CBCS SCHEME

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18MAT41

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Statistical Tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Derive Cauchy Riemann equations in Cartasian form. **(07 Marks)**

b. Find the analytic function whose real part is e^{2x} (xcos2y – ysin2y). **(07 Marks)**

c. Determine the analytic function w = u + iv, if $v = \log(x^2 + y^2) + x - 2y$ **(06 Marks)**

OR

2 a. Derive Cauchy Riemann equations in polar form. **(07 Marks)**

b. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and f(z) = u + iv is an analytic function of z = x + iy, find f(z) in terms of z. **(07 Marks)**

c. Find the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. **(06 Marks)**

Module-2

3 a. Discuss the transformation $w = e^z$ and show that it transforms the region between the real axis and a line parallel to real axis at y = π into upper half of W-plane. **(07 Marks)**

b. Find the bilinear transformation which maps the points Z = 1, i, -1 into the points W = i, 0, -i. **(07 Marks)**

c. Evaluate $\int_C \dfrac{Z^2 - Z + 1}{Z - 1} dz$, where C is the circle $|Z| = 1/2$. **(06 Marks)**

OR

4 a. Discuss the transformation $W = Z^2$. **(07 Marks)**

b. Find the bilinear transformation which maps the points Z = 0, i, ∞ onto the points W = 1, -i, -1 respectively. **(07 Marks)**

c. Evaluate $\oint_C \dfrac{\sin \pi Z^2 + \cos \pi Z^2}{(Z-1)(Z-2)} dz$ where C is the circle $|Z| = 3$. **(06 Marks)**

Module-3

5 a. The probability density function of a random variable X is as follows:

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K and i) $P(X < 4)$, ii) $P(3 < X \le 6)$. **(07 Marks)**

b. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that,

i) Exactly 2 will be defective

ii) None will be defective

iii) At least two will be defective (Use binomial distribution). **(07 Marks)**

c. Fit a Poisson distribution to the set of observations

x	0	1	2	3	4
f	122	60	15	2	1

(06 Marks)

OR

6 a. If X is a continuous random variable with probability function given by

$f(x) = kx, 0 \le x < 2$

$= 2k, 2 \le x \le 4$

$= -kx + 6k, 4 \le x < 6,$

Find the value of K and the mean value of X. (07 Marks)

b. If the probability of a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals more than two get a bad reaction. (Use Poisson distribution). (07 Marks)

c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i) more than 2150 hours and ii) less than 1950 hours. (06 Marks)

Module-4

7 a. Fit a straight line $y = a + bx$ to the following data:

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

(07 Marks)

b. Find the correlation for the data given below

x	1	2	3	4	5	6
y	6	4	3	5	4	2

(07 Marks)

c. Two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$, find i) mean value of x, mean value of y and ii) Correlation coefficient between x and y. (06 Marks)

OR

8 a. Fit a second degree parabola to the following data:

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

b. Ten competitors in a contest are ranked by two judges as follows:

x	1	6	5	10	3	2	4	9	7	8
y	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient. (07 Marks)

c. Find the regression equations y on x and x on y using the table of values given below.

x	16	24	32	40	48	56
y	0.39	0.75	1.23	1.91	2.77	3.81

(06 Marks)

Module-5

9 a. The joint probability distribution of two random variables X and Y is shown below:

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Find the marginal distributions of X and Y. Also verify that X and Y are stochastically independent. **(07 Marks)**

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? **(07 Marks)**

c. A certain stimulus administered to each of 12 patients resulted in the following increase in blood. Pressures: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be calculated that stimulus is accompanied by an increase in blood pressure given that for 11 degrees of freedom the value of $t_{0.05}$ is 2.201? **(06 Marks)**

OR

10 a. A joint probability distribution is given by the following table:

