CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. Use statistical table is permitted.

Module-1

1 a. Using Taylor's series method, solve $dy = (xy - 1)dx$, $y(1) = 2$ at $x = 1.02$ considering upto 3rd degree term. **(05 Marks)**

b. Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ at the point $x = 0.2$ by taking step length $h = 0.2$. **(05 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at $x = 0.8$ by Adams – Bashforth predictor – corrector method. **(06 Marks)**

OR

2 a. Using modified Euler's method, find an approximate value of y when $x = 0.1$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$. Take $h = 0.1$ and perform three iterations. **(05 Marks)**

b. Solve $\frac{dy}{dx} = 2y + 3e^x$ $y(0) = 0$ using Taylors series method an find $y(0.1)$. **(05 Marks)**

c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ the data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. **(06 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right)$ $y(0) = 1$, $y'(0.1) = 0.5$, evaluate $y(0.1)$ using 4th order – Runge – Kutta method. **(05 Marks)**

b. Express $f(x) = x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(05 Marks)**

c. If α and β are the roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Using the Milne's method obtain the approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0.1) = 0.1$. Given :

$y(0.1) = 1.03995$ $y'(0.1) = 0.6955$ $y(0.2) = 1.138036$
$y'(0.2) = 1.258$ $y(0.3) = 1.29865$ $y'(0.3) = 1.873$ **(05 Marks)**

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. State and prove Rodrigue's formula. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Derive Cauchy's Riemann equations in Cartesian form. (05 Marks)

b. Using Cauchy's residue theorem evaluate the integral $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$. (05 Marks)

c. Find the bilinear transformation which maps the points Z = 0, i, ∞ onto the points W = 1, –i, –1, respectively. Find the invariant points. (06 Marks)

OR

6 a. State and prove Cauchy's theorem. (05 Marks)

b. Given $u - v = (x - y)(x^2 + 4xy + y^2)$ find the analytic function f(z) = u + iv. (05 Marks)

c. Discuss the transformation $W = e^z$. (06 Marks)

Module-4

7 a. Derive mean and variance of the Binomial distribution. (05 Marks)

b. The probability that an individual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals more than 2 will get a bad reaction. (05 Marks)

c. The joint probability distribution of two random variable X and Y as follows :

x \ y	–2	–1	4	6
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine :
i) Marginal distribution of X and Y
ii) Covariance of X and Y
iii) Correlation of X and Y. (06 Marks)

OR

8 a. Derive mean and standard deviation of exponential distribution. (05 Marks)

b. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours. Out of 2500 bulbs find the number of bulbs that are likely to last between 1900 and 2100 hours. Given that $P(0 < z < 1.67) = 0.4525$. (05 Marks)

c. The joint probability distribution of two random variable X and Y as follows :

x \ y	–4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine :
i) Marginal distribution of X and Y
ii) Covariance of X and Y
iii) Correlation of X and Y. (06 Marks)

Module-5

9 a. Explain the following terms :
i) Null hypothesis
ii) Type I and Type II error
iii) Significance level. (05 Marks)

b. Find he student 't' for the following variables values in a sample of eight – 4, –2, –2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. (05 Marks)

c. Find the fixed probability vector of the regular stochastic matrix :

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}.$$ (06 Marks)

OR

10 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. (05 Marks)

b. A set of five similar coins is tossed 320 times and the result is

Number of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution for v = 5 we have $\chi^2_{0.05} = 11.07$. (05 Marks)

c. A students study habits are as follows. If he studies one night, he is 60% sure not study the next night. On the other hand if he does not study one night, he is 80% sure not to study the next night. In the long run how often does he study? (06 Marks)

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15MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix by elementary row transformations: $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$. (05 Marks)

b. Solve the following system of equations by Gauss elimination method
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$ (05 Marks)

c. Find all the eigen values and the corresponding eigen vectors for the matrix.
$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$. (06 Marks)

OR

2 a. Reduce the matrix to echelon form and find the rank of the matrix.
$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$. (05 Marks)

b. Solve the following system of equations by Gauss elimination method:
$x_1 - 2x_2 + 3x_3 = 2$
$3x_1 - x_2 + 4x_3 = 4$
$2x_1 + x_2 - 2x_3 = 5$ (05 Marks)

c. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ Find A^{-1}. (06 Marks)

