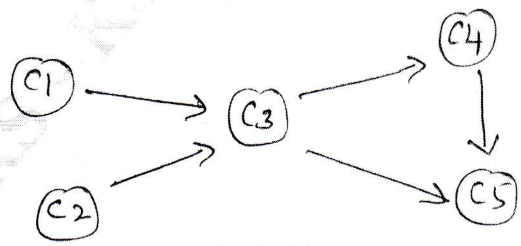

Fig.Q.4(c)

Module-3

5 a. Discuss the control abstraction for greedy method along with coin charge problem as an example. **(06 Marks)**

b. Give the Dijkstra's algorithm. Apply the algorithm to find shortest path by considering 'a' as source vertex. **(08 Marks)**

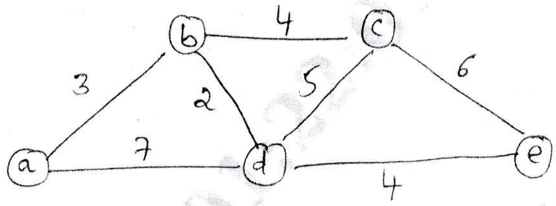

Fig.Q.5(b)

c. Construct Huffman tree for the following data and obtain its Huffman code.

Characters	A	B	C	D	E
Probability	0.1	0.1	0.2	0.2	0.4

(06 Marks)

OR

6 a. Apply Prim's algorithm to find minimum cost spanning tree for the following graph. **(06 Marks)**

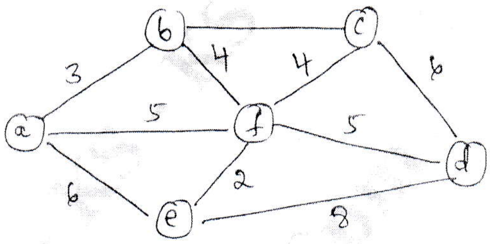

Fig.Q.6(a)

b. Write an algorithm for Greedy Knapsack problem. Consider the following instances of the Knapsack problem.
N = 3, m = 20, (P1, P2, P3) = (25, 24, 15) and (W1, W2, W3) = (18, 15, 10). Apply greedy method to find optimal solution. **(08 Marks)**

c. Construct a heap for the list 1, 8, 6, 5, 3, 7, 4 by successive key insertions. **(06 Marks)**

Module-4

7 a. What are multistage graphs? Write a pseudocode for forward approach corresponding to multistage graph. **(06 Marks)**

b. Write Bellman and Ford algorithm to compute shortest path. Apply the algorithm for the following graph.

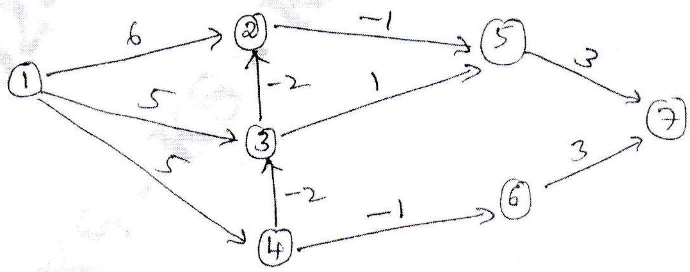

Fig.Q.7(b)

Consider ① as source vertex. **(08 Marks)**

c. Consider the following graph and the matrix showing the edge length.

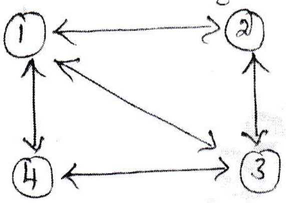

Fig.Q.7(c)

$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

Find the optimal Tour for the salesperson. **(06 Marks)**

OR

8 a. Solve the following Knapsack problem using memory function algorithm capacity W = 5.

Item	1	2	3	4
Weight	2	1	3	2
Value	12	10	20	15

(06 Marks)

b. Write the Floyd's algorithm. Apply the algorithm for the following graph and obtain the resultant matrix:

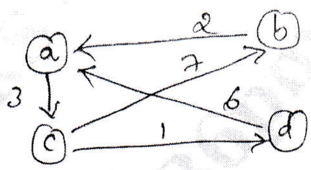

Fig.Q.8(b)

(08 Marks)

c. What is transitive closure? Give Warshall's algorithm and obtain the complexity. **(06 Marks)**

Module-5

9 a. Discuss the backtracking technique along with N-Queens problem as an example. **(06 Marks)**

b. Solve the following assignment problem using branch and bound method.

	Job1	Job2	Job3	Job4	
C =	9	2	7	8	person a
	6	4	3	7	person b
	5	8	1	8	person c
	7	6	9	4	person d

(08 Marks)

c. Write a note on non deterministic algorithm. **(06 Marks)**

OR

10 a. Construct state space tree of the backtracking algorithm applied to the instance S = {3, 5, 6, 7} and d = 15 of the subset sum problem. **(06 Marks)**

b. Define NP-complete problem. Discuss with an example. **(08 Marks)**

c. What is Hamiltonian cycle? How to find Hamiltonian cycle using Backtracking? Explain with an example. **(06 Marks)**

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