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find i) marginal distributions ii) COV (X, Y). **(07 Marks)**

b. The mean life time of a sample of 100 bulbs is 1570 hours with a standard deviation of 120 hours. The company claims that the average life of bulbs produced by it is 1600 hrs. Use appropriate test to verify the acceptance ($Z_{0.05} = 1.96$). **(07 Marks)**

c. Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05 ($\chi^2_{0.05} = 7.82$ for 3 degrees of freedom). Given that,

x	0	1	2	3	4
f	419	352	154	56	19

(06 Marks)

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CBCS SCHEME

USN | | | | | | | | | | **18CS42**

Fourth Semester B.E. Degree Examination, Feb./Mar.2022
Design and Analysis of Algorithm

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. What is an algorithm? Explain the criteria to be satisfied by algorithm. **(06 Marks)**

b. Algorithm Enigma (A[0…n-1, 0…n-1])

```
    for i ← 0 to n – 2 do
        for j ← i + 1 to n – 1 do
            if A[i, j] ≠ A[j, i]
                return false
            end for
        end for
    return true
end algorithm
```

(i) What does this algorithm compute?
(ii) What is its input size?
(iii) What is its basic operation?
(iv) How many times is the basic operation executed?
(v) What is the efficiency class of this algorithm? **(10 Marks)**

c. Prove the following theorem:
If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. **(04 Marks)**

OR

2 a. Design an algorithm for performing sequential search and compute best case, worst case and average case efficiency. **(10 Marks)**

b. The factorial function n! has value 1 when n<=1 and value $n*(n-1)!$ when n>1. Write both a recursive and an iterative algorithm to compute n! **(06 Marks)**

c. List the following functions according to their order of growth from the lowest to the highest. State proper reasons,
$(n-2)!$, $5\log(n+100)^{10}$, 2^{2n}, $0.001n^4 + 3n^3 + 1$, $\ln^2 n$, $\sqrt[3]{n}$, 3^n. **(04 Marks)**

Module-2

3 a. Design an algorithm for performing merge sort. Analyze its time efficiency. Apply the same to sort the following set of numbers 4, 9, 0, –1, 6, 8, 9, 2, 3, 12 **(10 Marks)**

b. Apply Strassen's multiplication to multiply the following matrices. Show the details of the computation.
$A = \begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$. **(10 Marks)**

OR

4 a. Apply topological sort on the following graph using source removal and DFS based methods. **(10 Marks)**

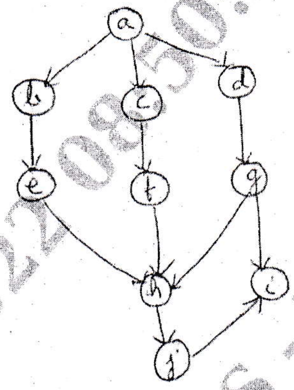

Fig. Q4 (a)

b. Desing an algoirhtm for performing quick sort, apply the same to sort the following set of numbers 5, 3, 1, 9, 8, 2, 4, 7 **(10 Marks)**

Module-3

5 a. Write an algorithm to solve the knapsack problem using greedy approach and apply the same to find an optimal solution to the knapsack instance, n = 5, m = 6,
$(p_1, p_2, p_3, p_4, p_5) = (25, 20, 15, 40, 50)$ and
$(w_1, w_2, w_3, w_4, w_5) = (3, 2, 1, 4, 5)$ using greedy approach. **(10 Marks)**

b. What is Dijkstra's algorithm used for? Apply Dijkstra's algorithm on the following graph. Initial node is G

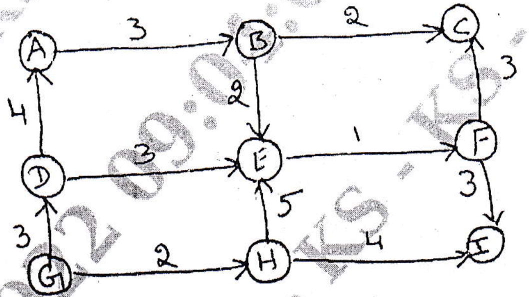

Fig. Q5 (b) **(10 Marks)**

OR

6 a. Define minimum spanning tree. Write Prim's algorithm to find minimum spanning tree. Apply the same on the following graph: **(10 Marks)**

