Fourth Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – II

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of a matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ by reducing to echelon form. (07 Marks)

b. Use Cayley-Hamilton theorem to find the inverse of a matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. (07 Marks)

c. Solve the following system of equation of Gauss Elimination method:
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$. (06 Marks)

OR

2 a. Test for consistency and solve
$5x_1 + x_2 + 3x_3 = 20$
$2x_1 + 5x_2 + 2x_3 = 18$
$3x_1 + 2x_2 + x_3 = 14$. (07 Marks)

b. Find all the Eigenvalues of the matrix
$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (07 Marks)

c. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix}$. (06 Marks)

Module-2

3 a. Solve $\dfrac{d^3y}{dx^3} - 2\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 8y = 0$. (07 Marks)

b. Solve $y'' - 4y' + 13y = \cos 2x$. (07 Marks)

c. Solve $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^3$. (06 Marks)

OR

4 a. Solve by the method of variation of parameters, $y'' - 2y' + y = e^x.\log x$. (07 Marks)

b. Solve by the method of undetermined coefficients $(D^2 + 1)y = \sin x$. (07 Marks)

c. Solve $\dfrac{d^2y}{dx^2} - 4y = 3^x$. (06 Marks)

Module-3

5 a. Find the Laplace transform of $\cos t \times \cos 2t . \cos 3t$. **(07 Marks)**
   b. Find the Laplace transform of $e^{3t} \sin 5t . \sin 3t$. **(07 Marks)**
   c. Find the Laplace transform of $t^3 \sin t$. **(06 Marks)**

OR

6 a. If f(t) is a periodic function of period T > 0, then prove that $L\{f(t)\} = \dfrac{1}{1 - e^{-sT}} \int\limits_0^T e^{-st} f(t) dt$. **(07 Marks)**

   b. Find the Laplace transform of $f(t) = E \sin wt,\ 0 < t < \pi/w$ having period π/w. **(07 Marks)**

   c. Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ as a unit step function and hence find its Laplace transform. **(06 Marks)**

Module-4

7 a. Find the Laplace of $\dfrac{1}{(s-1)(s+1)(s+2)}$. **(07 Marks)**

   b. Solve $y''' + 2y'' - y' - 2y = 0$ given y(0) = y'(0) = 0 and y''(0) = 6 by using Laplace transform. **(07 Marks)**

   c. Find: $L^{-1}\left[\dfrac{3s+2}{(s-2)(s+1)}\right]$. **(06 Marks)**

OR

8 a. Find $L^{-1}[\cot^{-1}(s/a)]$. **(07 Marks)**

   b. Employ Laplace transform to solve the equation $y'' + 5y' + 6y = 5e^{2x}$, y(0) = 2, y'(0) = 1. **(07 Marks)**

   c. Find the inverse Laplace transform of $\log\left[\dfrac{s+4}{s-4}\right]$. **(06 Marks)**

Module-5

9 a. State and prove Bayes theorem. **(07 Marks)**
   b. Prove that
      $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$. **(07 Marks)**
   c. A pair of dice is tossed twice. Find the probability of scoring 7 points
      i) Once   ii) atlesat once   iii) twice. **(06 Marks)**

OR

10 a. If A and B are two events having P(A) = 1/2, P(B) = 1/3 and $P(A \cap B) = 1/4$ compute
      i) P(A/B)   ii) P(B/A)   iii) $P(\overline{A}/\overline{B})$. **(07 Marks)**

   b. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(07 Marks)**

   c. In a school 25% of the students failed in first language, 15% of the students failed in second language and 10% of the students failed in both. If a student is selected at random find the probability that.
      i)   He failed in first language if he had failed in the second language.
      ii)   He failed in second language if he had failed in the first language.
      iii) He failed in either of the two languages. **(06 Marks)**

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17CS43

Fourth Semester B.E. Degree Examination, July/August 2022
Design and Analysis of Algorithms

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Design an algorithm to search an element in an array using sequential search. Discuss the worst case, best case and average case efficiency of this algorithm. **(08 Marks)**

   b. Give the recursive algorithm to solve towers of Hanoi problem. Show that the efficiency of this algorithm is exponential. **(06 Marks)**

   c. Define an algorithm. Explain the characteristics of an algorithm. **(06 Marks)**

OR

2 a. Give the general plan for analyzing time efficiency of an non recursive algorithm. Derive the worst case analysis for the algorithm to check whether all the elements in an array are distinct. **(08 Marks)**

   b. Explain the following types of problems:
      (i)   Combinatorial problem
      (ii) Graph problem **(06 Marks)**

   c. Explain asymptotic notation with example. **(06 Marks)**

Module-2

3 a. Write an algorithm to sort "n" numbers using Quicksort. Trace the algorithm to sort the following list in ascending order: 80, 60, 20, 40, 10, 30, 50, 20 **(08 Marks)**

   b. Apply DFS method and source removal method to find the Topological sequence for the graph shown in Fig.Q3(b).
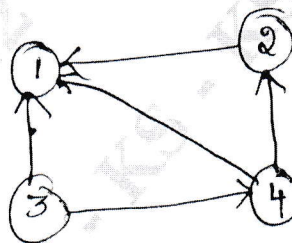