Module-2

3 a. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$. (06 Marks)

b. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ given that $y = 0$, $\frac{dy}{dx} = -1$ at $x = 1$. (05 Marks)

c. Solve by the method of undetermined coefficient $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{3x}$. (05 Marks)

OR

4 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$. (05 Marks)

b. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ subject to, $\frac{dy}{dx} = 2$, $y = 1$ at $x = 0$. (05 Marks)

c. Solve by the method of variation of parameters $y'' + a^2y = \sec ax$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Find: $L\{t \sin at\}$ **(05 Marks)**

b. Given $f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}$ where $f(t+a) = f(a)$. Show that $L\{f(t)\} = \frac{E}{S}\tanh\left(\frac{as}{4}\right)$. **(06 Marks)**

c. Find $L\{(3t^2 + 4t + 5)u(t-3)\}$. **(05 Marks)**

OR

6 a. Find $L\left\{\frac{1-e^{at}}{t}\right\}$. **(05 Marks)**

b. Prove that $L(\sin at) = \frac{a}{s^2 + a^2}$. **(05 Marks)**

c. Express the following function in terms of the unit step function and hence find their Laplace transform:

$$f(t) = \begin{cases} \sin t & 0 < t \le \pi/2 \\ \cos t & t > \pi/2 \end{cases}$$ **(06 Marks)**

Module-4

7 a. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)(s+3)}$. **(05 Marks)**

b. Find $L^{-1}\left\{\log\left(1 + \frac{a^2}{s^2}\right)\right\}$. **(05 Marks)**

c. Solve the differential equation $y'' - 3y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$ by Laplace transform techniques. **(06 Marks)**

OR

8 a. Find $L^{-1}\left\{\frac{s+5}{s^2 - 6s + 13}\right\}$. **(05 Marks)**

b. Find $L^{-1}\{\cot^{-1}(s/a)\}$. **(05 Marks)**

c. Solve, $y'' + a^2 y = \sin t$ with $y(0) = 0$, $y'(0) = 0$. Using Laplace transform. **(06 Marks)**

Module-5

9 a. The probability that 3 students A, B, C solve a problem are 1/2, 1/3, 1/4 respectively. If the problem is simultaneously assigned to all of them, what is the probability that the problem is solved? **(05 Marks)**

b. The probability that a team wins a match is 3/5. If this team play 3 matches in a tournament, what is the probability that the team i) win all the matches ii) loose all the matches. **(05 Marks)**

c. State and prove Baye's theorem. **(06 Marks)**

OR

10 a. Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$ **(06 Marks)**

b. A box contains 3 white, 5 black and 6 red balls. If a ball is drawn at random. What is the probability that it is entire red or white? **(05 Marks)**

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3%, 2% are defective. If a bolt drawn random was found defective what is the probability that it was manufactured by A. **(05 Marks)**

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15EC44

Fourth Semester B.E. Degree Examination, July/August 2022
Signals and Systems

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Determine whether the discrete-time signal,

$x(n) = \cos\left(\frac{n\pi}{4}\right)\sin\left(\frac{2\pi}{5}\right)$ is periodic. If periodic, find the fundamental period. **(05 Marks)**

b. Determine and sketch even and odd parts of the signal shown in the Fig.Q1(b).

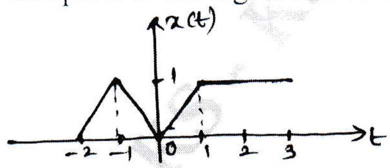


Fig.Q1(b) **(06 Marks)**

c. Prove the following properties of Impulse function:
i) $x(t) * \delta(t) = x(t)$ (ii) $x(t) * \delta(t - t_0) = x(t_0)$ **(05 Marks)**

OR

2 a. Determine whether the following systems are memoryless, causal, linear, time invariant and stable:
(i) $y(n) = n\,x(n)$ (ii) $y(t) = x(t/2)$ $|x(t)| \le M_x < \infty$ **(10 Marks)**

b. Sketch the waveforms of the following signals :
(i) $x(t) = u(t + 1) - 2u(t) + u(t - 1)$
(ii) $y(t) = r(t + 1) - r(t) + r(t - 2)$
(iii) $z(t) = -u(t + 3) + 2u(t + 1) - 2u(t - 1) + u(t - 3)$ **(06 Marks)**