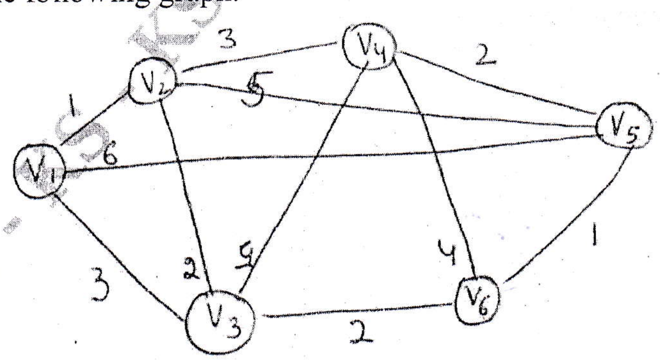

Fig. Q6 (a)

b. A message consisting of the characters given in the table below has to be transmitted over a network in a secured manner.

Character	A	M	R	_
Probability	0.4	0.2	0.3	0.1

(i) Construct Huffman tree for the given characters (Branch label : left (0), right(1))
(ii) Device Huffman codes for the given character.
(iii) Encode the text RAMA_RAMAR using Huffman codes.
(iv) Decode the text whose encoding is 1000101
(v) Compute the effectiveness of Huffman codes. **(10 Marks)**

Module-4

7 a. Design an algorithm to find all pairs of shortest paths given a weighted connected path using dynamic programming technique. Apply the same algorithm to compute all pairs of shortest path for the following weighted connected graph. (Refer Fig. Q7 (a)) **(10 Marks)**

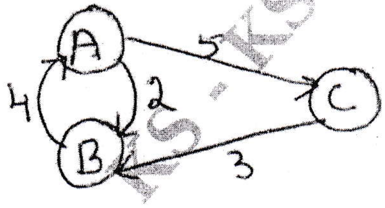

Fig. Q7 (a)

b. Design an algorithm to solve knapsack problem using dynamic programming. Apply the same to solve the following knapsack problem where W = 50.

Item	Weight	Value
1	10	60
2	20	100
3	30	120

(10 Marks)

OR

8 a. Define transitive closure of a directed graph. Write Warshall's algorithm to find transitive closure. Apply the same to find the transitive closure of the digraph given below in Fig. Q8 (a) : **(10 Marks)**

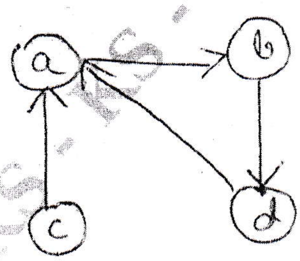

Fig. Q8 (a)

b. Define a multistage graph. Give an example. Explain the technique of finding the minimum cost path in a multistage graph. **(10 Marks)**

Module-5

9 a. What is backtracking? List out two advantages of backtracking strategy. Considering 4-Queens problem, provide two possible solutions to this problem using backtracking technique. **(10 Marks)**

b. Solve the following assignment problem using branch and bound technique.

	Job1	Job2	Job3	Job4
Person a	9	2	7	8
Person b	6	4	3	7
Person c	5	8	1	8
Person d	7	6	9	4

(10 Marks)

OR

10 a. Find a Hamiltonian circuit for the following graph shown in Fig. Q10 (a) using backtracking technique. **(10 Marks)**

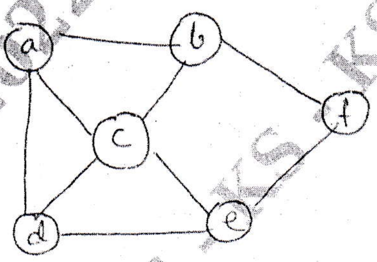

Fig. Q10 (a)

b. Explain the following concepts:
   (i) Tractable and intractable problems
   (ii) P problems
   (iii) Non deterministic algorithm.
   (iv) NP problem.
   (v) NP complete problems. **(10 Marks)**

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