Fig.Q3(b) **(06 Marks)**

   c. List out the advantages and disadvantages of Divide and Conquer technique. **(06 Marks)**

OR

4 a. Apply Strassen's matrix multiplication to multiply following matrices. Discuss how this method is better than direct matrix multiplication method.

$$\begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}$$

(08 Marks)

   b. Explain Divide and Conquer technique with its control abstraction. **(05 Marks)**

   c. Write an algorithm to sort 'n' numbers using Mergesort. Mention its time complexity. **(07 Marks)**

Module-3

5   a.   Write an algorithm to solve knapsack problem using Greedy technique. Find the optimal solution to the knapsack instance, n = 7, m = 15.
$(p_1 \ldots \ldots p_7) = (10, 5, 15, 7, 6, 18, 3)$
$(w_1 \ldots \ldots w_7) = (2, 3, 5, 7, 1, 4, 1)$ **(09 Marks)**

   b.   Apply Prim's algorithm to find the minimum cost spanning tree to the graph shown in Fig.Q5(b).
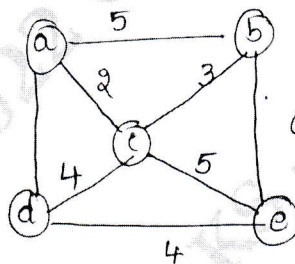

Fig.Q5(b)   **(05 Marks)**

   c.   Solve the graph given below in Fig.Q5(c) using single source shortest path algorithm with vertex 'a' as the source.
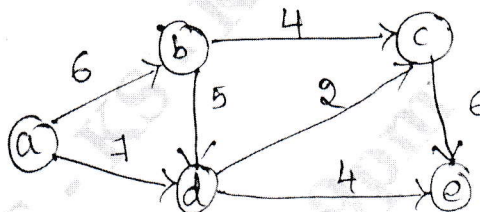

Fig.Q5(c)   **(06 Marks)**

OR

6   a.   Define heap. Write bottom up heap construction algorithm. Construct heap for the list 2, 6, 9, 8, 3, 7, 4 using bottom up algorithm.   **(08 Marks)**

   b.   State job sequencing with deadline problem. Find the solution generated by job sequencing algorithm for 7 jobs, given profits 3, 5, 20, 18, 1, 6, 30 and deadline 1, 3, 4, 3, 2, 1, 2 respectively.   **(06 Marks)**

   c.   Construct the Huffman tree for the following data:

Character	A	B	C	D	E	-
Probability	0.5	0.35	0.5	0.1	0.4	0.2

   (06 Marks)

Module-4

7   a.   Define transitive closure of a directed graph. Find the transitive closure matrix for the graph whose adjacency matrix is given below.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   (10 Marks)

   b.   Write an algorithm to construct optimal binary search tree for the following data:

Key	A	B	C	D
Probability	0.1	0.2	0.4	0.3

   (10 Marks)

OR

8 a. Apply dynamic programming method to solve the following instance of the knapsack problem. Knapsack capacity W = 10.

Item	Weight	Value
1	6	42
2	4	15
3	2	20
4	5	30

(10 Marks)

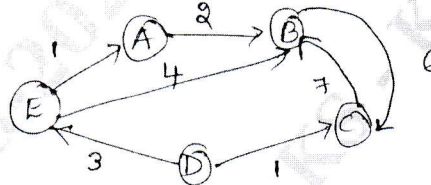b. Apply Floyd's algorithm to the graph given below in Fig.Q8(b). Show all necessary steps. Derive its time efficiency.

Fig.Q8(b) **(10 Marks)**

Module-5

9 a. Construct state space tree for solving 4 Queen's problem using backtracking. **(06 Marks)**

b. Apply Backtracking technique to solve the below instance of the sum of subset problem. S = {1, 3, 4, 6}, d = 7. **(08 Marks)**

c. Apply Branch and bound technique to the following instance of assignment problem.

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix} \begin{matrix} \text{person a} \\ \text{person b} \\ \text{person c} \\ \text{person d} \end{matrix}$$

job1   job2   job3   job4

(06 Marks)

OR

10 a. Discuss graph coloring problem. Find different solutions for 4 nodes and all possible 3 coloring problem. **(06 Marks)**

b. Solve the following instance of the knapsack problem using Branch and Bound technique. Given knapsack capacity = 10.

Item	Weight	Profit
1	4	40
2	7	42
3	5	25
4	3	12

(08 Marks)

c. Define Hamilton cycle. Check whether the Hamilton cycle exists for the graph given below in Fig.Q10(c).
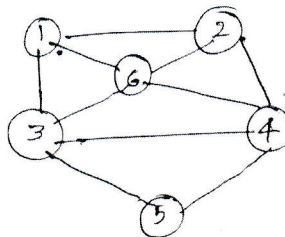

Fig.Q10(c) **(06 Marks)**