Module-2

3 a. An LTI system is characterized by an impulse response $h(n) = (1/2)^n u(n)$. Find the response of the system for the input $x(n) = (1/4)^n u(n)$. **(06 Marks)**

b. Find the convolution sum of the given two sequences $x(n) = \{1, 2, 3, 2\}$, $h(n) = \{1, 2, 2\}$ by using graphical convolution method. **(10 Marks)**

OR

4 a. Determine the convolution sum of the given sequences
$x(n) = \{3, 5, \underset{\uparrow}{-2}, 4\}$ and $h(n) = \{3, 1, 3\}$. **(08 Marks)**

b. Perform graphical convolution to determine the output of the system, when the input and impulse response are given by $x(t) = e^{-4t}[u(t) - u(t - 2)]$; $h(t) = e^{-2t} u(t)$. **(08 Marks)**

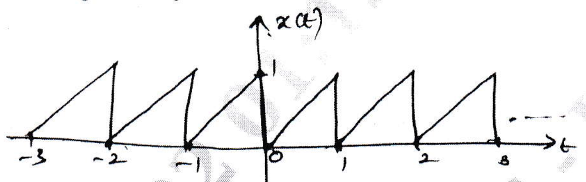
Module-3

5 a. For each impulse response listed below, determine whether the corresponding system is memoryless, causal and stable.
i) $h(n) = (0.99)^n u(n - 3)$ ii) $h(t) = e^{-3t} u(t - 1)$ **(08 Marks)**

b. Find the complex exponential fourier series representation of the following signals:
i) $x(t) = \sin(2t + \pi/4)$ ii) $x(t) = \cos^2(t)$ **(08 Marks)**

OR

6 a. Find the complex fourier series coefficients for the periodic waveform shown in Fig.Q6(a). Also draw the amplitude and phase spectra.



$x(t) = t;\ \ 0 < t < 1.$

Fig.Q6(a) **(08 Marks)**

b. Find the step response of an LTI system, whose impulse response is given by the following:
i) $h(t) = t^2 u(t)$ ii) $h(t) = e^{-t} u(t)$ **(08 Marks)**

Module-4

7 a. Show that the fourier transform of a rectangular pulse described by :

$$x(t) = 1 \quad ; \quad -T \le t \le T$$
$$\quad = 0 \quad ; \quad |t| > T$$

is a sinc function. Plot its magnitude and phase spectrum. **(08 Marks)**

b. If $x(t) \xleftrightarrow{FT} X(jw)$ or $X(e^{jw})$ and $y(t) \xleftrightarrow{FT} Y(jw)$ or $Y(e^{jw})$,
Show that $z(t) = x(t) * y(t) \xleftrightarrow{FT} X(jw)Y(jw)$ or $X(e^{jw})Y(e^{jw})$ **(08 Marks)**

OR

8 a. State sampling theorem and explain aliasing effect with relevant waveforms. **(04 Marks)**

b. Specify Nyquist rate and Nyquist interval for each of the following signals.
i) $x(t) = \sin c^2(2000t)$
ii) $y(t) = \sin c(200t) + \sin c^2(200t)$ **(06 Marks)**

c. Find the DTFT of the signal $a^n u(n)$ its magnitude and phase spectrum. **(06 Marks)**

Module-5

9 a. Using properties of z-transform, find the convolution of
$x(n) = \{\underset{\uparrow}{1}, 2, -1, 0, 3\}$ and $y(n) = \{\underset{\uparrow}{1}, 2, -1\}$ **(05 Marks)**

b. State and prove differentiation property of Z-transform. **(06 Marks)**

c. Find the z-transform of $x(n) = \alpha^{|n|}$, $|\alpha| \# 1$ and determine its ROC. **(05 Marks)**

OR

10 a. A causal discrete-time LTI system is described by

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$$

where x(n) and y(n) are the input and output of the system respectively.
i) Determine the system function, H(z)
ii) Find the impulse response, h(n)
iii) Find the step response of the system
iv) Find the frequency response of the system.
v) Find BIBO stability of the system. **(10 Marks)**

b. Find the inverse z-transform of the funciton

$$X[z] = \frac{z-4}{z^2 - 5z + 6}$$ **(06 Marks)**

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